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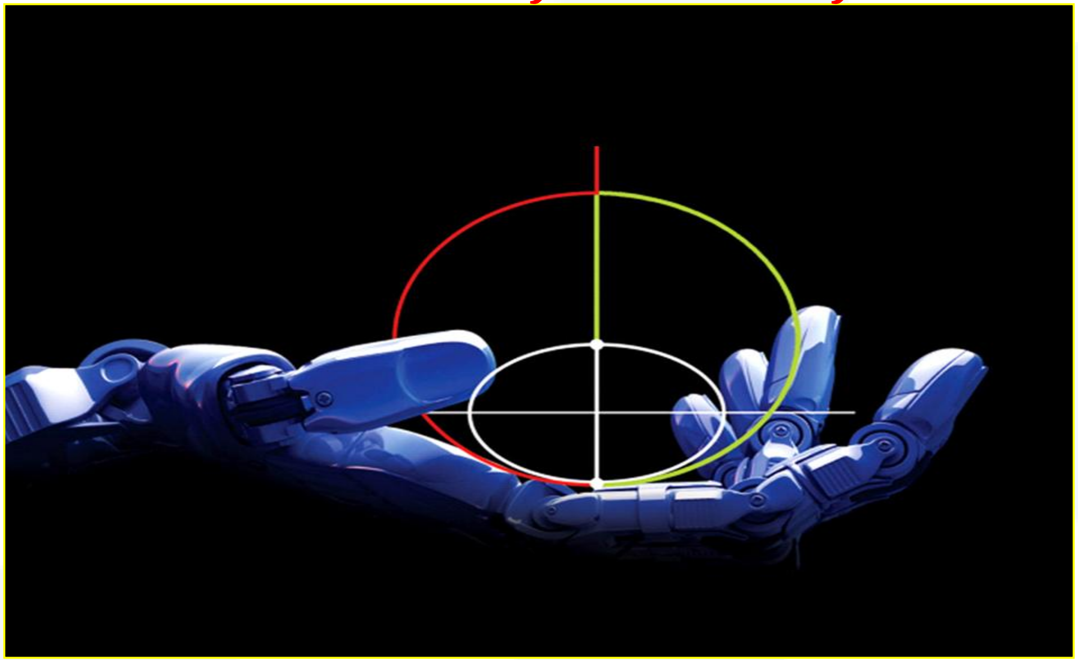
كلية الهندسة

قسم ميكاترونكس

مقرر النمذجة و المحاكاة

Planar Kinematics of a Rigid Body

Motion Analysis : Velocity



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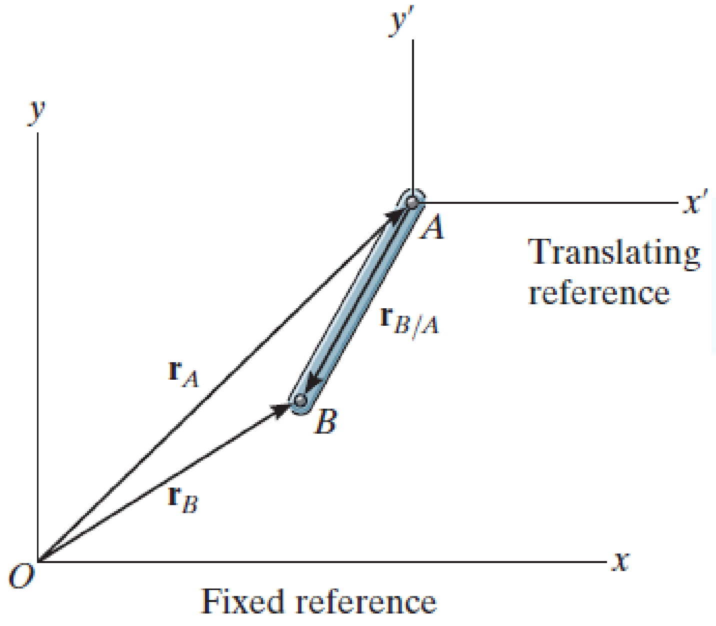
Contents

Relative-Motion Analysis: Velocity

The general plane motion of a rigid body can be described as a *combination* of translation and rotation. To view these "component" motions *separately* we will use a *relative-motion analysis* involving two sets of coordinate axes. The x, y coordinate system is fixed and measures the *absolute* position of two points A and B on the body, here represented as a bar. The origin of the x', y' coordinate system will be attached to the selected "base point" A, which generally has a *known* motion. The axes of this coordinate system *translate* with respect to the fixed frame but do not rotate with the bar.



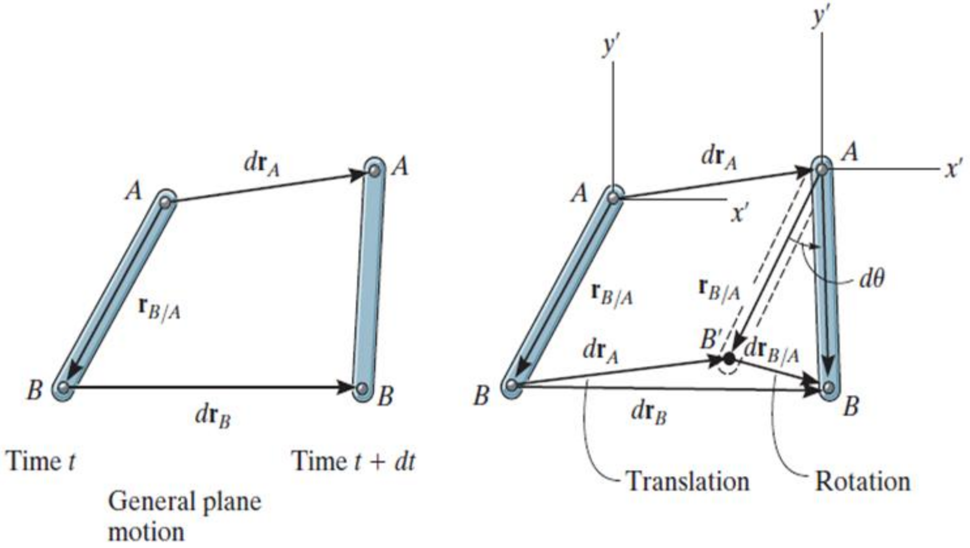
Position. The position vector $\mathbf{r}_A$ specifies the location of the "base point" A, and the relative-position vector $\mathbf{r}_{B/A}$ locates point B with respect to point A. By vector addition, the *position* of B is then

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Displacement. During an instant of time dt, points A and B undergo displacements $d\mathbf{r}_A$ and $d\mathbf{r}_B$ as shown. If we consider the general plane motion by its component parts then the *entire bar* first *translates* by an amount $d\mathbf{r}_A$ so that A, the base point, moves to its *final position* and point B moves to B'. The bar is then *rotated* about A by an amount $d\theta$ so that B' undergoes a *relative displacement* $d\mathbf{r}_{B/A}$ and thus moves to its final position B. Due to the rotation about A, $dr_{B/A} = r_{B/A}\, d\theta$, and the displacement of B is

$$d\mathbf{r}_B = d\mathbf{r}_A + d\mathbf{r}_{B/A}$$

- $d\mathbf{r}_{B/A}$: due to rotation about A
- $d\mathbf{r}_A$: due to translation of A
- $d\mathbf{r}_B$: due to translation and rotation



As slider block A moves horizontally to the left with a velocity $\mathbf{v}_A$, it causes crank CB to rotate counterclockwise, such that $\mathbf{v}_B$ is directed tangent to its circular path, i.e., upward to the left. The connecting rod AB is subjected to general plane motion, and at the instant shown it has an angular velocity $\boldsymbol{\omega}$.

Velocity. To determine the relation between the velocities of points A and B, it is necessary to take the time derivative of the position equation, or simply divide the displacement equation by dt. This yields

$$\frac{d\mathbf{r}_B}{dt} = \frac{d\mathbf{r}_A}{dt} + \frac{d\mathbf{r}_{B/A}}{dt}$$

The terms $d\mathbf{r}_B/dt = \mathbf{v}_B$ and $d\mathbf{r}_A/dt = \mathbf{v}_A$ are measured with respect to the fixed x, y axes and represent the *absolute velocities* of points A and B, respectively. Since the relative displacement is caused by a rotation, the magnitude of the third term is $dr_{B/A}/dt = r_{B/A}\, d\theta/dt = r_{B/A}\dot{\theta} = r_{B/A}\omega$, where ω is the angular velocity of the body at the instant considered. We will denote this term as the *relative velocity* $\mathbf{v}_{B/A}$, since it represents the

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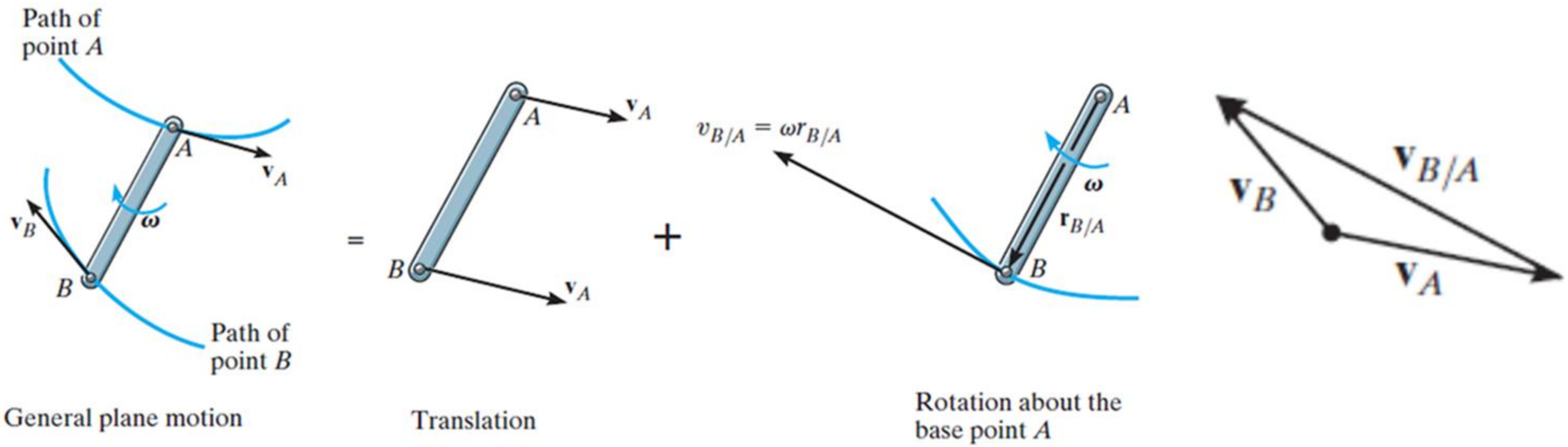
velocity of B with respect to A as measured by an observer fixed to the translating x', y' axes. In other words, *the bar appears to move as if it were rotating with an angular velocity* $\boldsymbol{\omega}$ *about the* z' *axis passing through* A. Consequently, $\mathbf{v}_{B/A}$ has a magnitude of $v_{B/A} = \omega r_{B/A}$ and a *direction* which is perpendicular to $\mathbf{r}_{B/A}$. We therefore have

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

where

$\mathbf{v}_B$ = velocity of point B
$\mathbf{v}_A$ = velocity of the base point A
$\mathbf{v}_{B/A}$ = velocity of B with respect to A

What the equation $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ states is that the velocity of B is determined by considering the entire bar to translate with a velocity of $\mathbf{v}_A$, and rotate about A with an angular velocity $\boldsymbol{\omega}$. Vector addition of these two effects, applied to B, yields $\mathbf{v}_B$.



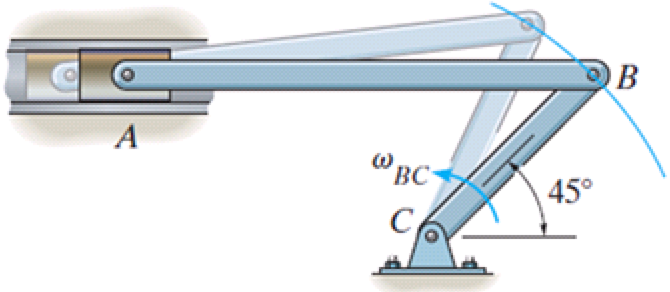
Since the relative velocity $\mathbf{v}_{B/A}$ represents the effect of *circular motion*, about A, this term can be expressed by the cross product $\mathbf{v}_{B/A} = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$ Hence, for application using Cartesian vector analysis.

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

where

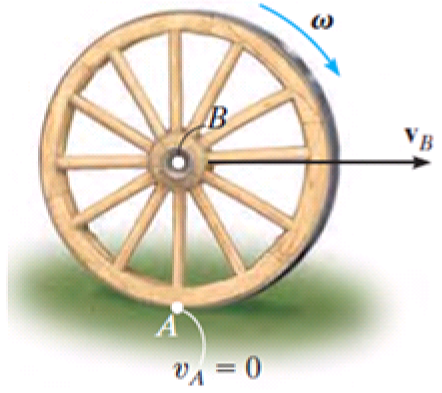
$\mathbf{v}_B$ = velocity of B
$\mathbf{v}_A$ = velocity of the base point A
$\boldsymbol{\omega}$ = angular velocity of the body
$\mathbf{r}_{B/A}$ = position vector directed from A to B

The velocity equation may be used in a practical manner to study the general plane motion of a rigid body which is either pin connected to or in contact with other moving bodies. When applying this equation, points A and B should generally be selected as points on the body which are pin-connected to other bodies, or as points in contact with adjacent bodies which have a *known motion*. For example, point A on link AB must move along a horizontal path, whereas point B moves on a circular path. The *directions* of $\mathbf{v}_A$ and $\mathbf{v}_B$ can therefore be established since they are always *tangent* to their paths of motion.



In the case of the wheel, which rolls *without slipping*, point A on the wheel can be selected at the ground.

Here A (momentarily) has zero velocity since the ground does not move. Furthermore, the center of the wheel, B, moves along a horizontal path so that $\mathbf{v}_B$ is horizontal.



Procedure for Analysis

The relative velocity equation can be applied either by using Cartesian vector analysis, or by writing the x and y scalar component equations directly. For application, it is suggested that the following procedure be used.

Vector Analysis

Kinematic Diagram.

- Establish the directions of the fixed x, y coordinates and draw a kinematic diagram of the body. Indicate on it the velocities $\mathbf{v}_A$, $\mathbf{v}_B$ of points A and B, the angular velocity $\boldsymbol{\omega}$, and the relative-position vector $\mathbf{r}_{B/A}$.

- If the magnitudes of $\mathbf{v}_A$, $\mathbf{v}_B$, or $\boldsymbol{\omega}$ are unknown, the sense of direction of these vectors can be assumed.

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Velocity Equation.

- To apply $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$, express the vectors in Cartesian vector form and substitute them into the equation. Evaluate the cross product and then equate the respective **i** and **j** components to obtain two scalar equations.
- If the solution yields a *negative* answer for an *unknown* magnitude, it indicates the sense of direction of the vector is opposite to that shown on the kinematic diagram.

Scalar Analysis

Kinematic Diagram.

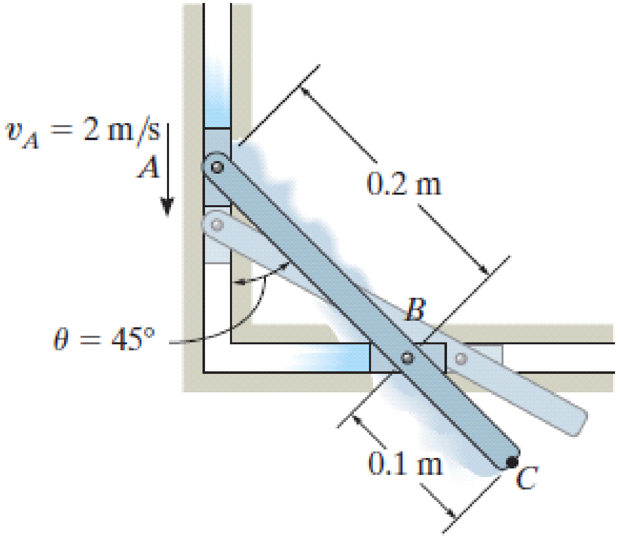
- If the velocity equation is to be applied in scalar form, then the magnitude and direction of the relative velocity $\mathbf{v}_{B/A}$ must be established. Draw a kinematic diagram such which shows the relative motion. Since the body is considered to be "pinned" momentarily at the base point A, the magnitude of $\mathbf{v}_{B/A}$ is $v_{B/A} = \omega r_{B/A}$. The sense of direction of $\mathbf{v}_{B/A}$ is always perpendicular to $\mathbf{r}_{B/A}$ in accordance with the rotational motion $\boldsymbol{\omega}$ of the body.

Velocity Equation.

- Write in symbolic form, $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$, and underneath each of the terms represent the vectors graphically by showing their magnitudes and directions. The scalar equations are determined from the x and y components of these vectors.

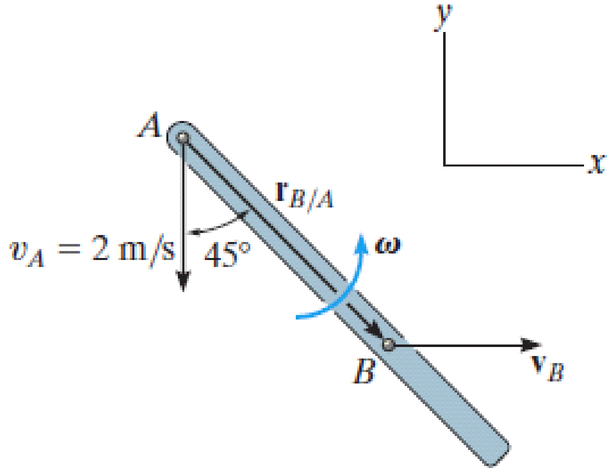
EXAMPLE

The link shown is guided by two blocks at A and B, which move in the fixed slots. If the velocity of A is 2 m/s downward, determine the velocity of B at the instant $\theta = 45°$.



SOLUTION I (VECTOR ANALYSIS)

Kinematic Diagram. Since points A and B are restricted to move along the fixed slots and $\mathbf{v}_A$ is directed downward, then velocity $\mathbf{v}_B$ must be directed horizontally to the right. This motion causes the link to rotate counterclockwise; that is, by the right-hand rule the angular velocity $\boldsymbol{\omega}$ is directed outward, perpendicular to the plane of motion.



Velocity Equation. Expressing each of the vectors in terms of their $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components, the base point, and B, we have

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$
$$v_B\mathbf{i} = -2\mathbf{j} + [\omega\mathbf{k} \times (0.2 \sin 45°\mathbf{i} - 0.2 \cos 45°\mathbf{j})]$$
$$v_B\mathbf{i} = -2\mathbf{j} + 0.2\omega \sin 45°\mathbf{j} + 0.2\omega \cos 45°\mathbf{i}$$

Equating the **i** and **j** components gives

$$v_B = 0.2\omega \cos 45° \quad 0 = -2 + 0.2\omega \sin 45°$$

Thus,

$$\omega = 14.1 \text{ rad/s} \circlearrowleft \qquad v_B = 2 \text{ m/s} \rightarrow$$ *Ans.*

SOLUTION II (SCALAR ANALYSIS)

The kinematic diagram of the relative "circular motion" which produces $\mathbf{v}_{B/A}$ is shown. Here $v_{B/A} = \omega(0.2\text{ m})$.

Thus,

$$v_B = v_A + v_{B/A}$$

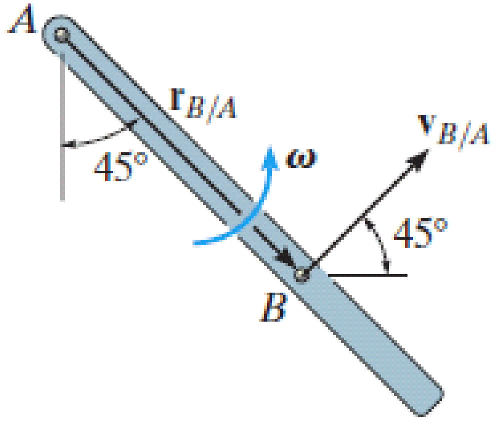
$$\begin{bmatrix} v_B \\ \rightarrow \end{bmatrix} = \begin{bmatrix} 2\text{ m/s} \\ \downarrow \end{bmatrix} + \begin{bmatrix} \omega(0.2\text{ m}) \\ \angle 45^\circ \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $$v_B = 0 + \omega(0.2)\cos 45^\circ$$

$(+\uparrow)$ $$0 = -2 + \omega(0.2)\sin 45^\circ$$

The solution produces the above results.

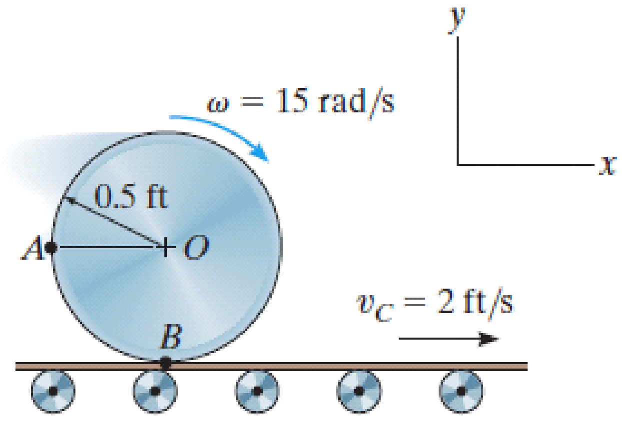
It should be emphasized that these results are *valid only* at the instant $\theta = 45^\circ$. A recalculation for $\theta = 44^\circ$ yields $v_B = 2.07\text{ m/s}$ and $\omega = 14.4\text{ rad/s}$; whereas when $\theta = 46^\circ$, $v_B = 1.93\text{ m/s}$ and $\omega = 13.9\text{ rad/s}$, etc.



Relative motion

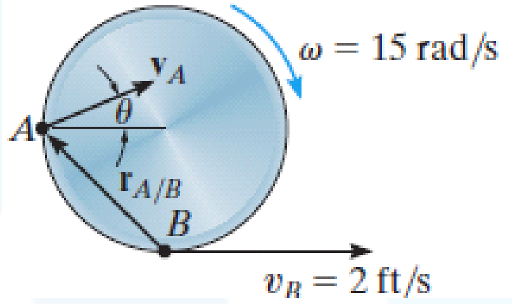
EXAMPLE

The cylinder shown rolls without slipping on the surface of a conveyor belt which is moving at 2 ft/s. Determine the velocity of point A. The cylinder has a clockwise angular velocity $\omega = 15$ rad/s at the instant shown.



SOLUTION I (VECTOR ANALYSIS)

Kinematic Diagram. Since no slipping occurs, point B on the cylinder has the same velocity as the conveyor. Also, the angular velocity of the cylinder is known, so we can apply the velocity equation to B, the base point, and A to determine $\mathbf{v}_A$.



Velocity Equation

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = 2\mathbf{i} + (-15\mathbf{k}) \times (-0.5\mathbf{i} + 0.5\mathbf{j})$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} = 2\mathbf{i} + 7.50\mathbf{j} + 7.50\mathbf{i}$$

so that

$$(v_A)_x = 2 + 7.50 = 9.50 \text{ ft/s} \quad (1)$$

$$(v_A)_y = 7.50 \text{ ft/s} \quad (2)$$

Thus,

$$v_A = \sqrt{(9.50)^2 + (7.50)^2} = 12.1 \text{ ft/s} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{7.50}{9.50} = 38.3^\circ \ \measuredangle \qquad \textit{Ans.}$$

SOLUTION II (SCALAR ANALYSIS)

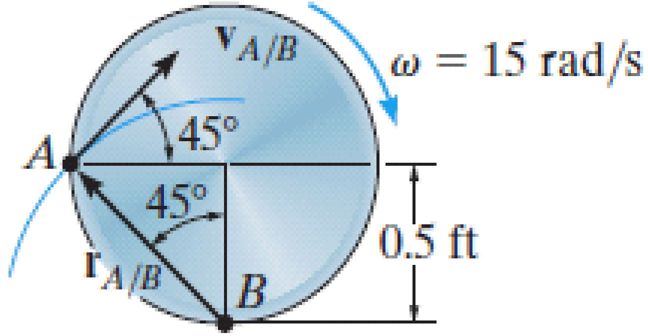
As an alternative procedure, the scalar components of $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$ can be obtained directly. From the kinematic diagram showing the relative "circular" motion which produces $\mathbf{v}_{A/B}$, we have

$$v_{A/B} = \omega r_{A/B} = (15 \text{ rad/s})\left(\frac{0.5 \text{ ft}}{\cos 45^\circ}\right) = 10.6 \text{ ft/s}$$

Thus,

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\begin{bmatrix} (v_A)_x \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (v_A)_y \\ \uparrow \end{bmatrix} = \begin{bmatrix} 2 \text{ ft/s} \\ \rightarrow \end{bmatrix} + \begin{bmatrix} 10.6 \text{ ft/s} \\ \measuredangle 45^\circ \end{bmatrix}$$



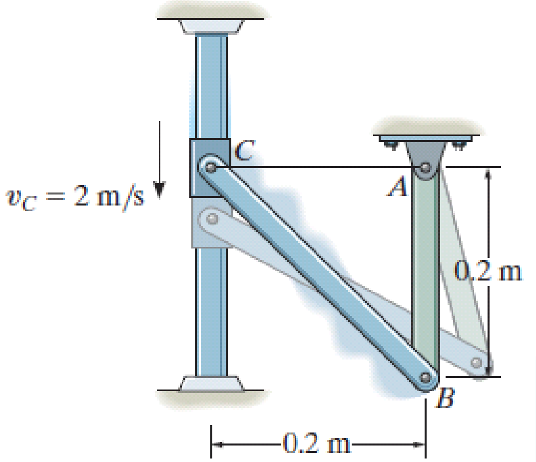
Equating the x and y components gives the same results as before, namely,

$(\overset{+}{\rightarrow})$ $$(v_A)_x = 2 + 10.6\cos 45^\circ = 9.50\ \text{ft/s}$$

$(+\uparrow)$ $$(v_A)_y = 0 + 10.6\sin 45^\circ = 7.50\ \text{ft/s}$$

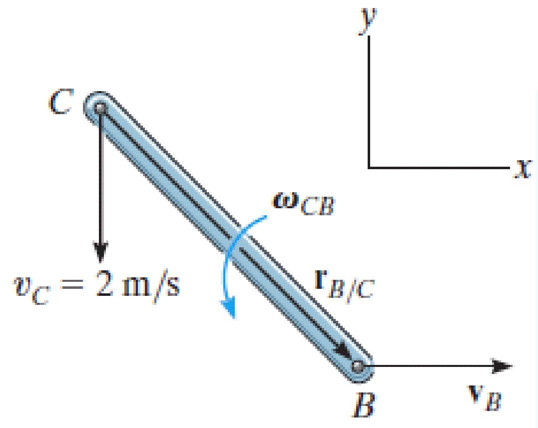
EXAMPLE

The collar C is moving downward with a velocity of 2 m/s. Determine the angular velocity of CB at this instant.



SOLUTION I (VECTOR ANALYSIS)

Kinematic Diagram. The downward motion of C causes B to move to the right along a curved path. Also, CB and AB rotate counterclockwise.



Velocity Equation. Link CB (general plane motion):

$$\mathbf{v}_B = \mathbf{v}_C + \boldsymbol{\omega}_{CB} \times \mathbf{r}_{B/C}$$

$$v_B\mathbf{i} = -2\mathbf{j} + \omega_{CB}\mathbf{k} \times (0.2\mathbf{i} - 0.2\mathbf{j})$$

$$v_B\mathbf{i} = -2\mathbf{j} + 0.2\omega_{CB}\mathbf{j} + 0.2\omega_{CB}\mathbf{i}$$

$$v_B = 0.2\omega_{CB} \qquad (1)$$

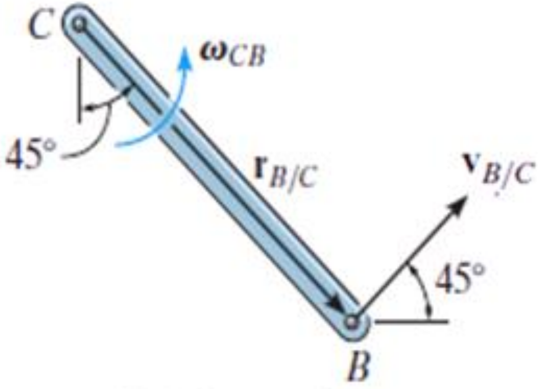
$$0 = -2 + 0.2\omega_{CB} \qquad (2)$$

$$\omega_{CB} = 10 \text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

$$v_B = 2 \text{ m/s} \rightarrow$$

SOLUTION II (SCALAR ANALYSIS)

The scalar component equations of $\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$ can be obtained directly. The kinematic diagram shows the relative "circular" motion which produces $\mathbf{v}_{B/C}$. We have



Relative motion

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$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

$$\begin{bmatrix} v_B \\ \rightarrow \end{bmatrix} = \begin{bmatrix} 2\text{ m/s} \\ \downarrow \end{bmatrix} + \begin{bmatrix} \omega_{CB}(0.2\sqrt{2}\text{ m}) \\ \measuredangle 45^\circ \end{bmatrix}$$

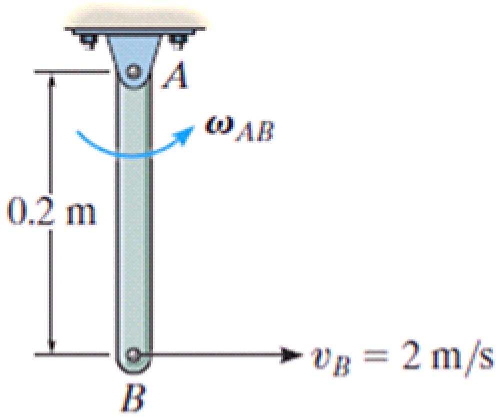
Resolving these vectors in the *x* and *y* directions yields

$$(\overset{+}{\rightarrow}) \qquad v_B = 0 + \omega_{CB}(0.2\sqrt{2}\cos 45^\circ)$$

$$(+\uparrow) \qquad 0 = -2 + \omega_{CB}(0.2\sqrt{2}\sin 45^\circ)$$

which is the same as Eqs. 1 and 2.

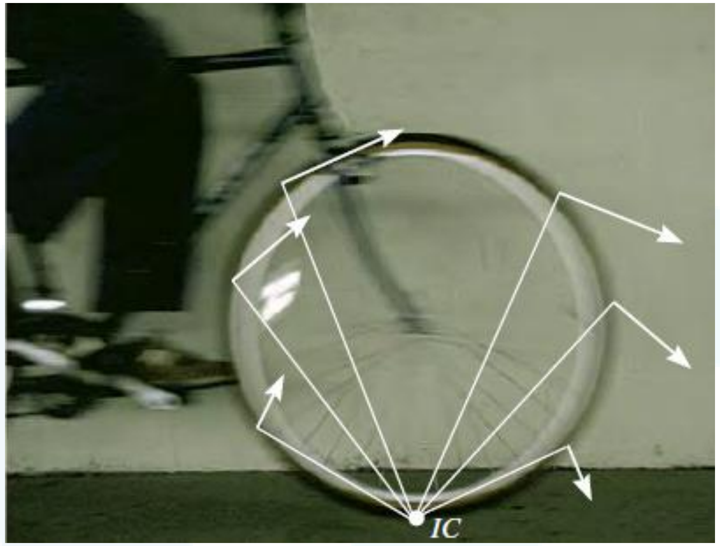
NOTE: Since link *AB* rotates about a fixed axis and v_B is known, its angular velocity is found from $v_B = \omega_{AB} r_{AB}$ or $2\text{ m/s} = \omega_{AB}(0.2\text{ m})$, $\omega_{AB} = 10\text{ rad/s}$.



Instantaneous Center of Zero Velocity

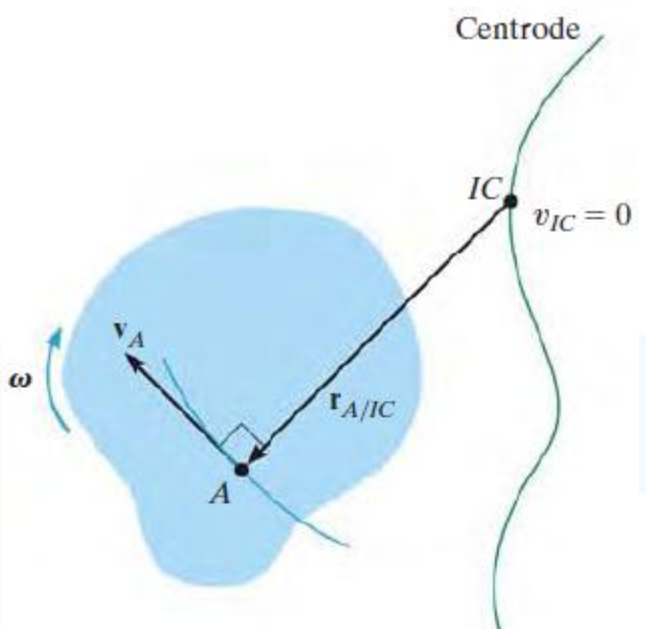
The velocity of any point B located on a rigid body can be obtained in a very direct way by choosing the base point A to be a point that has *zero velocity* at the instant considered. In this case, $\mathbf{v}_A = \mathbf{0}$, and therefore the velocity equation, $\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$, becomes $\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A}$. For a body having general plane motion, point A so chosen is called the *instantaneous center of zero velocity (IC)*, and it lies on the *instantaneous axis of zero velocity*. This axis is always perpendicular to the plane of motion, and the intersection of the axis with this plane defines the location of the IC. Since point A coincides with the IC, then $\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/IC}$ and so point B moves momentarily about the IC in a *circular path;* in other words, the body appears to rotate about the instantaneous axis. The *magnitude* of $\mathbf{v}_B$ is simply $v_B = \omega r_{B/IC}$, where ω is the angular velocity of the body. Due to the circular motion, the *direction* of $\mathbf{v}_B$ must always be *perpendicular* to $\mathbf{r}_{B/IC}$.

For example, the IC for the bicycle wheel is at the contact point with the ground. There the spokes are somewhat visible, whereas at the top of the wheel they become blurred. If one imagines that the wheel is momentarily pinned at this point, the velocities of various points can be found using $v = \omega r$. Here the radial distances shown in the photo, must be determined from the geometry of the wheel.



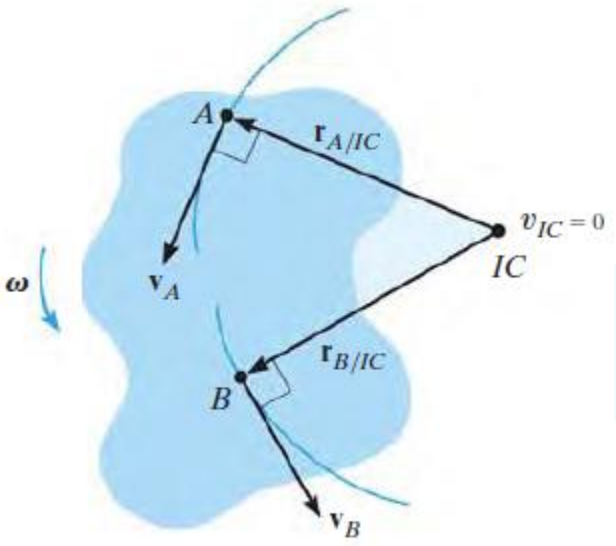
Location of the IC. To locate the IC we can use the fact that the *velocity* of a point on the body is *always perpendicular* to the *relative-position vector* directed from the IC to the point. Several possibilities exist:

- *The velocity* $\mathbf{v}_A$ *of a point A on the body and the angular velocity* $\boldsymbol{\omega}$ *of the body are known*. In this case, the IC is located along the line drawn perpendicular to $\mathbf{v}_A$ at A, such that the distance from A to the IC is $r_{A/IC} = v_A/\omega$. Note that the IC lies up and to the right of A since $\mathbf{v}_A$ must cause a clockwise angular velocity $\boldsymbol{\omega}$ about the IC.



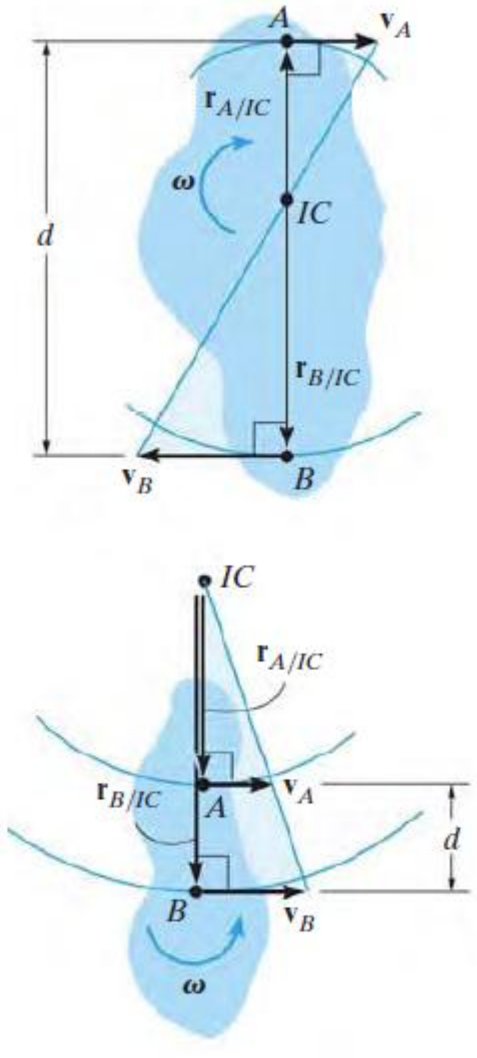
Location of IC knowing $\mathbf{v}_A$ and $\boldsymbol{\omega}$

- *The lines of action of two nonparallel velocities* $\mathbf{v}_A$ and $\mathbf{v}_B$ *are known* . Construct at points A and B line segments that are perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$. Extending these perpendiculars to their *point of intersection* as shown locates the IC at the instant considered.



Location of IC knowing the directions of $\mathbf{v}_A$ and $\mathbf{v}_B$

- *The magnitude and direction of two parallel velocities* $\mathbf{v}_A$ *and* $\mathbf{v}_B$ *are known*. Here the location of the *IC* is determined by proportional triangles. Examples are shown . In both cases $r_{A/IC} = v_A/\omega$ and $r_{B/IC} = v_B/\omega$. If d is a known distance between points A and B, then , $r_{A/IC} + r_{B/IC} = d$ and $r_{B/IC} - r_{A/IC} = d$.



Location of *IC* knowing $\mathbf{v}_A$ and $\mathbf{v}_B$

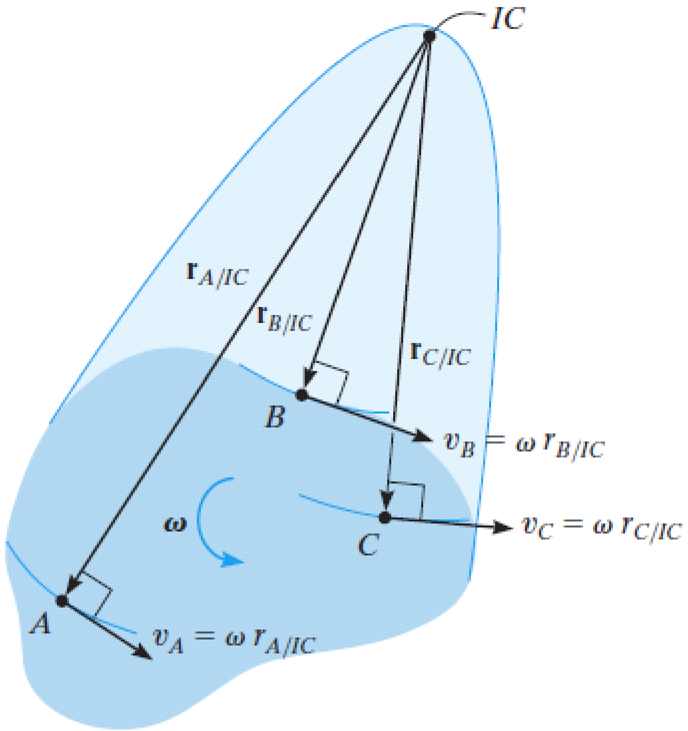
Realize that the point chosen as the instantaneous center of zero velocity for the body *can only be used at the instant considered* since the body changes its position from one instant to the next. The locus of points which define the location of the *IC* during the body's motion is called a *centrode*, and so each point on the centrode acts as the *IC* for the body only for an instant.

Although the *IC* may be conveniently used to determine the velocity of any point in a body, it generally *does not have zero acceleration* and therefore it *should not* be used for finding the accelerations of points in a body.

Procedure for Analysis

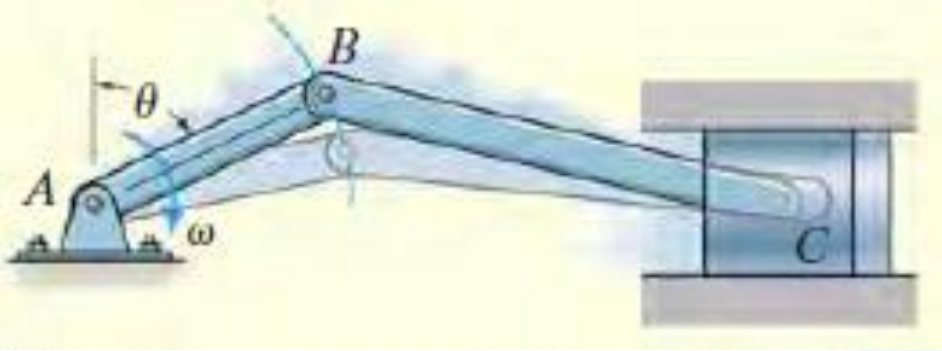
The velocity of a point on a body which is subjected to general plane motion can be determined with reference to its instantaneous center of zero velocity provided the location of the *IC* is first established using one of the three methods described above.

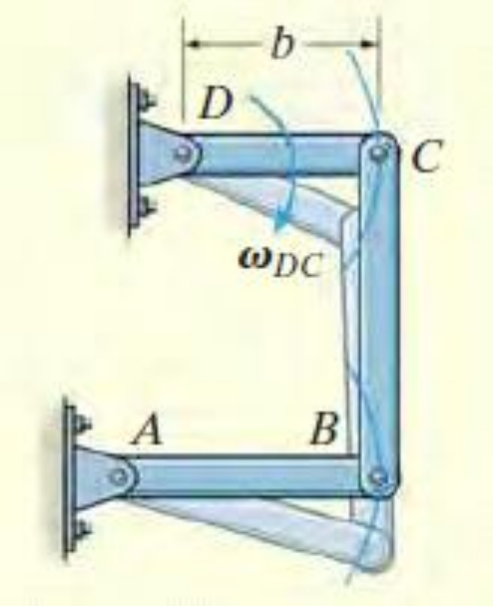
- As shown on the kinematic diagram, the body is imagined as "extended and pinned" at the *IC* so that, at the instant considered, it rotates about this pin with its angular velocity $\boldsymbol{\omega}$.
- The *magnitude* of velocity for each of the arbitrary points *A*, *B*, and *C* on the body can be determined by using the equation $v = \omega r$, where r is the radial distance from the *IC* to each point.
- The line of action of each velocity vector $\mathbf{v}$ is *perpendicular* to its associated radial line $\mathbf{r}$, and the velocity has a *sense of direction* which tends to move the point in a manner consistent with the angular rotation $\boldsymbol{\omega}$ of the radial line



EXAMPLE

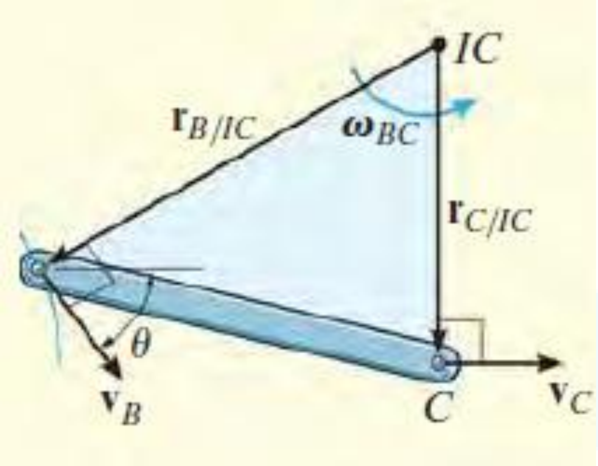
Show how to determine the location of the instantaneous center of zero velocity for (a) member BC and (b) the link CB



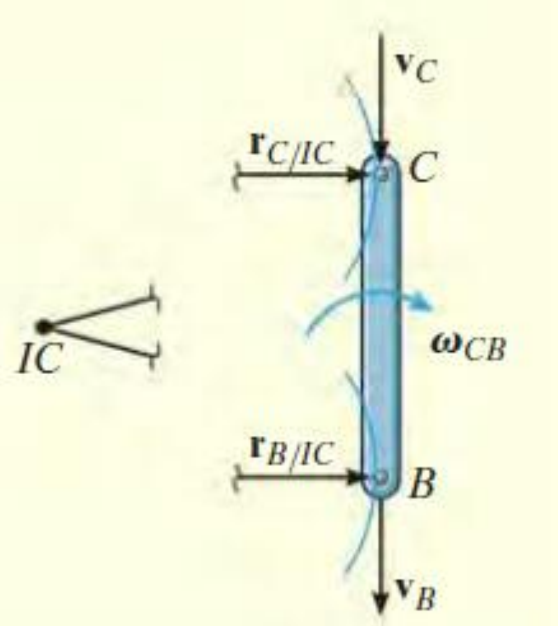


SOLUTION

Part (a). As shown, point B moves in a circular path such that such that $\mathbf{v}_B$ is perpendicular to AB. Therefore, it acts at an angle θ from the horizontal as shown. The motion of point B causes the piston to move forward *horizontally* with a velocity $\mathbf{v}_C$.
When lines are drawn perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_C$, they intersect at the IC.

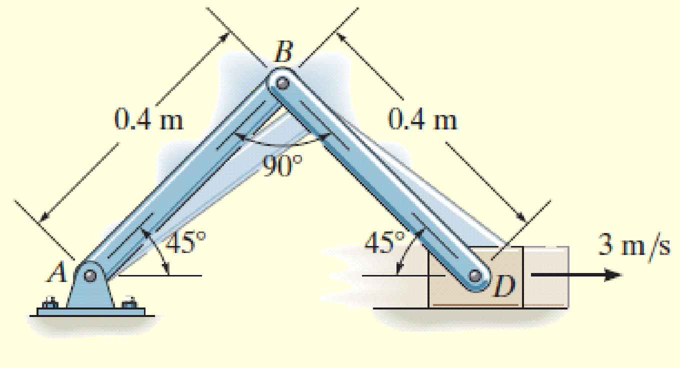


Part (b). Points B and C follow circular paths of motion since links AB and DC are each subjected to rotation about a fixed axis. Since the velocity is always tangent to the path, at the instant considered, $\mathbf{v}_C$ on rod DC and $\mathbf{v}_B$ on rod AB are both directed vertically downward, along the axis of link CB, Fig. Radial lines drawn perpendicular to these two velocities form parallel lines which intersect at "infinity;" i.e., $r_{C/IC} \to \infty$ and $r_{B/IC} \to \infty$. Thus, $\omega_{CB} = (v_C/r_{C/IC}) \to 0$. As a result, link CB momentarily *translates*. An instant later, however, CB will move to a tilted position, causing the IC to move to some finite location.



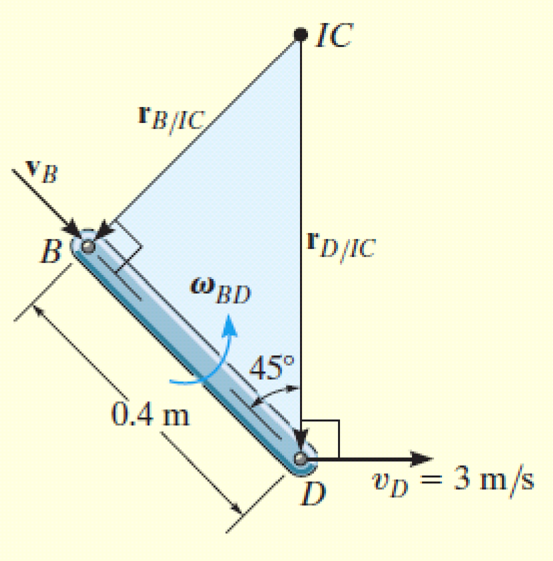
EXAMPLE

Block D moves with a speed of 3 m/s. Determine the angular velocities of links BD and AB, at the instant shown.



SOLUTION

As D moves to the right, it causes AB to rotate clockwise about point A. Hence, $\mathbf{v}_B$ is directed perpendicular to AB. The instantaneous center of zero velocity for BD is located at the intersection of the line segments drawn perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_D$. From the geometry,



$$r_{B/IC} = 0.4 \tan 45^\circ \text{ m} = 0.4 \text{ m}$$

$$r_{D/IC} = \frac{0.4 \text{ m}}{\cos 45^\circ} = 0.5657 \text{ m}$$

Since the magnitude of $\mathbf{v}_D$ is known, the angular velocity of link BD is

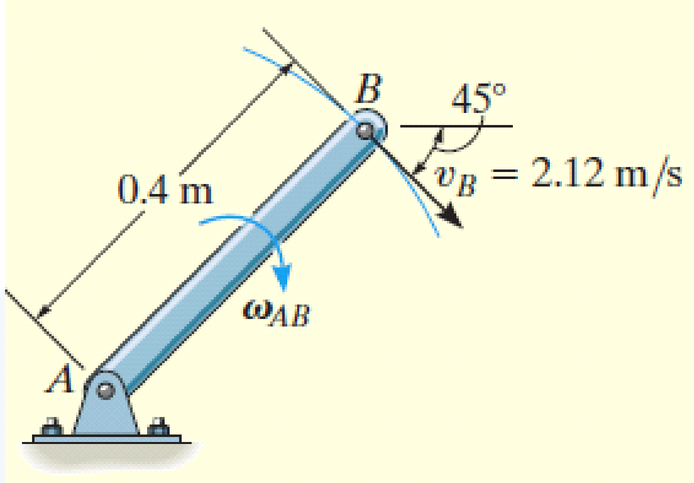
$$\omega_{BD} = \frac{v_D}{r_{D/IC}} = \frac{3 \text{ m/s}}{0.5657 \text{ m}} = 5.30 \text{ rad/s} \circlearrowleft \qquad \textit{Ans.}$$

The velocity of B is therefore

$$v_B = \omega_{BD}(r_{B/IC}) = 5.30 \text{ rad/s } (0.4 \text{ m}) = 2.12 \text{ m/s} \quad \measuredangle 45^\circ$$

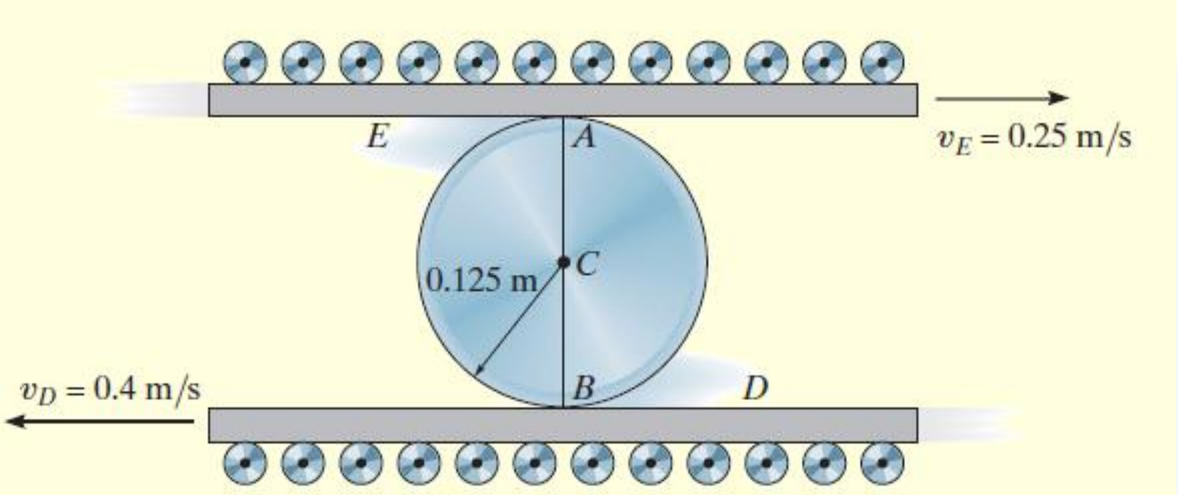
The angular velocity of AB is

$$\omega_{AB} = \frac{v_B}{r_{B/A}} = \frac{2.12 \text{ m/s}}{0.4 \text{ m}} = 5.30 \text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$



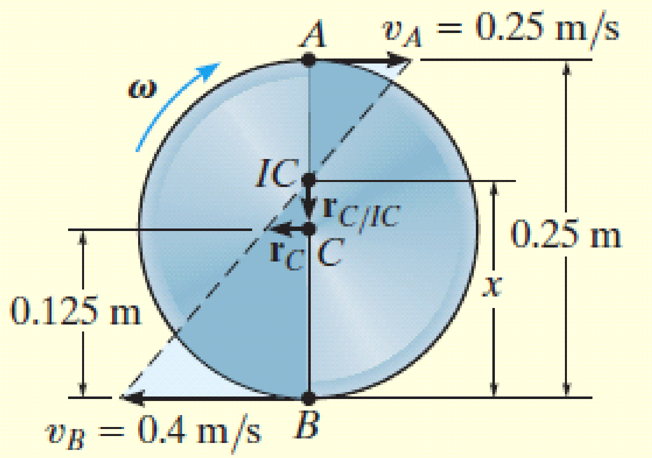
EXAMPLE

The cylinder shown rolls without slipping between the two moving plates E and D. Determine the angular velocity of the cylinder and the velocity of its center C.



SOLUTION

Since no slipping occurs, the contact points A and B on the cylinder have the same velocities as the plates E and D, respectively. Furthermore, the velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ are *parallel*, so that by the proportionality of right triangles the IC is located at a point on line AB Assuming this point to be a distance x from B, we have



$$v_B = \omega x; \qquad 0.4 \text{ m/s} = \omega x$$

$$v_A = \omega(0.25 \text{ m} - x); \qquad 0.25 \text{ m/s} = \omega(0.25 \text{ m} - x)$$

Dividing one equation into the other eliminates ω and yields

$$0.4(0.25 - x) = 0.25x$$

$$x = \frac{0.1}{0.65} = 0.1538 \text{ m}$$

Hence, the angular velocity of the cylinder is

$$\omega = \frac{v_B}{x} = \frac{0.4 \text{ m/s}}{0.1538 \text{ m}} = 2.60 \text{ rad/s} \circlearrowright \qquad \textit{Ans.}$$

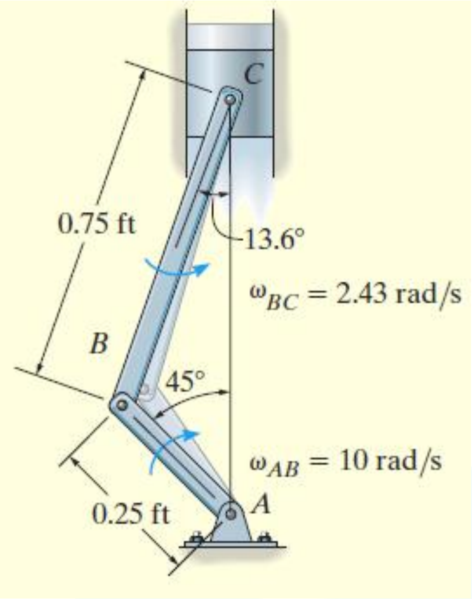
The velocity of point C is therefore

$$v_C = \omega r_{C/IC} = 2.60 \text{ rad/s } (0.1538 \text{ m} - 0.125 \text{ m})$$

$$= 0.0750 \text{ m/s} \leftarrow \qquad \textit{Ans.}$$

EXAMPLE

The crankshaft AB turns with a clockwise angular velocity of 10 rad/s, Determine the velocity of the piston at the instant shown.

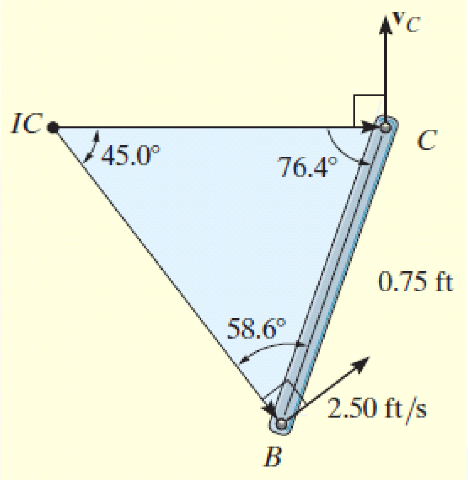


SOLUTION

The crankshaft rotates about a fixed axis, and so the velocity of point B is

$$v_B = 10 \text{ rad/s } (0.25 \text{ ft}) = 2.50 \text{ ft/s} \measuredangle 45^\circ$$

Since the directions of the velocities of B and C are known, then the location of the IC for the connecting rod BC is at the intersection of the lines extended from these points, perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_C$, The magnitudes of $\mathbf{r}_{B/IC}$ and $\mathbf{r}_{C/IC}$ can be obtained from the geometry of the triangle and the law of sines, i.e.,



$$\frac{0.75\text{ ft}}{\sin 45^\circ} = \frac{r_{B/IC}}{\sin 76.4^\circ}$$

$$r_{B/IC} = 1.031\text{ ft}$$

$$\frac{0.75\text{ ft}}{\sin 45^\circ} = \frac{r_{C/IC}}{\sin 58.6^\circ}$$

$$r_{C/IC} = 0.9056\text{ ft}$$

The rotational sense of $\boldsymbol{\omega}_{BC}$ must be the same as the rotation caused by $\mathbf{v}_B$ about the IC, which is counterclockwise. Therefore,

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{2.5\text{ ft/s}}{1.031\text{ ft}} = 2.425\text{ rad/s}$$

Using this result, the velocity of the piston is

$$v_C = \omega_{BC} r_{C/IC} = (2.425\text{ rad/s})(0.9056\text{ ft}) = 2.20\text{ ft/s} \qquad \textit{Ans.}$$

انتهت المحاضرة