Digital Image Processing

Chapter 9:
Morphological Image Processing

What are Morphological Operations?

Morphological operations come from the word "morphing" in Biology which means "changing a shape".

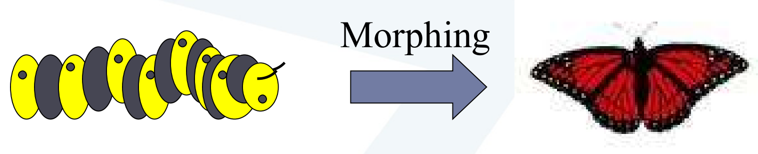


Image morphological operations are used to manipulate object shapes such as thinning, thickening, and filling.

Binary morphological operations are derived from set operations.

Basic Set Operations

Concept of a set in binary image morphology:

Each set may represent one object. Each pixel (x,y) has its status: belong to a set or not belong to a set.

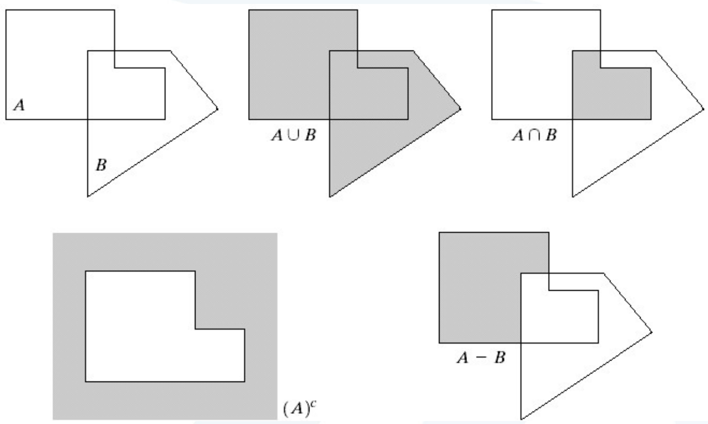


FIGURE 9.1 (a) Two sets *A* and *B*. (b) The union of *A* and *B*. (c) The intersection of *A* and *B*. (d) The complement of *A*. (e) The difference between *A* and *B*.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

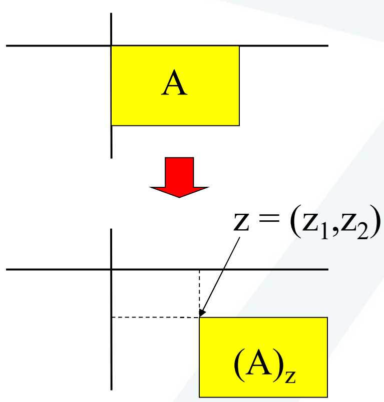
Translation and Reflection Operations

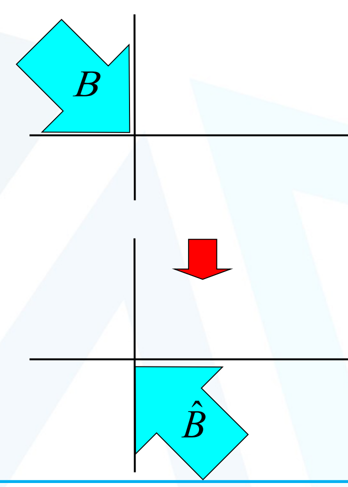
Translation

$$(A)_z = \{c \mid c = a + z, \quad \text{for } a \in A\}$$

Reflection

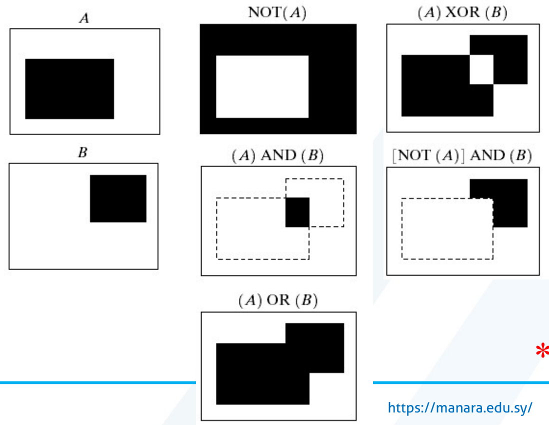
$$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$$





Logical Operations*

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0



*For binary images only

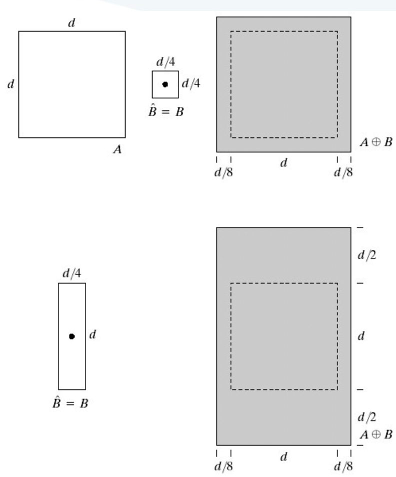
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Dilation Operations

$$A \oplus B = \left\{ z \mid \left(\hat{B}\right)_z \cap A \neq \phi \right\}$$

ϕ = Empty set

Dilate means "extend"

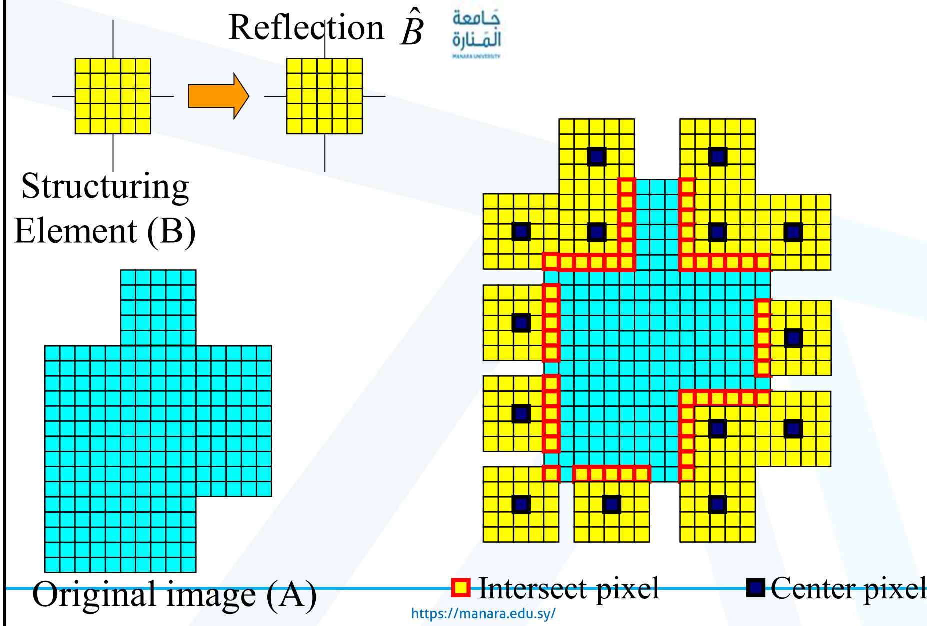


A = Object to be dilated

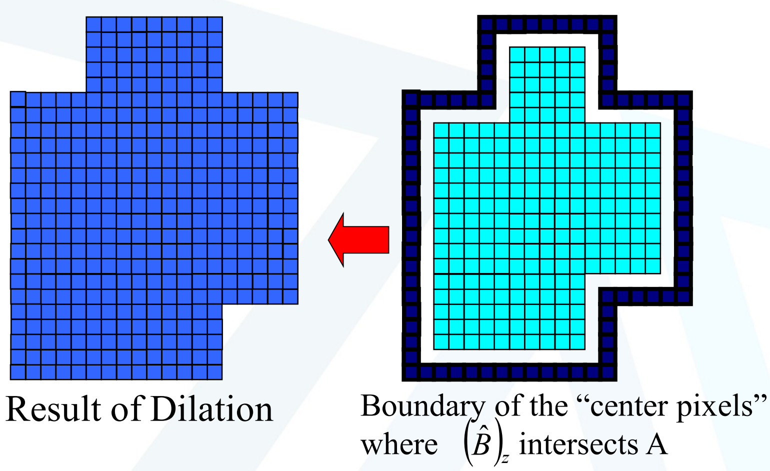
B = Structuring element

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

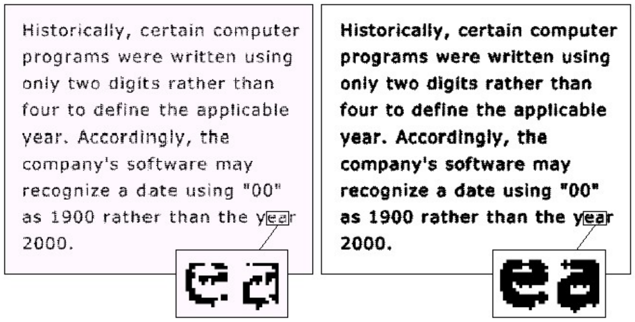
Dilation Operations (cont.)



Dilation Operations (cont.)



Example: Application of Dilation



0	1	0
1	1	1
0	1	0

a c
b

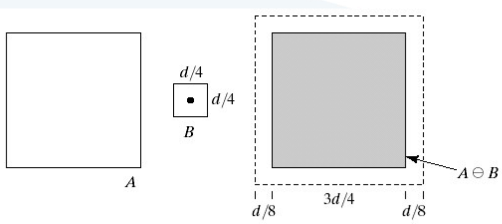
FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

"Repair" broken characters

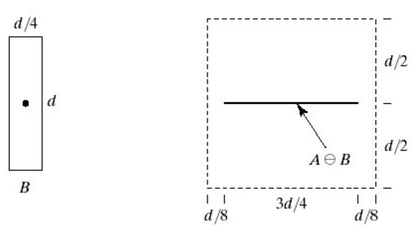
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Erosion Operation

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



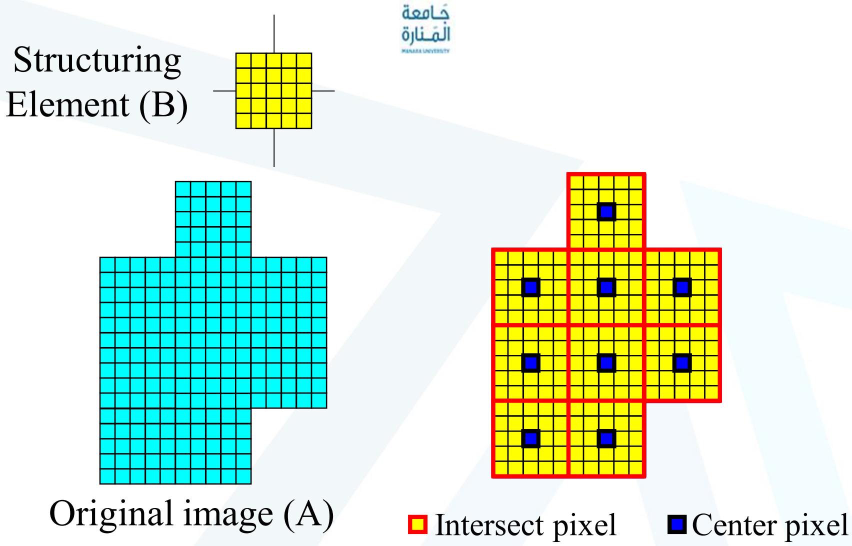
Erosion means "trim"



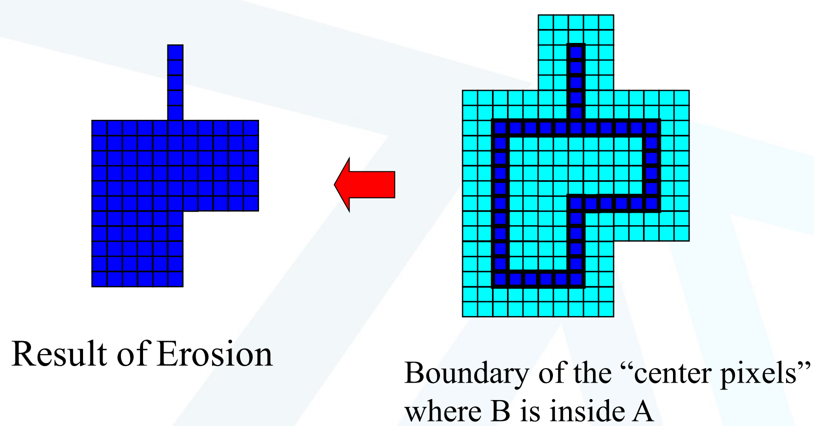
A = Object to be eroded
B = Structuring element

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Erosion Operations (cont.)



Erosion Operations (cont.)



Example: Application of Dilation and Erosion

a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Remove small objects such as noise

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Duality Between Dilation and Erosion

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

where c = complement

Proof:

$$\begin{aligned}(A \ominus B)^c &= \{z \mid (B)_z \subseteq A\}^c \\ &= \{z \mid (B)_z \cap A^c = \phi\}^c \\ &= \{z \mid (B)_z \cap A^c \neq \phi\} \\ &= A^c \oplus \hat{B}\end{aligned}$$

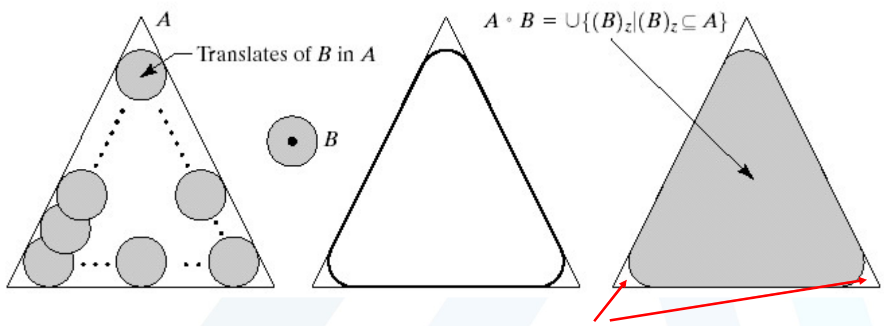
Opening Operation

$$A \circ B = (A \ominus B) \oplus B$$

or

$$A \circ B = \cup\{(B)_z | (B)_z \subseteq A\}$$

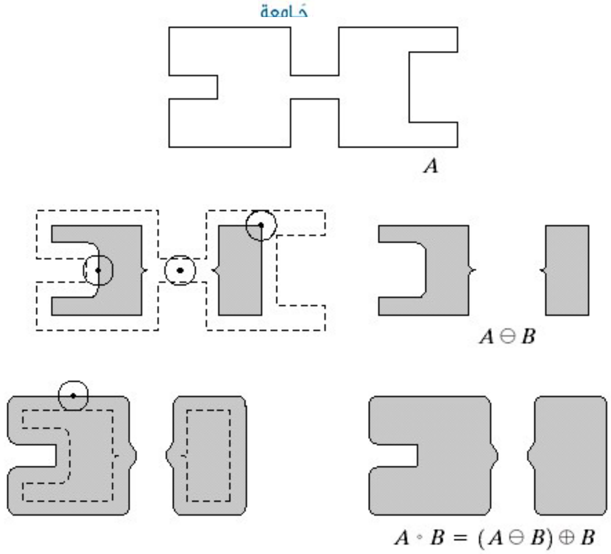
= Combination of all parts of A that can completely contain B



Opening eliminates narrow and small details and corners.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

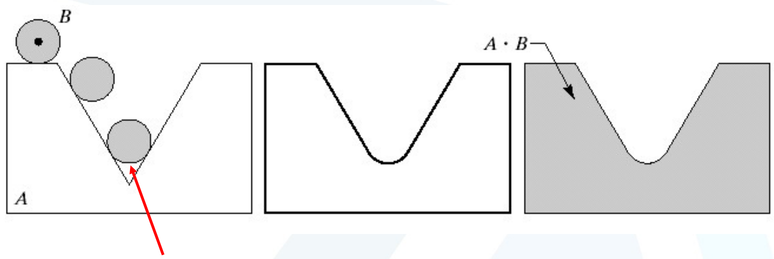
Example of Opening



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Closing Operation

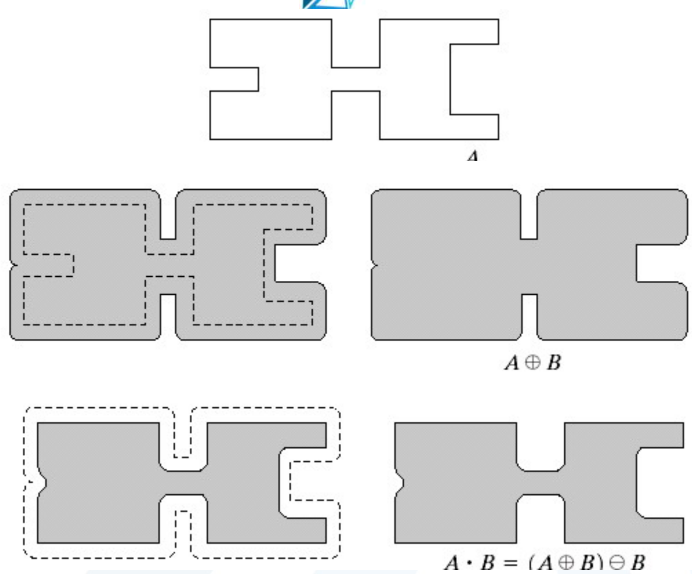
$$A \bullet B = (A \oplus B) \ominus B$$



Closing fills narrow gaps and notches

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example of Closing



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Duality Between Opening and Closing

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

Properties Opening

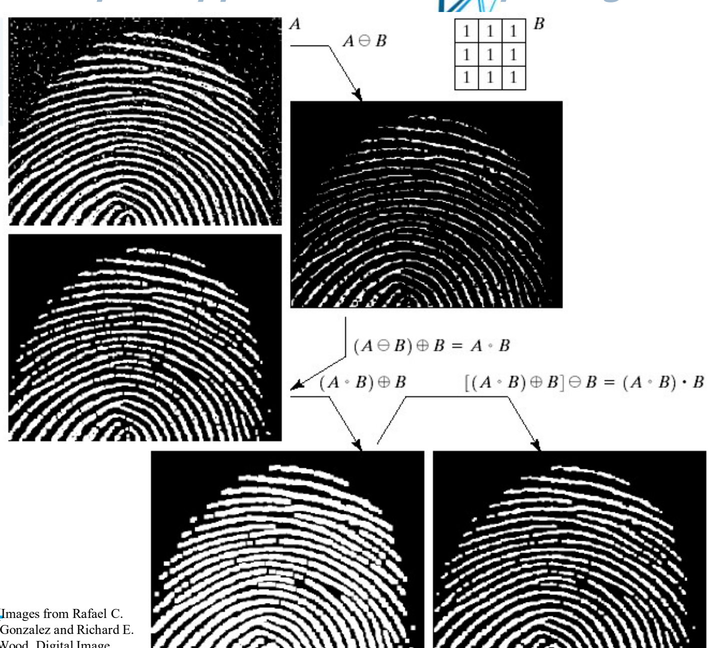
1. $A \circ B \subseteq A$
2. If $C \subset D$ then $C \circ B \subset D \circ B$
3. $(A \circ B) \circ B = A \circ B$

Properties Closing

1. $A \subseteq A \bullet B$
2. If $C \subset D$ then $C \bullet B \subset D \bullet B$
3. $(A \bullet B) \bullet B = A \bullet B$

Idem potent property: can't change any more

Example: Application of Morphological Operations



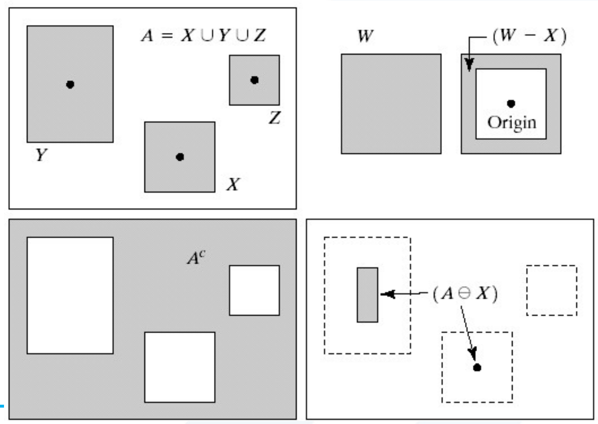
Finger print enhancement

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Hit-or-Miss Transformation

$$A \circledast X = (A \ominus X) \cap [A^c \ominus (W - X)]$$

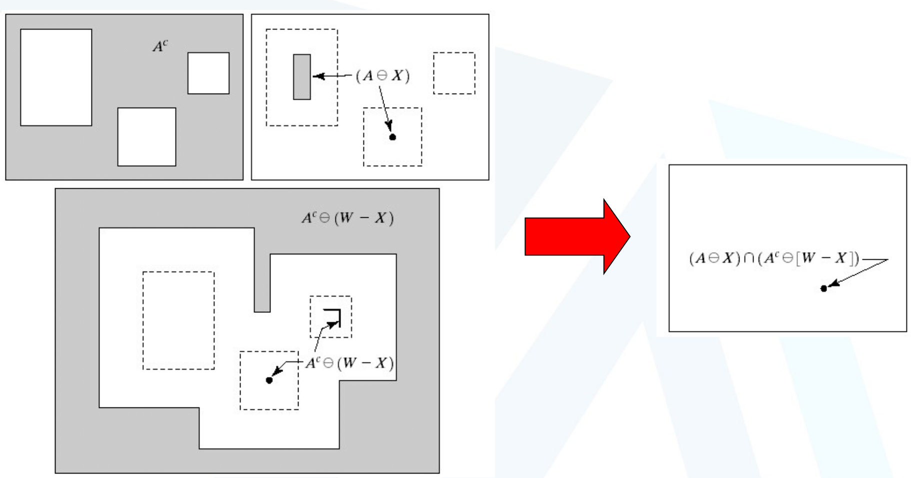
where X = shape to be detected
W = window that can contain X



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Hit-or-Miss Transformation (cont.)

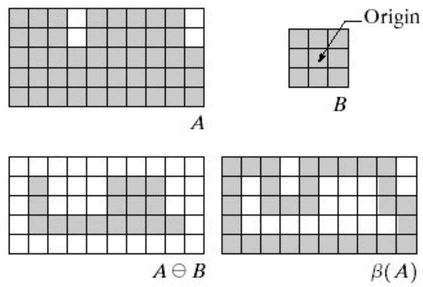
$$A \circledast B = (A \ominus B) \cap [A^c \ominus (W - X)]$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$



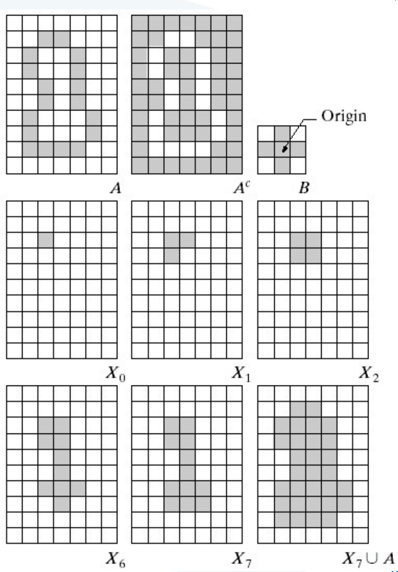
Original image

Boundary

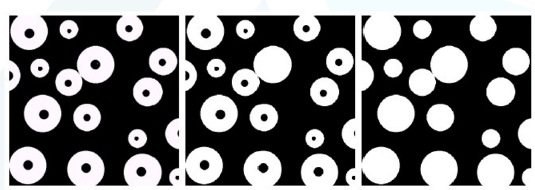
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c$$



where X_0 = seed pixel p



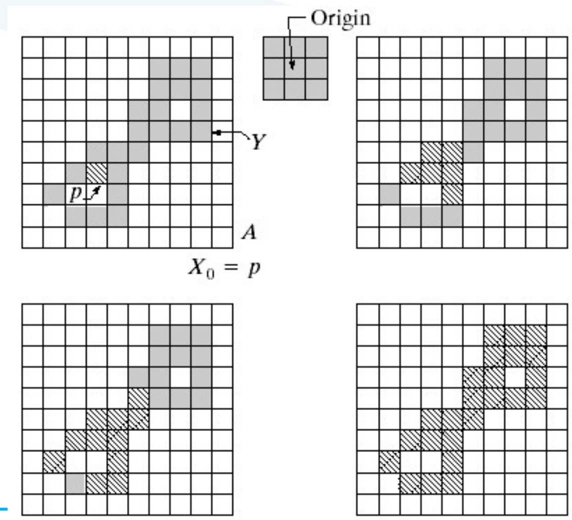
Original image

Results of region filling

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Extraction of Connected Components

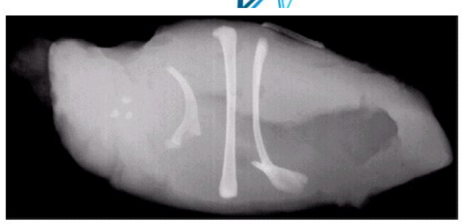
$$X_k = \left(X_{k-1} \oplus B\right) \cap A \qquad \text{where } X_0 = \text{seed pixel } p$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example: Extraction of Connected Components

X-ray image of bones



Thresholded image

Connected components

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

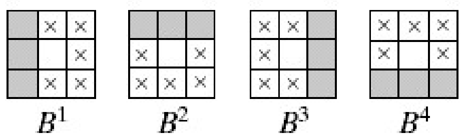
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Convex Hull

Convex hull has no concave part.

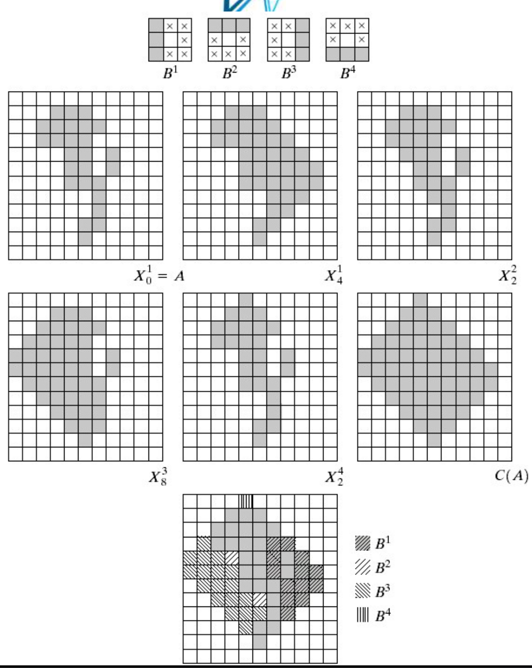
Algorithm: $C(A) = \bigcup_{i=1}^{4} D^i$ where $D^i = X^i_{conv}$

$$X^i_k = \left(X_{k-1} \circledast B^i\right) \cup A, \quad i = 1,2,3,4$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example: Convex Hull

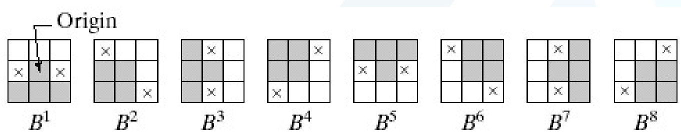


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Thinning

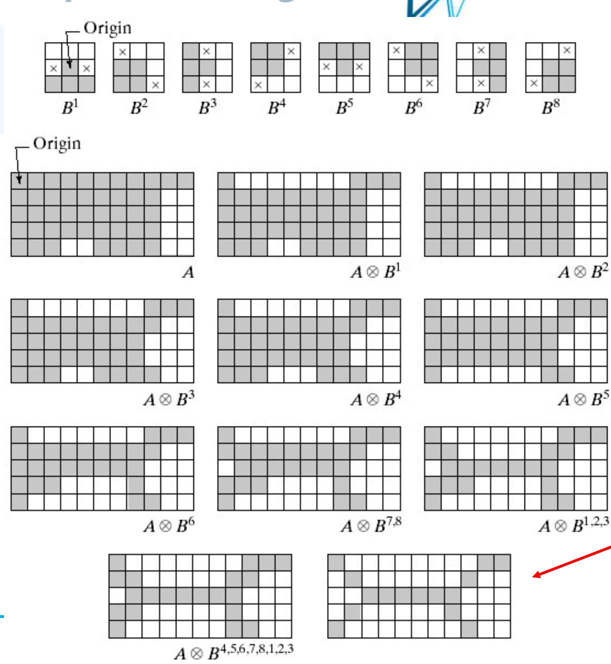
$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example: Thinning



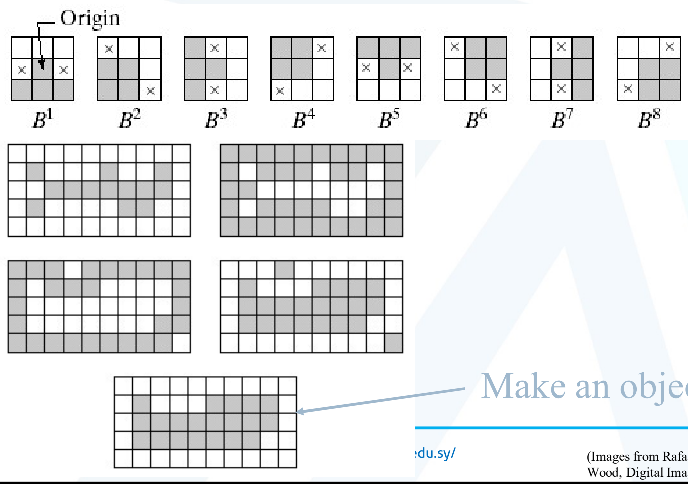
Make an object thinner.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Thickening

$$A \odot B = A \cup (A \circledast B)$$

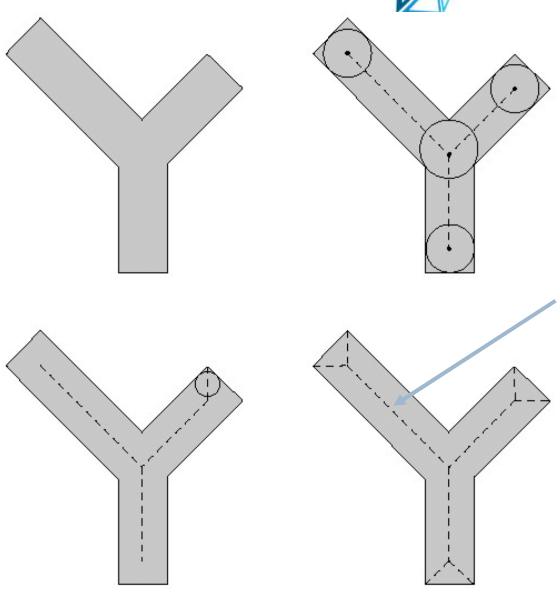
$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$



Make an object thicker

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Skeletons



Dot lines are skeletons of this structure

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Skeletons (cont.)

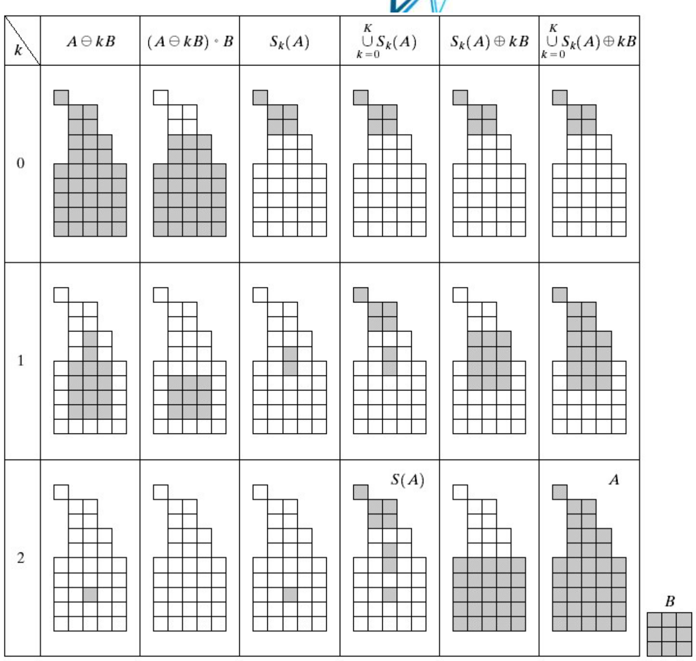
$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$

where $(A \ominus kB) = \underbrace{(\ldots(A \ominus B) \ominus B) \ominus \ldots) \ominus B}_{k \text{ times}}$

and $K = \max\{k \mid (A \ominus kB) \neq \phi\}$

Skeletons



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

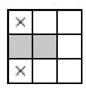
Pruning

$$X_1 = A \otimes \{B\}$$ thinning

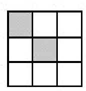
$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$ = finding end points

$$X_3 = (X_2 \oplus H) \cap A$$ = dilation at end points

$$X_4 = X_1 \cup X_3$$ = Pruned result



B^1, B^2, B^3, B^4 (rotated 90°)



B^5, B^6, B^7, B^8 (rotated 90°)

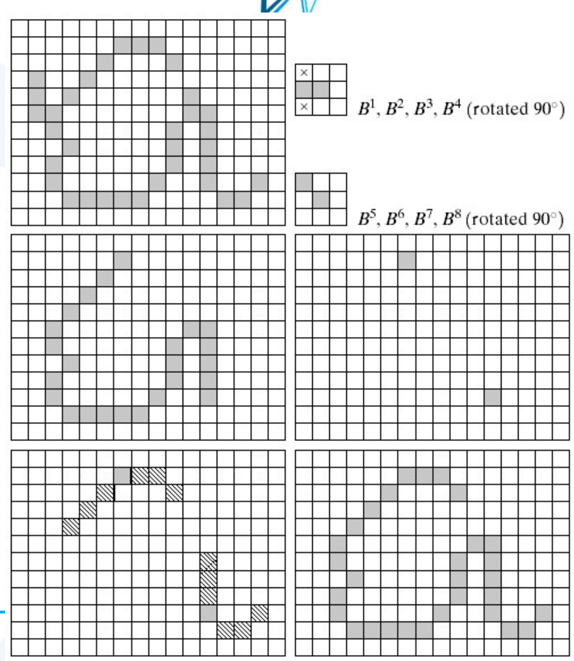
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.)

Example: Pruning

Original image

After Thinning 3 times

Dilation of end points



End points

Pruned result

Summary of Binary Morphological Operations

TABLE 9.2 Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Summary of Binary Morphological Operations (cont.)

Operation	Equation	Comments
Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Summary of Binary Morphological Operations (cont.)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} = ((\dots((A \otimes B^1) \otimes B^2)\dots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} = ((\dots(A \odot B^1) \odot B^2 \dots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

TABLE 9.2 Summary of morphological results and their properties. *(continued)*

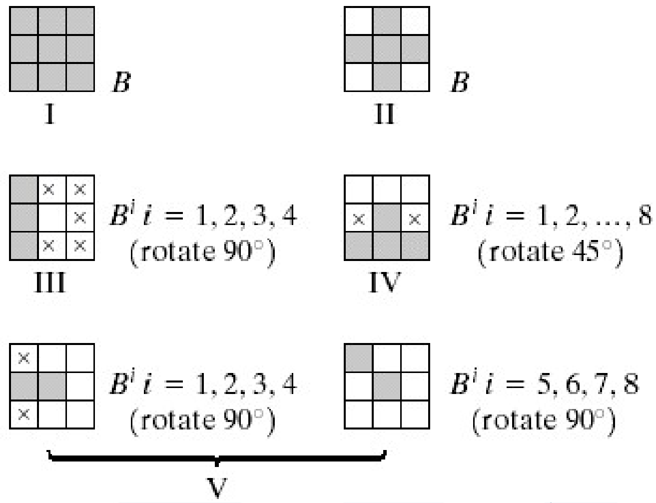
(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Summary of Binary Morphological Operations (cont.)

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Basic Types of Structuring Elements



x = don't care

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

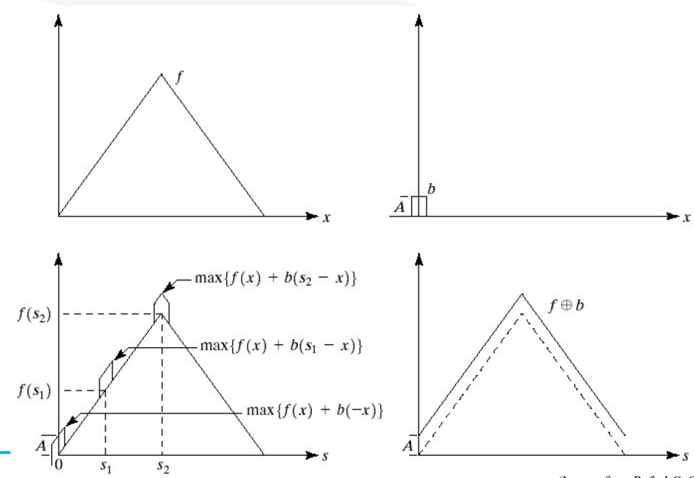
Gray-Scale Dilation

1-D Case

$$f \oplus b = \max\left\{ f(s-x) + b(x) \mid (s-x) \in D_f \text{ and } x \in D_b \right\}$$

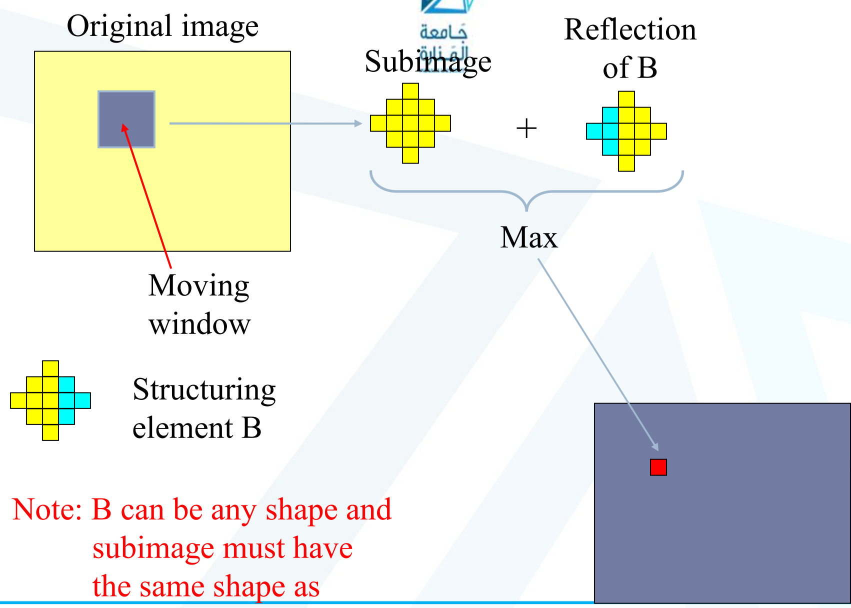
2-D Case

$$f \oplus b = \max\left\{ f(s-x, t-y) + b(x, y) \mid (s-x), (t-y) \in D_f ; (x, y) \in D_b \right\}$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gray-Scale Dilation (cont.)



Note: B can be any shape and subimage must have the same shape as reflection of B.

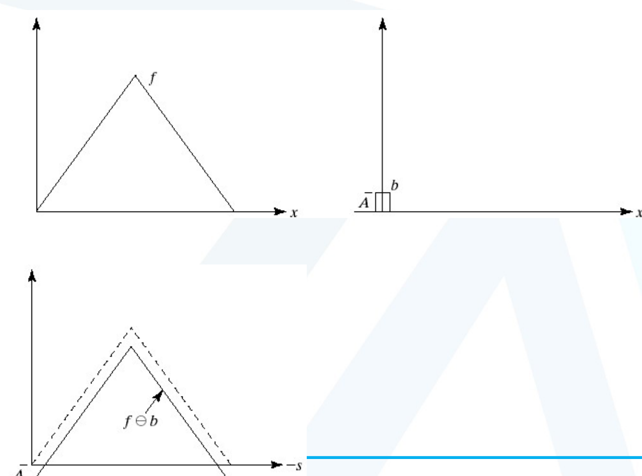
Gray-Scale Erosion

1-D Case

$$f \ominus b = \min\{f(s+x) - b(x) \mid (s+x) \in D_f \text{ and } x \in D_b\}$$

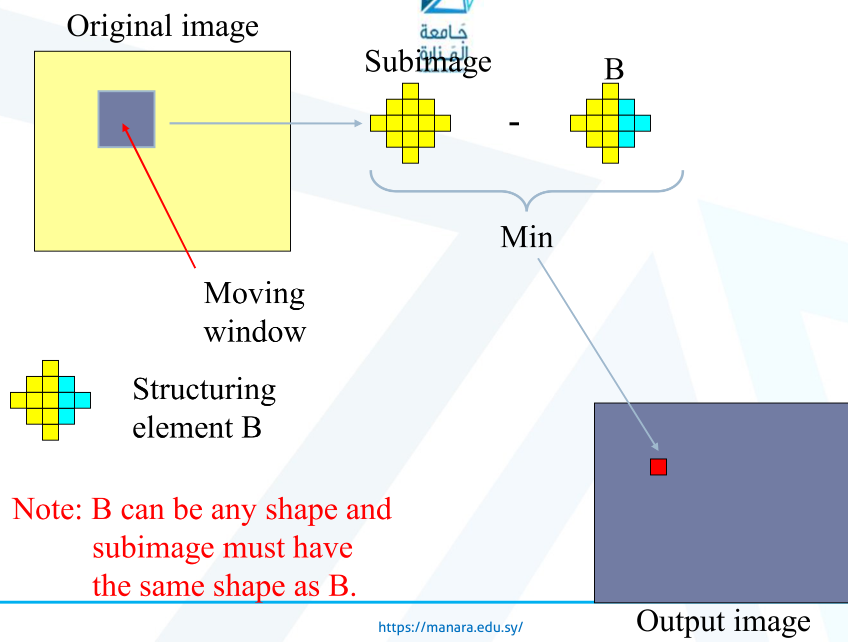
2-D Case

$$f \ominus b = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$



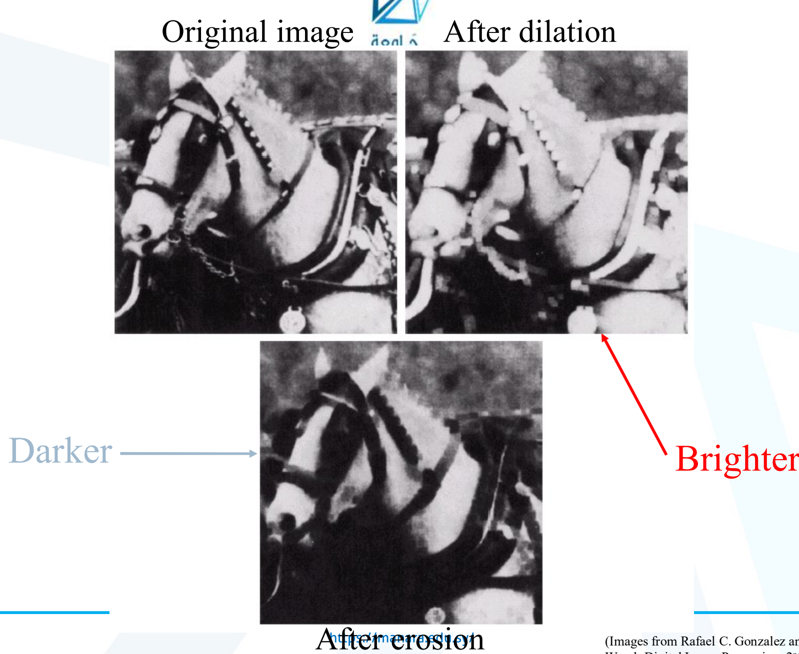
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gray-Scale Erosion (cont.)



Note: B can be any shape and subimage must have the same shape as B.

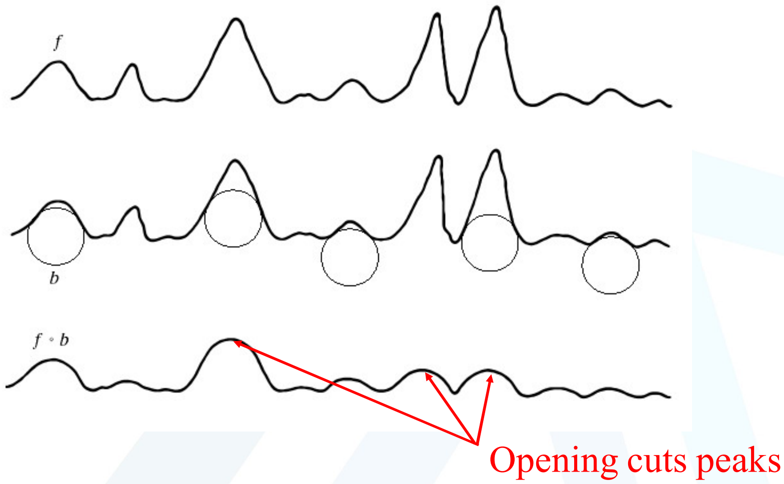
Example: Gray-Scale Dilation and Erosion



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gray-Scale Opening

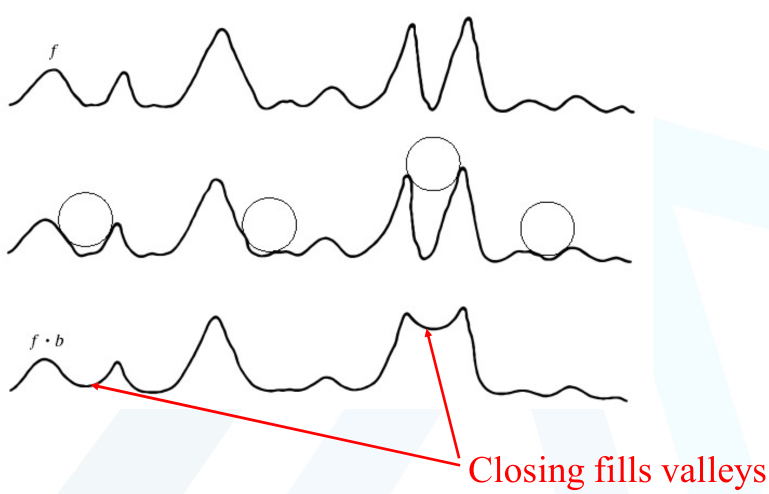
$$f \circ b = (f \ominus b) \oplus b$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

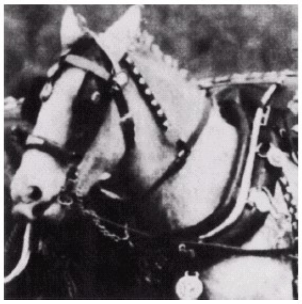
Gray-Scale Closing

$$f \bullet b = (f \oplus b) \ominus b$$

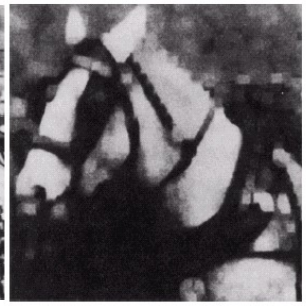


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

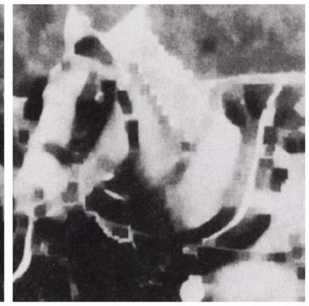
Example: Gray-Scale Opening and Closing



Original image



After opening



After closing

Reduce white objects

Reduce dark objects

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gray-scale Morphological Smoothing

Smoothing: Perform opening followed by closing

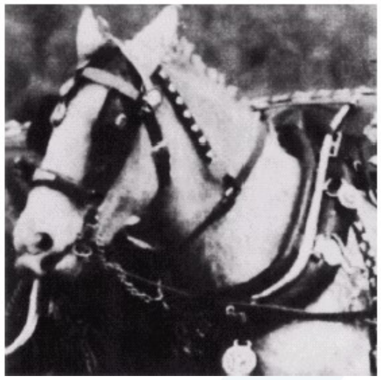
Original image

After smoothing

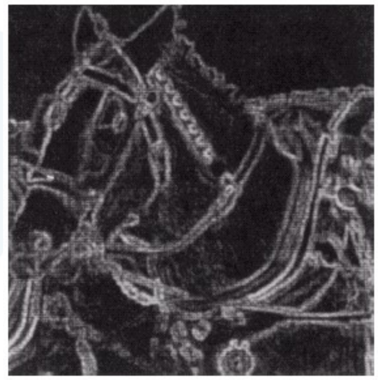
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Morphological Gradient

$$g = (f \oplus b) - (f \ominus b)$$



Original image



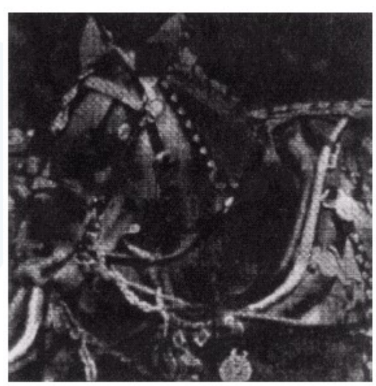
Morphological Gradient

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Top-Hat Transformation

$$h = f - (f \circ b)$$

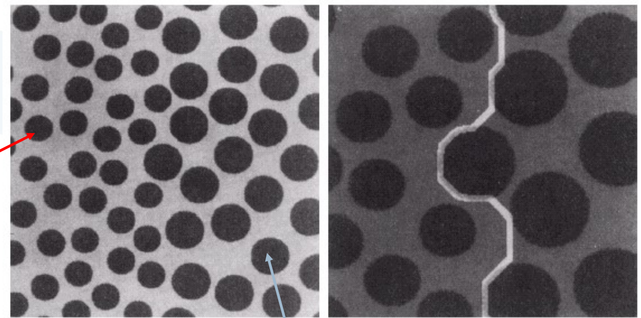
Original image



Results of top-hat transform

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example: Texture Segmentation Application



Small blob

Original image | Segmented result

Large blob

Algorithm:

1. Perform closing on the image by using successively larger structuring elements until small blobs are vanished.
2. Perform opening to join large blobs together
3. Perform intensity thresholding

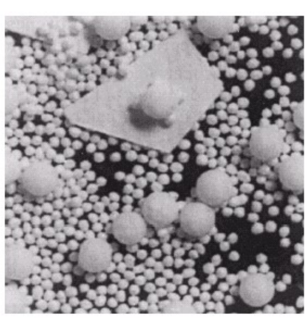
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example: Granulometry

Objective: to count the number of particles of each size

Method:

1. Perform opening using structuring elements of increasing size
2. Compute the difference between the original image and the result after each opening operation
3. The differenced image obtained in Step 2 are normalized and used to construct the size-distribution graph.



Original image

Size distribution graph

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

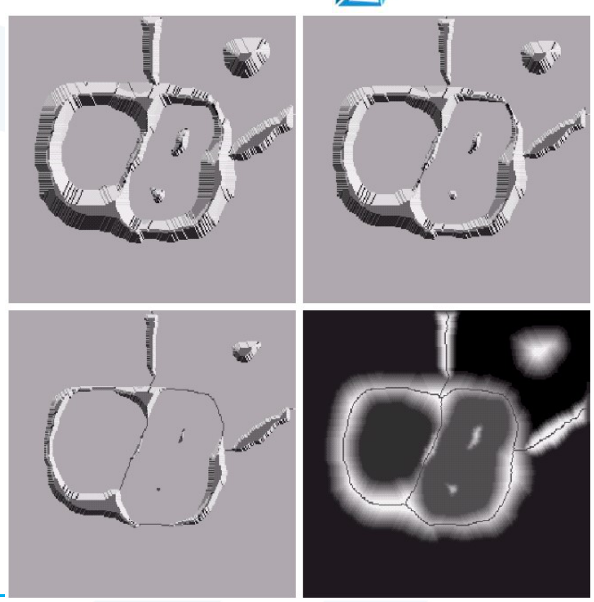
Morphological Watershads

a b
c d

FIGURE 10.44
(a) Original image.
(b) Topographic view. (c)–(d) Two stages of flooding.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Morphological Watershads

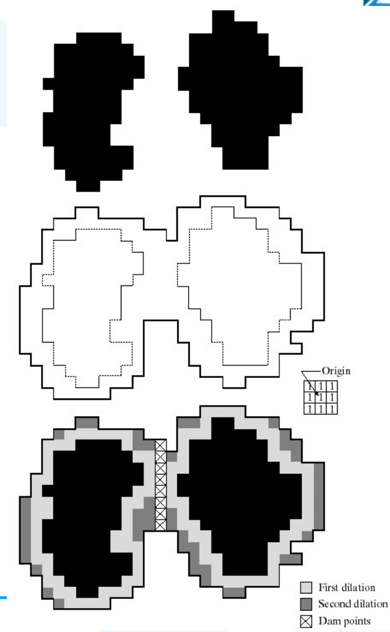


e f
g h

FIGURE 10.44
(Continued)
(e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Morphological Watersheds

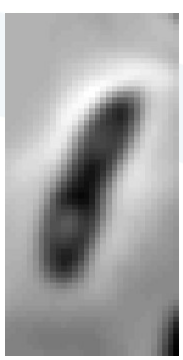


a
b c
d

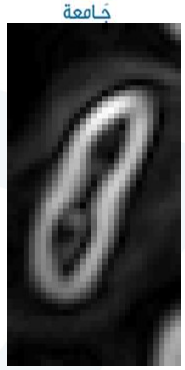
FIGURE 10.45 (a) Two partially flooded catchment basins at stage $n - 1$ of flooding. (b) Flooding at stage n, showing that water has spilled between basins (for clarity, water is shown in white rather than black). (c) Structuring element used for dilation. (d) Result of dilation and dam construction.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

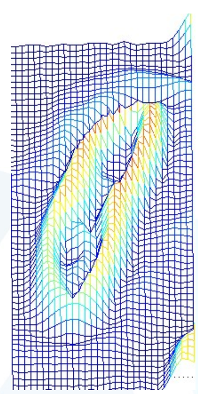
Gradient Image



Original image



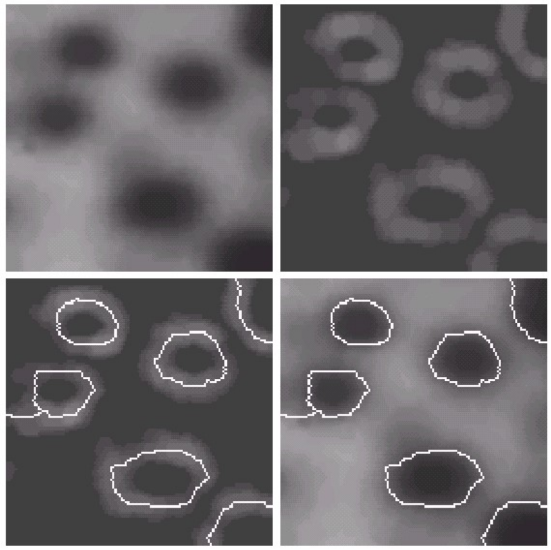
$|\nabla P|$



Surface of $|\nabla P|$

$|\nabla P|$ at edges look like mountain ridges.

Morphological Watershads

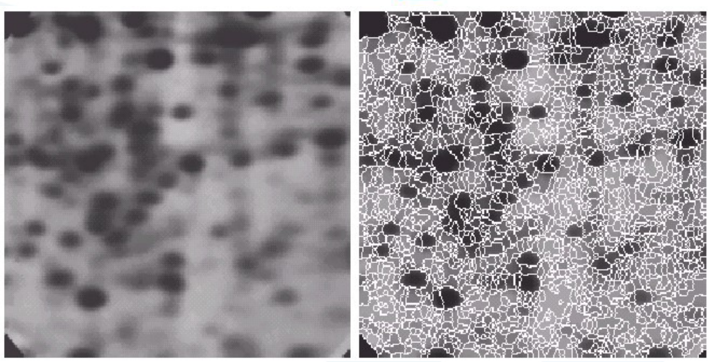


a b
c d

FIGURE 10.46 (a) Image of blobs. (b) Image gradient. (c) Watershed lines. (d) Watershed lines superimposed on original image. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Morphological Watershads

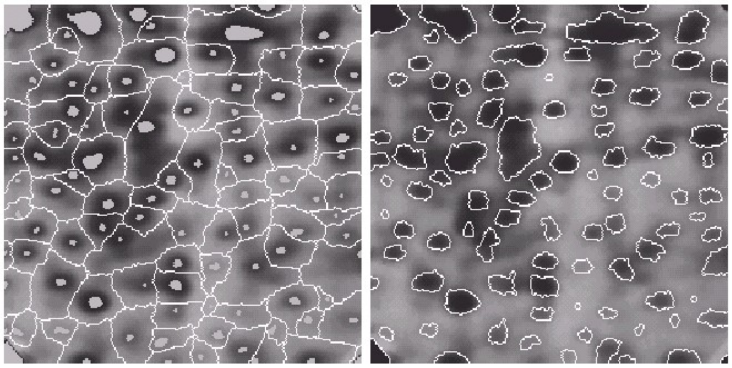


a b

FIGURE 10.47 (a) Electrophoresis image. (b) Result of applying the watershed segmentation algorithm to the gradient image. Oversegmentation is evident. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Morphological Watershads



a b

FIGURE 10.48 (a) Image showing internal markers (light gray regions) and external markers (watershed lines). (b) Result of segmentation. Note the improvement over Fig. 10.47(b). (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Convex Hull

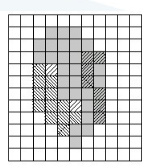


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.