جامعة المنارة

كلية: الهندسة

اسم المقرر: الرياضيات المتقطعة

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تجميع العلاقات

Combining Relations

Combining relations via Boolean operators using matrices

Suppose that the relations R1 and R2 on a set A={1,2,3}

R1={(1,1),(1,3), (2,1),(3,2)}

R2={(1,1),(1,3),(2,2),(2,3),(3,1)}

Find R1∪ R2 , R1∩ R2

Solution:

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad , \quad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 1 \vee 1 & 0 \vee 0 & 1 \vee 1 \\ 1 \vee 0 & 0 \vee 1 & 0 \vee 1 \\ 0 \vee 1 & 1 \vee 0 & 0 \vee 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow R1 \cup R2 = \{(1,1),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2)\}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 1 \wedge 1 & 0 \wedge 0 & 1 \wedge 1 \\ 1 \wedge 0 & 0 \wedge 1 & 0 \wedge 1 \\ 0 \wedge 1 & 1 \wedge 0 & 0 \wedge 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow R1 \cap R2 = \{(1,1),(1,3)\}$$

Combining relations via relation Composition using matrices

Exercise: Suppose that the relations S and R on a set A={1,2,3} ,
R={(1,1),(1,3), (2,1),(2,2)}
S={(1,2),(2,3),(3,1),(3,3)}
Find $S \circ R$

Solution:

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$S \circ R$={(1,1),(1,2),(1,3),(2,2),(2,3)}

Note:

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

$1 = (1 \wedge 0) \vee (0 \wedge 0) \vee (1 \wedge 1)$

Combining relations using matrices

Let R_1 and R_2 be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent

a) $R_1 \cup R_2$. **b)** $R_1 \cap R_2$. **c)** $R_2 \circ R_1$.

d) $R_1 \circ R_1$.

Solution:

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_2 \circ R_1} = M_{R_1} \odot M_{R_2} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{R_1 \circ R_1} = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$