Structural Mechanics (1)

Week No-05

Part-01

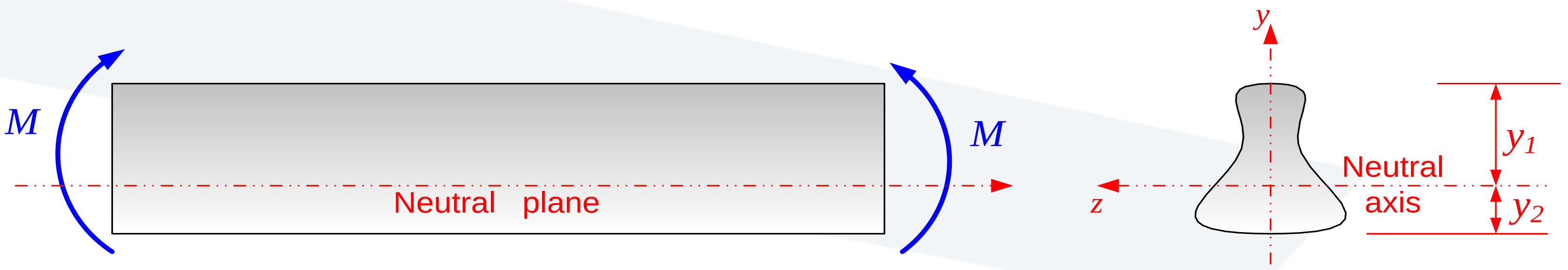
Deflection in Determinate Structures

Deflections of Trusses, Beams, & Frames: Work-Energy Methods

- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

Strain energy in a beam element

Bending Strain energy in a beam element



$$U=\iiint_V \frac{1}{2}\sigma_x\varepsilon_x dV=\iiint_V \frac{\sigma_x^2}{2E}dV=\iiint_V \frac{1}{2E}\left(\frac{M^2}{I^2}y^2\right)dxdA=\int_0^L \frac{1}{2E}dx\left(\frac{M^2}{I^2}\right)\iint_A y^2dA=\int_0^L \frac{M^2}{2EI}dx$$

Shear Strain energy in a beam element,

$$U=\iiint_V \frac{1}{2}\tau\gamma dV=\iiint_V \frac{\tau^2}{2G}dV=k\int_0^L \frac{S^2}{2GA}dx$$

$$k=\begin{cases}1.2 & \text{forarectangle}\\ 1.1 & \text{foracircle}\\ 1.2 & \text{forathin circular}\end{cases}$$

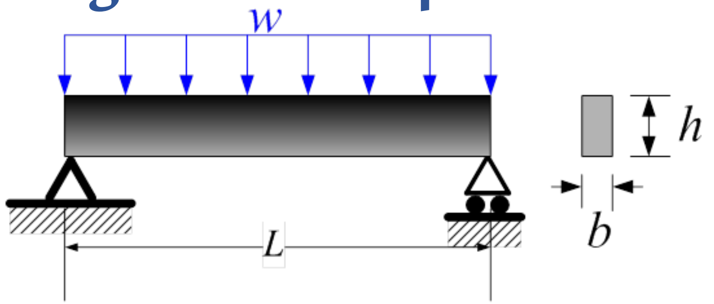
Comparison of bending and shear strain energies in a simple beam

$$U_b = \int_0^L \frac{M^2}{2EI} dx$$

$$M(x) = -\tfrac{1}{2}wx^2 + \tfrac{1}{2}wLx$$

$$U_S = 1.2\int_0^L \frac{S^2}{2GA} dx$$

$$S(x) = -wx + \tfrac{1}{2}wL$$



$$I = bh^3/12 \quad \& \quad A = bh$$

$$U_b = \int_0^L \frac{M^2}{2EI} dx = \frac{6}{Ebh^3}\int_0^L (-\tfrac{1}{2}wx^2 + \tfrac{1}{2}wLx)^2 dx$$

$$= \frac{6w^2}{Ebh^3}\int_0^L (\tfrac{1}{4}x^4 - \tfrac{1}{2}Lx^3 + \tfrac{1}{4}L^2x^2)dx$$

$$= \frac{6w^2}{Ebh^3}\left[\tfrac{1}{20}x^5 - \tfrac{1}{8}Lx^4 + \tfrac{1}{12}L^2x^3\right]_0^L = \frac{0.05w^2L^5}{Ebh^3}$$

$$U_S = 1.2\int_0^L \frac{S^2}{2GA} dx = \frac{0.6}{Gbh}\int_0^L (wx - \tfrac{1}{2}wL)^2 dx$$

$$= \frac{0.6w^2}{Gbh}\int_0^L (x^2 - Lx + \tfrac{1}{4}L^2)dx$$

$$= \frac{0.6w^2}{Gbh}\left[\tfrac{1}{3}x^3 - \tfrac{1}{2}Lx^2 + \tfrac{1}{4}L^2x\right]_0^L = \frac{0.05w^2L^3}{Gbh}$$

$$U_S / U_b = \left(\frac{0.05w^2L^3}{Gbh}\right) / \left(\frac{0.05w^2L^5}{Ebh^3}\right)$$

$$= (E/G)\left(h^2/L^2\right) \approx 2h^2/L^2 \ll 1.$$

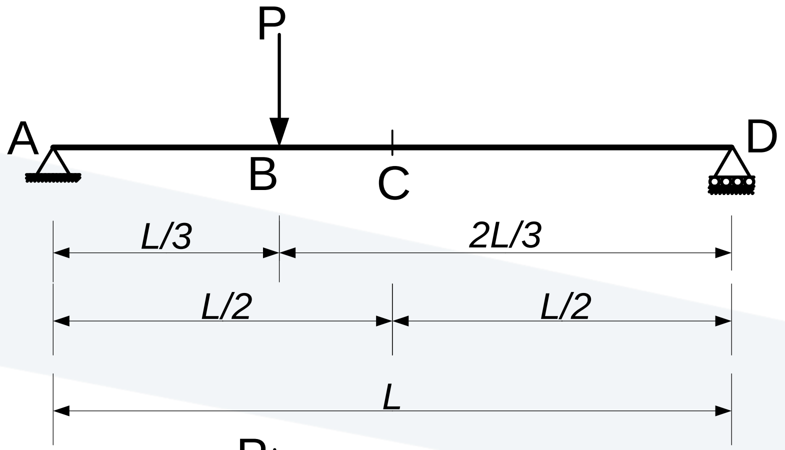
Shear and Axial Strain energies are negligible in comparison with the bending moment energy

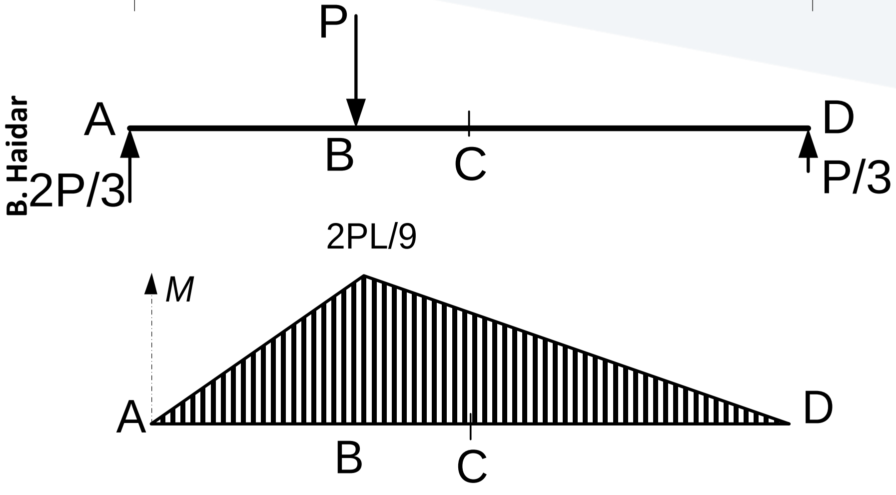
DEFLECTIONS OF BEAMS BY THE V. W. M.

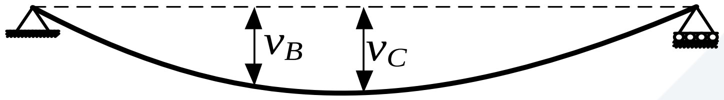
01/12/2024

B. Haidar

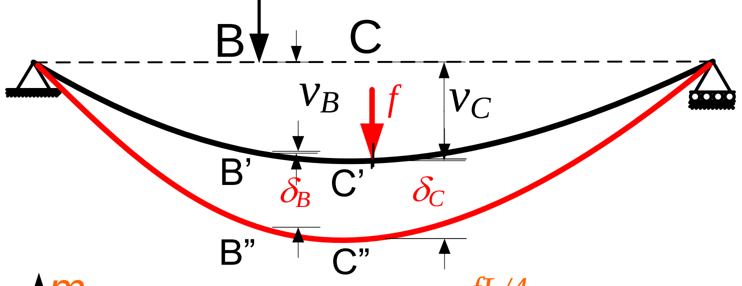
Structural Mechanics (1)

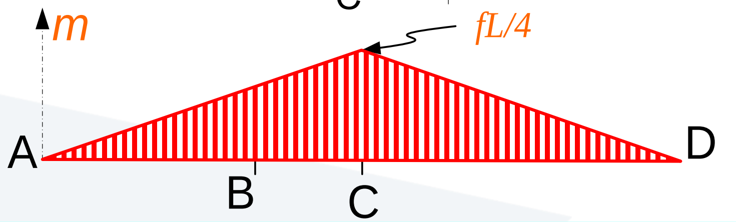




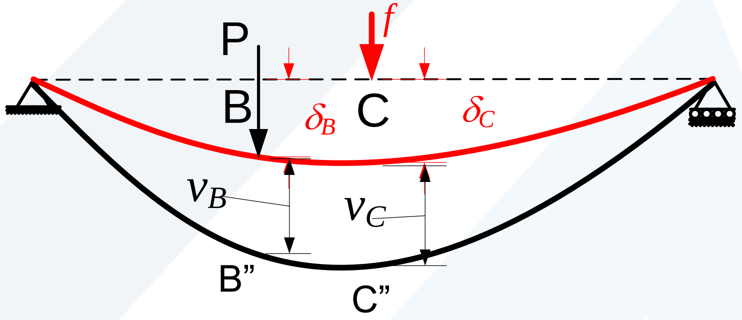


$$\frac{1}{2}Pv_B = \int_0^L \frac{M^2}{2EI}dx$$





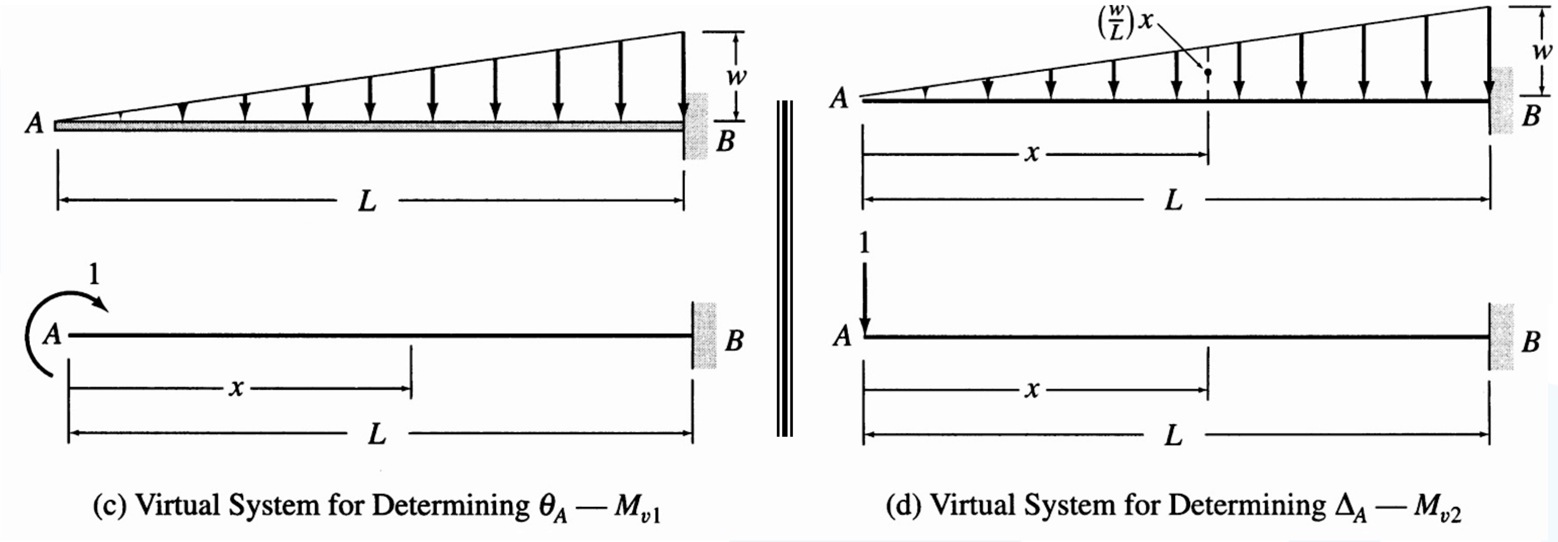
$$\frac{1}{2}Pv_B + P\delta_B + \frac{1}{2}f\delta_C = \int_0^L \frac{(M+m)^2}{2EI}dx$$



$$\frac{1}{2}f\delta_C + f v_C + \frac{1}{2}Pv_B = \int_0^L \frac{(m+M)^2}{2EI}dx$$

$$P\delta_B = f v_C = \int_0^L \frac{mM}{EI}dx$$

Example-01. Determine the slope and deflection at point A of the beam shown in the figure, by the virtual work method



Solution

Real System: The real B. M. is: $\text{for}, 0 < x < L,\ M(x) = -\frac{1}{2}(x)(\frac{wx}{L})(\frac{x}{3}) = -\frac{wx^3}{6L}$

Virtual System 1: The V. B. M. is: $\text{for}, 0 < x < L,\ M_{v1}(x) = 1$

Virtual System 2: The V. B. M. is: $\text{for}, 0 < x < L,\ M_{v2}(x) = -1(x) = -x$

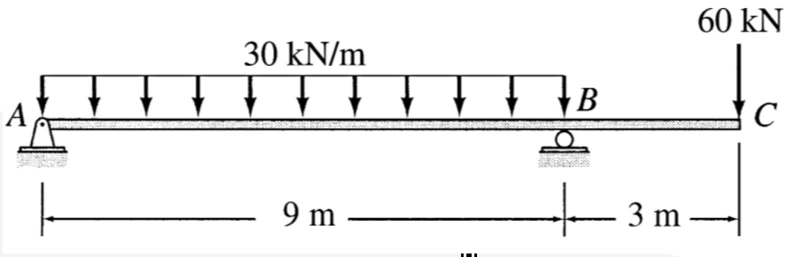
$$\theta_A = \int_0^L \frac{M_{v1}M}{EI}dx = \int_0^L 1(-\frac{wx^3}{6LEI})dx = -\frac{wL^3}{24EI}$$

$$\theta_A = \frac{wL^3}{24EI}\ (\downarrow\uparrow)$$

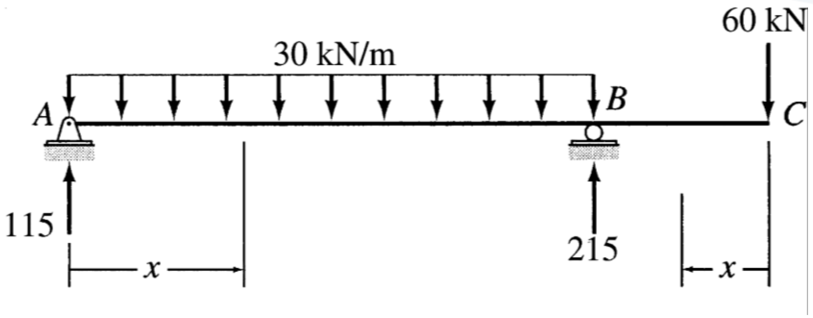
$$\Delta_A = \int_0^L \frac{M_{v2}M}{EI}dx = \int_0^L (-x)(-\frac{wx^3}{6LEI})dx = \frac{wL^4}{30EI}$$

$$\Delta_A = \frac{wL^4}{30EI}(\downarrow)$$

Example-02. Determine the deflection at point C of the beam shown in the figure, by the virtual work method. EI=const. E=200 GPa, I=800(10^6) mm^4



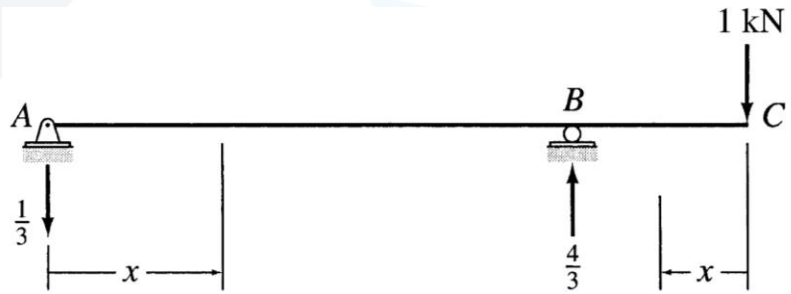
Real System



$$Segment\ AB:\ 0 < x < 9, M(x) = 115x - 15x^2$$

$$Segment\ BC:\ 0 < x < 3, M(x) = -60x$$

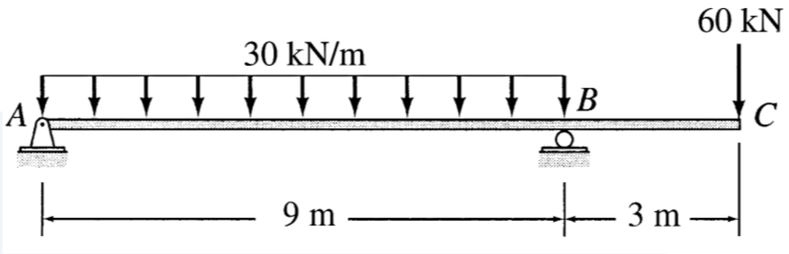
Virtual System



$$Segment\ AB:\ 0 < x < 9, m(x) = -\frac{x}{3}$$

$$Segment\ BC:\ 0 < x < 3, m(x) = -x$$

Example-02. Determine the deflection at point C of the beam shown in the figure, by the virtual work method. EI=const. E=200 GPa, I=800(10^6) mm^4



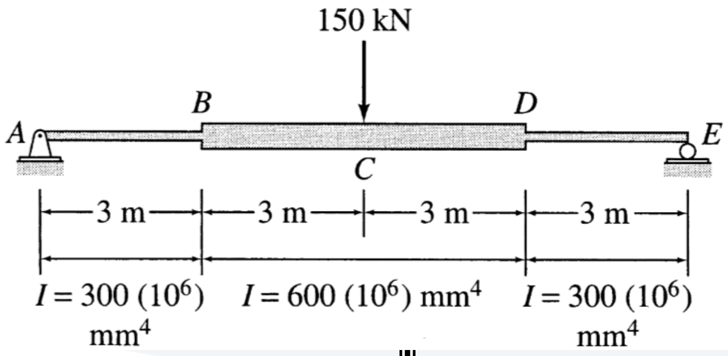
Segment	x Coordinate: Origin	x Coordinate: Limits (m)	M (kN-m)	M_v (kN-m)
AB	A	0–9	$(115x - 15x^2)$	$-\frac{x}{3}$
CB	C	0–3	$-60x$	$-x$

$$\Delta_C = \int_0^L \frac{M(x).m(x)}{EI}dx = \frac{1}{EI}\int_0^9 (115x - 15x^2)\left(-\frac{x}{3}\right)dx + \frac{1}{EI}\int_0^3 (-60x)(-x)dx = -\frac{933.75}{EI}$$

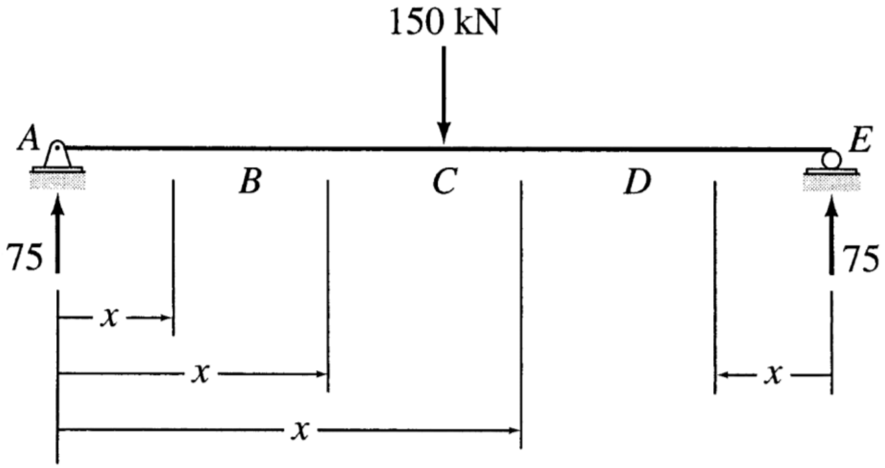
$$\Delta_C = -\frac{933.75}{EI} = -\frac{933.75}{200\,(10^6)800\,(10^{-6})} = -0.005836\,m$$

$$\boldsymbol{\Delta_C = 5.836\,mm} \uparrow$$

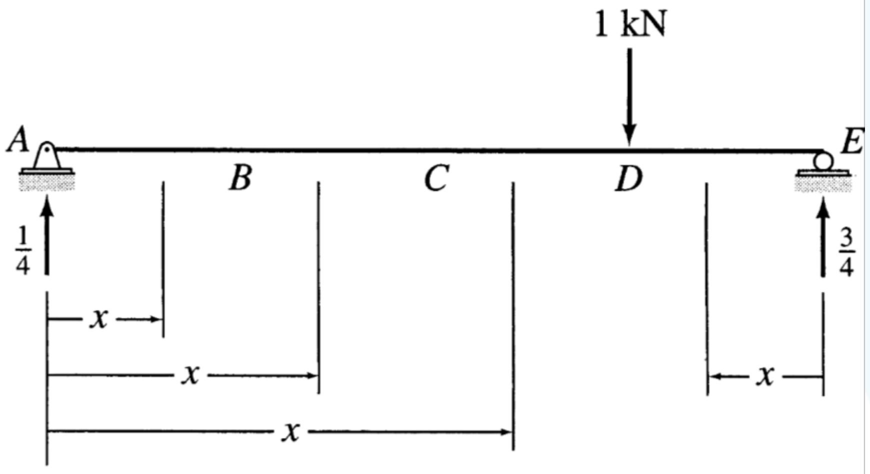
Example-03. Determine the deflection at point D of the beam shown in the figure, by the virtual work method. E=200 GPa.



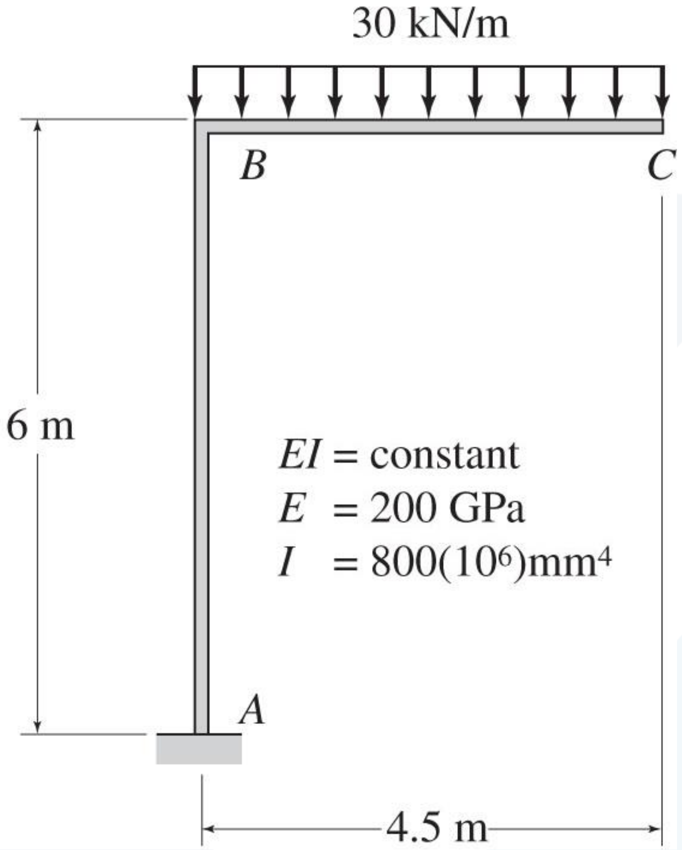
Real System



Virtual System

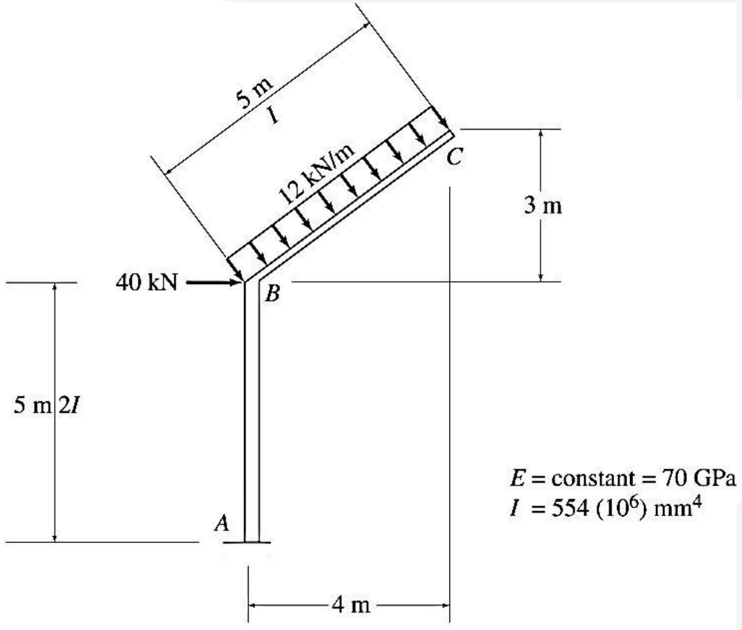


Example-04. Use the virtual work method to determine the deflection at joint C of the following frame.

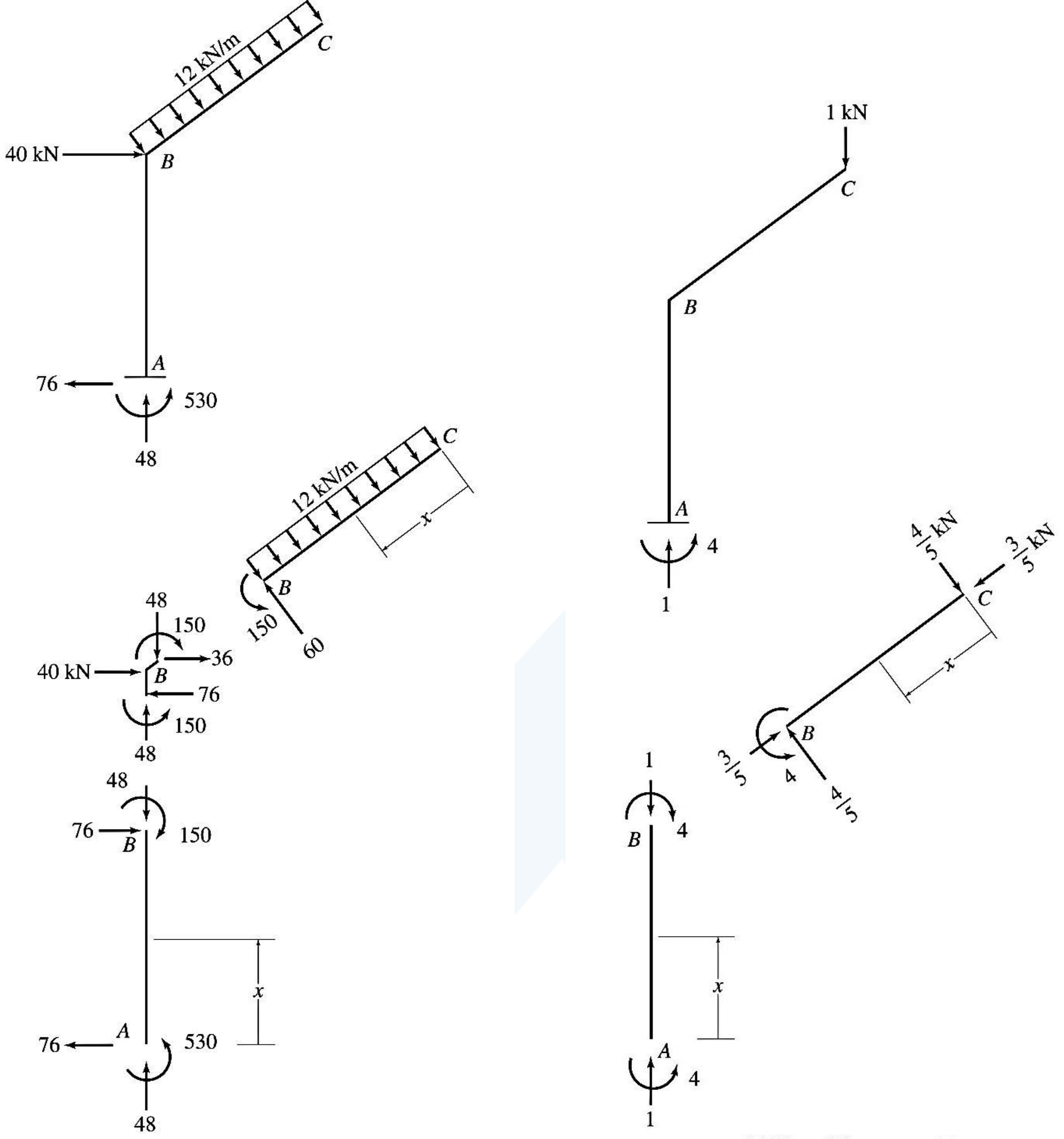


$$\Delta_C = 60.9\ mm \downarrow$$

Ex.6. Compute the vertical deflection at joint C of the shown frame



(a)



(b) Real System — M

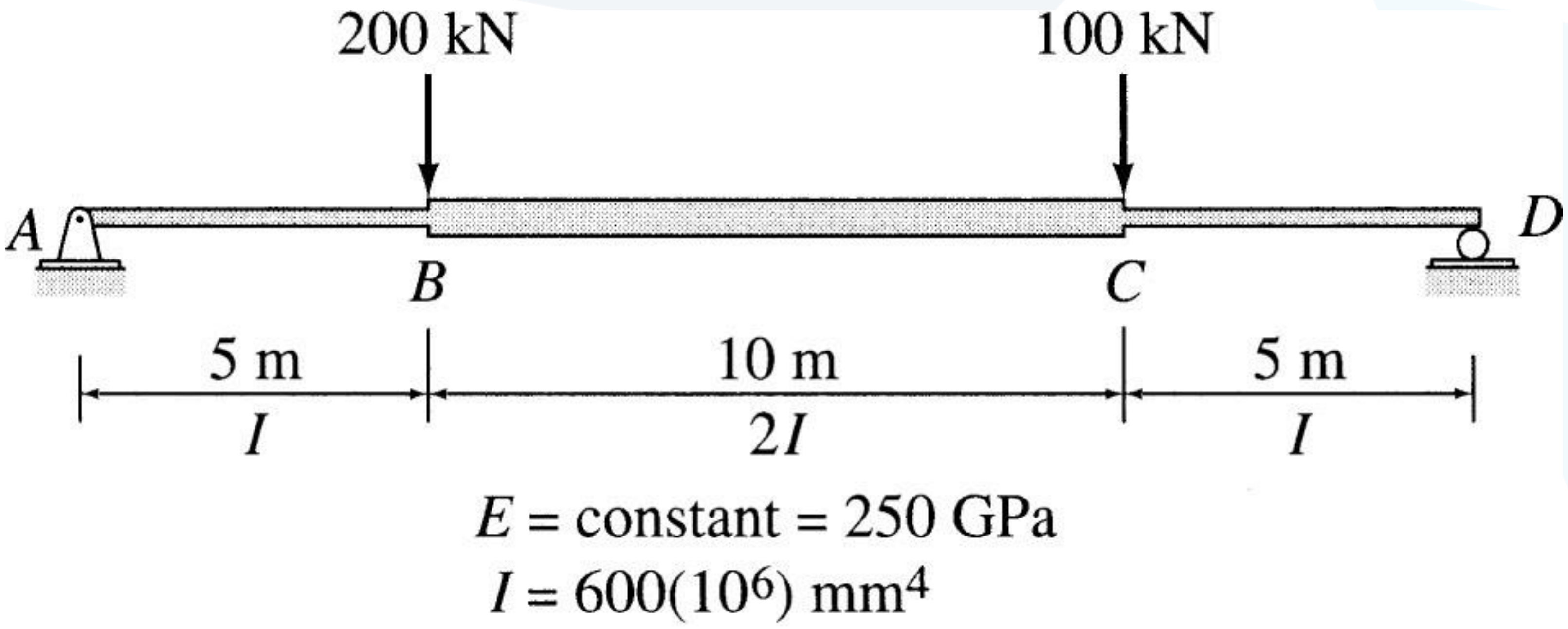
(c) Virtual System — M_v

Segment	x Coordinate		EI ($I = 554 \times 10^6$ mm^4)	M (kN–m)	M_v (kN–m)
	Origin	Limits (m)			
AB	A	0–5	$2EI$	$76x - 530$	-4
CB	C	0–5	EI	$-12\frac{x^2}{2}$	$-\frac{4}{5}x$

$$\Delta_C = 107 \text{ mm} \downarrow$$

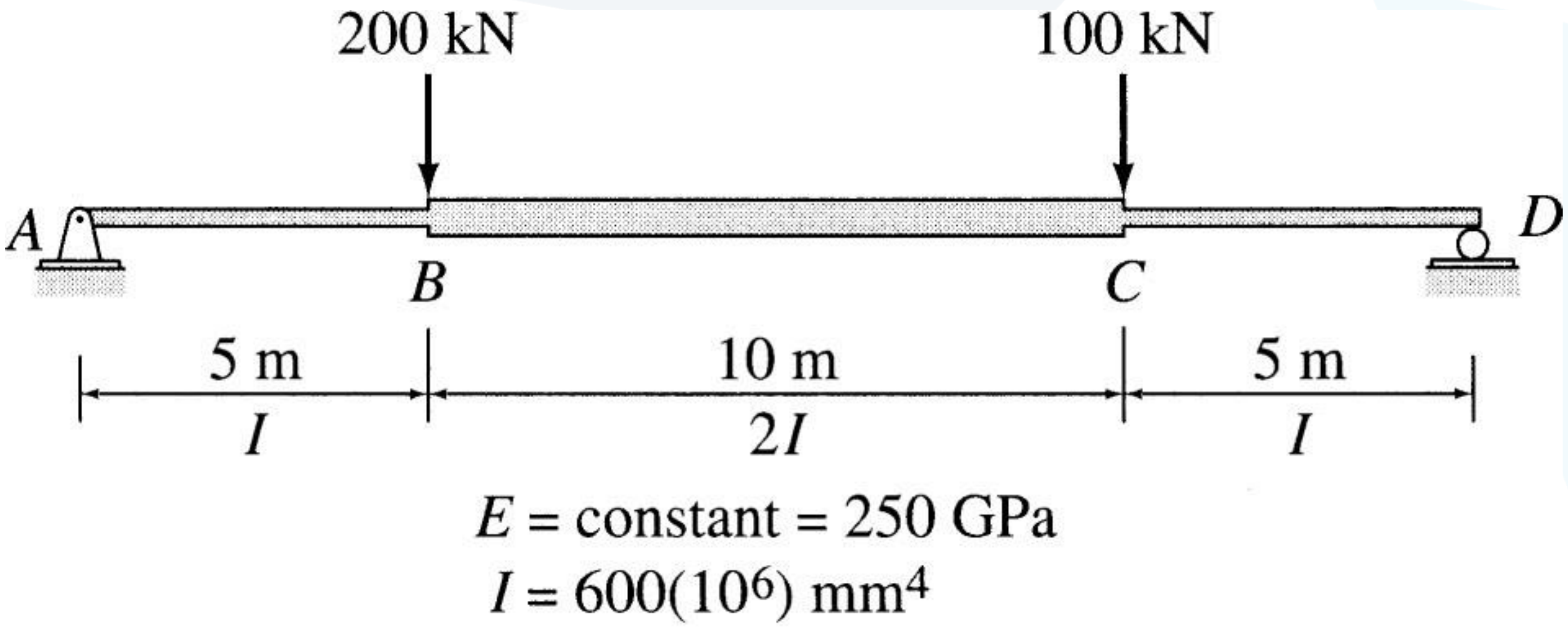
Homework

Problem.01: Use the virtual work method to determine the deflection at joint C of the following beam.



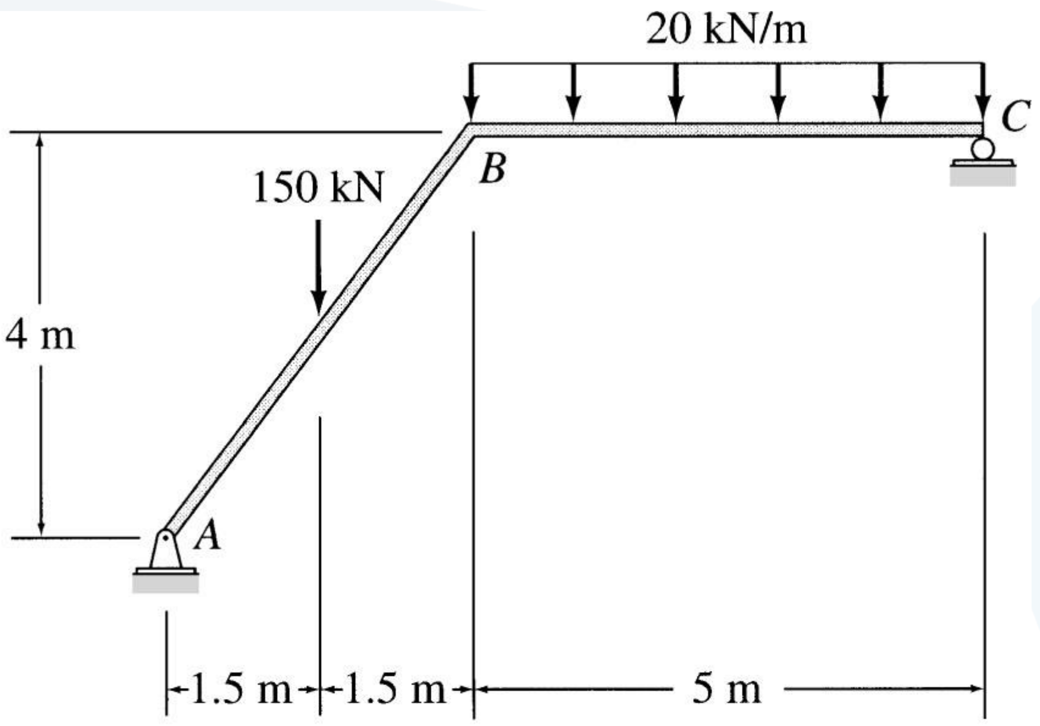
Homework

Problem.01: Use the virtual work method to determine the deflection at joint C of the following beam.



Homework

Problem.02: Use the virtual work method to determine the deflection at joint B of the following frame.



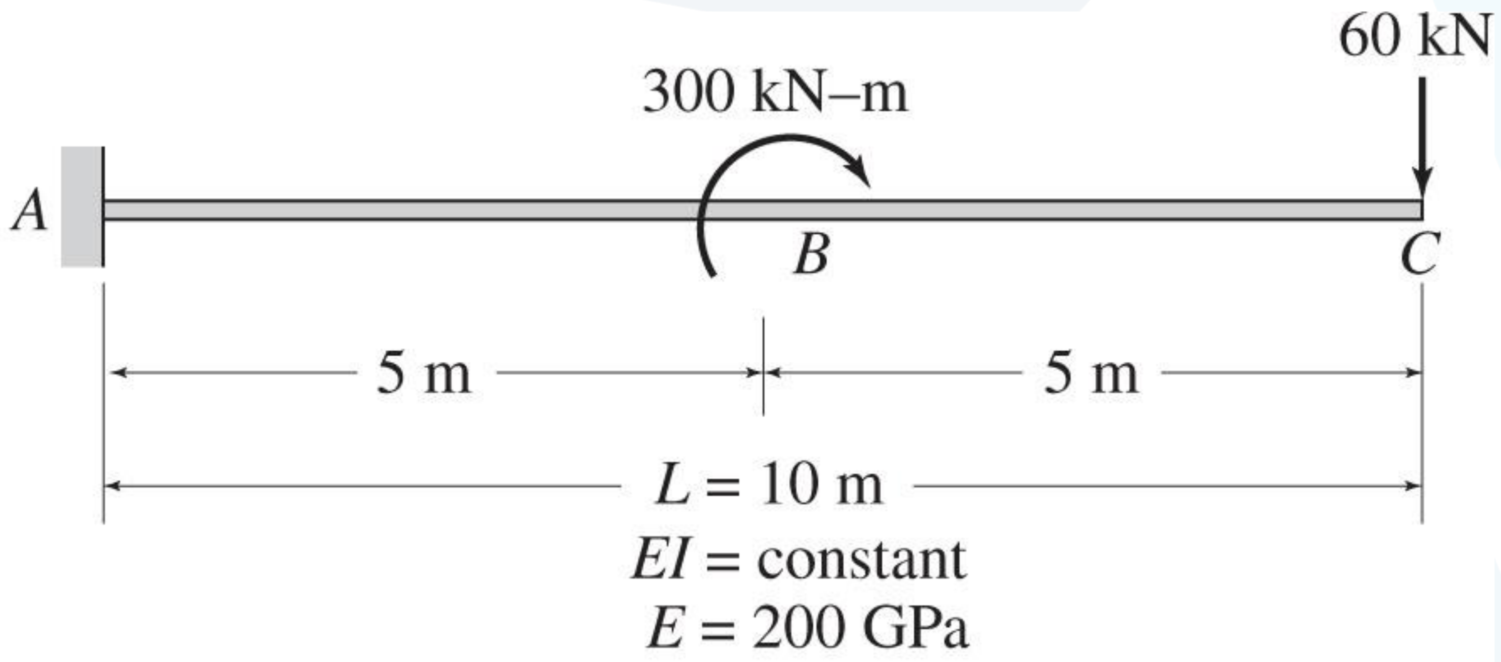
EI = constant

$E = 200$ GPa

$I = 500(10^6)$ mm^4

Homework

Problem.03: Determine the smallest moment of inertia (I) required for the beam shown, so that its maximum deflection does not exceed the limit of 1/360 of the span length (i.e., $\Delta_{max} \leq L/360$). Use the method of virtual work.



Homework

Problem.04: Determine the smallest moment of inertia (I) required for the members of the frame shown, so that the horizontal deflection at joint C does not exceed 20 mm. Use the method of virtual work.