Trusses الجيزان الشبكية

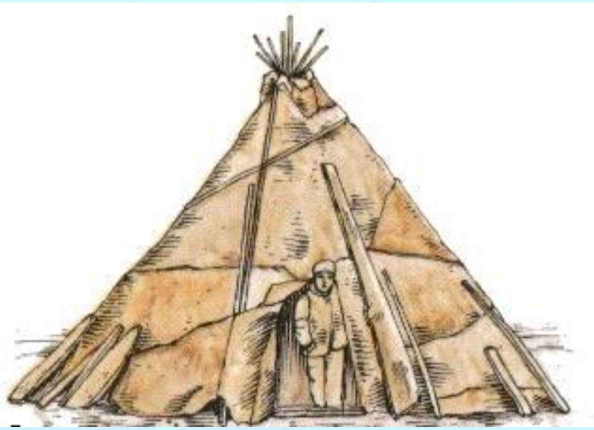
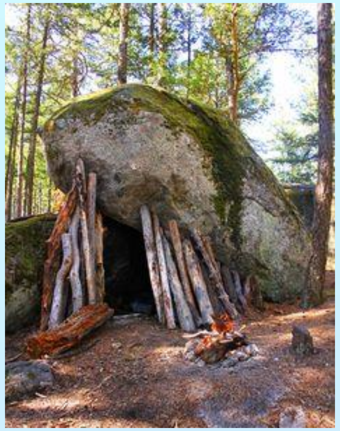
1. Statically Determinate Trusses
2. Determination of the Internal Forces
2.1. Method of Joints
2.2. Method of Sections
3. Supplementary Examples

1. الجائز الشبكي المقرر سكونيا
2. تحديد القوى الداخلية
2 . 1. طريقة توازن العقد
2. 2. طريقة توازن المقطع
3. أمثلة إضافية.

Definition: A truss is a structure composed of straight slender members that are connected at their ends by pin joints. The truss is loaded only at its joints, making its members working axially. The truss is one of the oldest and most important structures in engineering applications.

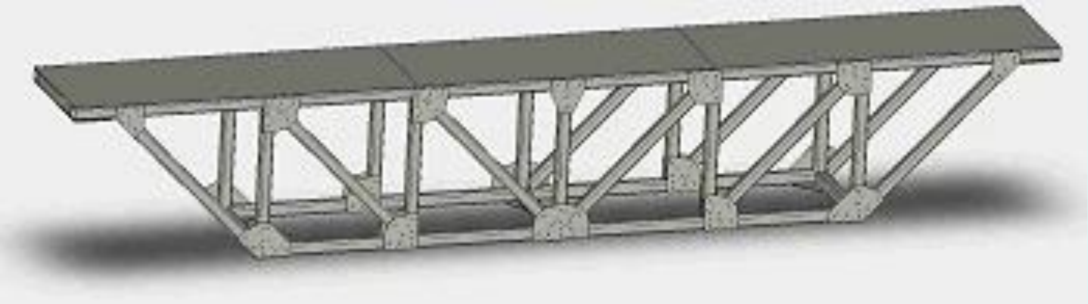
تعريف: يتكون الجائز الشبكي من عناصر مستقيمة نحيلة تتصل ببعضها عند نهاياتها بواسطة عقد مفصلية. تُحصر الحمولات في العقد مما يجعل العناصر تعمل محورياً.يعد الجائز الشبكي واحدا من أقدم الجمل الإنشائية وأكثرها أهمية وفعالية.







Using DragonPlate 1 "carbon fiber square tubes and gussets", we built a carbon fiber truss that is very light and incredibly strong. The truss measures 8 feet (2.44m) long, 16 inches (41 cm) high with a depth of 12 inches (30 cm). Weighing in at just 14lbs. (6.35 kgf), the truss is light enough for a six year old child to lift and carry!

Combination of tension members with Compression members in the same Structure

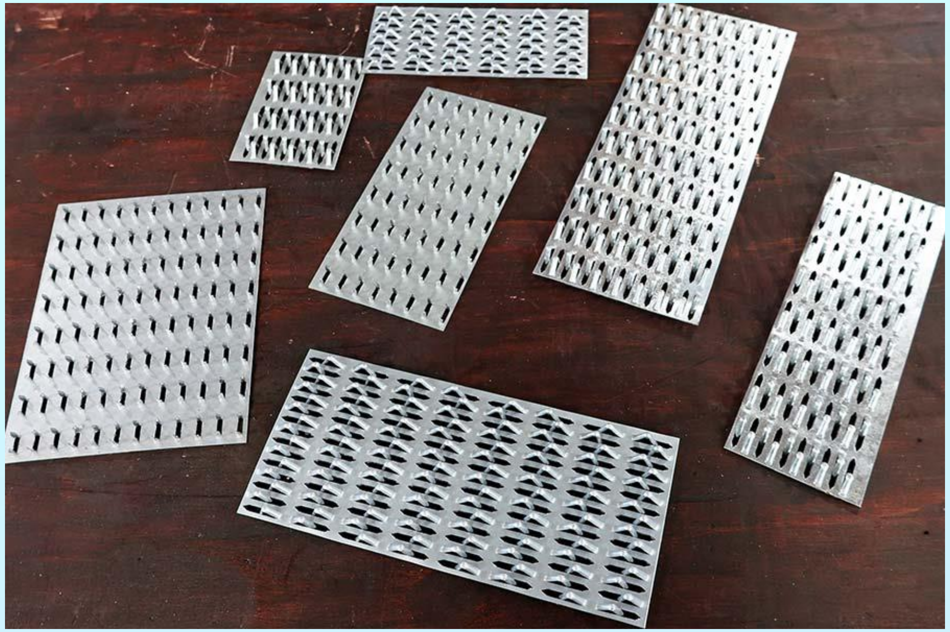
Trusses

Weighing in at just 14lbs. (6.35 kgf), the truss is light enough for a six year old child to lift and carry!

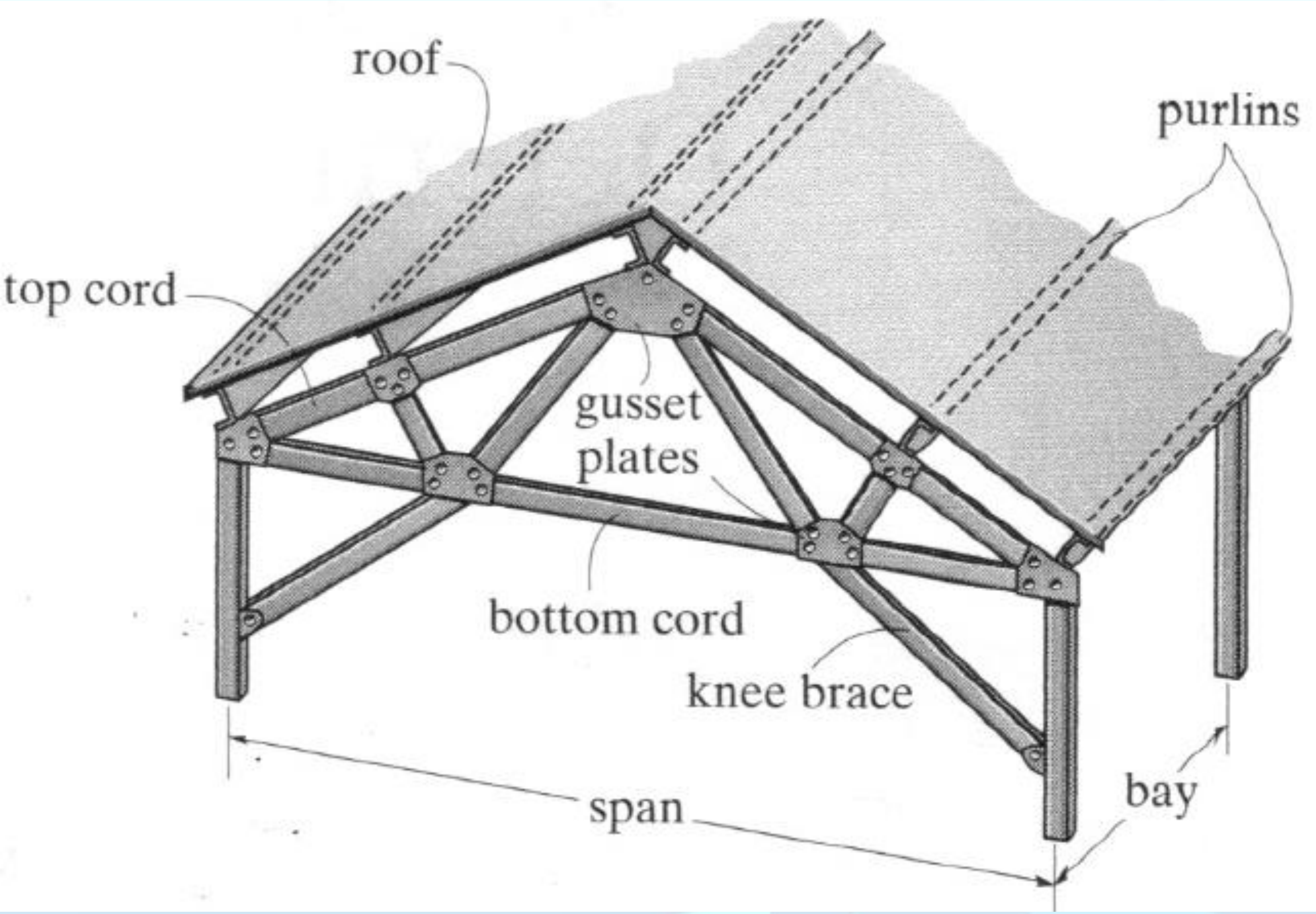
The truss is loaded with 35×80lb bags of concrete mixture, totaling a whopping weight of 2800 (1270 kgf)!

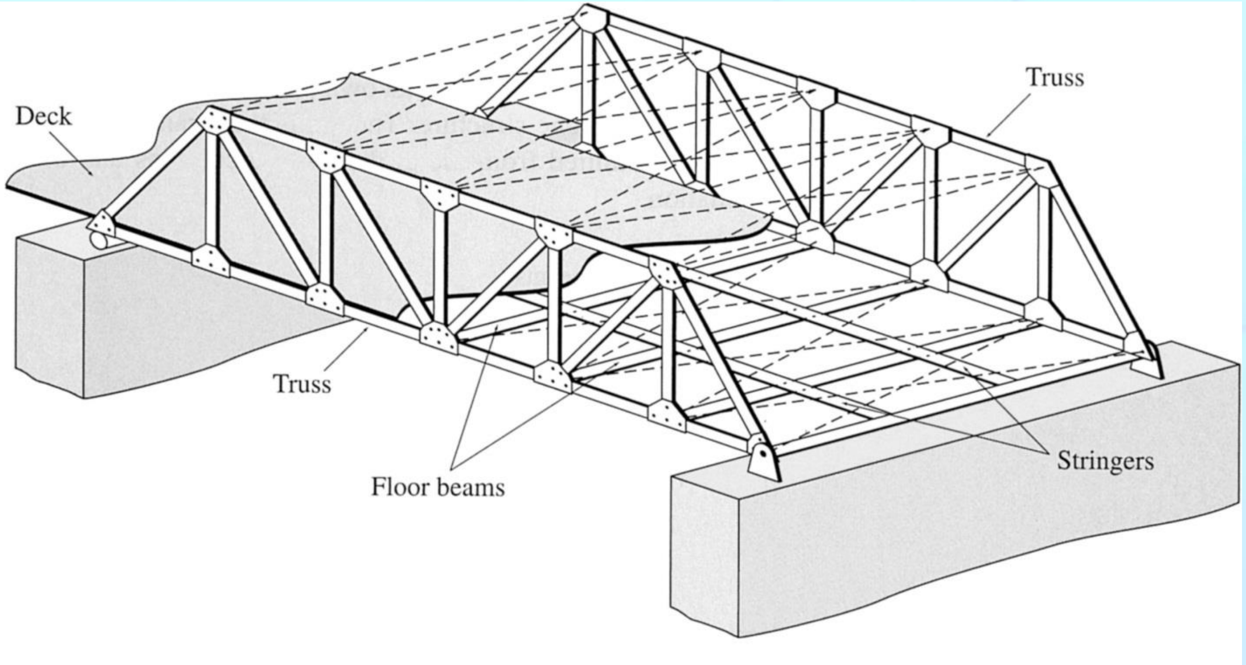
Trusses are very efficient structures

The ratio of the self weight to the load bearing capacity, is very small



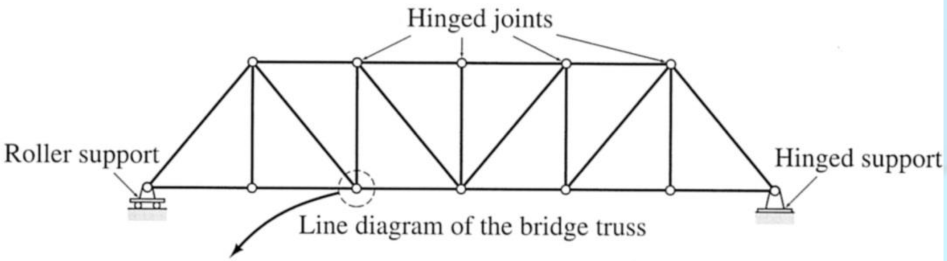

wikiHow


roof
purlins
top cord
gusset
plates
bottom cord
knee brace
span
bay



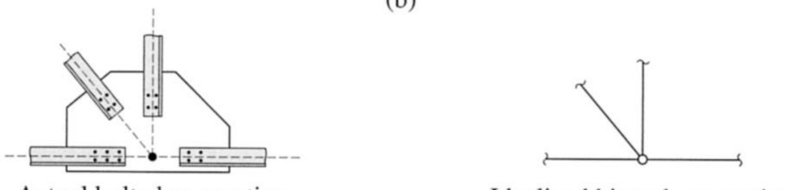
Deck
Truss
Truss
Floor beams
Stringers

(a)



Hinged joints
Roller support
Hinged support
Line diagram of the bridge truss

(b)



Actual bolted connection
Idealized hinged connection

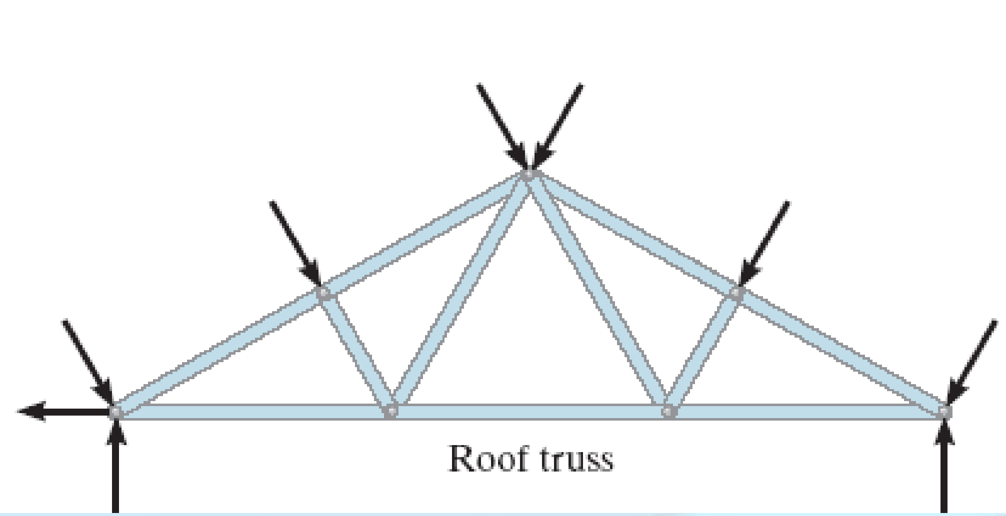
(c)

1. Statically Determinate Trusses

To be able to determine the internal forces in the individual members of an ideal truss, the following assumptions are made:

1. The members are connected through smooth pins (frictionless joints).

2. External forces are applied at the pins only.

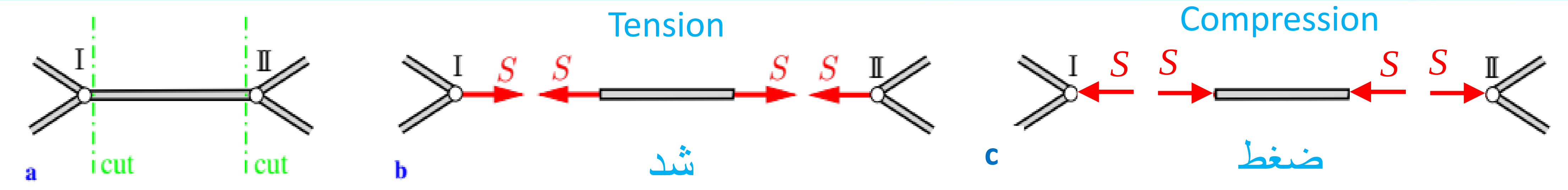


In a plane truss m members connected through j joints, & supported by r reactions.

In order to be able to determine the $m + r$ unknown forces from the $2j$ equilibrium conditions, the number of unknowns has to be equal to the number of equations: $2j = m + r$.

For space truss, the condition becomes: $3j = m + r$.

Tension member or Compression member



It is not always possible to determine by inspection whether a member is subject to tension or compression. Therefore, we shall always assume that all the members of a truss are under tension. If the analysis gives a negative value for the force in a member, this member is in reality subject to compression.

In practice, it may be more convenient to determine first the support reactions from the free-body diagram of the complete truss. Then three other equilibrium equations within the method of joints will serve as checks.

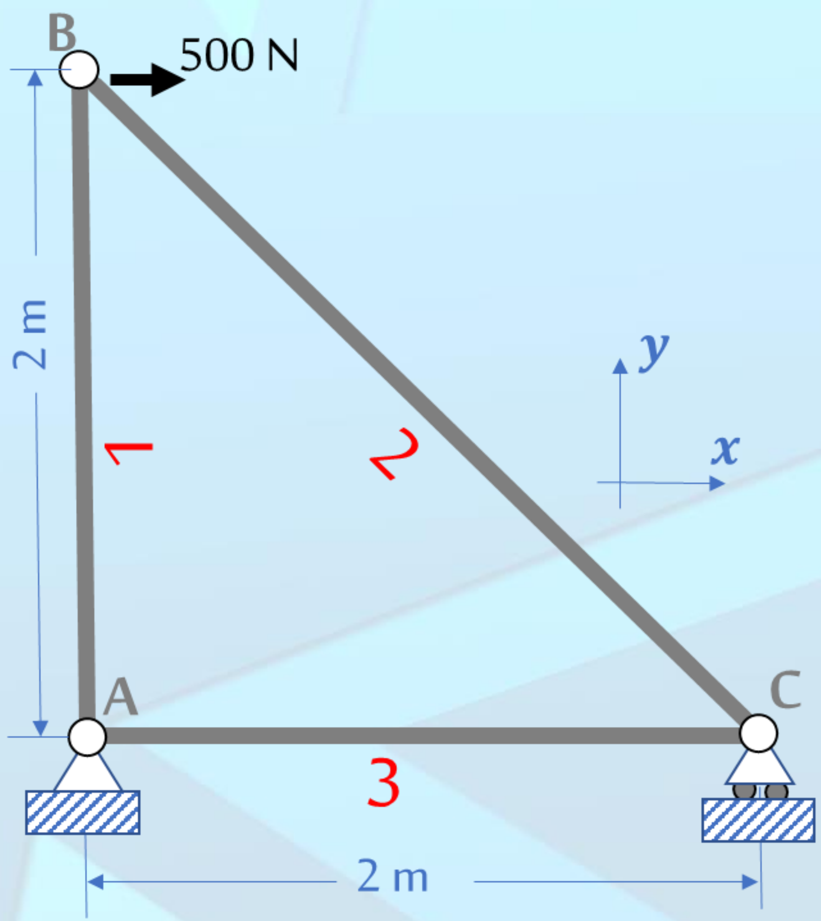
2. Determination of the Internal Forces

.2 تحديد القوى الداخلية

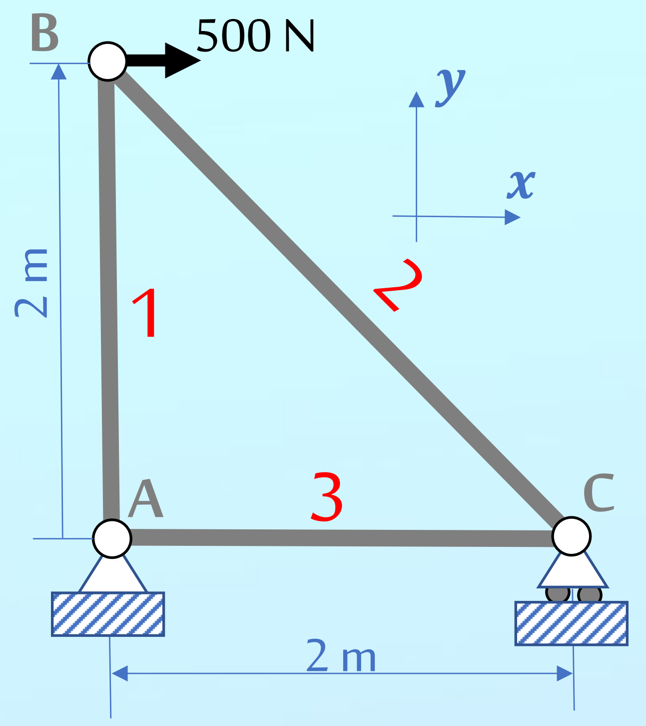
2.1. Method of Joints

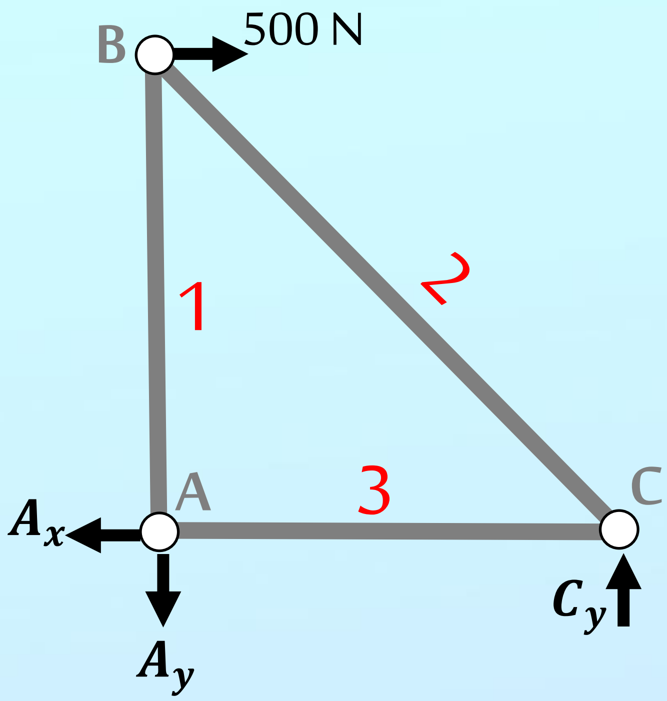
2 . 1. طريقة توازن العقد

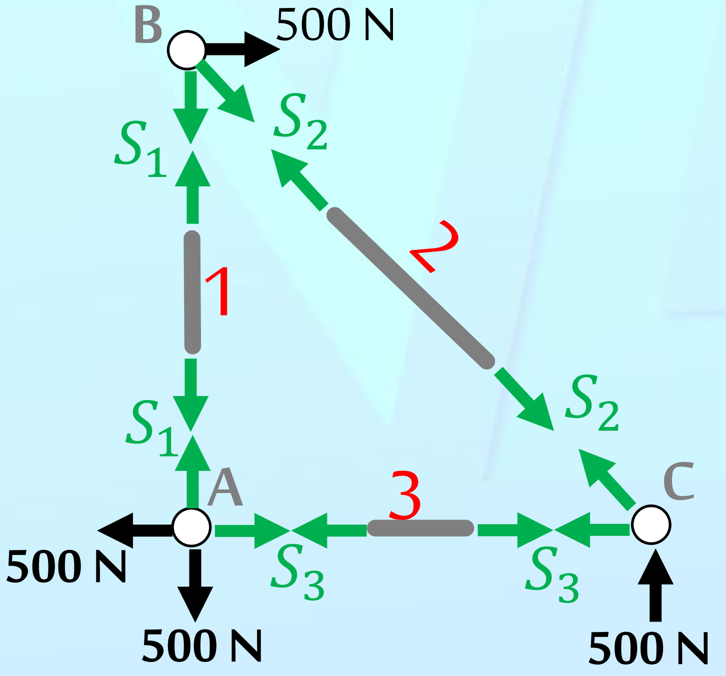
The method of joints consists of applying the equilibrium conditions to the free-body diagram of each joint of the truss. It is a systematic method and can be used for every statically determinate truss.



Illustrative Example. Determine the force in each member of the truss and indicate whether the members are in tension (T) or compression (C).







Eq. Eqs. of the truss:

$\rightarrow$: $A_x = 500$ N

$\curvearrowleft^{A}=0: -2(500) + 2C_y = 0$

$\Rightarrow C_y = 500$ N

$\curvearrowleft^{C}=0: -2(500) + 2A_y = 0$

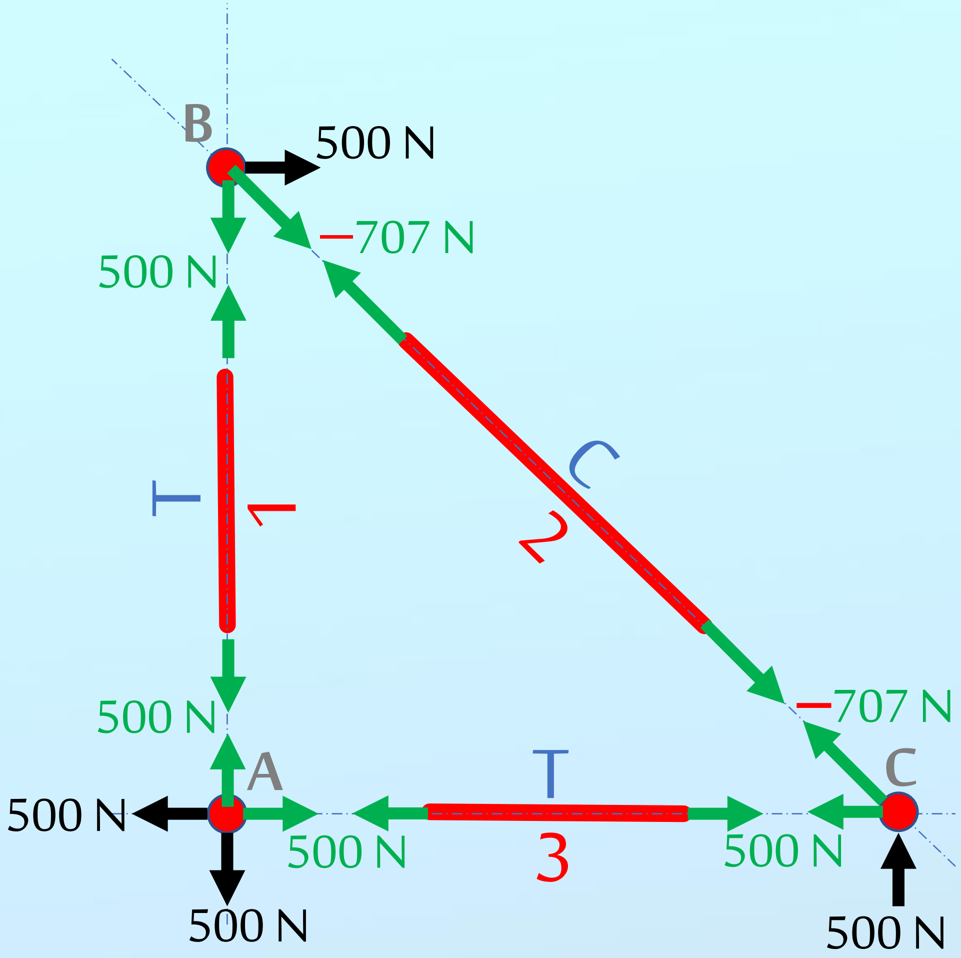
$\Rightarrow A_y = 500$ N

Eq. Eqs. of joint A: $S_1 = 500\ N, S_3 = 500\ N$

Eq. Eqs. of joint B: $S_2 = -707\ N, S_1 = 500\ N$

Eq. Eqs. of joint C: $S_2 = -707\ N, S_3 = 500\ N$

$S_1 = S_{AB} = S_{BA}, \ldots.$



Member	Internal Force [N]
1 or AB	500
2 or BC	−707
3 or AC	500

2. Determination of the Internal Forces

2. تحديد القوى الداخلية

2.1. Method of Joints

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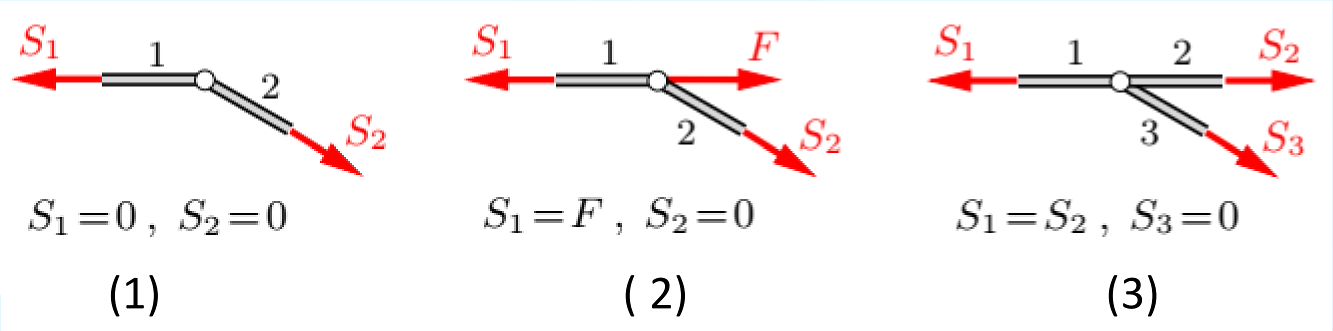
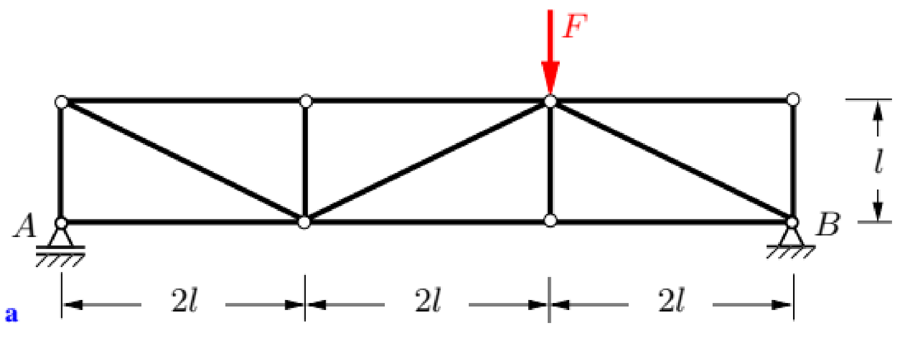
The method of joints consists of applying the equilibrium conditions to the free-body diagram of each joint of the truss. It is a systematic method and can be used for every statically determinate truss.

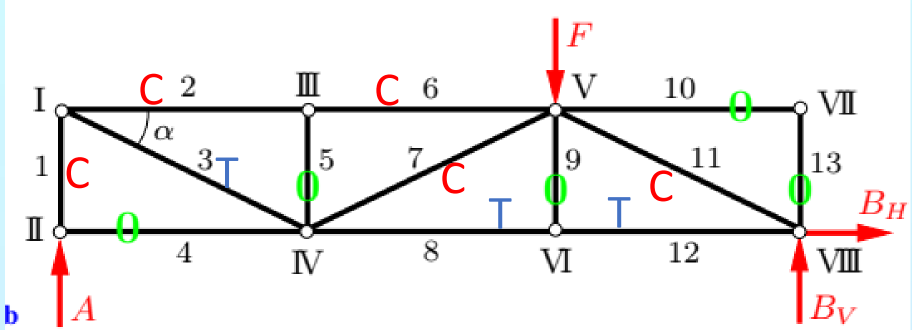
Identification of zero-force members

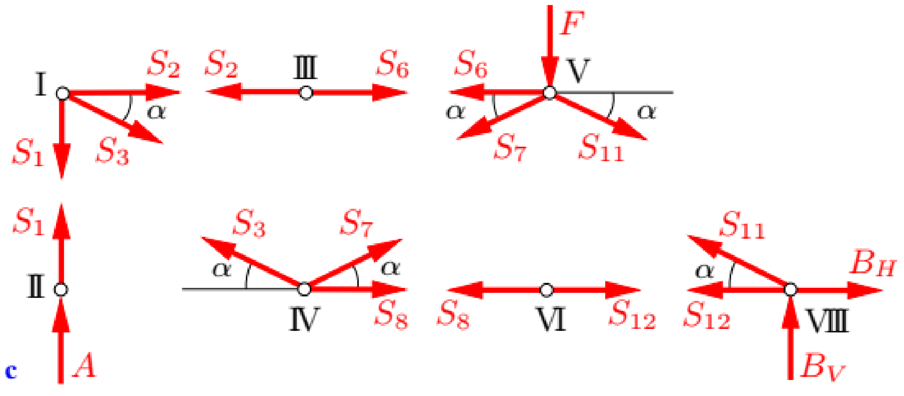


تنعدم القوة الداخلية في عناصر الجائز الشبكي في أي من الحالات التالية:

1. عنصران متلاقيان في عقدة غير محملة لا تحوي غيرهما.
2. عنصر مائل على حمولة في عقدة تحوي عنصرين فقط ومحملة باتجاه أحدهما.
3. عنصر مائل في عقدة غير محملة تحوي ثلاثة عناصر اثنان منهما على استقامة وتحدة.







Example 1. The truss shown in Fig. a is loaded by an external force F. Determine the forces at the supports and in the members of the truss.

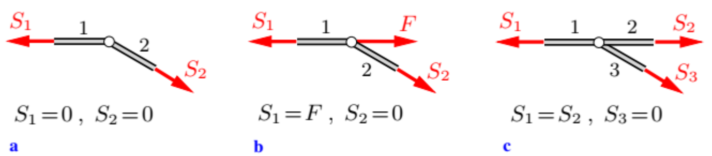
Solution: numbering joints & members: $2j = m + r$.

1) Fig. b, F. B. D. of the truss. Eq. Eqs.:

→: $B_H = 0$

↶^{A}=0: $-4lF+6lB_V=0 \Rightarrow B_V=(2/3)F$

↶^{B}=0: $2\,lF-6lA=0 \Rightarrow A=(1/3)F$



2) Identification of zero-force members.:

3) Joint Equilibrium: F. B. Ds. for joints

II) ↑: $S_1 + A = 0 \Rightarrow S_1 = -A = -(1/3)F$

I) ↑: $-S_1 - S_3\sin\alpha = 0 \Rightarrow S_3 = -S_1/\sin\alpha = (1/3)F/\sin\alpha$

→: $S_3\cos\alpha + S_2 = 0 \Rightarrow S_2 = -S_3\cos\alpha = -(1/3)F/\tan\alpha$

III) →: $-S_2 + S_6 = 0 \Rightarrow S_6 = S_2 = -(1/3)F/\tan\alpha$

IV) ↑: $S_3\sin\alpha + S_7\sin\alpha = 0 \Rightarrow S_7 = -S_3 = -(1/3)F/\sin\alpha$

→: $-S_3\cos\alpha + S_7\cos\alpha + S_8 = 0 \Rightarrow S_8 = S_3\cos\alpha - S_7\cos\alpha = (2/3)F/\tan\alpha$

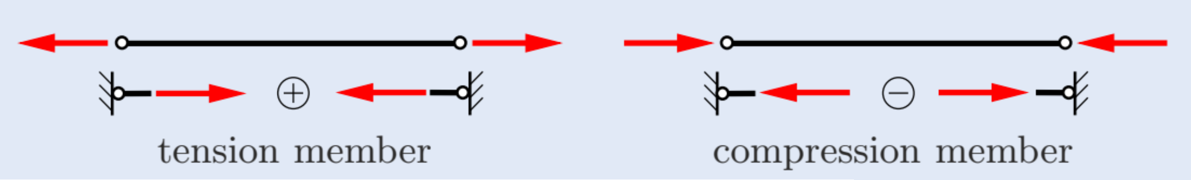
V) ↑: $-S_7\sin\alpha - S_{11}\sin\alpha - F = 0 \Rightarrow S_{11} = -S_7 - F/\sin\alpha = -(2/3)F/\sin\alpha$

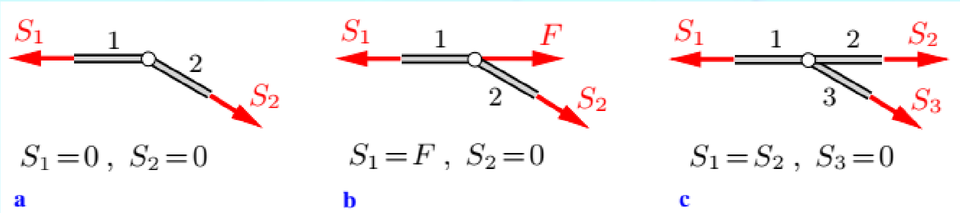
→: $-S_6 - S_7\cos\alpha + S_{11}\cos\alpha = 0 \Rightarrow [(1/3) + (1/3) - (2/3)]F/\tan\alpha \equiv 0$

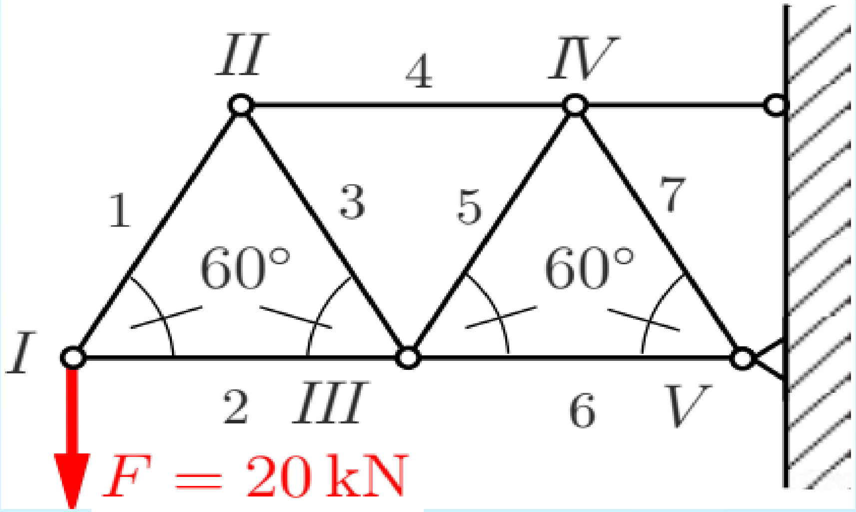
VI) →: $-S_8 + S_{12} = 0 \Rightarrow S_{12} = S_8 = (2/3)F/\tan\alpha$

VIII) ↑: $B_V + S_{11}\sin\alpha = 0 \Rightarrow [(2/3) - (2/3)]F \equiv 0$

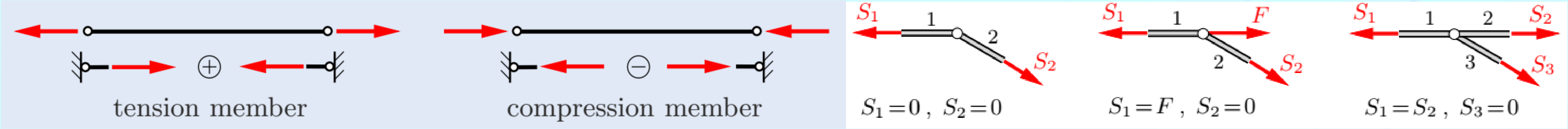
→: $-S_{12} - S_{11}\cos\alpha + B_H = 0 \Rightarrow [-(2/3) + (2/3)]F/\tan\alpha + 0 \equiv 0$

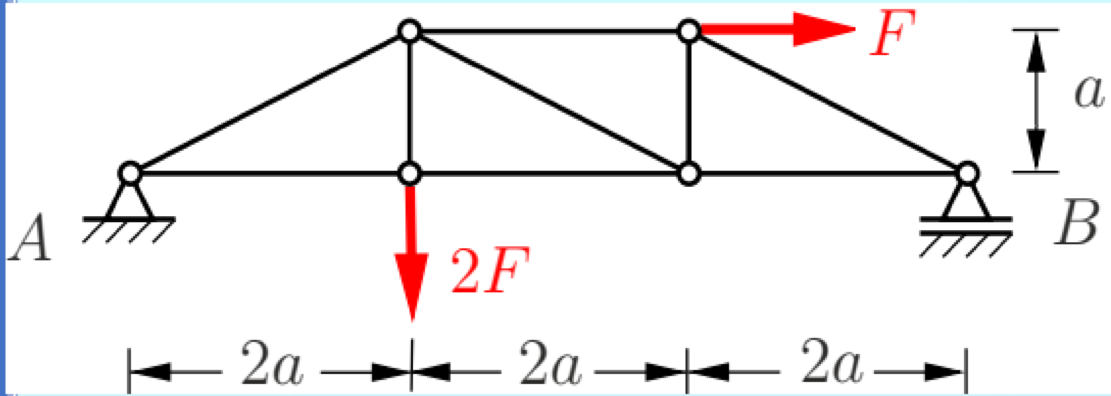






Example 3. For the given truss, all bar forces have to be determined.





Example 2. For the given truss, the forces in the bars shall be determined. Take $F = 12\ kN$.

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Definition: A truss is a structure composed of straight slender members that are connected at their ends by pin joints. The truss is one of the oldest and most important structures in engineering applications.

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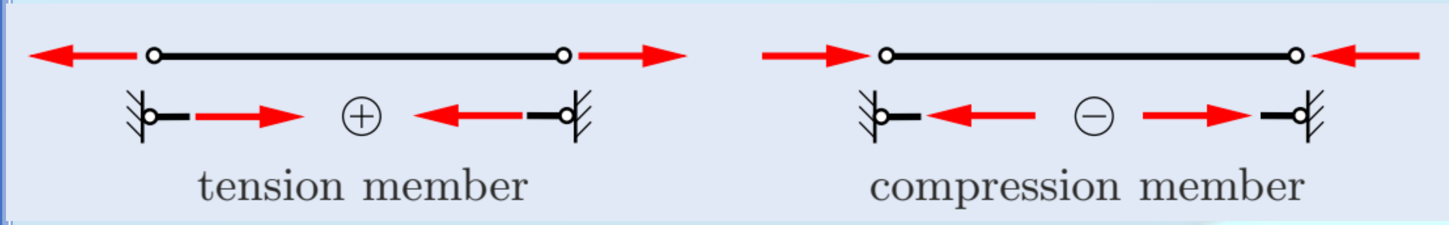
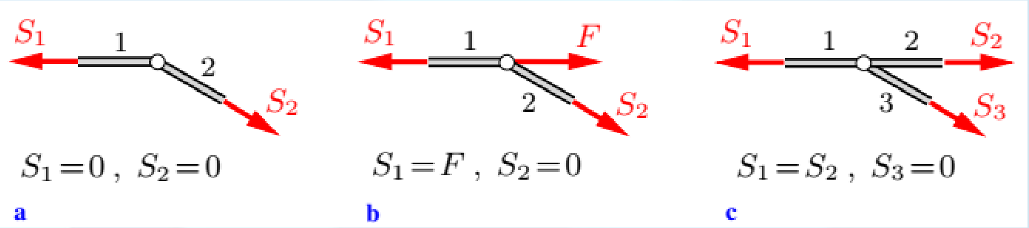




2.2. Method of Sections

It is not always necessary to determine the forces in all of the members of a truss.

If several forces only are of interest, it may be advantageous to use the **method of sections** instead of the method of joints.

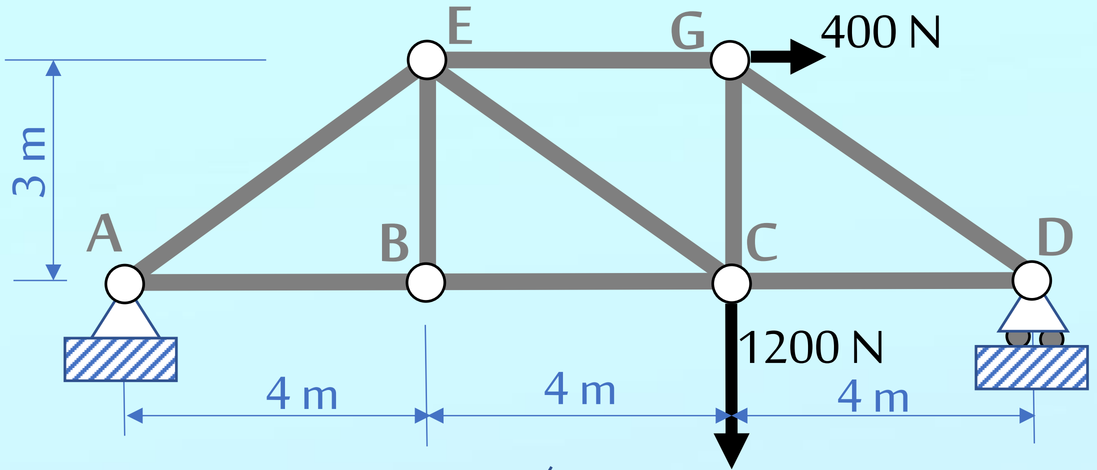
In this case, the truss is divided by a cut **(an imaginary)** into two parts.

The cut has to be made in such a way that it either goes through three members that do not all belong to the same joint.

If the support reactions are computed in advance, the F. B. D. for each part of the truss contains only 3 unknown forces that can be determined by the 3 conditions of equilibrium.

Illustrative Example .

Determine the force in members EG, EC, and BC of the truss. Indicate whether the members are in tension or compression.



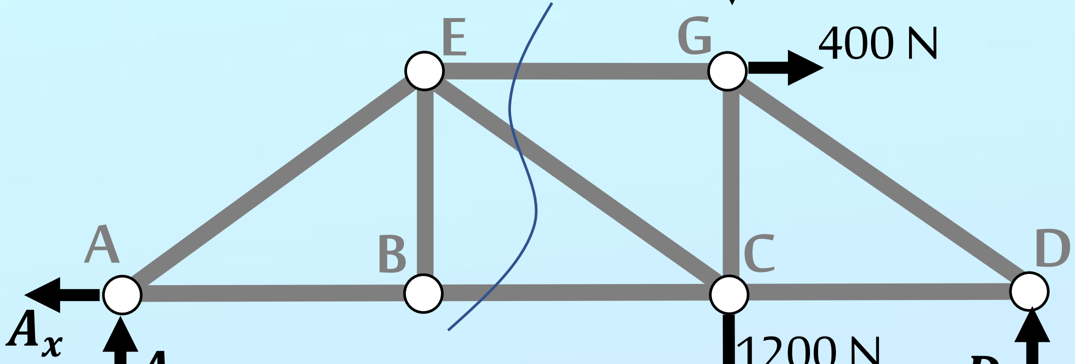
Solution:

1- Reactions: F. B. D. of the Truss

$\rightarrow: -A_x + 400 = 0 \Rightarrow A_x = 400 \text{ N}$

$\circlearrowleft_B: -3(400) + 4(1200) - 12A_y = 0 \Rightarrow A_y = 300 \text{ N}$

$\uparrow: +300 - 1200 + D_y = 0 \Rightarrow D_y = 900 \text{ N}$



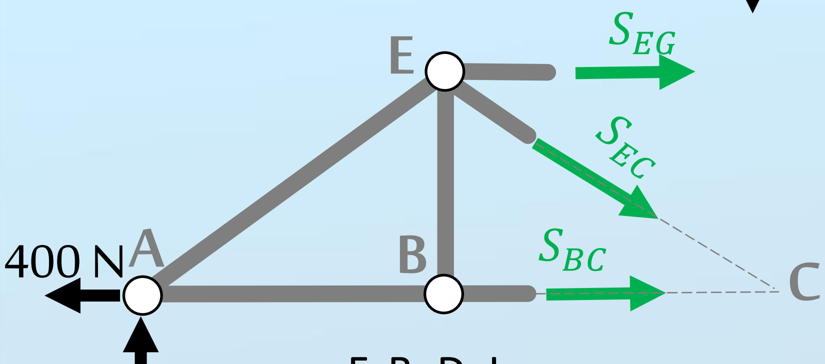
F. B. D.

2- Section Internal Forces: F. B. D. L.

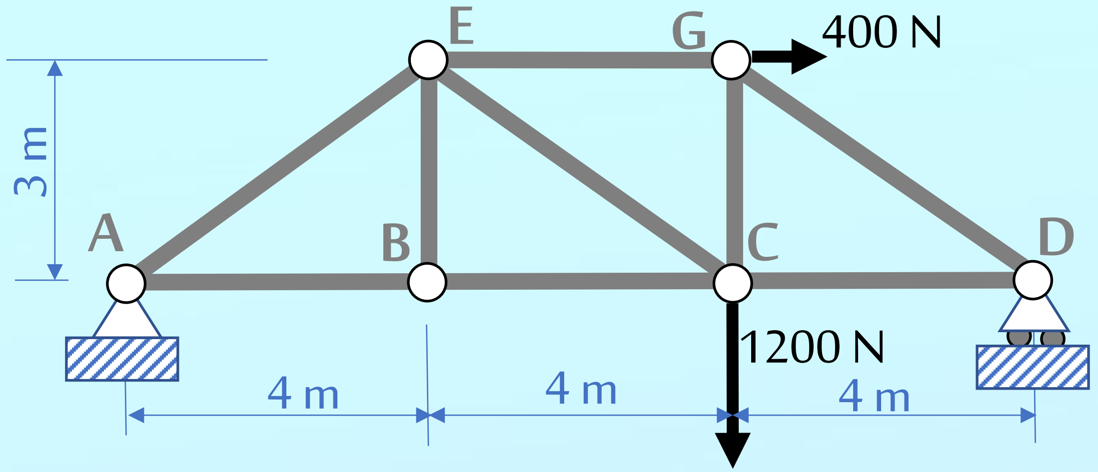
$\circlearrowleft_E: -3(400) - 4(300) + 3S_{BC} = 0 \Rightarrow S_{BC} = 800 \text{ N (T)}$

$\circlearrowleft_C: -8(300) - 3S_{EG} = 0 \quad \Rightarrow S_{EG} = -800 \text{ N (C)}$

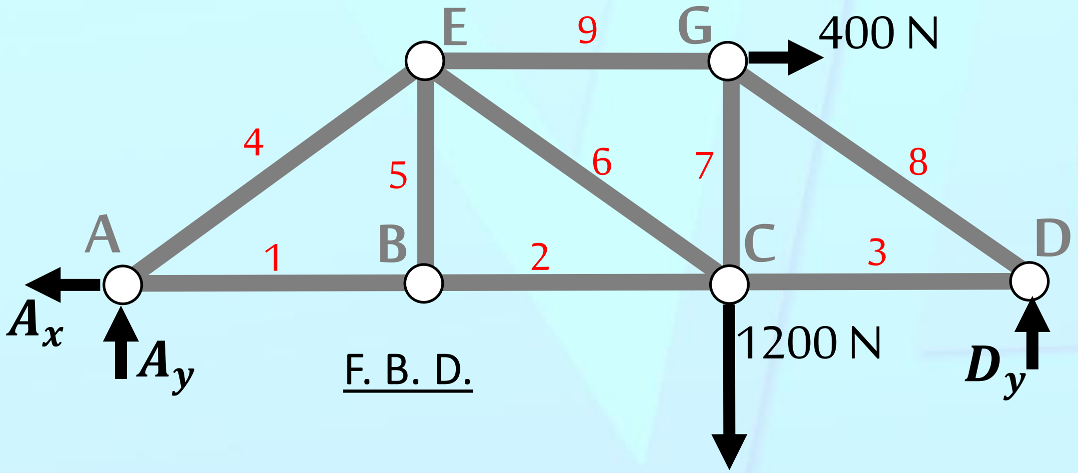
$\uparrow: +300 - S_{EC}\left(\frac{3}{5}\right) = 0 \quad \Rightarrow S_{EC} = 500 \text{ N (T)}$



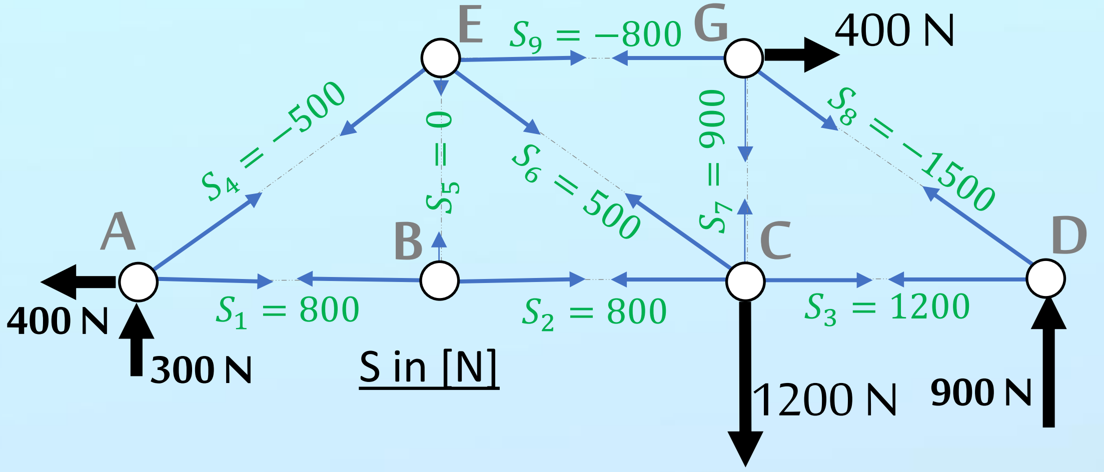
F. B. D. L.



E
G
400 N
3 m
A
B
C
D
1200 N
4 m
4 m
4 m



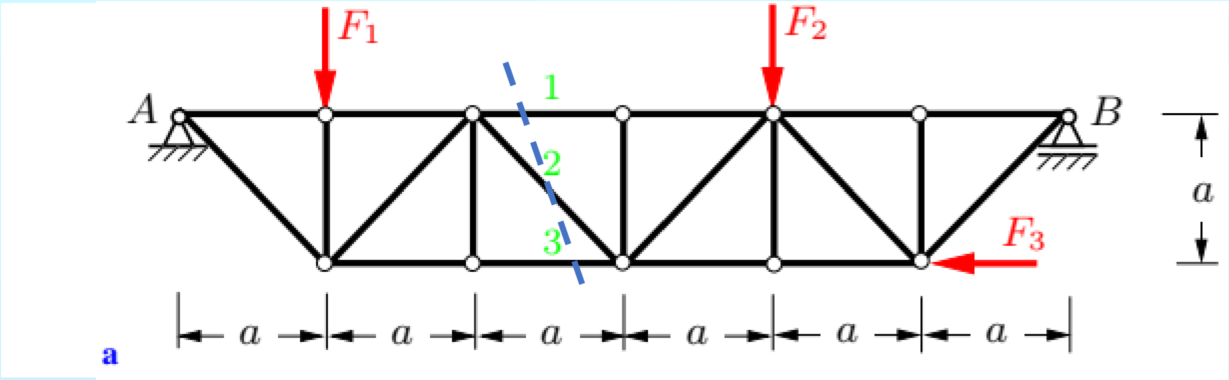
E
9
G
400 N
4
5
6
7
8
A
B
C
D
1
2
3
A_x
A_y
F. B. D.
1200 N
D_y



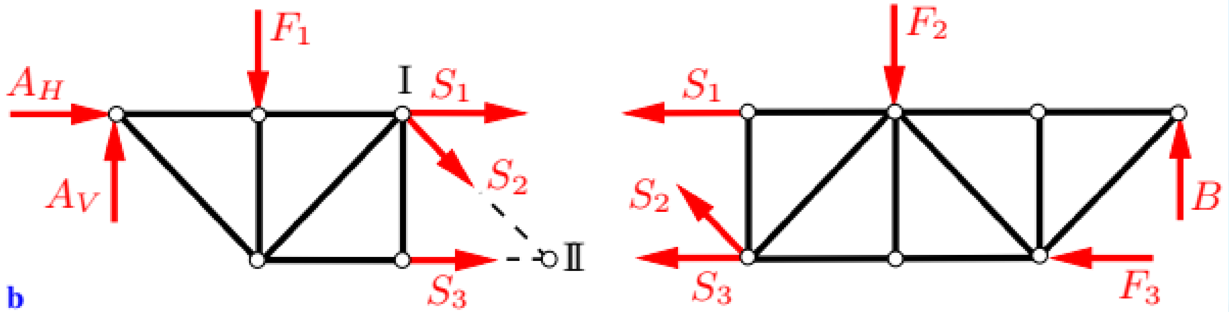
E
$S_9 = -800$
G
400 N
$S_4 = -500$
$S_5 = 0$
$S_6 = 500$
$S_7 = 900$
$S_8 = -1500$
A
B
C
D
400 N
300 N
$S_1 = 800$
$S_2 = 800$
$S_3 = 1200$
S in [N]
1200 N
900 N

Example 1 To illustrate the method, we consider the truss shown in Fig.a with the aim of determining the forces in members 1,2 &3.

As a first step, the reactions at supports A & B are computed by applying the Eq. Eqs. to the free-body diagram of the whole truss



In the second step, we pass an imaginary section through the members 1,2 & 3, cutting the truss into two parts. Fig.b shows the free-body diagrams of the two parts of the truss. The internal forces in members 1,2 & 3 act as external forces in the free-body diagrams; they are assumed to be tensile forces.



Both parts of the truss in Fig.b are rigid bodies in equilibrium. Therefore, either part may be used for the analysis.

$\overset{\curvearrowleft}{\mathrm{I}}: aF_1 - 2aA_V + aS_3 = 0 \Rightarrow S_3 = 2aA_V - F_1$

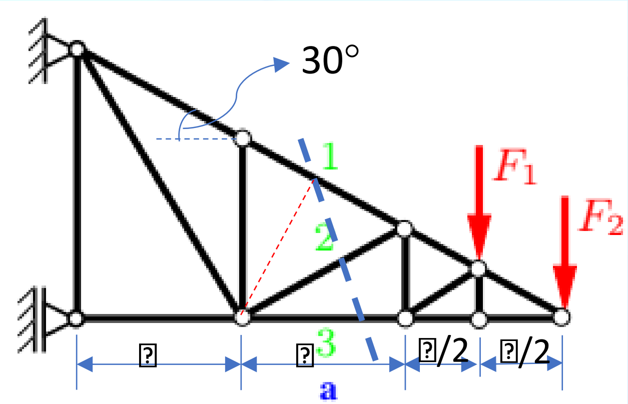
$\overset{\curvearrowleft}{\mathrm{II}}: -aA_H - 3aA_V + 2aF_1 - aS_1 = 0 \Rightarrow S_1 = 2F_1 - 3\,A_V - A_H$

$\uparrow: \; A_V - F_1 - S_2 \sin 45^\circ = 0 \Rightarrow S_2 = \sqrt{2}(A_V - F_1)$

We shall apply the Eq. Eqs. to the free-body diagram on the left-hand side of Fig.b.

Example 2 In many cases, the method of sections can be applied without having to determine the forces at the supports.

Consider, for example, the truss in Fig.a. The forces in members 1,2 & 3 can be obtained immediately from the Eq. Eqs. for the part of the truss on the right as shown in Fig.b.

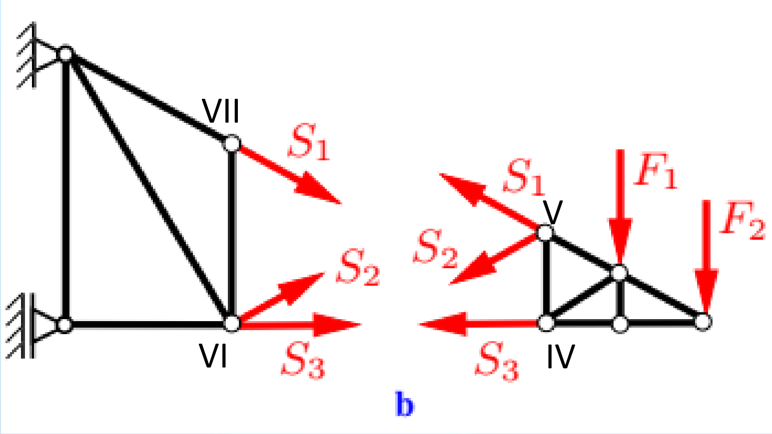


Solution

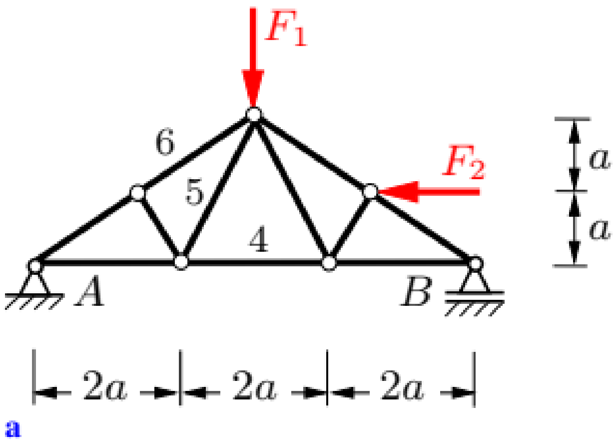
$\circlearrowleft$ V: $$-lF_2 - (l/2)F_1 - (l\tan 30^{\circ})S_3 = 0 \Rightarrow S_3 = -(2F_2 + F_1)/2\tan 30^{\circ}$$

$\circlearrowleft$ VI: $$-2lF_2 - (3l/2)F_1 + l\,S_1 = 0 \Rightarrow S_1 = (4F_2 + 3F_1)/2$$

$\uparrow$: $$-F_2 - F_1 + S_1\sin 30° - S_2\sin 30° = 0 \Rightarrow S_2 = -F_1/2$$



Example 3 A truss is loaded by two forces, $F_1 = 2F$ and $F_2 = F$, as shown in Fig.a. Determine the force S_4.



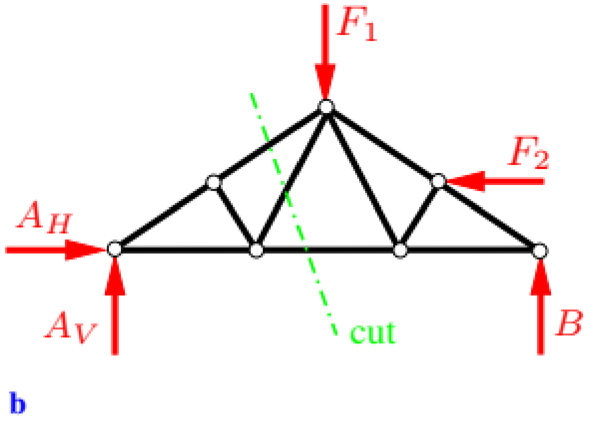
Solution First, we determine the forces at the supports.

Applying the equilibrium conditions to the free-body diagram of the whole truss (Fig.b) yields

$$\overset{\curvearrowleft}{A}: \quad -3aF_1 + aF_2 + 6aB = 0 \Rightarrow B = \frac{3F_1 - F_2}{6} = 5F/6$$

$$\overset{\curvearrowleft}{B}: \quad 3aF_1 + aF_2 - 6A_V = 0 \Rightarrow A_V = (3aF_1 + aF_2)/6 = 7F/6$$

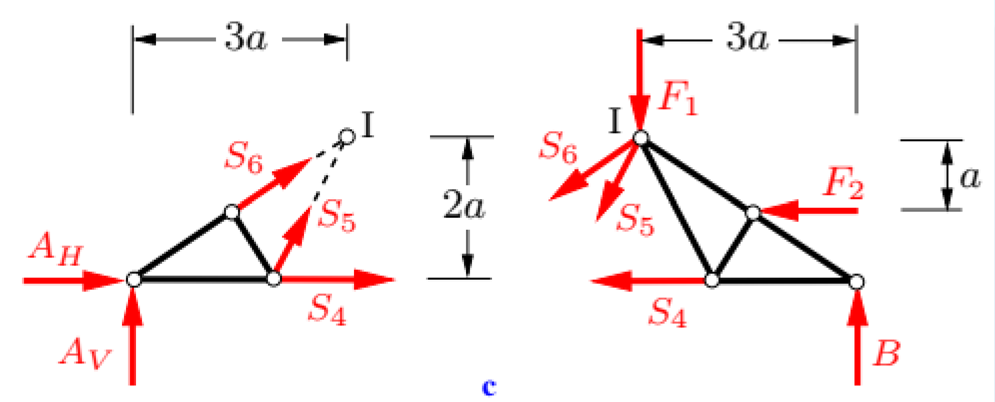
$$\longrightarrow: \quad A_H - F_3 = 0 \Rightarrow A_H = F_2 = F$$

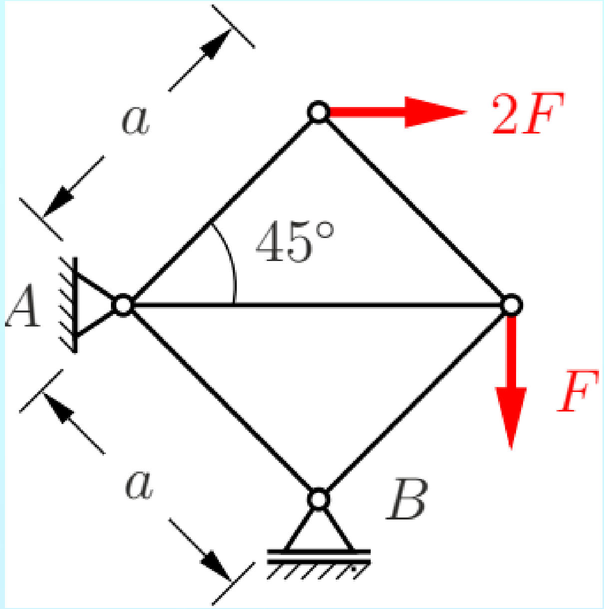


Then we pass an imaginary section through the members 4, 5 & 6 (Fig.c).

The unknown force S_4 follows from the moment equation about point I

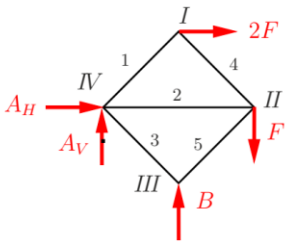
$$\overset{\curvearrowleft}{I}: \quad 2S_4 + 2A_H - 3A_V = 0 \Rightarrow S_4 = \frac{3}{4}F$$





Problem 1. For the given truss, the bar forces have to be determined with the Method of Joints.

Solution The reaction forces result from the equilibrium conditions for the entire system

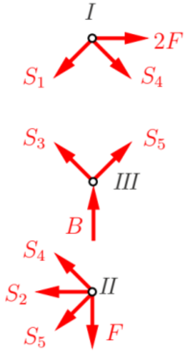


$$\rightarrow: \quad A_H + 2F = 0\,,$$
$$\uparrow: \quad A_V + B - F = 0\,,$$
$$\overset{\curvearrowleft}{A}: \quad \sqrt{2}\,aF + \frac{\sqrt{2}}{2}a2F - \frac{\sqrt{2}}{2}aB = 0$$

to

$$\underline{\underline{A_V = -3F}}\,, \quad \underline{\underline{A_H = -2F}}\,, \quad \underline{\underline{B = 4F}}\,.$$

Equilibrium at nodes I, III and II yields:



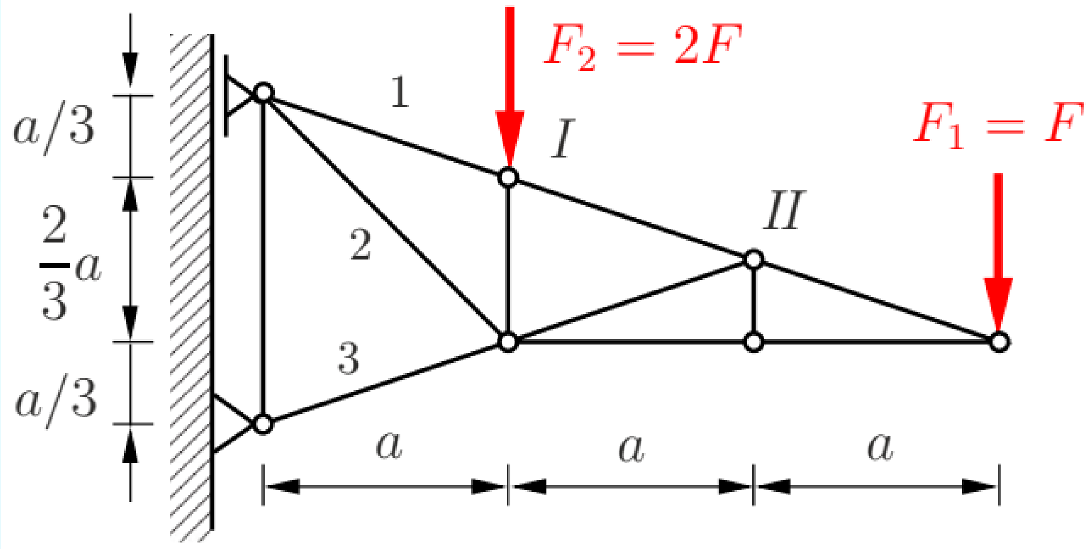
$$I \swarrow: \quad S_1 - 2F\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_1 = \sqrt{2}\,F}}\,,$$
$$\searrow: \quad S_4 + 2F\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_4 = -\sqrt{2}\,F}}\,,$$
$$III \nwarrow: \quad S_3 + B\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_3 = -2\sqrt{2}\,F}}\,,$$
$$\nearrow: \quad S_5 + B\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_5 = -2\sqrt{2}\,F}}\,,$$
$$II \leftarrow: \quad S_2 + \frac{\sqrt{2}}{2}S_4 + \frac{\sqrt{2}}{2}S_5 = 0\,,$$
$$\underline{\underline{S_2 = 3F}}\,.$$

To check, we make sure that the equilibrium conditions at node IV are fulfilled:

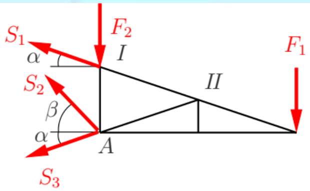
$$IV \rightarrow: \quad A_H + \frac{\sqrt{2}}{2}S_1 + S_2 + \frac{\sqrt{2}}{2}S_3 = -2F + F + 3F - 2F = 0\,,$$
$$\uparrow: \quad A_V + \frac{\sqrt{2}}{2}S_1 - \frac{\sqrt{2}}{2}S_3 = -3F + F + 2F = 0\,.$$

Table:

i	1	2	3	4	5
S_i	$\sqrt{2}F$	$3F$	$-2\sqrt{2}F$	$-\sqrt{2}F$	$-2\sqrt{2}F$



Problem 2. For the given truss, the bar forces have to be determined with the Method of Joints.



Solution The equilibrium conditions for the separated system follow with the help of angle α and β:

$$\leftarrow: \quad S_1 \cos\alpha + S_2 \cos\beta + S_3 \cos\alpha = 0\,,$$

$$\uparrow: \quad S_1 \sin\alpha + S_2 \sin\beta - S_3 \sin\alpha - F_1 - F_2 = 0\,,$$

$$\overset{\curvearrowleft}{A}: \quad 2aF_1 - \frac{2}{3}aS_1 \cos\alpha = 0\,.$$

With

$$\sin\alpha = \frac{1}{\sqrt{10}}\,, \quad \cos\alpha = \frac{3}{\sqrt{10}}\,, \quad \sin\beta = \cos\beta = \frac{\sqrt{2}}{2}\,,$$

it follows

$$\underline{\underline{S_1 = \sqrt{10}F = 3.16\ F}}\,, \quad \underline{\underline{S_2 = \frac{3\sqrt{2}}{4}F = 1.06\ F}}\,,$$

$$\underline{\underline{S_3 = -\frac{5\sqrt{10}}{4}F = -3.95\ F}}\,.$$

If load F_2 is moved to node II, only the moment equilibrium condition changes:

$$\overset{\curvearrowleft}{A}: \quad 2aF_1 + aF_2 - \frac{2}{3}aS_1 \cos\alpha = 0\,.$$

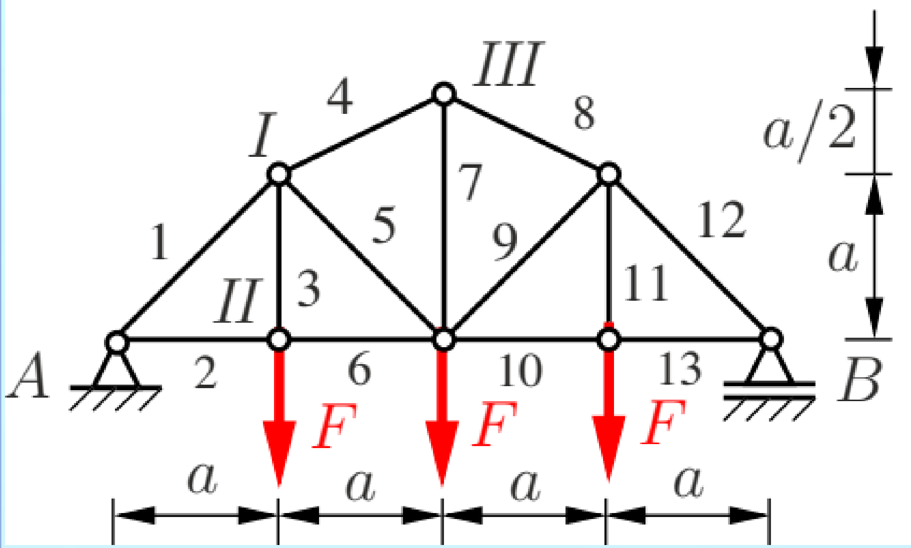
Thus, the bar forces result as

$$\underline{\underline{S_1 = 2\sqrt{10}F = 6.32\ F}}\,, \quad \underline{\underline{S_2 = -\frac{3\sqrt{2}}{4}F = -1.06\ F}}\,,$$

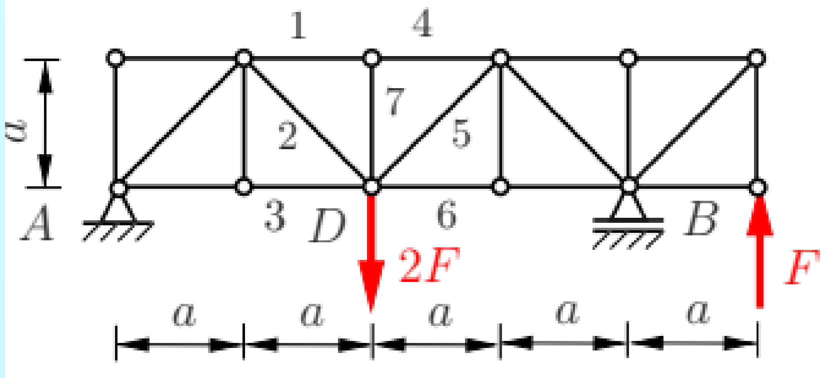
$$\underline{\underline{S_3 = -\frac{7\sqrt{10}}{4}F = -5.53\ F}}\,.$$

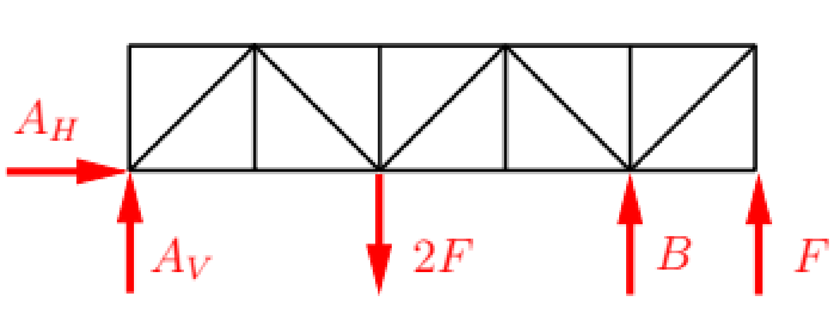
Remark: With the larger moment, S_1 and S_3 become larger and the tension bar changes into a compression bar.

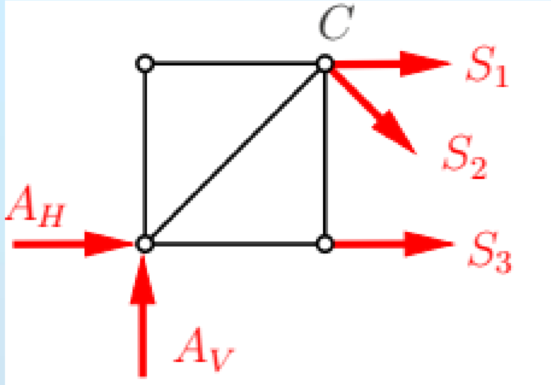
Problem 3. Determine the bar forces for the given truss.

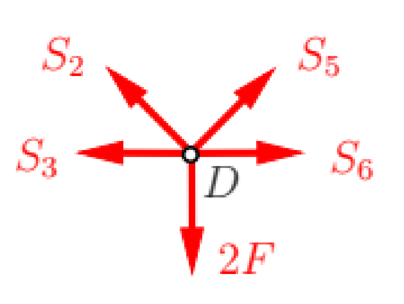


Problem 1 For the given truss, the forces in the bars 1 through 7 shall be determined

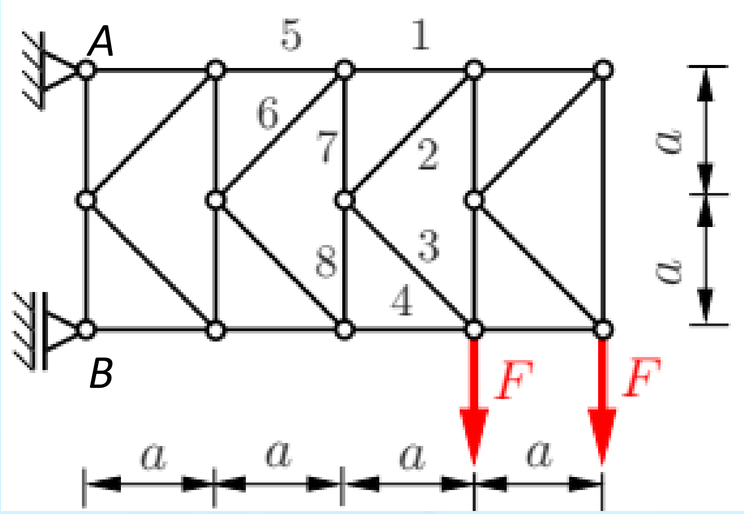








Problem 2 Determine the bar forces for the given truss.



Problem 3 Determine the bar forces for the given truss.