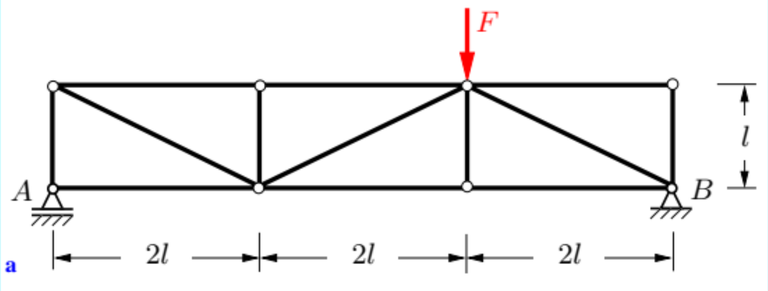
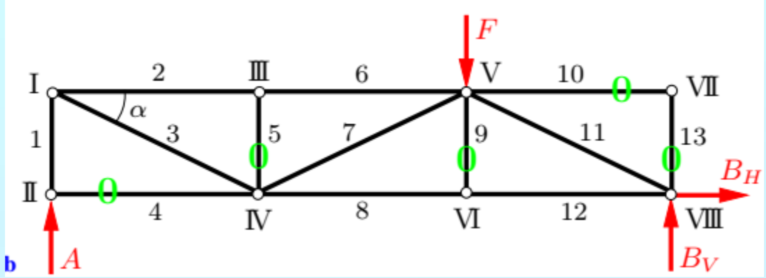
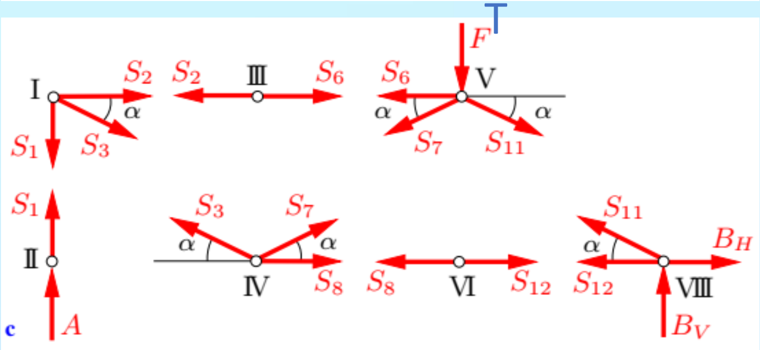
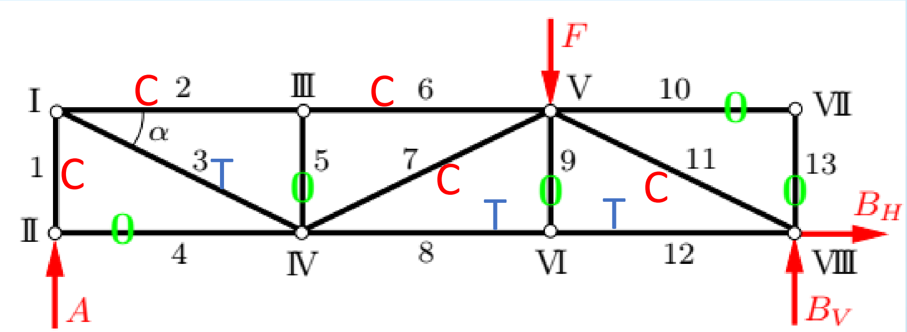








Example 1. The truss shown in Fig. a is loaded by an external force F. Determine the forces at the supports and in the members of the truss.

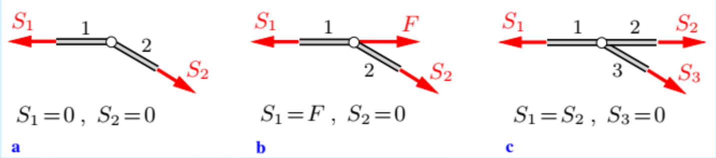
Solution: numbering joints & members: $2j = m + r$.

1) Fig. b, F. B. D. of the truss. Eq. Eqs.:

→: $B_H = 0$

$\curvearrowleft^{A}$=0: $-4lF+6lB_V=0 \Rightarrow B_V=(2/3)F$

$\curvearrowleft^{B}$=0: $2\,lF-6lA=0 \Rightarrow A=(1/3)F$



2) Identification of zero-force members.:

3) Joint Equilibrium: F. B. Ds. for joints

II) ↑: $S_1 + A = 0 \Rightarrow S_1 = -A = -(1/3)F$

I) ↑: $-S_1 - S_3\sin\alpha = 0 \Rightarrow S_3 = -S_1/\sin\alpha = (1/3)F/\sin\alpha$

→: $S_3\cos\alpha + S_2 = 0 \Rightarrow S_2 = -S_3\cos\alpha = -(1/3)F/\tan\alpha$

III) →: $-S_2 + S_6 = 0 \Rightarrow S_6 = S_2 = -(1/3)F/\tan\alpha$

IV) ↑: $S_3\sin\alpha + S_7\sin\alpha = 0 \Rightarrow S_7 = -S_3 = -(1/3)F/\sin\alpha$

→: $-S_3\cos\alpha + S_7\cos\alpha + S_8 = 0 \Rightarrow S_8 = S_3\cos\alpha - S_7\cos\alpha = (2/3)F/\tan\alpha$

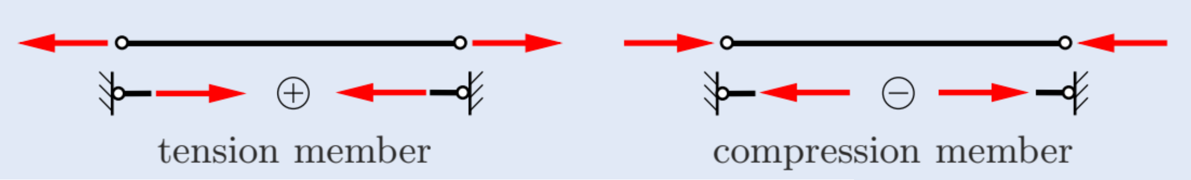
V) ↑: $-S_7\sin\alpha - S_{11}\sin\alpha - F = 0 \Rightarrow S_{11} = -S_7 - F/\sin\alpha = -(2/3)F/\sin\alpha$

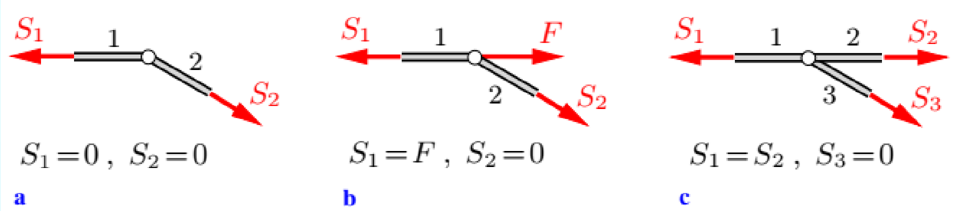
→: $-S_6 - S_7\cos\alpha + S_{11}\cos\alpha = 0 \Rightarrow [(1/3) + (1/3) - (2/3)]F/\tan\alpha \equiv 0$

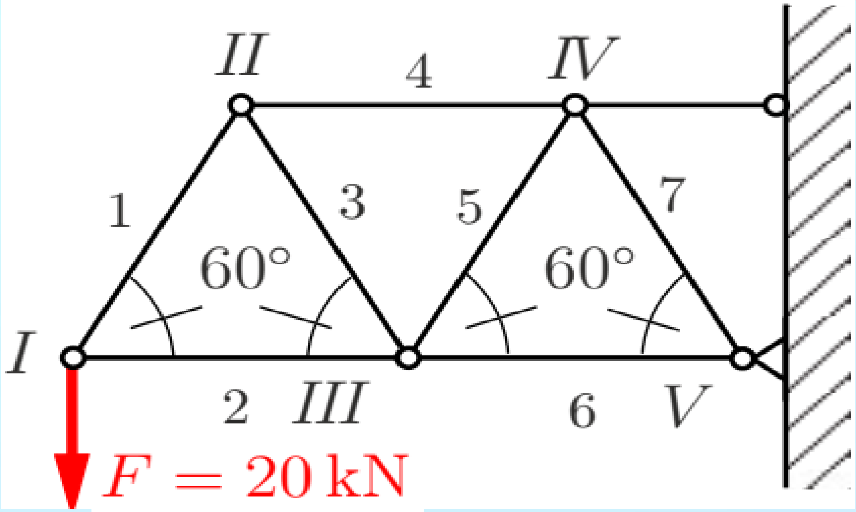
VI) →: $-S_8 + S_{12} = 0 \Rightarrow S_{12} = S_8 = (2/3)F/\tan\alpha$

VIII) ↑: $B_V + S_{11}\sin\alpha = 0 \Rightarrow [(2/3) - (2/3)]F \equiv 0$

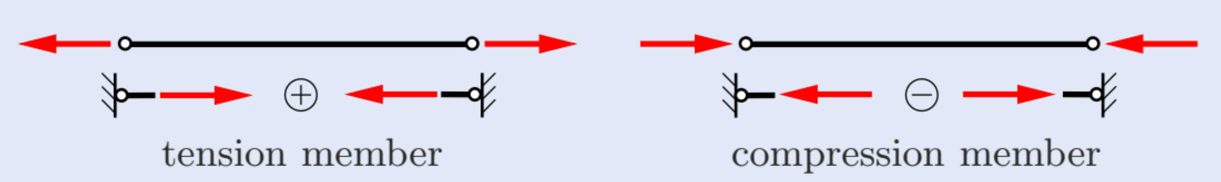
→: $-S_{12} - S_{11}\cos\alpha + B_H = 0 \Rightarrow [-(2/3) + (2/3)]F/\tan\alpha + 0 = 0$

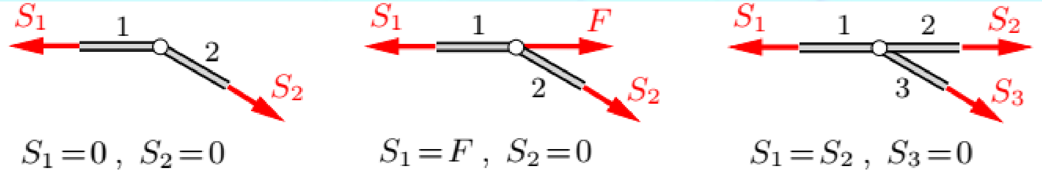


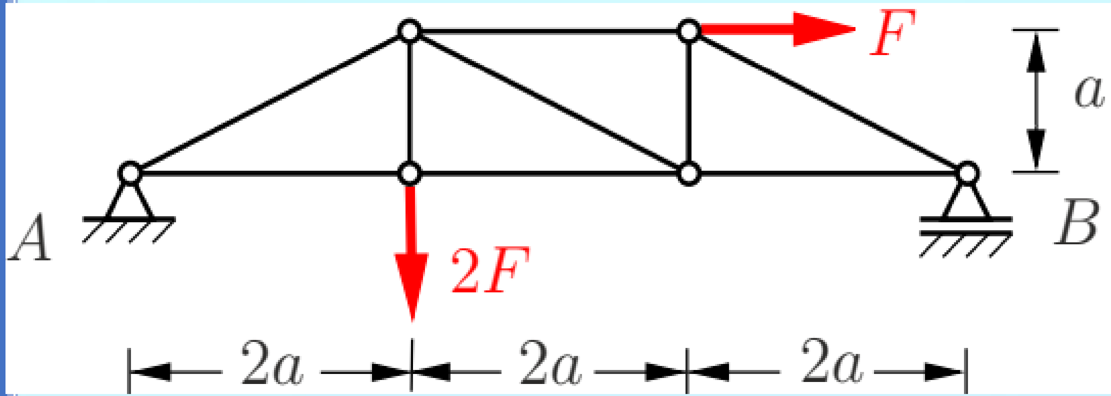




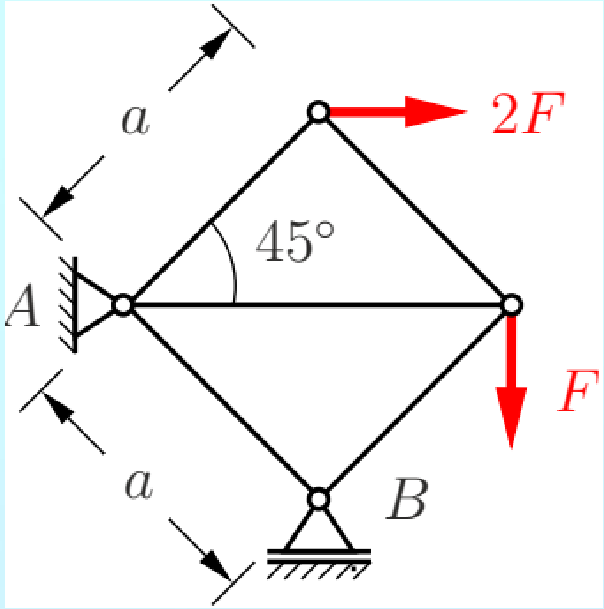
Example 2. For the given truss, all bar forces have to be determined.





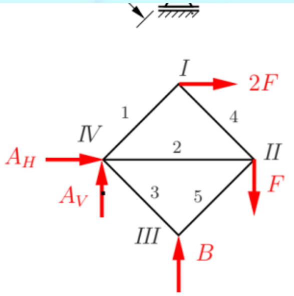


Example 3. For the given truss, the forces in the bars shall be determined. Take $F = 12\ kN$.



Problem 1. For the given truss, the bar forces have to be determined with the Method of Joints.

Solution The reaction forces result from the equilibrium conditions for the entire system



$$\rightarrow: \quad A_H + 2F = 0\,,$$

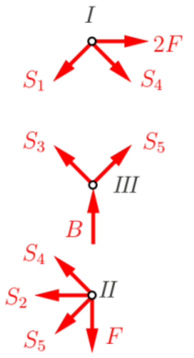
$$\uparrow: \quad A_V + B - F = 0\,,$$

$$\overset{\curvearrowleft}{A}: \quad \sqrt{2}\,aF + \frac{\sqrt{2}}{2}\,a2F - \frac{\sqrt{2}}{2}\,aB = 0$$

to

$$\underline{\underline{A_V = -3F}}\,, \quad \underline{\underline{A_H = -2F}}\,, \quad \underline{\underline{B = 4F}}\,.$$

Equilibrium at nodes I, III and II yields:



$$I \swarrow: \quad S_1 - 2F\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_1 = \sqrt{2}\,F}}\,,$$

$$\searrow: \quad S_4 + 2F\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_4 = -\sqrt{2}\,F}}\,,$$

$$III \nwarrow: \quad S_3 + B\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_3 = -2\sqrt{2}\,F}}\,,$$

$$\nearrow: \quad S_5 + B\frac{\sqrt{2}}{2} = 0 \quad \leadsto \quad \underline{\underline{S_5 = -2\sqrt{2}\,F}}\,,$$

$$II \leftarrow: \quad S_2 + \frac{\sqrt{2}}{2}S_4 + \frac{\sqrt{2}}{2}S_5 = 0\,,$$

$$\underline{\underline{S_2 = 3F}}\,.$$

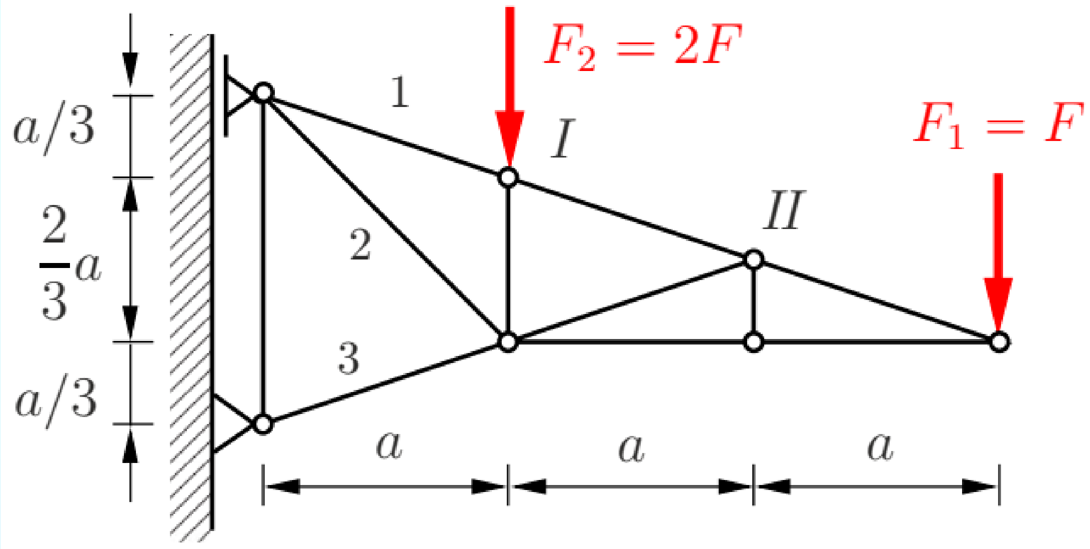
To check, we make sure that the equilibrium conditions at node IV are fulfilled:

$$IV \rightarrow: \quad A_H + \frac{\sqrt{2}}{2}S_1 + S_2 + \frac{\sqrt{2}}{2}S_3 = -2F + F + 3F - 2F = 0\,,$$

$$\uparrow: \quad A_V + \frac{\sqrt{2}}{2}S_1 - \frac{\sqrt{2}}{2}S_3 = -3F + F + 2F = 0\,.$$

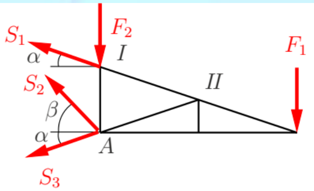
Table:

i	1	2	3	4	5
S_i	$\sqrt{2}F$	$3F$	$-2\sqrt{2}F$	$-\sqrt{2}F$	$-2\sqrt{2}F$



Problem 2. For the given truss, the bar forces have to be determined with the Method of Joints.

Solution The equilibrium conditions for the separated system follow with the help of angle α and β:



$$\leftarrow: \quad S_1 \cos\alpha + S_2 \cos\beta + S_3 \cos\alpha = 0\,,$$

$$\uparrow: \quad S_1 \sin\alpha + S_2 \sin\beta - S_3 \sin\alpha - F_1 - F_2 = 0\,,$$

$$\overset{\curvearrowleft}{A}: \quad 2aF_1 - \frac{2}{3}aS_1 \cos\alpha = 0\,.$$

With

$$\sin\alpha = \frac{1}{\sqrt{10}}\,, \quad \cos\alpha = \frac{3}{\sqrt{10}}\,, \quad \sin\beta = \cos\beta = \frac{\sqrt{2}}{2}\,,$$

it follows

$$\underline{\underline{S_1 = \sqrt{10}F = 3.16\ F}}\,, \quad \underline{\underline{S_2 = \frac{3\sqrt{2}}{4}F = 1.06\ F}}\,,$$

$$\underline{\underline{S_3 = -\frac{5\sqrt{10}}{4}F = -3.95\ F}}\,.$$

If load F_2 is moved to node II, only the moment equilibrium condition changes:

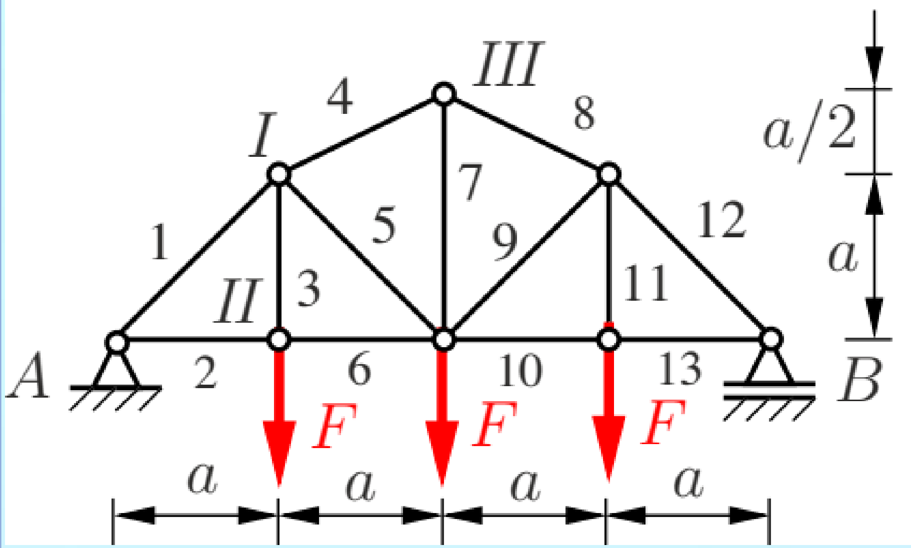
$$\overset{\curvearrowleft}{A}: \quad 2aF_1 + aF_2 - \frac{2}{3}aS_1 \cos\alpha = 0\,.$$

Thus, the bar forces result as

$$\underline{\underline{S_1 = 2\sqrt{10}F = 6.32\ F}}\,, \quad \underline{\underline{S_2 = -\frac{3\sqrt{2}}{4}F = -1.06\ F}}\,,$$

$$\underline{\underline{S_3 = -\frac{7\sqrt{10}}{4}F = -5.53\ F}}\,.$$

Remark: With the larger moment, S_1 and S_3 become larger and the tension bar changes into a compression bar.



Problem 3. Determine the bar forces for the given truss.