جامعة المنارة

كلية: الهندسة

اسم المقرر: الرياضيات المتقطعة

رقم الجلسة (السادسة)

عنوان الجلسة

العلاقات

Relations

Relations

Let $C = \{2, 3, 4, 5\}$ and $D = \{3, 4\}$ and define a binary relation S from C to D as follows:

$$\text{For all } (x, y) \in C \times D, \quad (x, y) \in S \quad \Leftrightarrow \quad x \geq y.$$

a. Is $2\ S\ 4$? Is $4\ S\ 3$? Is $(4, 4) \in S$? Is $(3, 2) \in S$?
b. Write S as a set of ordered pairs.

Solution:

a. $2\ S\ 4$:False

$4\ S\ 3$:True

$(4,4) \in S$

$(3,2) \notin S$

b. $S=\{(3,3),(4,3),(4,4),(5,3),(5,4)\}$

Relations

Define a binary relation R from $\mathbf{R}$ to $\mathbf{R}$ as follows:

$$\text{For all } (x, y) \in \mathbf{R} \times \mathbf{R}, \quad x\,R\,y \quad \Leftrightarrow \quad y = x^2.$$

a. Is $(2, 4) \in R$? Is $(4, 2) \in R$? Is $(-3)\,R\,9$? Is $9\,R\,(-3)$?

Solution:

a. $(2,4) \in R$

$(4,2) \notin R$

$(-3)\ R\ 9$:True

$9\ R\ (-3)$:False

Relations

Let S be the set of all strings of a's and b's of length 4. Define a relation R on S as follows:

For all $s, t \in S$,

$$s \; R \; t \quad \Leftrightarrow \quad s \text{ has the same first two characters as } t.$$

a. Is $abaa \; R \; abba$?
b. Is $aabb \; R \; bbaa$?
c. Is $aaaa \; R \; aaab$?

Solution:

a. True
b. False
c. True

Relations

- Let A = {1,2,3,4} , Define a binary relation *S* from A to A as follows:

  $$S = \{ (a,b) : a \text{ divides } b \}$$

  Write *S* as a set of ordered pairs.

Solution:

$S = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

- Let A = {1,2,3} and B = {0,1,2,4} , Define a binary relation *S* from A to B as follows:

  $$S = \{ (a,b) : a = b \}$$

  Write *S* as a set of ordered pairs.

Solution:

$S = \{(1,1), (2,2)\}$

Relations

جَامعة المَنارة

MANARA UNIVERSITY

Consider the following relations on $\{1, 2, 3, 4\}$:

Determine the properties of these relations

$$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\},$$
$$R_2 = \{(1,1), (1,2), (2,1)\},$$
$$R_3 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\},$$
$$R_4 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\},$$
$$R_5 = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\},$$

	reflexive	irreflexive	symmetric	antisymmetric	asymmetric	transitive
R1						
R2			×			
R3	×		×			
R4		×		×	×	×
R5	×			×		×

Properties of Relations

Consider these relations on the set of integers: Determine the properties of these relations

$$R_1 = \{(a, b) \mid a \le b\},$$
$$R_2 = \{(a, b) \mid a > b\},$$
$$R_3 = \{(a, b) \mid a = b \text{ or } a = -b\},$$
$$R_4 = \{(a, b) \mid a = b\},$$
$$R_5 = \{(a, b) \mid a = b + 1\},$$
$$R_6 = \{(a, b) \mid a + b \le 3\}.$$

	reflexive	irreflexive	symmetric	antisymmetric	asymmetric	transitive
R1	×			×		×
R2		×		×	×	×
R3	×		×			×
R4	×		×	×		×
R5		×		×	×	
R6			×			

Note :

- R5 is not transitive because ∃(2,1)∈R3 and (1,0) ∈R3 but (2,0)∉ R3
- R6 is not transitive because ∃(2,0) ∈ R6:2+0<=3 and (0,3) ∈ R6 :0+3<=3 but (2,3)∉ R6: 2+3>3

جامعة المنارة
MANARA UNIVERSITY

Represent relations using matrices

Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order).

a) $\{(1, 1), (1, 2), (1, 3)\}$
b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$
c) $\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$
d) $\{(1, 3), (3, 1)\}$

Solution:

a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Representing relations using matrices

جـامعة
المَـنارة
MANARA UNIVERSITY

Represent each of these relations on {1, 2, 3, 4} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
b) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}
c) {(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)}
d) {(2, 4), (3, 1), (3, 2), (3, 4)}

Solution:

a) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Representing relations using matrices

List the ordered pairs in the relations on {1, 2, 3} corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Solution:

a) {(1,1),(1,3),(2,2),(3,1),(3,3)}

b) {(1,2),(2,2),(3,2)}

c) {(1,1),(1,2),(1,3),(2,1),(2,3),(3,1),(3,2),(3,3)}

Properties of Relations

Represent each of these relations on {1,2,3,4} with a matrix ,and determine its properties.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
c) {(2, 4), (4, 2)}
d) {(1, 2), (2, 3), (3, 4)}

Solution:

$$a)\ \begin{bmatrix} 0&0&0&0 \\ 0&1&1&1 \\ 0&1&1&1 \\ 0&0&0&0 \end{bmatrix} \quad b)\ \begin{bmatrix} 1&1&0&0 \\ 1&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix} \quad c)\ \begin{bmatrix} 0&0&0&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&1&0&0 \end{bmatrix}$$

$$d)\ \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \end{bmatrix}$$

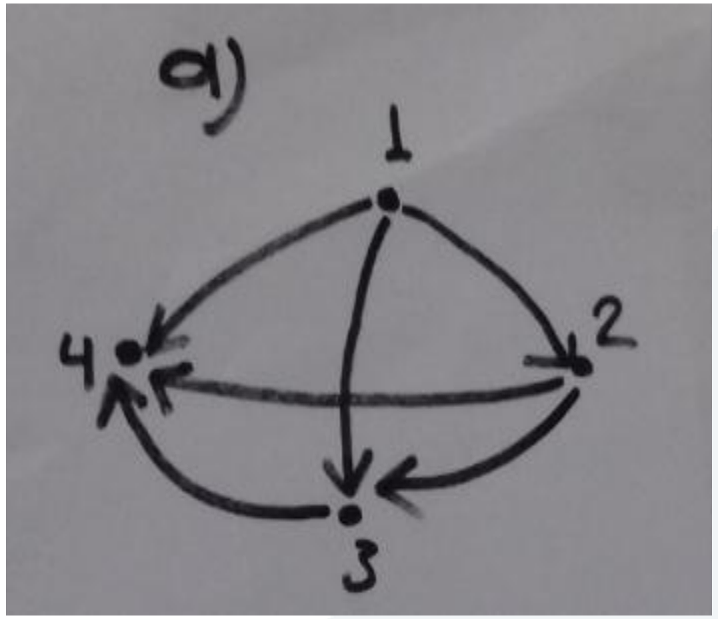
	reflexive	irreflexive	symmetric	antisymmetric	asymmetric	transitive
a)						×
b)	×		×			×
c)		×	×			
d)		×		×	×	

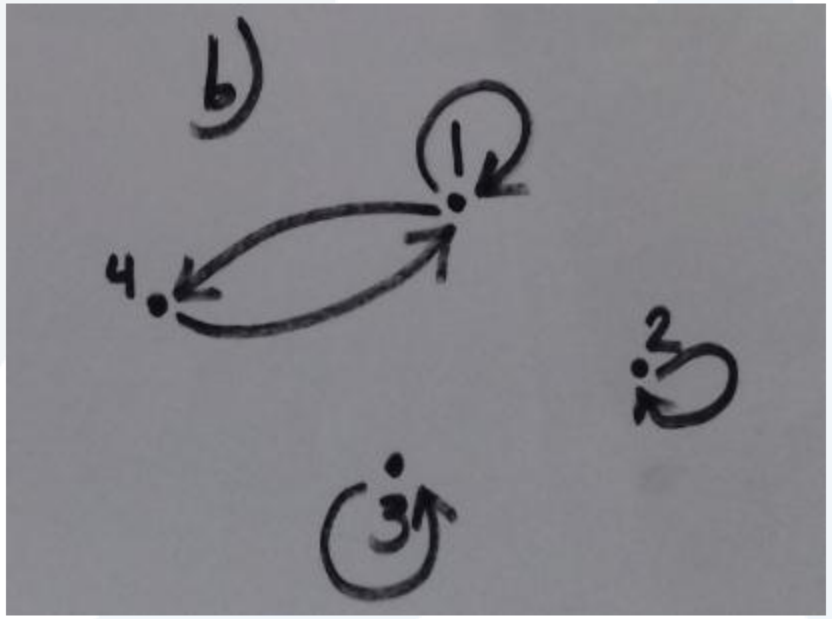
Representing relations with graph

represent these relations with graph.

a) $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$
b) $\{(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)\}$

Solution:





Properties of relation

Let A={-3,-1,2,4,5} and relation R is defined from A to A as follows R={ (a,b) : ab $\geq$ 0}

1- write R as a set of ordered pairs.

2- represent the relation with matrix.

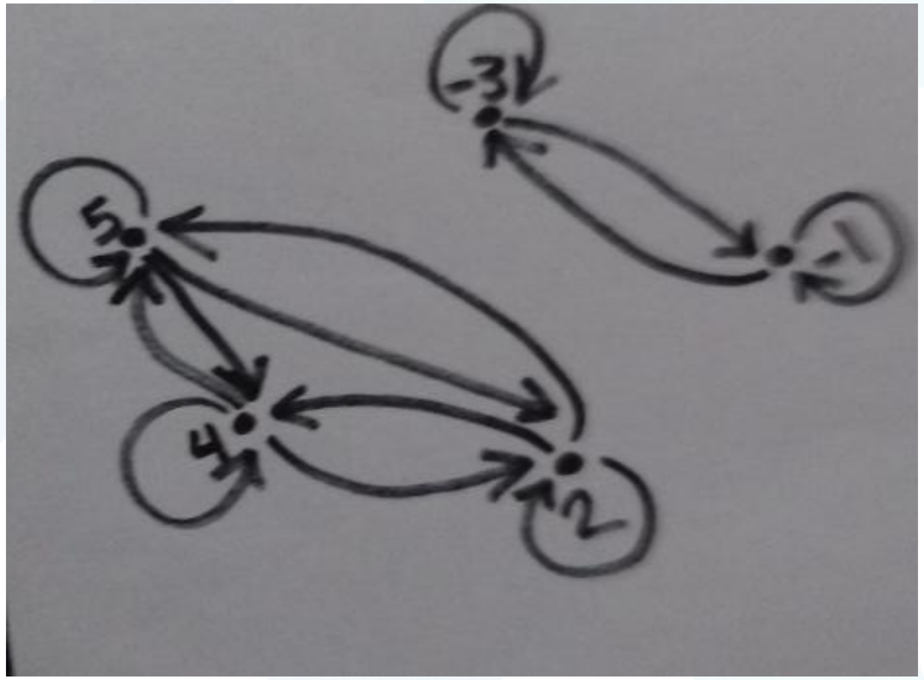
3- represent the relation with graph.

4- determine if R is equivalence relation

Solution:

R={(-3,-3),(-3,-1),(-1,-3),(-1,-1),(2,2),(2,4),(2,5),(4,2),(4,4),(4,5),(5,2),(5,4),(5,5)}

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$



R is equivalence relation because it is reflexive, symmetric and transitive.

Properties of relation

Let A={-1,1,2,3,4} and relation R is defined from A to A as follows R={ (x,y): x = y^2}

1- write R as a set of ordered pairs.

2- represent the relation with matrix.

3-represent the relation with graph.

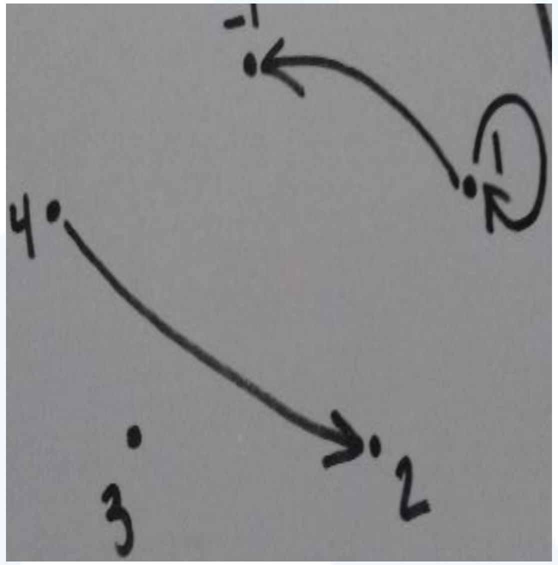
4-determine if R is equivalence relation

5-determine if R is partial ordering relation

Solution:

R={(1,-1),(1,1),(4,2)}

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$



R is not reflexive , antisymmetric , transitive

R is not equivalence relation

R is not partial ordering relation

homework

جَامعة المَنارة
MANARA UNIVERSITY

In 1–5 a number of binary relations are defined on the set $A = \{0, 1, 2, 3\}$. For each relation:

a. Draw the directed graph.
b. Determine whether the relation is reflexive.
c. Determine whether the relation is symmetric.
d. Determine whether the relation is transitive.

Give a counterexample in each case in which the relation does not satisfy one of the properties.

1. $R_1 = \{(0, 0), (0, 1), (0, 3), (1, 1), (1, 0), (2, 3), (3, 3)\}$
2. $R_2 = \{(0, 0), (0, 1), (1, 1), (1, 2), (2, 2), (2, 3)\}$
3. $R_3 = \{(2, 3), (3, 2)\}$
4. $R_4 = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$
5. $R_5 = \{(0, 0), (0, 1), (0, 2), (1, 2)\}$

homework

Let A={0,1,2,3,4} and relation R is defined from A to A as follows

$$R = \{(0,0), (0,4), (1,1), (1,3), (2,2), (3,1), (3,3), (4,0), (4,4)\}$$

1- represent the relation with matrix.

2- represent the relation with graph.

3- determine if R is equivalence relation