Structural Mechanics (2)

Lecture No-04

Part-01

Displacement Method for Beams and Frames or Slope-Deflection Method

Displacement Method for Beams and Frames (Slope – Deflection Method)

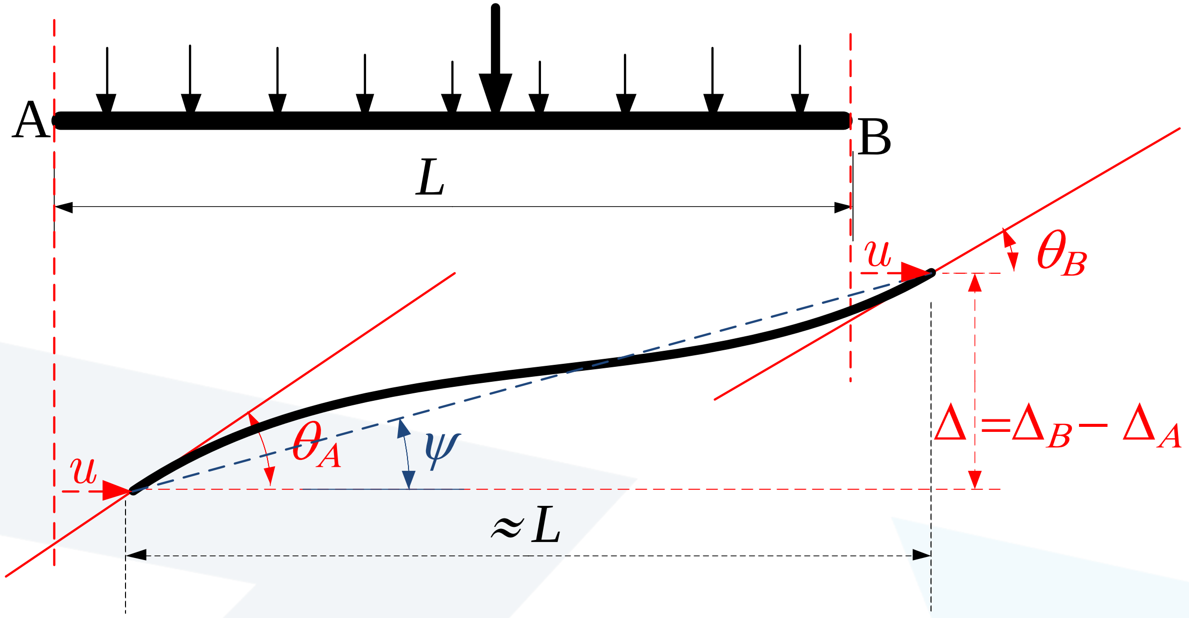
- Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.
- Analysis of Continuous Beams.
- Analysis of Frames without Sidesway.
- Analysis of Frames with Sidesway.

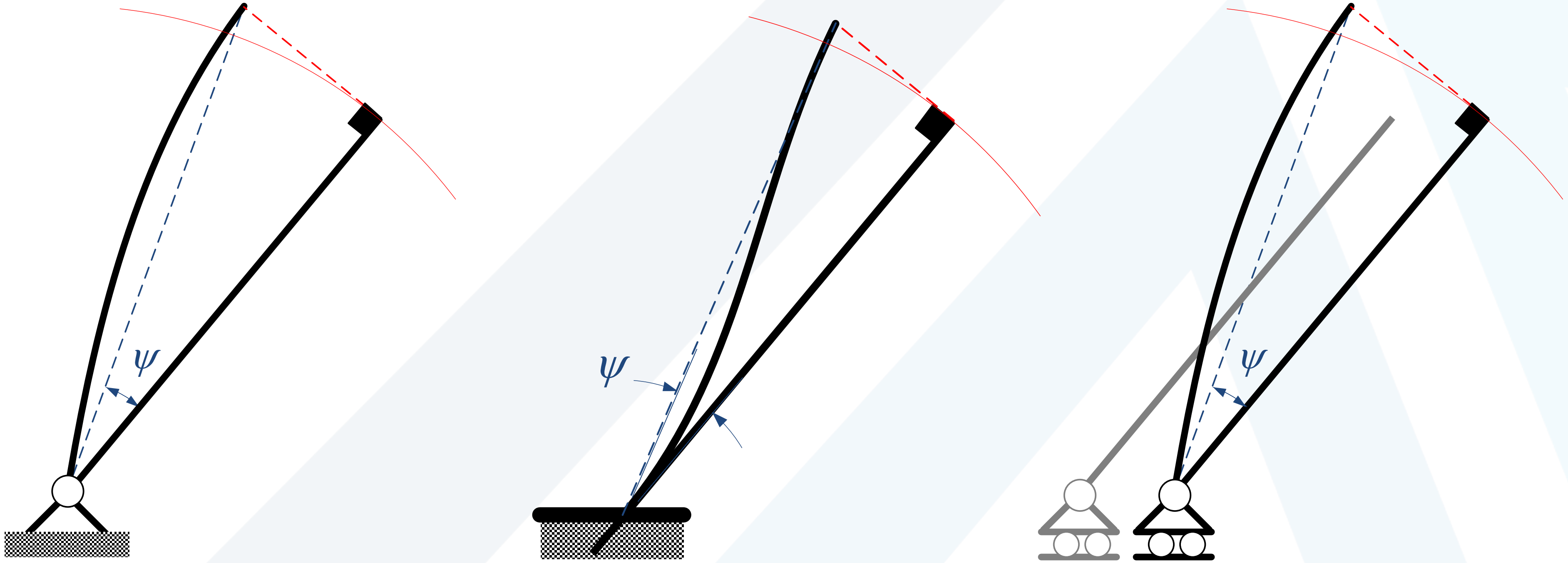
Beam Displacements

Δ Deflection

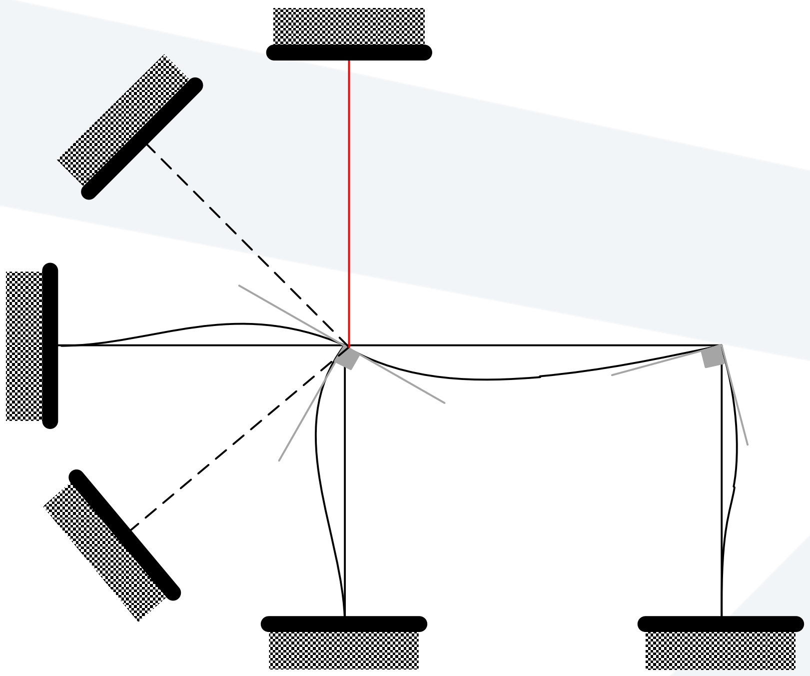
u Axial displacement

$$\theta \ll 1, \sin\theta \approx \theta, \cos\theta \approx 1$$
$$\tan\psi \approx \sin\psi \approx \Delta/L$$
$$\psi \ll 1, \sin\psi \approx \psi, \cos\psi \approx 1$$

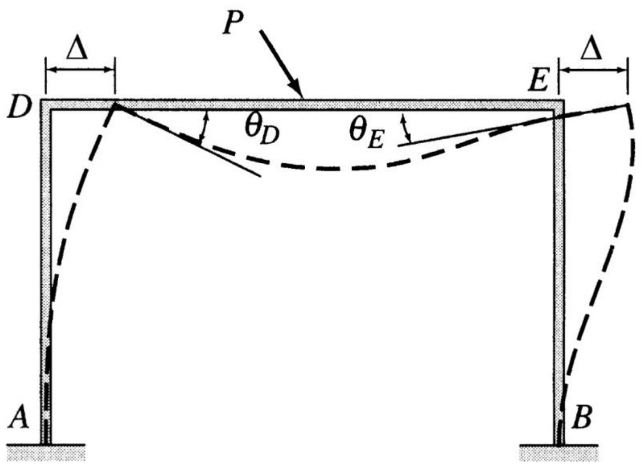




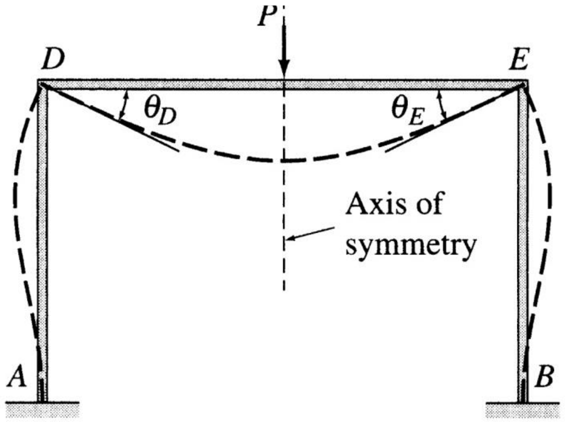
Analysis of Frames with Sidesway



(a) Frame without Sidesway



(b) Frame with Sidesway



EI = constant

(c) Symmetric Frame Subjected to Symmetric Loading — No Sidesway

$$N_{ss} = 2j - [2(f+h)+r+m]$$

j: Number of joints

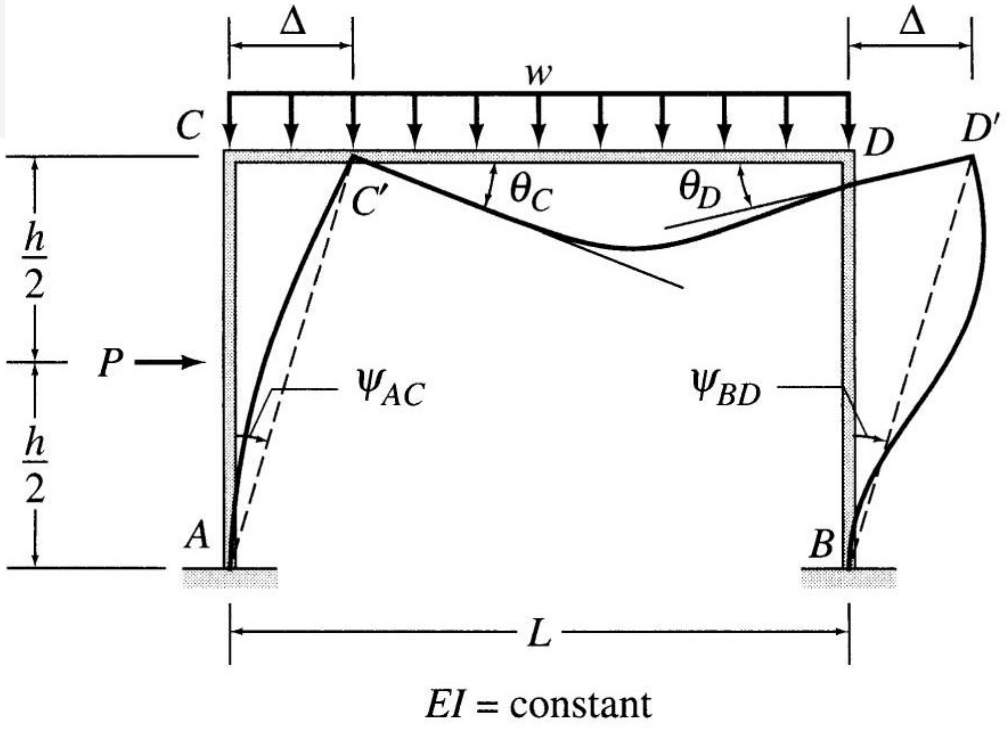
f: Number of fixed supports

h: Number of hinged supports

r: Number of roller supports

m: Number of effective inextensible members

Analysis of Frames with Sidesway



$$M_{AC} = \frac{2EI}{h}\left(\theta_C + \frac{3\Delta}{h}\right) + FEM_{AC}$$

$$M_{CA} = \frac{2EI}{h}\left(2\theta_C + \frac{3\Delta}{h}\right) + FEM_{CA}$$

$$M_{BD} = \frac{2EI}{h}\left(\theta_D + \frac{3\Delta}{h}\right)$$

$$M_{DB} = \frac{2EI}{h}\left(2\theta_D + \frac{3\Delta}{h}\right)$$

$$M_{CD} = \frac{2EI}{L}\left(2\theta_C + \theta_D\right) + FEM_{CD}$$

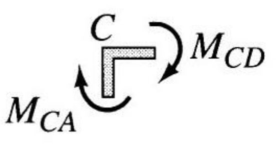
$$M_{DC} = \frac{2EI}{L}\left(2\theta_D + \theta_C\right) + FEM_{DC}$$

3 kinematic unknowns

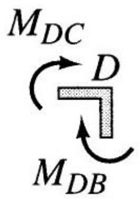
θ_C, θ_D & Δ.

Analysis of Frames with Sidesway

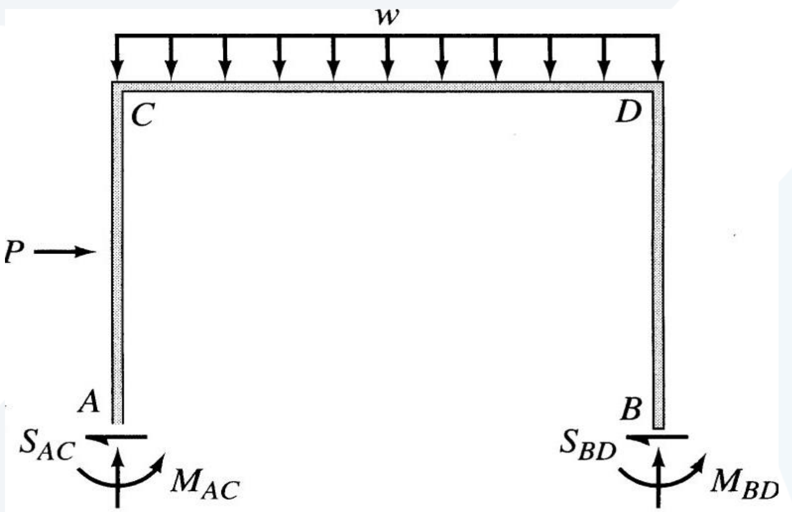
$$M_{CA} + M_{CD} = 0 \qquad (1)$$

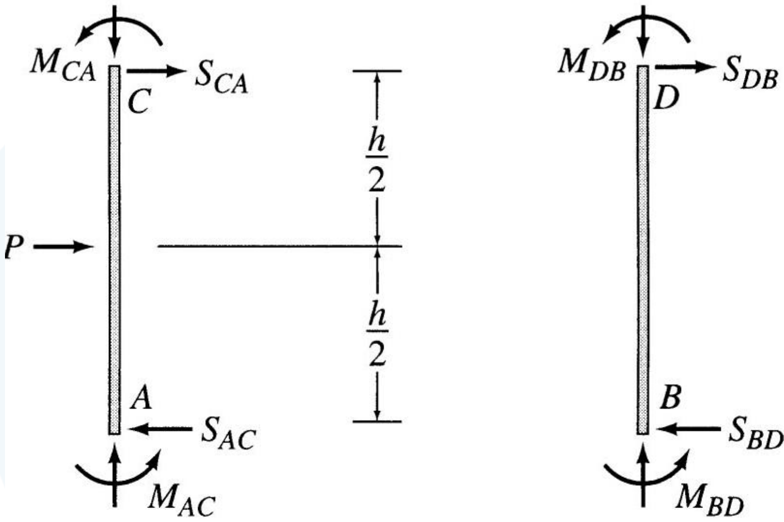


$$M_{DB} + M_{DC} = 0 \qquad (2)$$



$$P - S_{AC} - S_{BD} = 0$$





$$\downarrow\uparrow + \sum M_C^{AC} = 0$$

$$M_{AC} - S_{AC}(h) + P(h/2) + M_{CA} = 0$$

$$S_{AC} = \frac{M_{AC} + M_{CA}}{h} + \frac{P}{2}$$

$$\downarrow\uparrow + \sum M_D^{BD} = 0$$

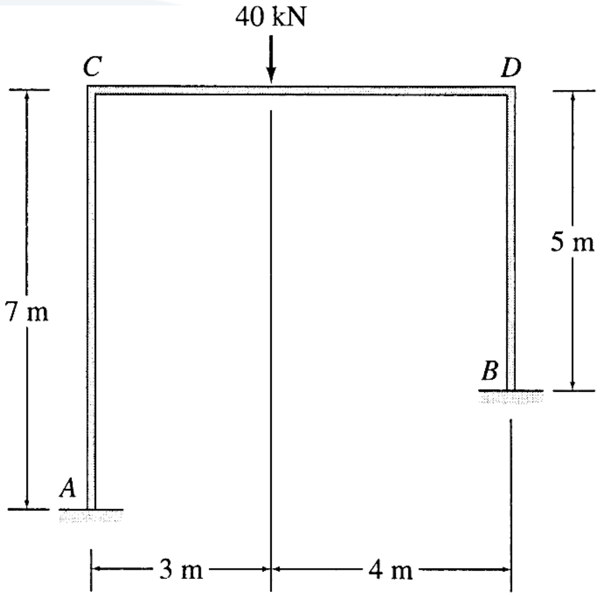
$$M_{BD} - S_{BD}(h) + M_{DB} = 0$$

$$S_{BD} = \frac{M_{BD} + M_{DB}}{h}$$

$$M_{AC} + M_{CA} + M_{BD} + M_{DB} = ph/2 \qquad (3)$$

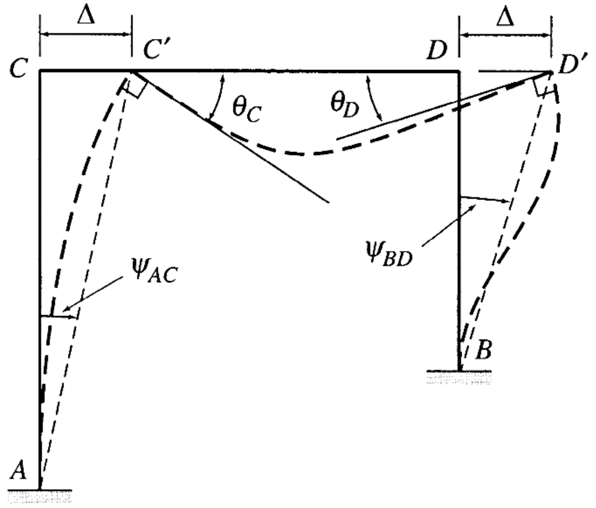
(1), (2) & (3) three equations with three unknowns θ_C, θ_D & Δ.

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



EI = constant

(a) Frame



(b) Qualitative Deflected Shape of the Frame

Degrees of freedom:

$$\theta_A = 0 \;\; \& \;\; \theta_B = 0$$

$$\theta_C \neq 0 \; , \; \theta_D \neq 0 \; , \Delta \neq 0$$

Chord rotations:

$$\psi_{AC} = -\frac{\Delta}{7} \qquad \psi_{BD} = -\frac{\Delta}{5}$$

$$\psi_{BD} = -\frac{\Delta}{5}$$

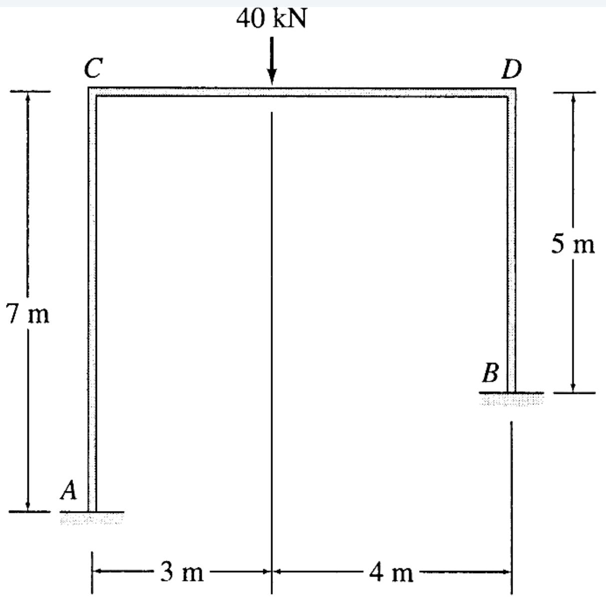
Fixed-End Moments:

$$FEM_{AC} = FEM_{CA} = FEM_{DB} = FEM_{BD} = 0$$

$$FEM_{CD} = \frac{40(3)(4)^2}{(7)^2} = 39.2\; kN.m \curvearrowleft or + 39.2\; kN.m$$

$$FEM_{DC} = -\frac{40(3)^2(4)}{(7)^2} = 29.4\; kN.m \curvearrowright or - 29.4\; kN.m$$

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



EI = constant

(a) Frame

Slope Deflection Equations:

$$M_{AC} = \frac{2EI}{7}\left[\theta_C - 3\left(-\frac{\Delta}{7}\right)\right] = 0.286EI\theta_C + 0.122EI\Delta$$

$$M_{CA} = \frac{2EI}{7}\left[2\theta_C - 3\left(-\frac{\Delta}{7}\right)\right] = 0.571EI\theta_C + 0.122EI\Delta$$

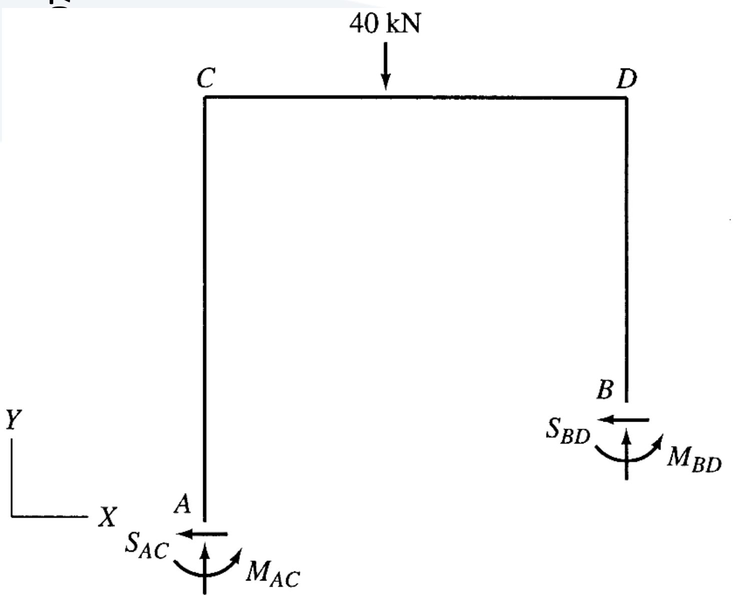
$$M_{BD} = \frac{2EI}{5}\left[\theta_D - 3\left(-\frac{\Delta}{5}\right)\right] = 0.4EI\theta_D + 0.24EI\Delta$$

$$M_{DB} = \frac{2EI}{5}\left[2\theta_D - 3\left(-\frac{\Delta}{5}\right)\right] = 0.8EI\theta_D + 0.24EI\Delta$$

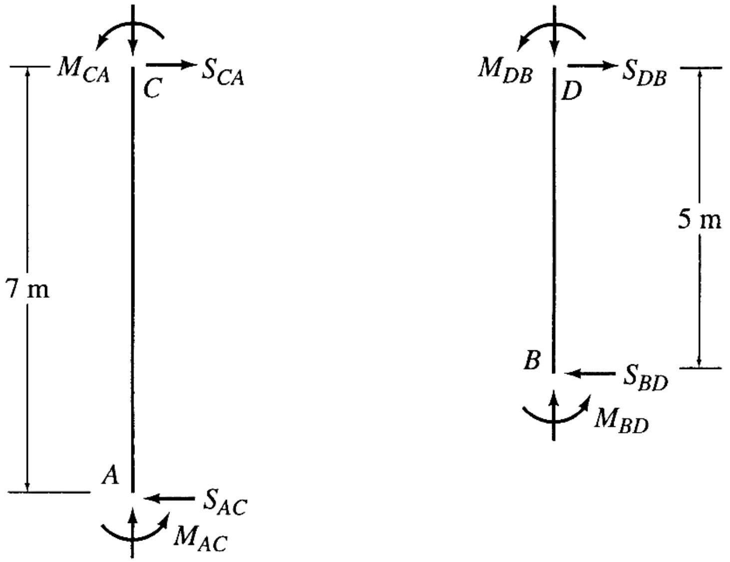
$$M_{CD} = \frac{2EI}{7}(2\theta_C + \theta_D) + 39.2 = 0.571EI\theta_C + 0.286EI\theta_D + 39.2$$

$$M_{DC} = \frac{2EI}{7}(\theta_C + 2\theta_D) - 29.4 = 0.286EI\theta_C + 0.571EI\theta_D - 29.4$$

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



(c) Free-Body Diagram of the Entire Frame



(d) Free-Body Diagrams of Columns *AC* and *BD*

Equilibrium Equations:

$$M_{CA} + M_{CD} = 0$$

$$M_{DC} + M_{DB} = 0$$

$$S_{AC} + S_{BD} = 0$$

$$S_{AC} = \frac{M_{AC} + M_{CA}}{7} \quad \textbf{and} \; S_{BD} = \frac{M_{BD} + M_{DB}}{5}$$

$$\frac{M_{AC} + M_{CA}}{7} + \frac{M_{BD} + M_{DB}}{5} = 0$$

$$5(M_{AC} + M_{CA}) + 7(M_{BD} + M_{DB}) = 0$$

Joint Displacements:

$$1.142EI\theta_C + 0.286EI\theta_D + 0.122EI\Delta = -39.2$$

$$0.286EI\theta_C + 1.371EI\theta_D + 0.24EI\Delta = 29.4$$

$$4.285EI\theta_C + 8.4EI\theta_D + 4.58EI\Delta = 0$$

$$EI\theta_C = -40.211 \; kN.m^2$$

$$EI\theta_D = 34.24 \; kN.m^2$$

$$EI\Delta = -25.177 \; kN.m^2$$

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method

$EI\theta_C = -40.211\ kN.m^2$

$EI\theta_D = 34.24\ kN.m^2$

$EI\Delta = -25.177\ kN.m^2$

Member End Moments:

$M_{AC} = -14.6$ kN.m (**14.6 kN.m ↷**)

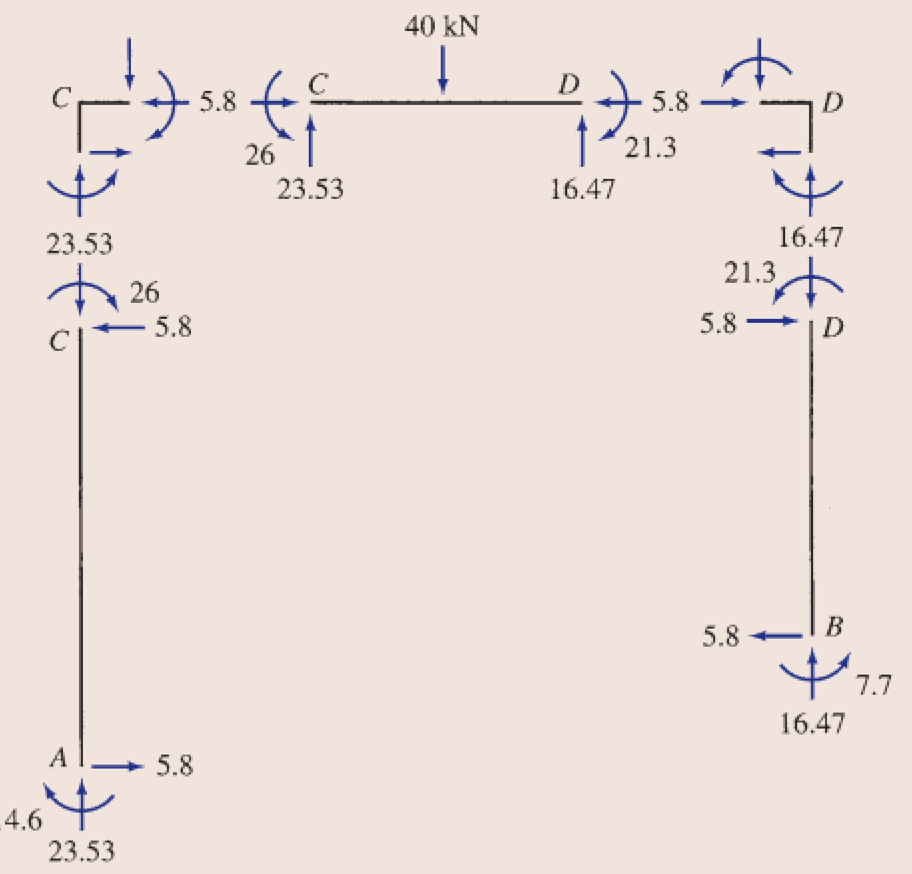
$M_{CA} = -26$ kN.m (**26 kN.m ↷**)

$M_{BD} = 7.7$ kN.m (**7.7 kN.m ↶**)

$M_{DB} = 21.3$ kN.m (**21.3 kN.m ↶**)

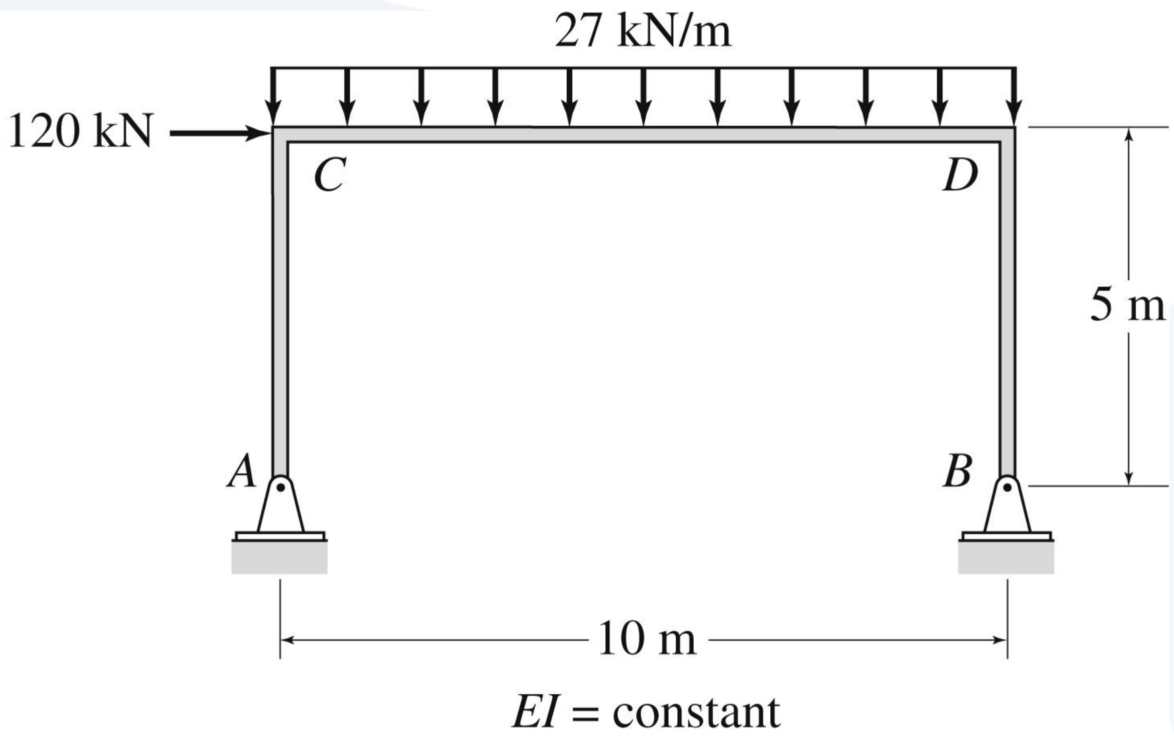
$M_{CD} = 26$ kN.m (**26 kN.m ↶**)

$M_{DC} = -21.3$ kN.m (**21.3 kN.m ↷**)



(e) Member End Moments, Shears, and Axial Forces

Ex.3: Determine the member end moments and reactions for the frame shown using slope-deflection method



Ex.3: Determine the member end moments and reactions for the frame shown using slope-deflection method

Fixed-end moments: $FEM_{AC} = FEM_{CA} = 0$

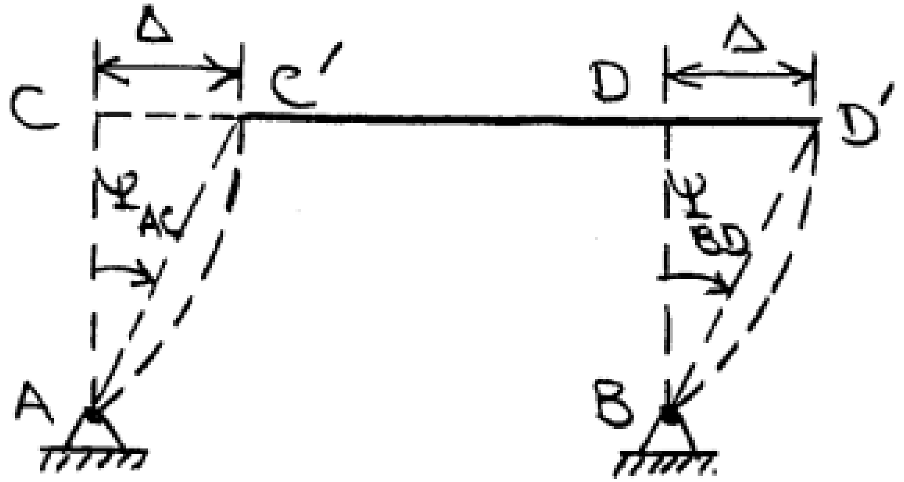
$FEM_{BD} = FEM_{DB} = 0$; $FEM_{CD} = \frac{27(10)^2}{12} = 225$ kN·m;

$FEM_{DC} = -225$ kN·m.

Chord rotations:

$\Psi_{AC} = \Psi_{BD} = -\frac{\Delta}{5}$

$\Psi_{CD} = 0$



Ex.3: Determine the member end moments and reactions for the frame shown using slope-deflection method

Slope-deflection equations: $M_{AC} = M_{BD} = 0$

$$M_{CA} = 0.6\,EI\theta_C + 0.12\,EI\Delta$$

$$M_{DB} = 0.6\,EI\theta_D + 0.133\,EI\Delta$$

$$M_{CD} = 0.4\,EI\theta_C + 0.2\,EI\theta_D + 225$$

$$M_{DC} = 0.2\,EI\theta_C + 0.4\,EI\theta_D - 225$$

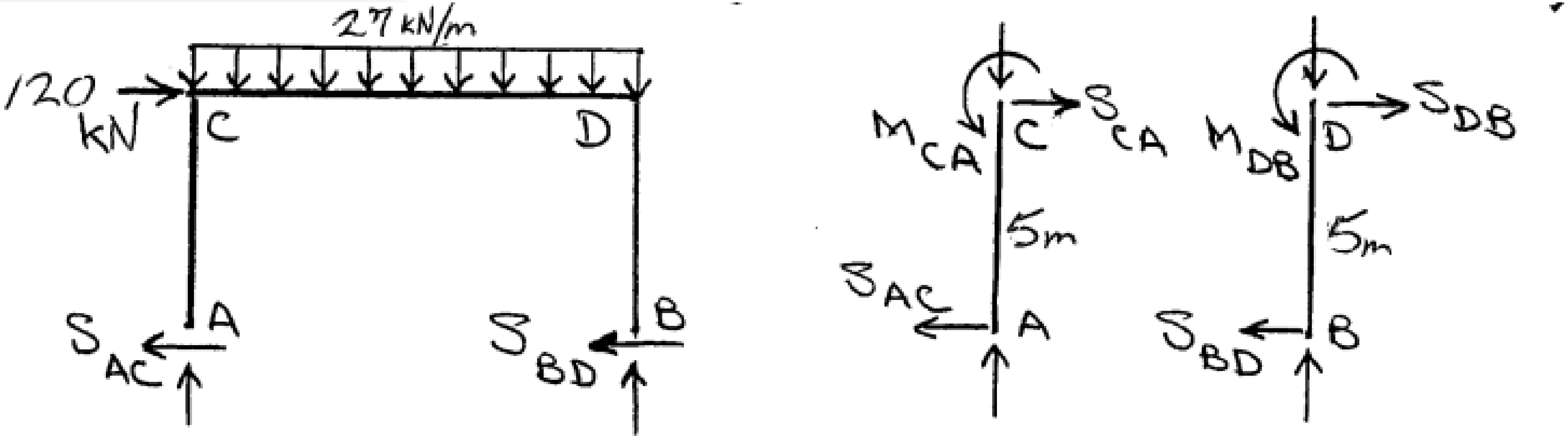
Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$EI\theta_C + 0.2\,EI\theta_D + 0.12\,EI\Delta = -225 \quad (1)$$

$$M_{DB} + M_{DC} = 0$$

$$0.2\,EI\theta_C + \quad EI\theta_D + 0.12\,EI\Delta = 225 \quad (2)$$

Ex.3: Determine the member end moments and reactions for the frame shown using slope-deflection method



$$\Sigma F_x = 0 \qquad S_{AC} + S_{BD} = 40 \qquad \frac{M_{CA}}{5} + \frac{M_{DB}}{5} = 120$$

$$0.12\, EI\theta_C + 0.12\, EI\theta_D + 0.048\, \Delta = 120 \qquad (3)$$

Ex.3: Determine the member end moments and reactions for the frame shown using slope-deflection method

By solving Eqs. (1) thru (3) simultaneously, we obtain:

$EI\theta_C = -781.3 \text{ kN}\cdot\text{m}^2$; $EI\theta_D = -218.8 \text{ kN}\cdot\text{m}^2$

$EI\Delta = 5000 \text{ kN}\cdot\text{m}^3$.

Member end moments:

$M_{CA} = 131.25 \text{ kN}\cdot\text{m}$; $M_{DB} = 468.75 \text{ kN}\cdot\text{m}$

$M_{CD} = -131.25 \text{ kN}\cdot\text{m}$; $M_{DC} = -468.75 \text{ kN}\cdot\text{m}$

Ex.3: Determine the member end moments and reactions for the frame shown using slope-deflection method