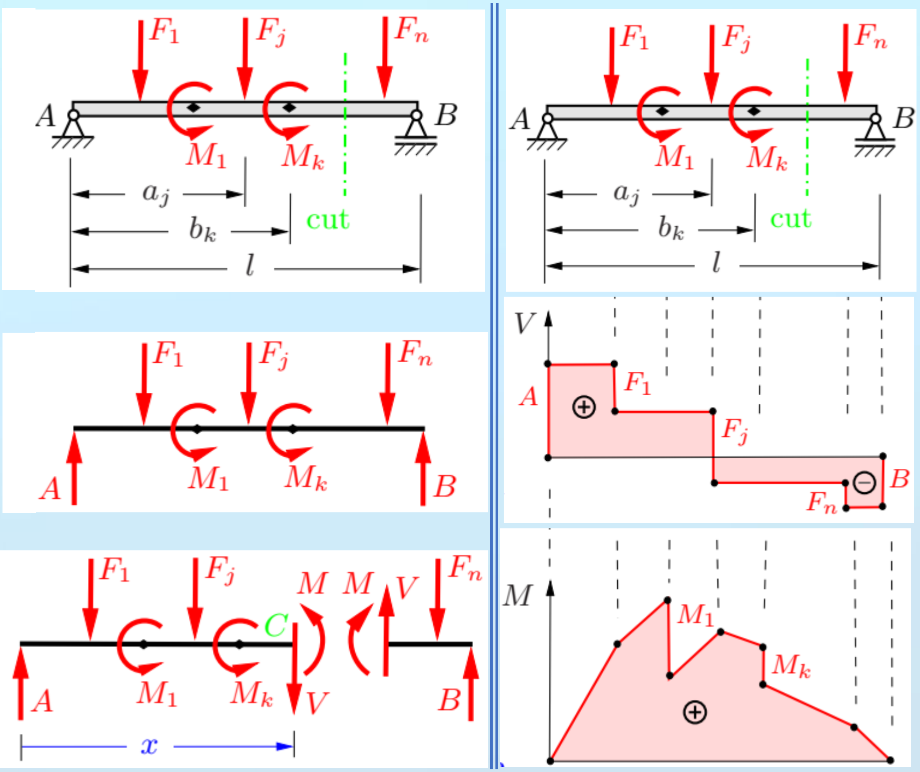
Stress Resultants in Straight Beams

Beams are usually subjected to forces perpendicular to their axes. If there is no loading (external or reactions) in the direction of the beam axis, the normal force vanishes $N = 0$.

تنعدم القوة الناظمية في الجيزان المستقيمة المحملة عمودياً على محاورها.

Beams under Concentrated Loads

To determine V & M choose a coordinate system and cut at an arbitrary x. Represent V & M with their positive directions in the F. B. Ds.; use Eq. Eqs. for either portion of the beam. Results are a shear-force and a bending-moment diagram. جملة احدتثيات، قطع، معادلات توازن لأي من الجزئين



1. Reactions

$\overset{\curvearrowleft}{A}$: $lB - \sum a_i F_i + \sum M_i = 0 \quad \rightarrow B = \frac{1}{l}[\sum a_i F_i - \sum M_i]$

$\overset{\curvearrowleft}{B}$: $-lA + \sum (l - a_i) F_i + \sum M_i = 0 \quad \rightarrow A = \frac{1}{l}[\sum (l - a_i) F_i + \sum M_i]$

2. Cut at X

$\uparrow$: $-V + A - \sum F_i = 0 \quad \rightarrow V = A - \sum F_i$

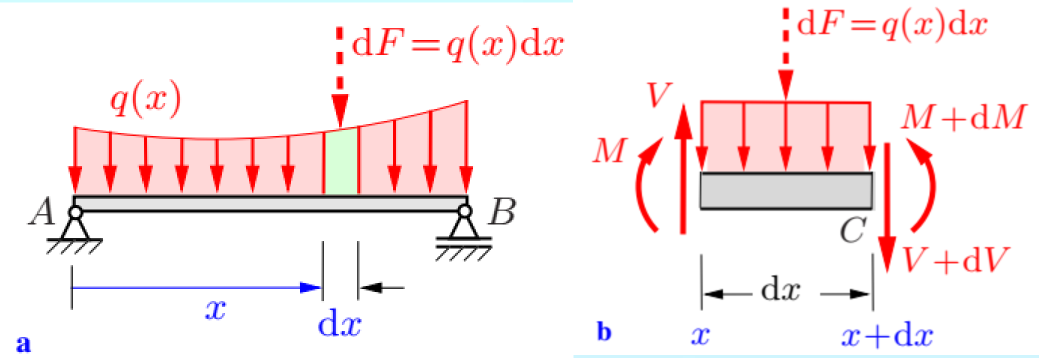
$\overset{\curvearrowleft}{X}$: $M - xA + \sum (x - a_i) F_i + \sum M_i = 0$

$\rightarrow M = xA - \sum (x - a_i) F_i - \sum M_i$

$\frac{dM}{dx} = A - \sum F_i = V$ مشتق تابع عزم الانعطاف يساوي تابع قوة القص

عند القوى أو العزوم المركزة توجد قفزة مساوية عكسا في المخطط المقابل

Relationship between distributed Loading and Stress Resultants (General case)



q	V	M
0	constant	linear
constant	linear	quadratic parabola
linear	quadratic parabola	cubic parabola

المساند	support	V	M
مفصل	pin /roller	$\neq 0$	0
وثاقة	fixed end	$\neq 0$	$\neq 0$
طرف حر	free end	0	0

Any part of the beam is in Equilibrium

Eq. Eqs. of [**dx**]:

$$\uparrow: V - q(x)dx - (V + dV) = 0 \Rightarrow \boxed{\frac{dV}{dx} = -q(x)}$$

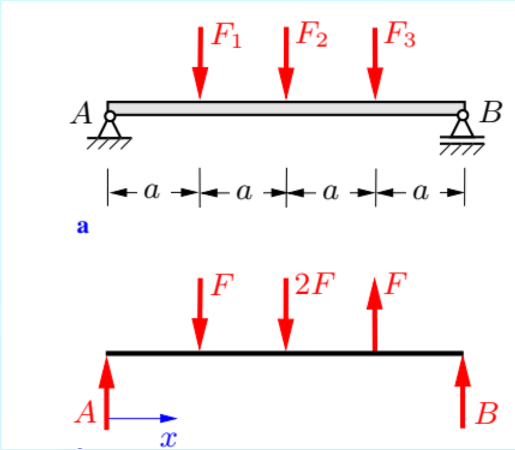
$$\circlearrowleft_C: (M + dM) + \left(\frac{dx}{2}\right) q(x)dx - dxV - M = 0$$

with $dx \to 0, \Rightarrow \boxed{\frac{dM}{dx} = V(x)}$ & $\boxed{\frac{d^2M}{dx^2} = -q(x)}$

القص موجب فالعزم متزايد، القص سالب فالعزم متناقص،
القص معدوم فالعزم عند نهاية حدية (كبرى أو صغرى).

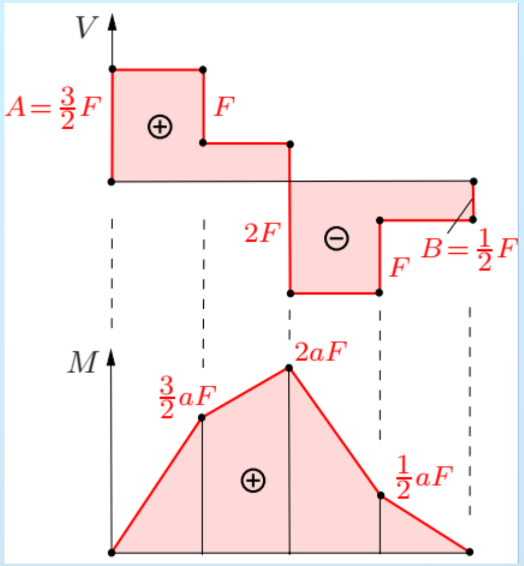
إذا كانت الحمولة كثيرة حدود (معدومة، ثابتة، خطية....)
فالقص كثيرة حدود (ثابتة، خطية، درجة ثانية: قطع مكافئ...)
والعزم كثيرة حدود (خطية، درجة ثانية: مكافئ، درجة ثالثة...)

Example 1 The simply supported beam in Fig.a. is subjected to the three forces $F_1 = F$, $F_2 = 2\,F$ and $F_3 = -F$. Draw the shear-force and bending-moment diagrams.

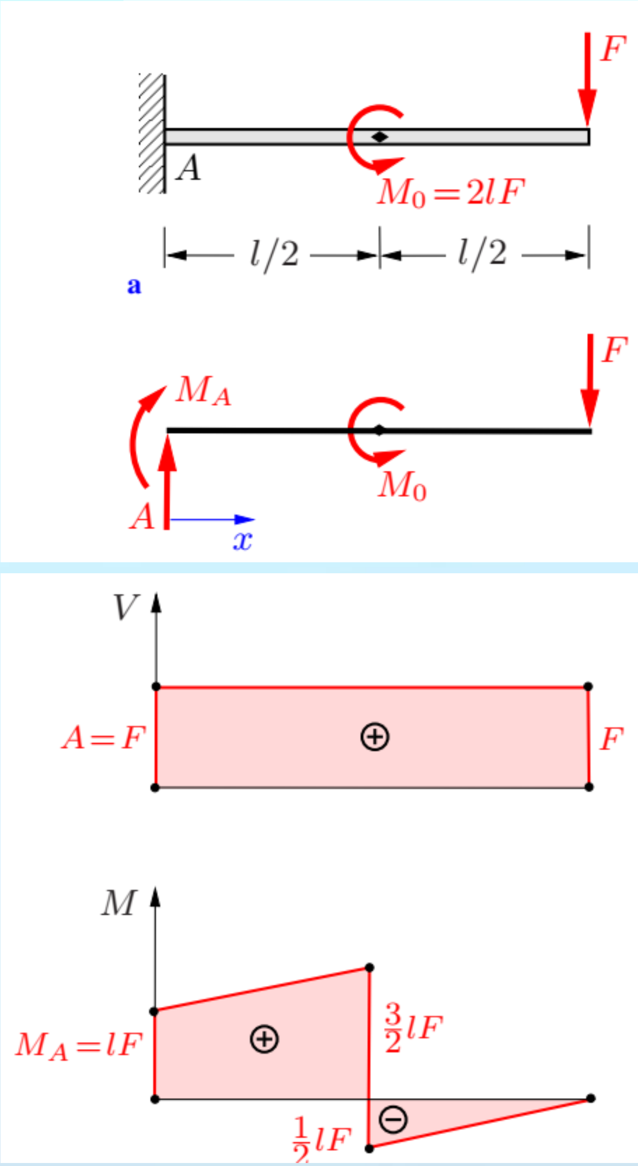


Solution:

0. Reactions

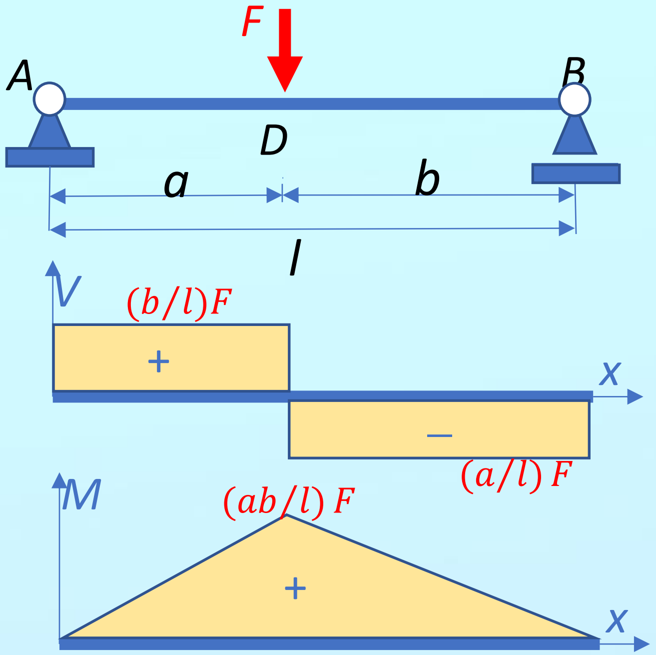


Example 2 Determine the shear-force and bending-moment diagrams for the cantilever beam shown in Fig.a.

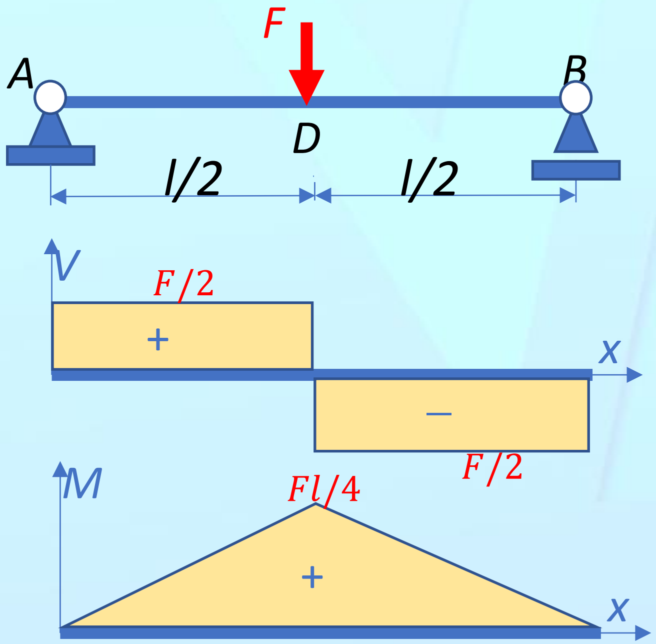


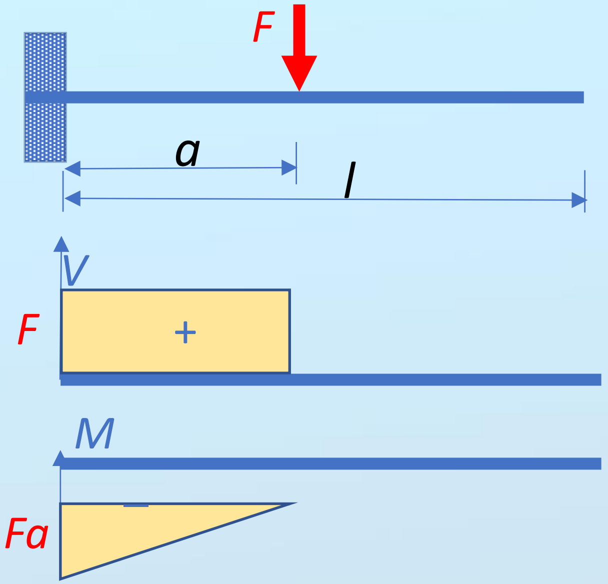
Solution:

By Heart

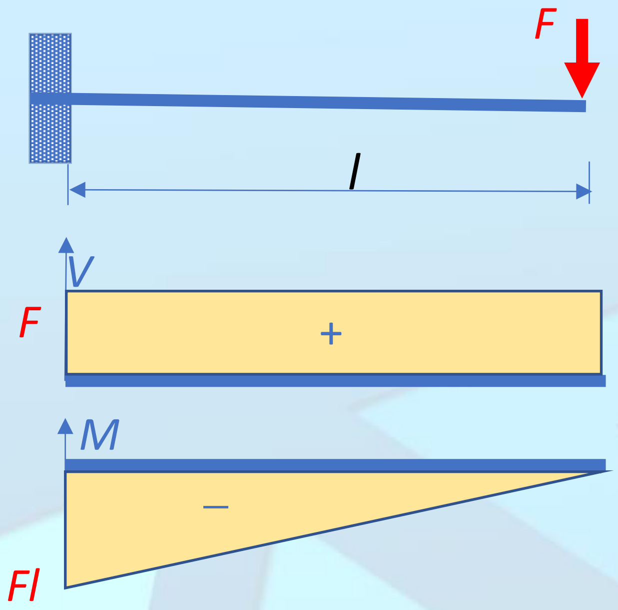


If $a=b=l/2 \Rightarrow$

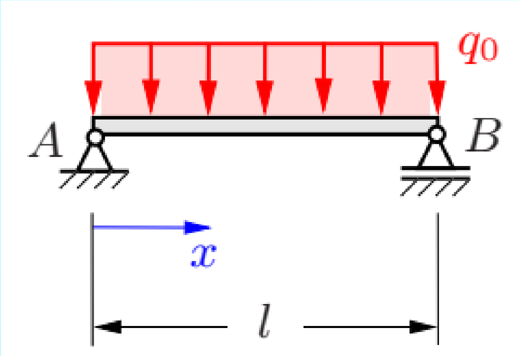


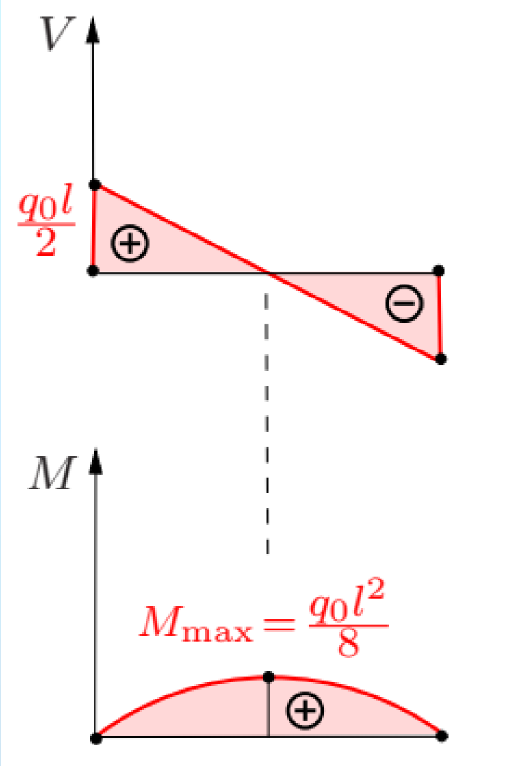


If $a=l \Rightarrow$



Example 3 Determine the shear-force and bending-moment diagrams for the beam shown in Fig. using the section method and the integration method.



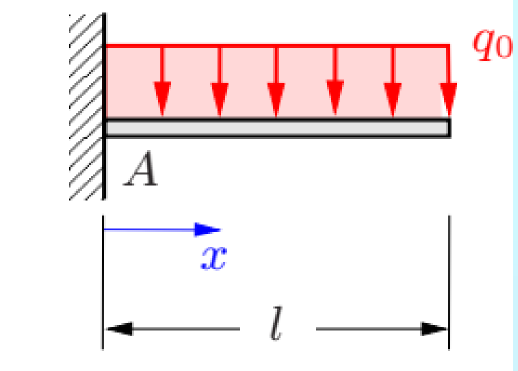


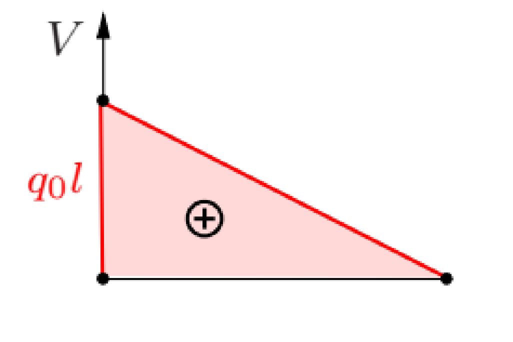
Solution:

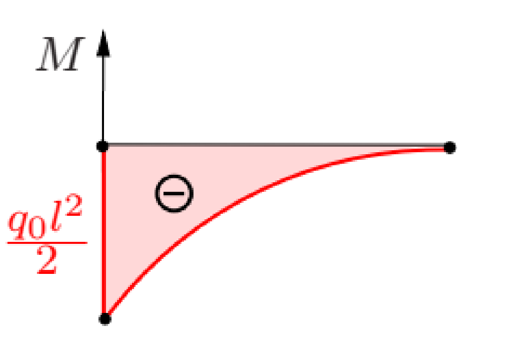
Reactions:

Cut or Cuts:

Example 4 Determine the shear-force and bending-moment diagrams for the cantilever beam shown in Fig. using the section method and the integration method.





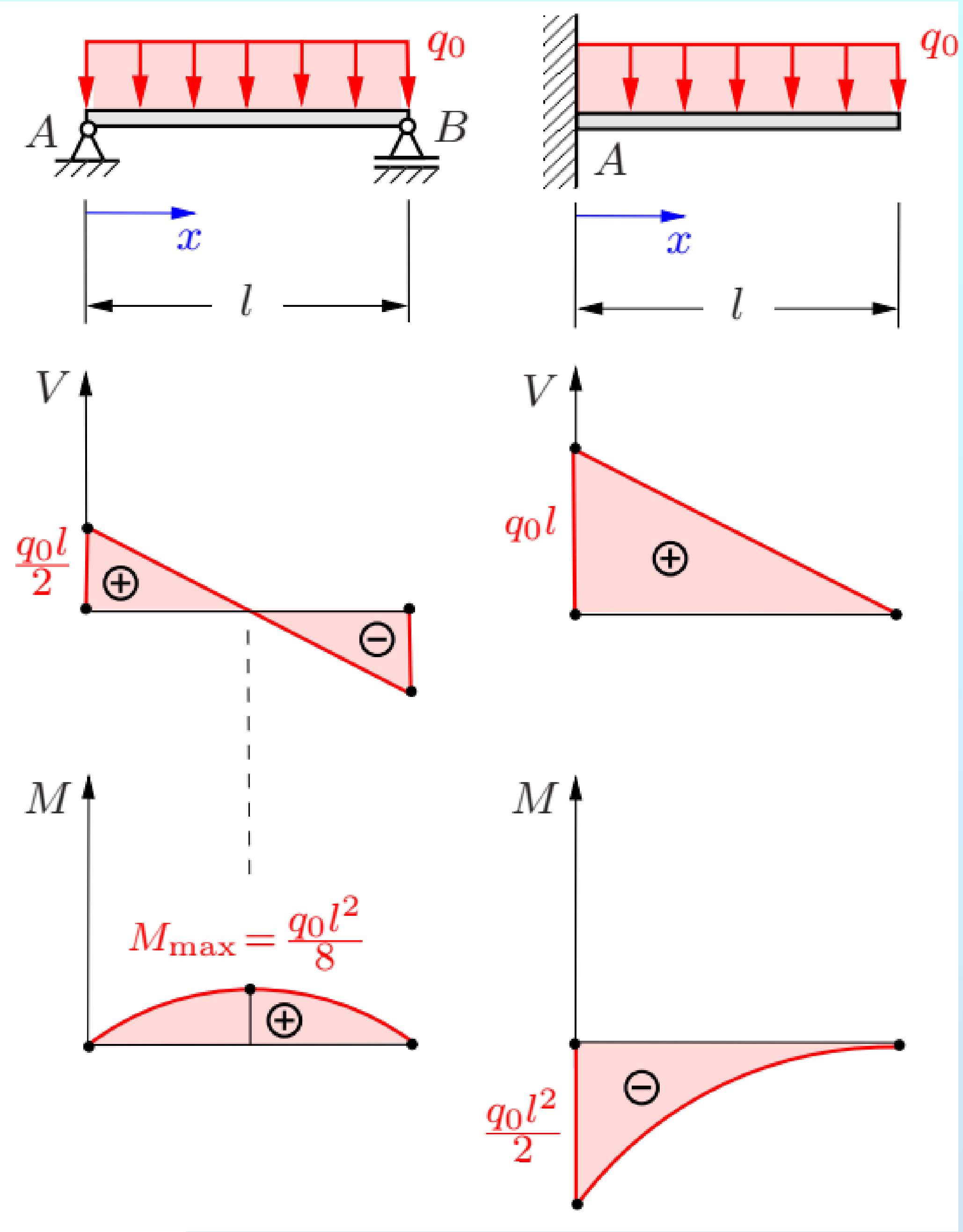


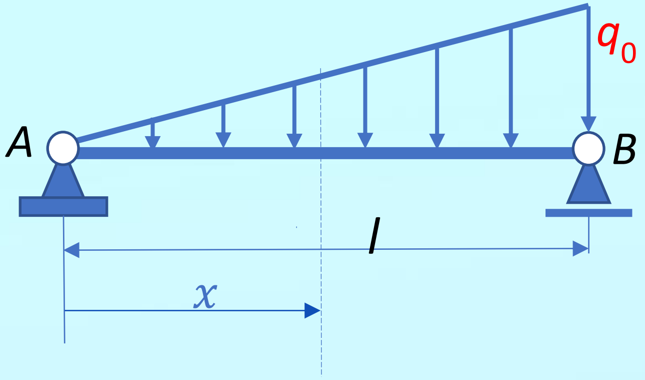
Solution:

Reactions:

Cut or Cuts:

By Heart





Example 5 Determine the shear-force & bending-moment diagrams for the shown simple beam. using the integration method.

Solution:

1) Find the function of the distributed load: $q(x) = \frac{q_0}{l}x$

2) Integrate twice the equation: $\frac{d^2M}{dx^2} = -q(x)$ To get:

$$\frac{d^2M}{dx^2} = -\frac{q_0}{l}x \Rightarrow V = \frac{dM}{dx} = -\frac{q_0}{2l}x^2 + C_1 \Rightarrow M = -\frac{q_0}{6l}x^3 + C_1x + C_2$$

3) Determine the two constants C_1 & C_2 from the two boundary conditions:

1- At x =0 (pin support at A) M = 0: $0 = -\frac{q_0}{6l}(0)^3 + C_1(0) + C_2 \Rightarrow C_2 = 0$

2- At x =l (roller support at B) M = 0: $0 = -\frac{q_0}{6}l^2 + C_1 l \Rightarrow C_1 = \frac{q_0}{6}l$

4) Write the final expressions of V & M as:

$$V = -\frac{q_0}{2l}x^2 + \frac{q_0}{6}l = \frac{q_0}{6l}(-3x^2 + l^2)$$

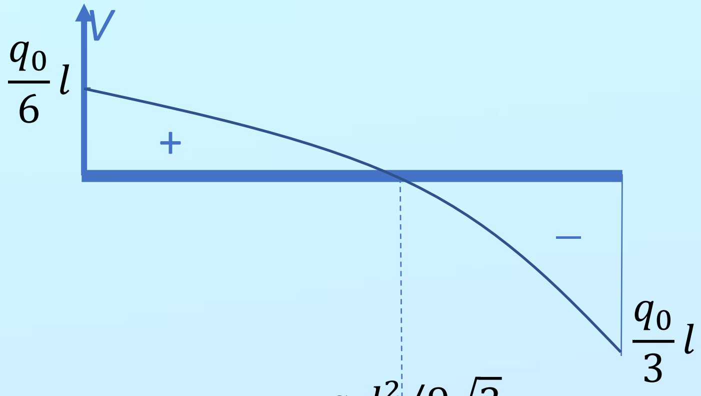
$x = 0: V = \frac{q_0}{6}l$ & $x = l: V = -\frac{q_0}{3}l$

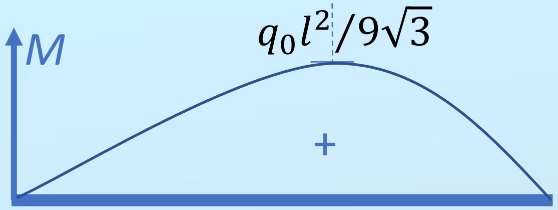
$V = 0 \Rightarrow x = \frac{l}{\sqrt{3}} = 0.577l$

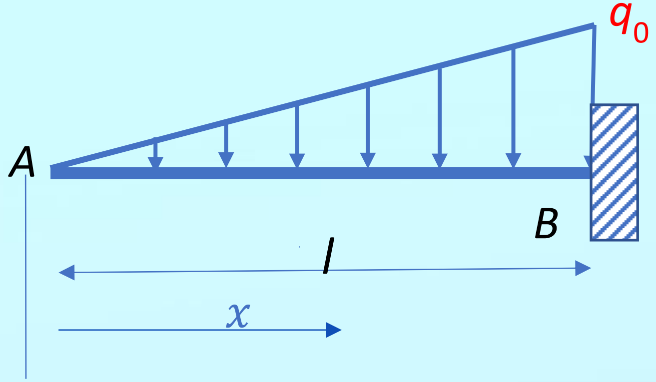
$$M = -\frac{q_0}{6l}x^3 + \frac{q_0}{6}lx = \frac{q_0}{6l}(-x^3 + l^2x)$$

$x = 0: M = 0$ & $x = l: M = 0$

$x = \frac{l}{\sqrt{3}} = 0.577l \Rightarrow M_{max} = \frac{q_0l^2}{9\sqrt{3}} = \frac{q_0l^2}{15.6}$







Example 6. Determine the shear-force & bending-moment diagrams for the shown cantilever beam, using the integration method.

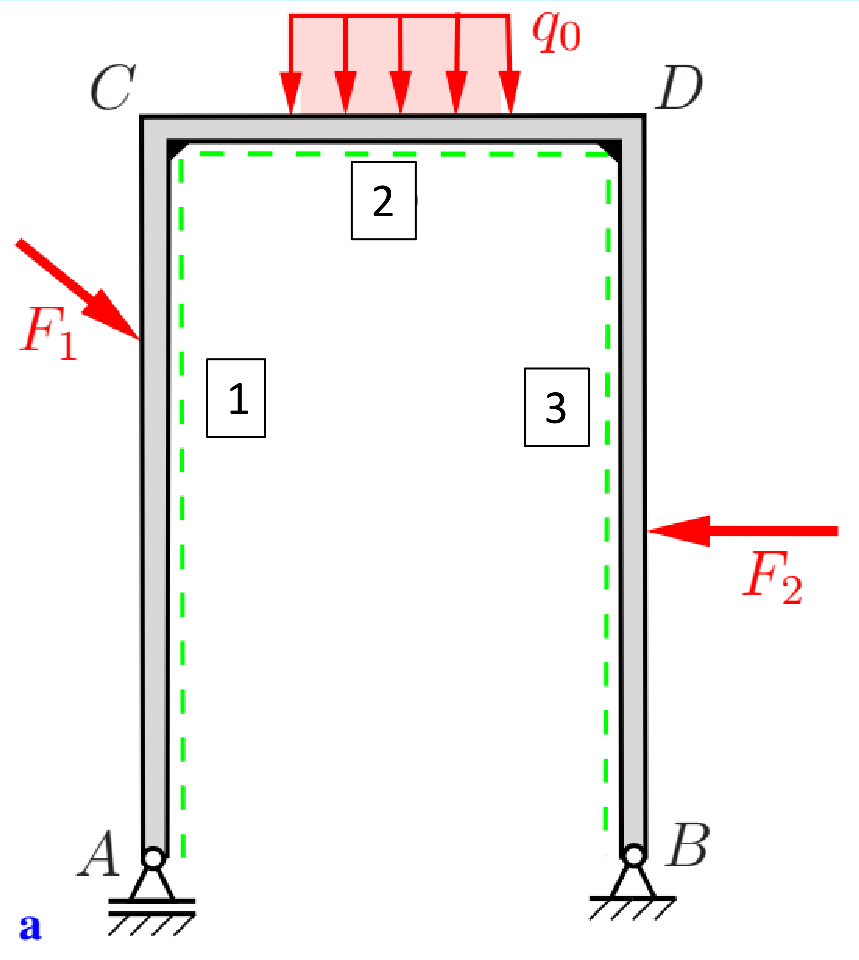
Solution:

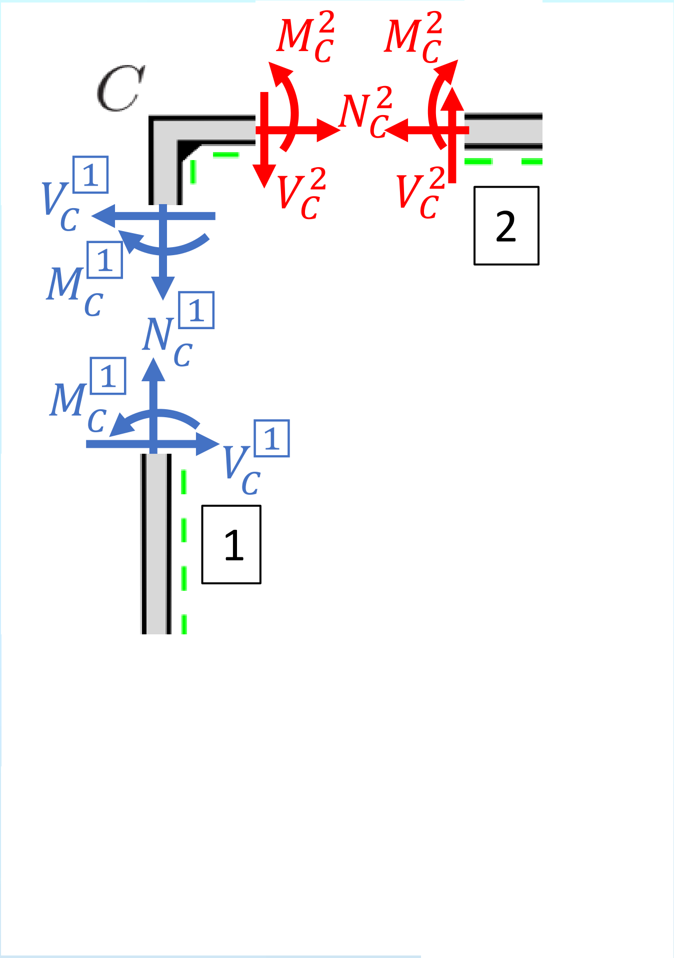
Reactions: No need

Cut or Cuts:

Stress Resultants (Internal Forces) in Frames.

The methods for determining the stress resultants will now be generalized to frames





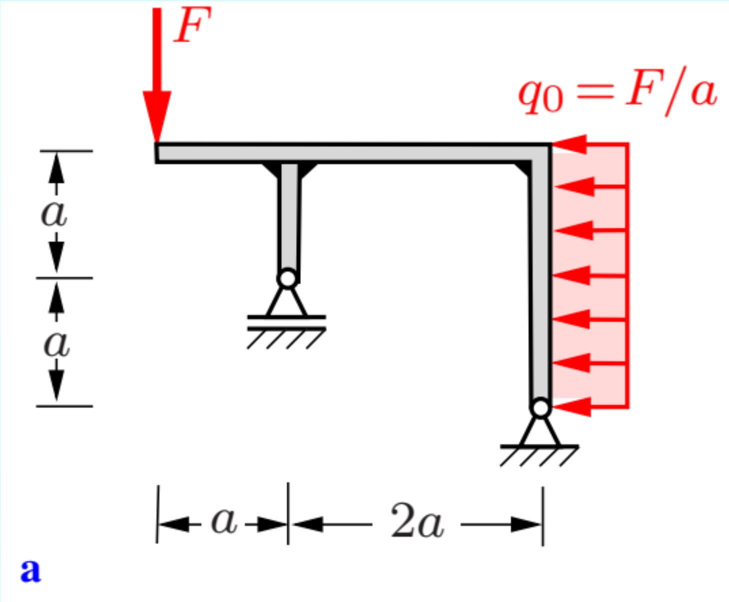
C is a rigid joint. So

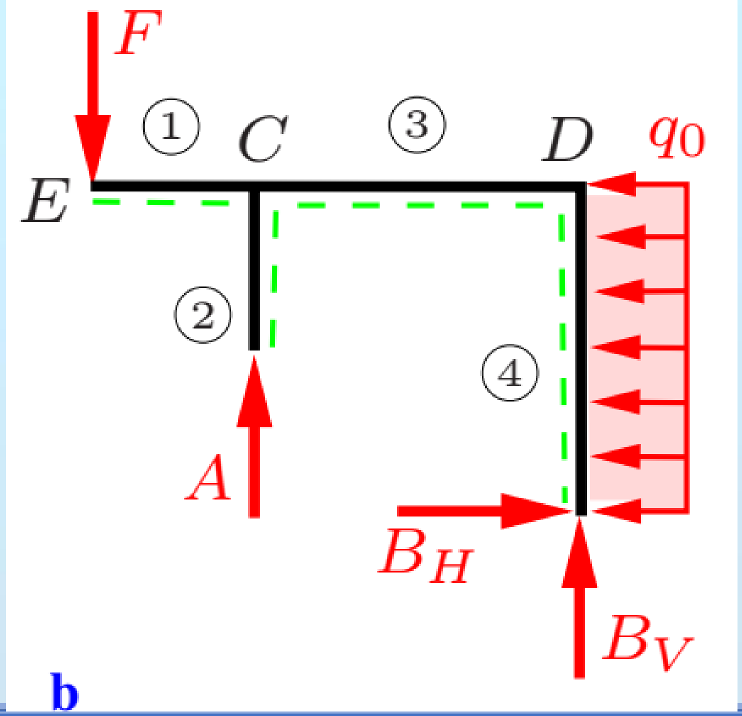
3Eq. Eqs. of Joint C

$$N_C^{\boxed{1}} = -V_C^{\boxed{2}}$$

$$V_C^{\boxed{1}} = N_C^{\boxed{2}}$$

$$M_C^{\boxed{1}} = M_C^{\boxed{2}}$$





Example 7. Determine the Stress resultant (Normal Force, Shear Force and Bending Moment) in the frame shown in the fig. a.

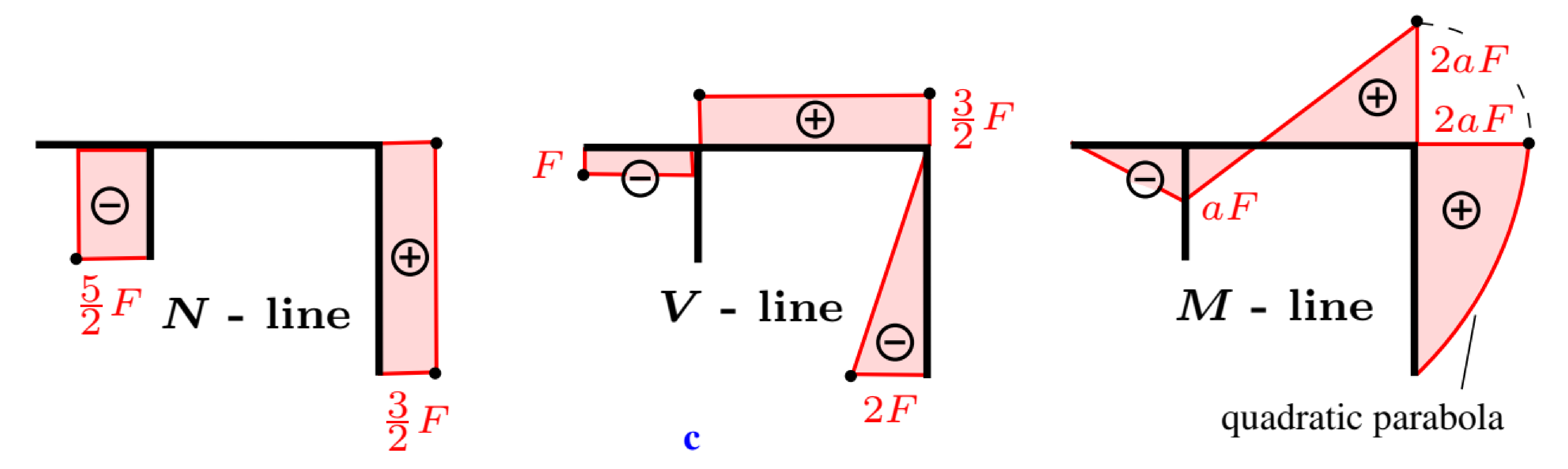
Solution The support reactions are obtained from the equilibrium conditions for the frame as a whole

$$\overset{\curvearrowleft}{B}: + a(2aq_0) + 3aF - 2aA = 0 \qquad \Rightarrow A = \frac{5}{2}F$$

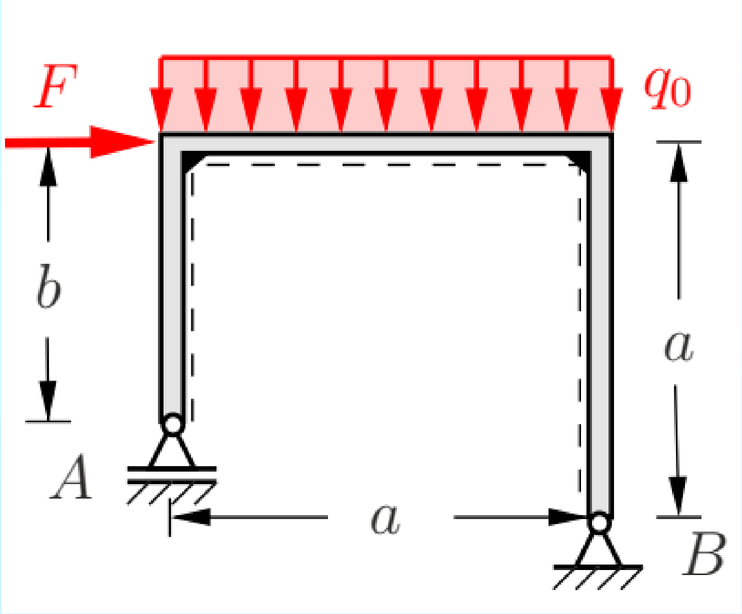
$$\uparrow: -F + \frac{5}{2}F + B_V = 0 \qquad \Rightarrow B_V = -\frac{3}{2}F$$

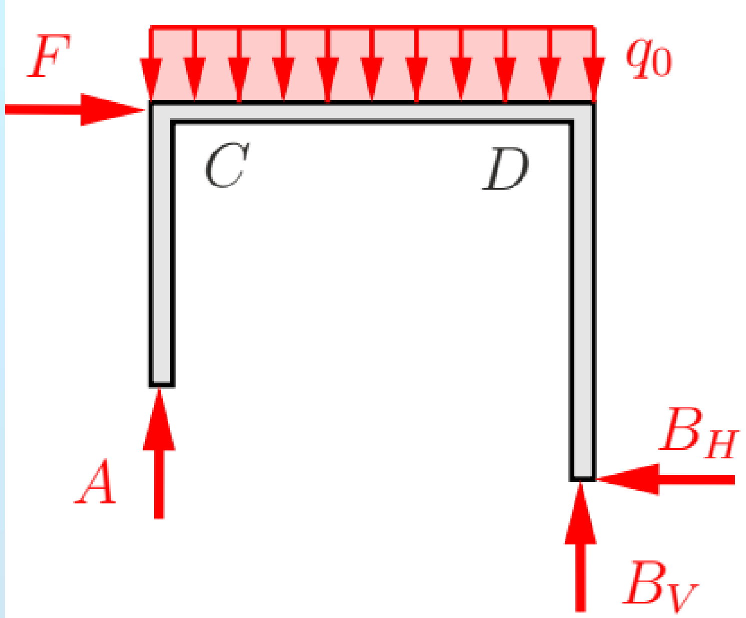
$$\rightarrow: B_H - 2aq_0 = 0 \qquad \Rightarrow B_H = 2F$$

In order to define the algebraic signs of the stress resultants, we introduce the dashed lines according to Fig. b. The following s

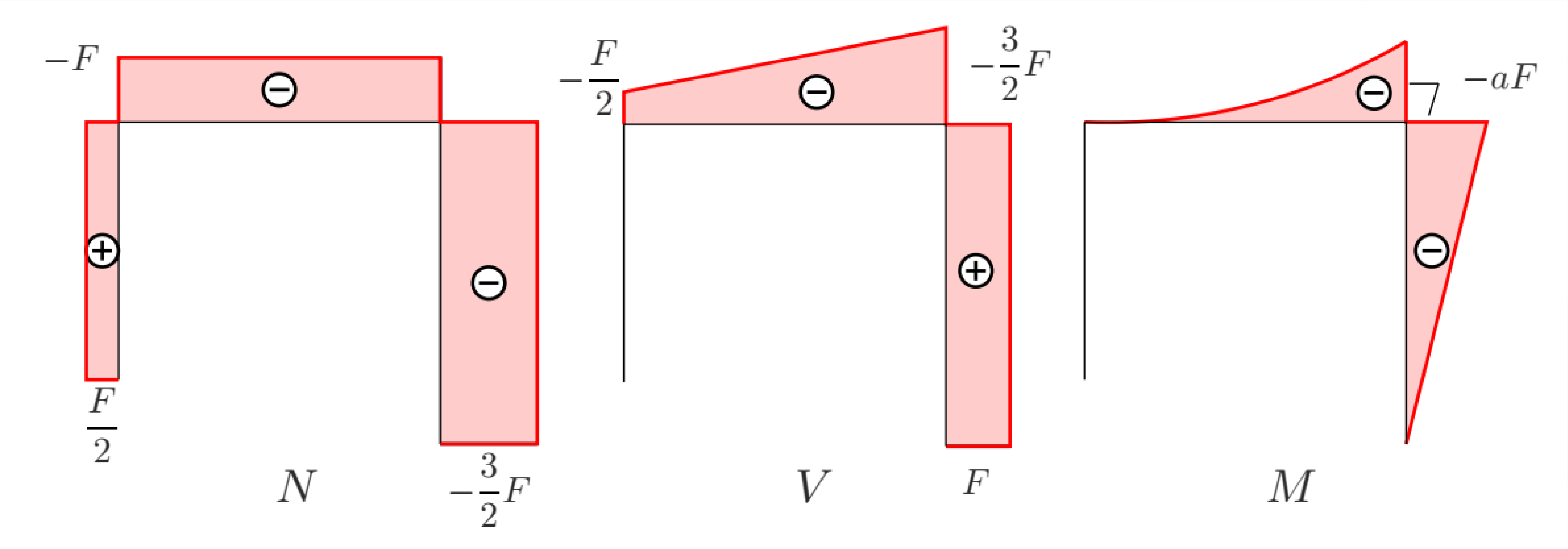


$\frac{5}{2}F$
N - line
$\frac{3}{2}F$
F
$\frac{3}{2}F$
V - line
2F
c
2aF
2aF
aF
M - line
quadratic parabola

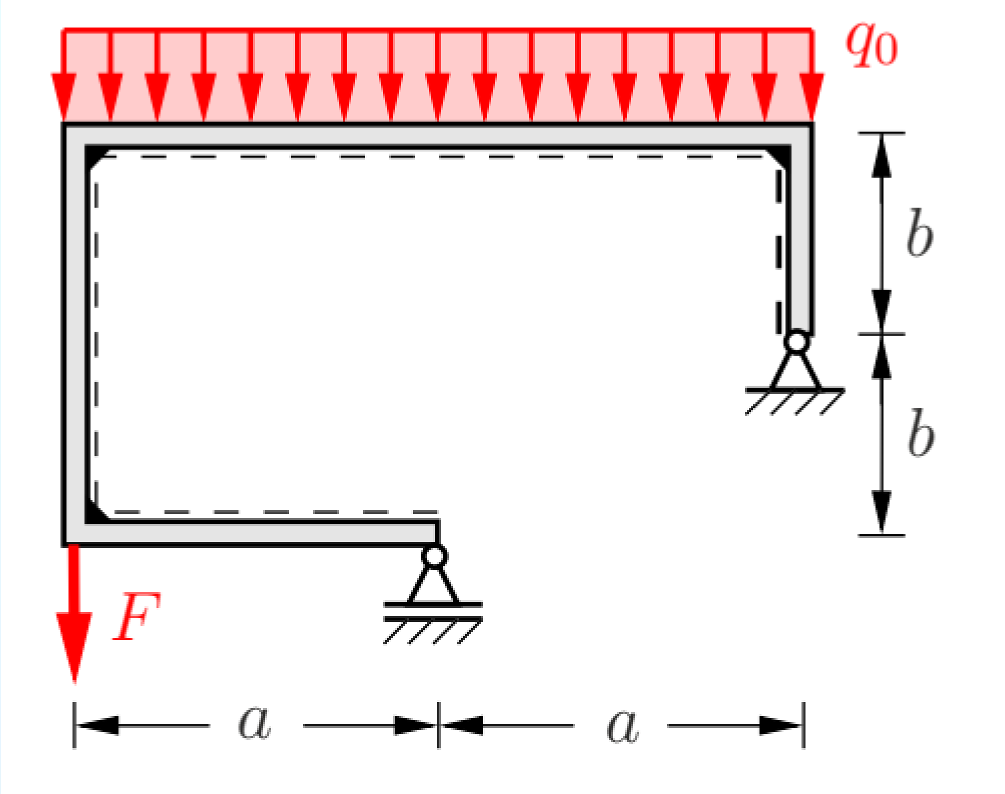




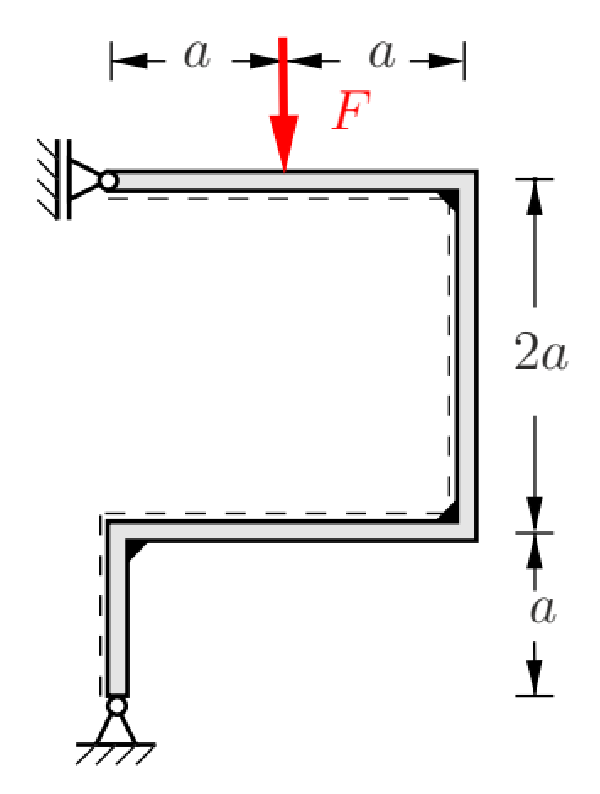
Example 8. The depicted frame is loaded by a force F and a uniform line load $q_0 = F/a$. Determine the Normal force, shear force and bending moment diagrams.

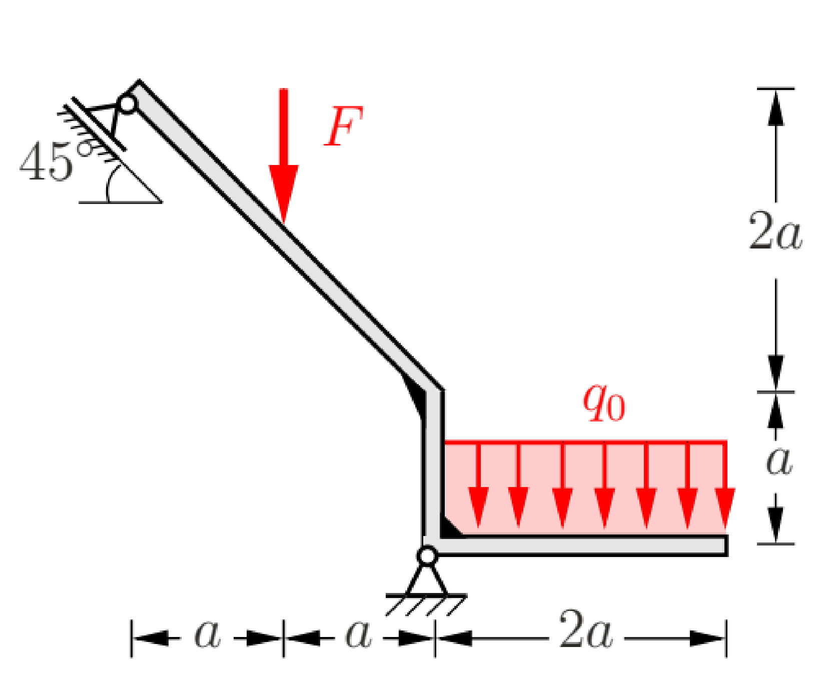

−F
F/2
N
−3/2 F
F/2
−3/2 F
V
F
−aF
M

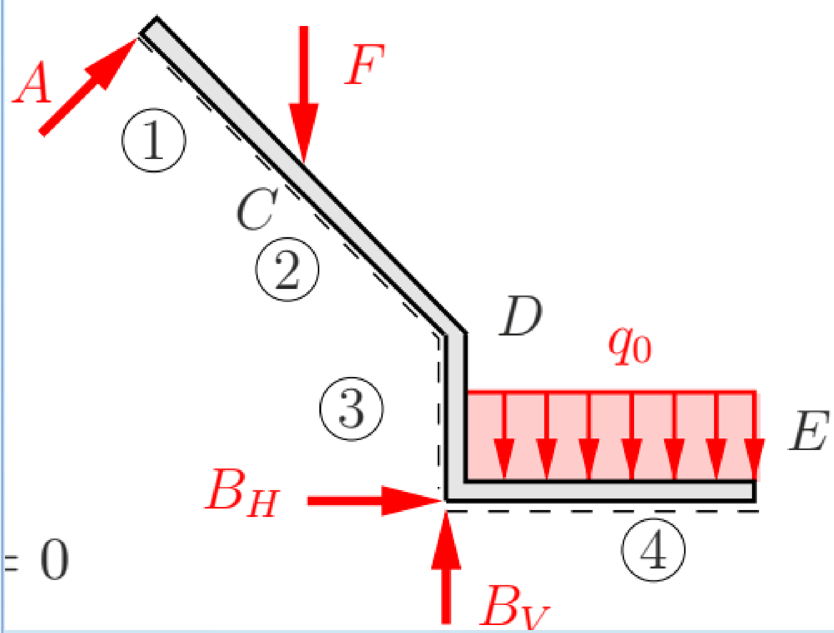
Example 9.



Example 10.







Example 11. The depicted frame is loaded by a force F and a uniform line load $q_0 = F/a$. Determine the Normal force, shear force and bending moment diagrams.

Solution

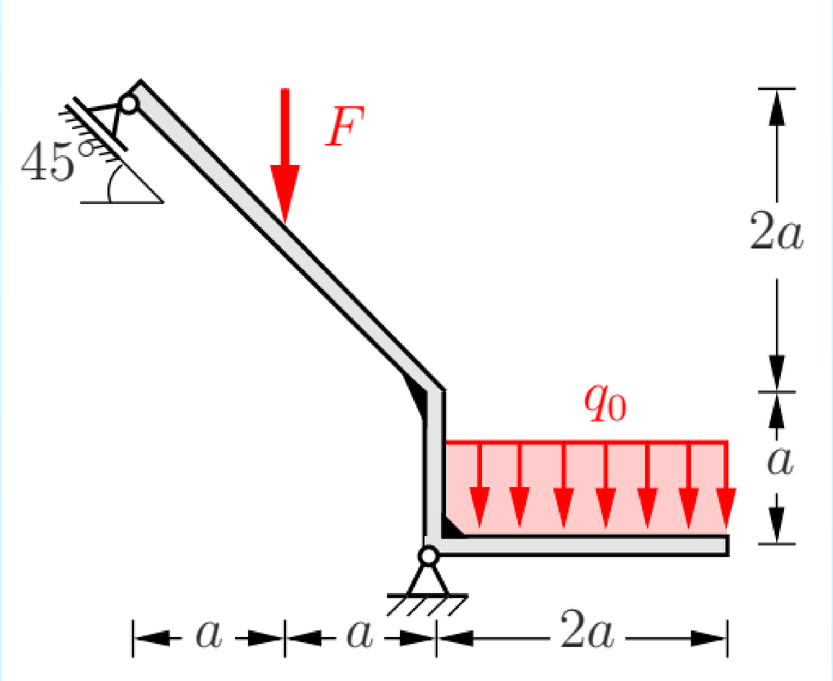
Reactions: A, B_V, B_H

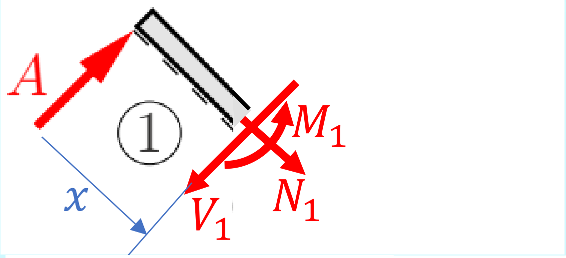
$\rightarrow: A\cos 45^\circ + 0 + B_H = 0$

$\uparrow: A\sin 45^\circ + B_V + 0 = 3F$

$\curvearrowleft_A: 0 + 2aB_V + 3aB_H - aF - 3a(2aq_0) = 0 \;\Rightarrow: 0 + 2B_V + 3B_H = 7F$

$$A = -0.283F, B_V = 3.2F, B_H = 0.2F$$

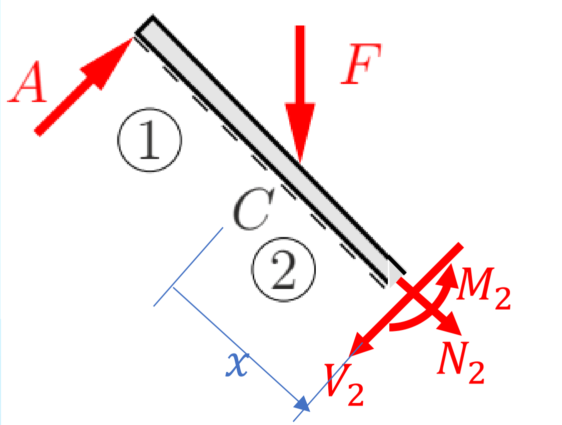




$N_1 = 0,$

$V_1 = -0.283F,$

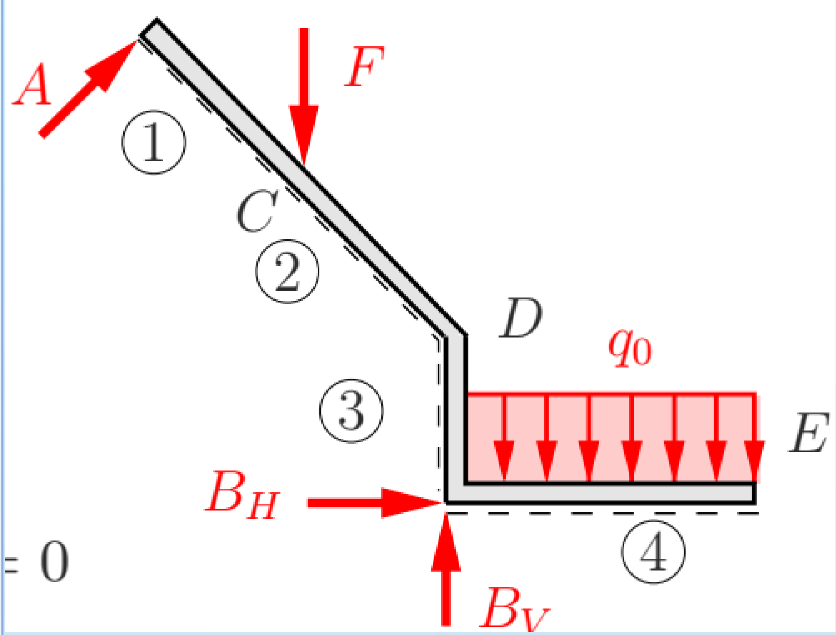
$M_1 = -0.283Fx\text{: } M_1^A = 0, M_1^C = -0.4aF$

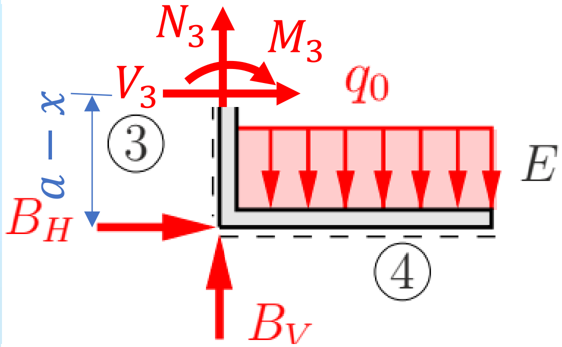


$N_2 = -0.707F,$

$V_2 = -0.990F,$

$M_2 = -0.4aF - 0.990Fx\text{: } M_2^C = -0.4Fa, M_2^D = -1.8Fa$

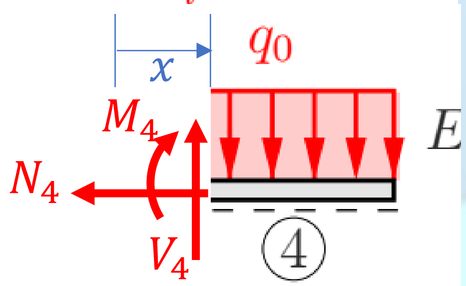




$N_3 = -1.2F,$

$V_3 = -0.2F,$

$M_3 = -1.8Fa - 0.2Fx\text{: } M_3^D = -1.8Fa, M_3^B = -2Fa$



$N_4 = 0,$

$V_4 = (2a - x)q_0 = (2a - x)\frac{F}{a}\text{: } V_4^B = 2F, V_4^E = 0$

$M_4 = -\frac{1}{2}(2a - x)^2\frac{F}{a}, M_4^B = -2Fa, M_4^E = 0$