Stress Resultants in Straight Beams

القوى الداخلية في الجيزان المستقيمة

Beams are usually subjected to forces perpendicular to their axes. If there is no loading (external or reactions) in the direction of the beam axis, the normal force vanishes $N = 0$.

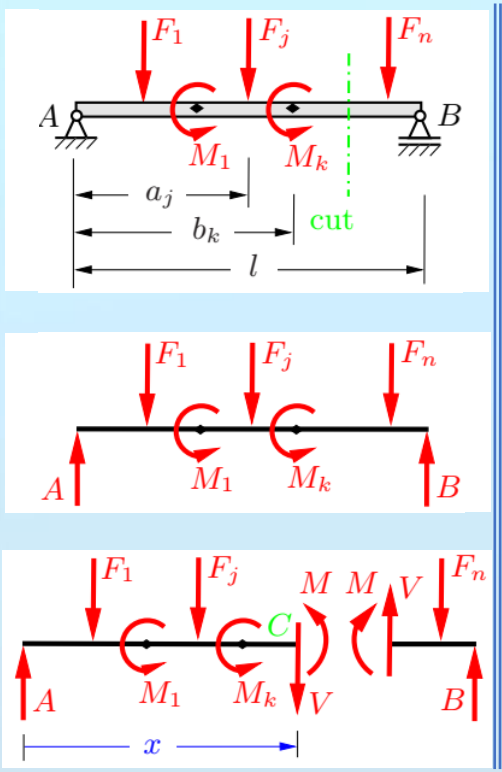
تنعدم القوة الناظمية في الجيزان المستقيمة المحملة عمودياً على محاورها.

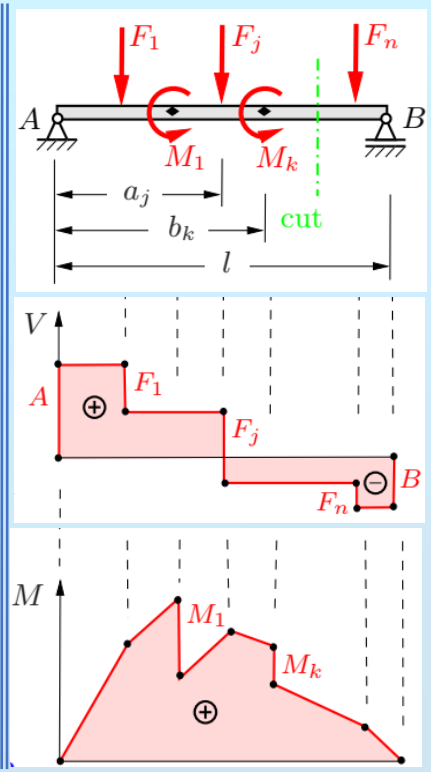
Beams under Concentrated Loads

To determine V & M choose a coordinate system and cut at an arbitrary x. Represent V & M with their positive directions in the F. B. Ds.; use Eq. Eqs. for either portion of the beam.

Results are a shear-force & a bending-moment diagram.

جملة احداثيات، قطع، معادلات توازن لأي من الجزئين





1. Reactions

$$\curvearrowleft_A: lB - \sum a_i F_i + \sum M_i = 0 \quad \rightarrow B = \frac{1}{l}[\sum a_i F_i - \sum M_i]$$

$$\curvearrowleft_B: -lA + \sum (l - a_i) F_i + \sum M_i = 0 \quad \rightarrow A = \frac{1}{l}[\sum (l - a_i) F_i + \sum M_i]$$

2. Cut at X

$$\uparrow: -V + A - \sum F_i = 0 \quad \rightarrow V = A - \sum F_i$$

$$\curvearrowleft_X: M - xA + \sum (x - a_i) F_i + \sum M_i = 0$$

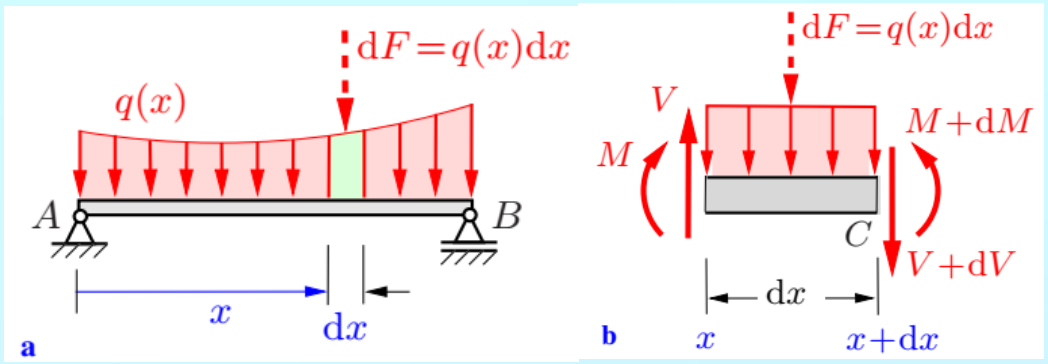
$$\rightarrow M = xA - \sum (x - a_i) F_i - \sum M_i$$

$$\frac{dM}{dx} = A - \sum F_i = V$$

مشتق تابع عزم الانعطاف يساوي تابع قوة القص

عند القوى أو العزوم المركزة توجد قفزة مساوية عكسا في المخطط المقابل

Relationship between distributed Loading and Stress Resultants (General case)



Any part of the beam is in Equilibrium

Eq. Eqs. of [**dx**]:

$$\uparrow: V - q(x)\mathrm{d}x - (V + \mathrm{d}V) = 0 \Rightarrow \boxed{\frac{\mathrm{d}V}{\mathrm{d}x} = -q(x)}$$

$$\curvearrowleft_C: (M + \mathrm{d}M) + \left(\frac{\mathrm{d}x}{2}\right) q(x)\mathrm{d}x - \mathrm{d}xV - M = 0$$

$$\text{with } \mathrm{d}x \to 0, \Rightarrow \boxed{\frac{\mathrm{d}M}{\mathrm{d}x} = V(x)} \;\&\; \boxed{\frac{d^2M}{dx^2} = -q(x)}$$

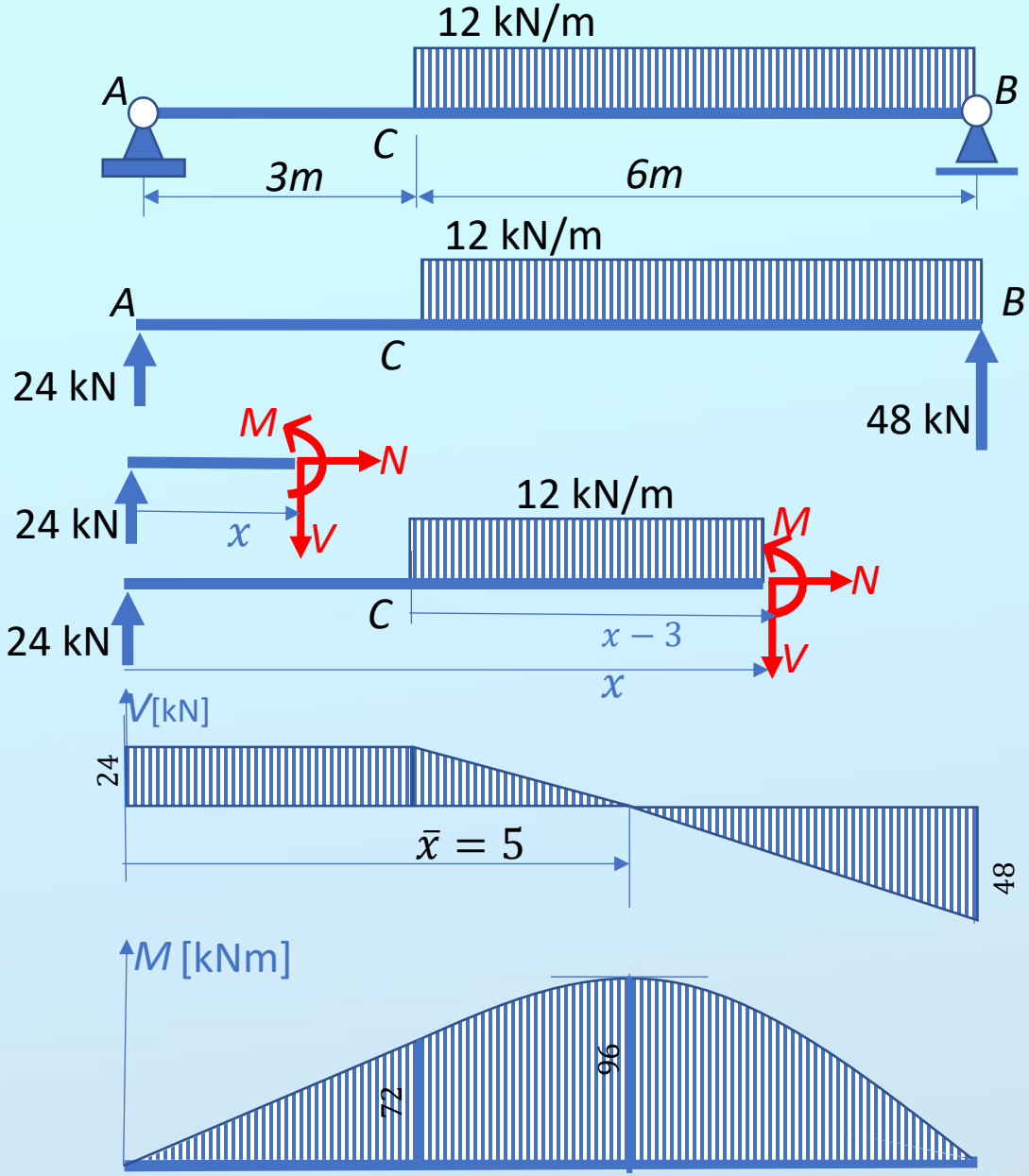
q	V	M
0	constant	linear
constant	linear	quadratic parabola
linear	quadratic parabola	cubic parabola

القص موجب فالعزم متزايد، القص سالب فالعزم متناقص،
القص معدوم فالعزم عند نهاية حدية (كبرى أو صغرى).

إذا كانت الحمولة كثيرة حدود (معدومة، ثابتة، خطية....)
فالقص كثيرة حدود (ثابتة، خطية، درجة ثانية: قطع مكافئ...)
والعزم كثيرة حدود (خطية، درجة ثانية: مكافئ، درجة ثالثة...)

المساند	support	V	M
مفصل	pin /roller	$\neq 0$	**0**
وثاقة	fixed end	$\neq 0$	$\neq 0$
طرف حر	free end	**0**	**0**

Problem 2. Draw the shear force and bending moment diagrams for the shown beam.



0. Reactions:

$\rightarrow: A_x = 0$

$\overset{\curvearrowleft}{B}: +3(6)(12) - 9A_z = 0 \Rightarrow A_z = 24 \text{ kN}$

$\overset{\curvearrowleft}{A}: -6(6)(12) + 9B_z = 0 \Rightarrow B_z = 48 \text{ kN}$

1. Cut: A…C, $0 < x < 3\text{m}$:

$\rightarrow: N = 0; \quad \uparrow: V = 24 \text{ kN}$

$\overset{\curvearrowleft}{X}: M - x(24) = 0 \Rightarrow M = 24x,$

$x = 0: M = 0; x = 3: M = 72 \text{ kNm}.$

2. Cut: C…B, $3 < x < 9\text{m}$:

$\rightarrow: N = 0;$

$\uparrow: V = 24 - 12(x - 3) = -12x + 60$

$x = 3: V = 24 \text{ kN}, x = 9: V = -48 \text{ kN};$

$V = 0: \bar{x} = 60/12 = 5 \text{ m}$

$\overset{\curvearrowleft}{X}: M - x(24) + \frac{1}{2}(x - 3)(12)(x - 3) = 0$

$\Rightarrow M = -6x^2 + 60x - 54,$

$x = 3: M = 72 \text{ kNm}; x = 9: M = 0.$

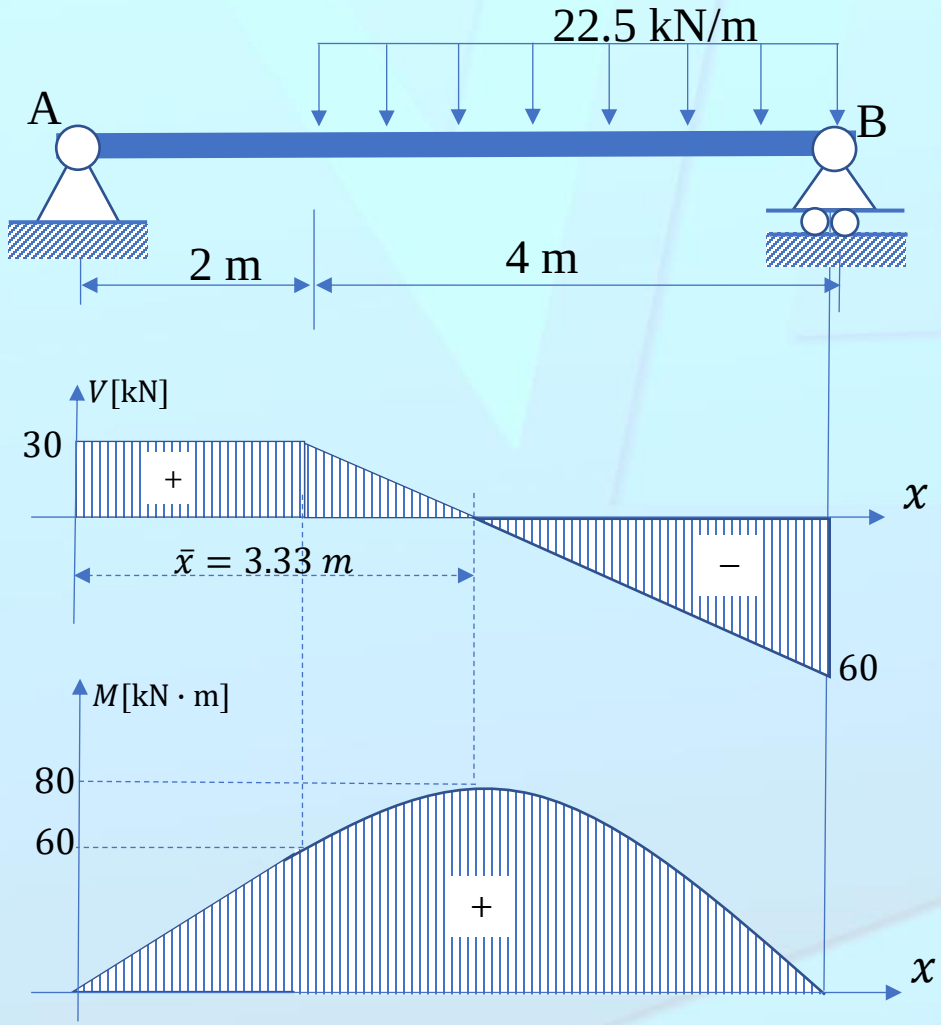
$M_{max} = M_{|V=0} = M_{|\bar{x}=5} = 96 \text{ kNm}.$

Problem 3. For the shown beam find

(a) Reactions in supports A and B.

(b) Shear force and bending moment diagrams.

(c) Determine the value and the location of the maximum bending moment.



Solution:

(a) Reactions:

$\sum M_A = 0:\ -22.5(4)(4) + 6B_y = 0 \Rightarrow B_y = 60\ \text{kN}(\uparrow)$

$\sum F_y = 0:\ A_y + 60 - 22.5(4) = 0 \Rightarrow A_y = 30\ \text{kN}(\uparrow)$

(b) Shear force and bending moment diagrams

For: $0 < x < 2: V(x) = 30,$

$$M = 30x, M(0) = 0, M(2) = 60\ kNm$$

For: $2 < x < 6: V(x) = 30 - 22.5(x-2) = -22.5x + 75,\ \ V(2) = 30\ kN, V(6) = -60\ kN$

$$M(x) = 30x - 22.5\frac{(x-2)^2}{2} = -11.25x^2 + 75x - 45,\ \ M(2) = 60\ kNm, M(6) = 0$$

(c) Maximum bending moment:

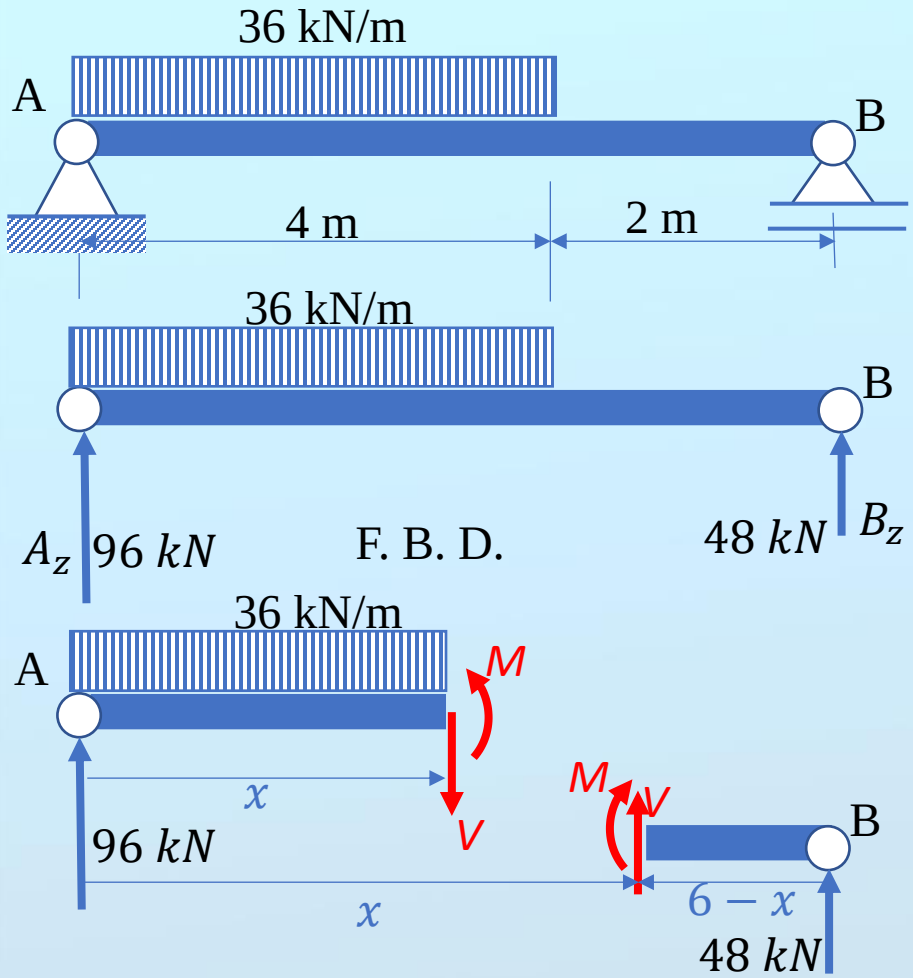
$$V(x) = 0\ if\ x = 75 \div 22.5 = 10/3 = \bar{x}\ ,\qquad M_{max} = M(10/3) = 80\ kNm$$

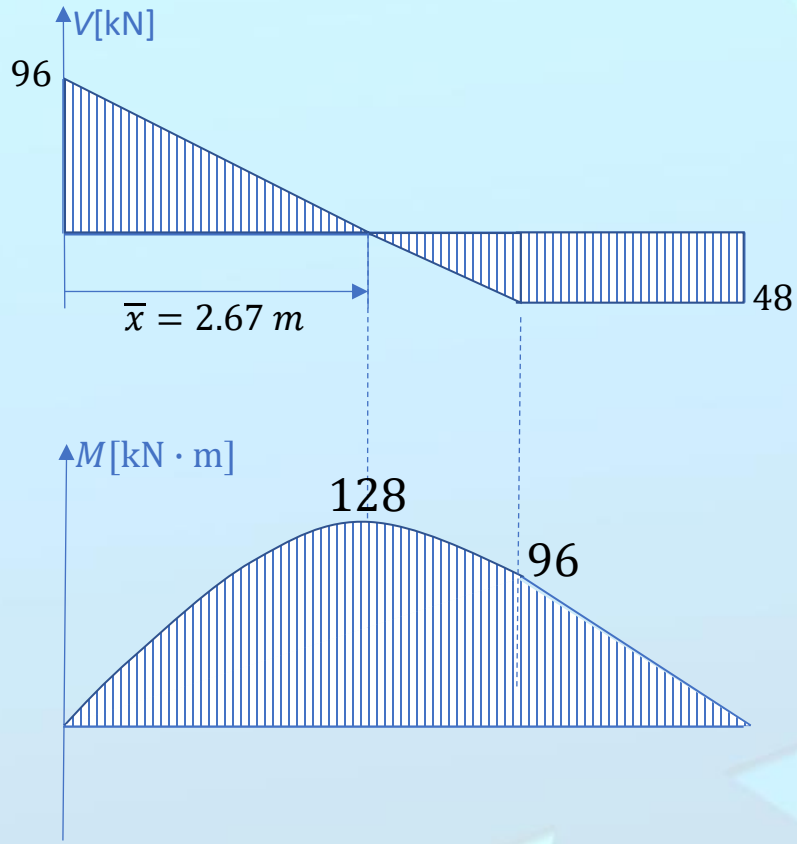
Problem 4. For the shown beam find

(a) Reactions in supports A and B.

(b) Shear force and bending moment diagrams.

(c) Determine the value and the location of the maximum bending moment.





Cut 1: $0 < x < 4$:

$V(x) = 96 - 36x.$

$x = 0: V = 96\ kN;$

$x = 4: V = -48\ kN;$

$V = 0: \overline{x} = 2.67\ m$

$M(x) = 96x - 18x^2.$

$x = 0: M = 0;$

$x = 4: M = 96kN \cdot m;$

$\overline{x} = 2.67\ m: M_{max} = 128\ kN \cdot m$

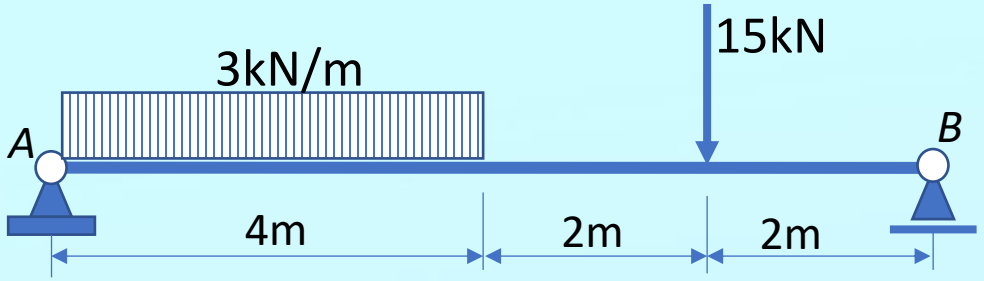
Cut 2: $4 < x < 6$:

$V = -48\ kN.$

$M(x) = 48(6 - x).$

$x = 4: M = 96kN \cdot m.$

$x = 0: M = 0.$



Problem 5. Determine the shear-force and bending-moment diagrams for the simple beam shown in Fig. using the section method.

Problem 6. Determine the shear-force and bending-moment diagrams for the simple beam shown in Fig. using the section method. Then determine the location and the value of the maximum bending moment.