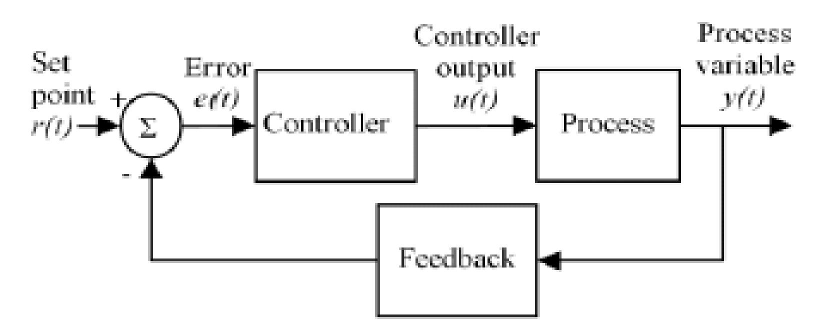


Control Systems

Lecture (2)

An Introduction to

Mathematical Modeling of Mechanical and Electrical Systems

Mechatronics Engineering Department
Assistant Professor Isam Asaad

References

- *Modern Control Systems, Richard C. Dorf and Robert H. Bishop*, Prentice Hall, 12th edition, 2010, ISBN-10: 0-13-602458-0
- Modelling, Dynamics and Control, University of Sheffield, John Anthony Rossiter.
- Control Systems Course, professor Aniket Khandekar, Zeal college of engineering and Research, Pune.

Goals and Objectives:

- The lecture focuses on mathematical models of physical systems.
- After completing this chapter, you should be able to:
  - Describe a physical system using differential equations.
  - Understand how these equations are derived based on physical laws governing the system.
  - Recognize the significance of deriving mathematical models in control systems analysis.

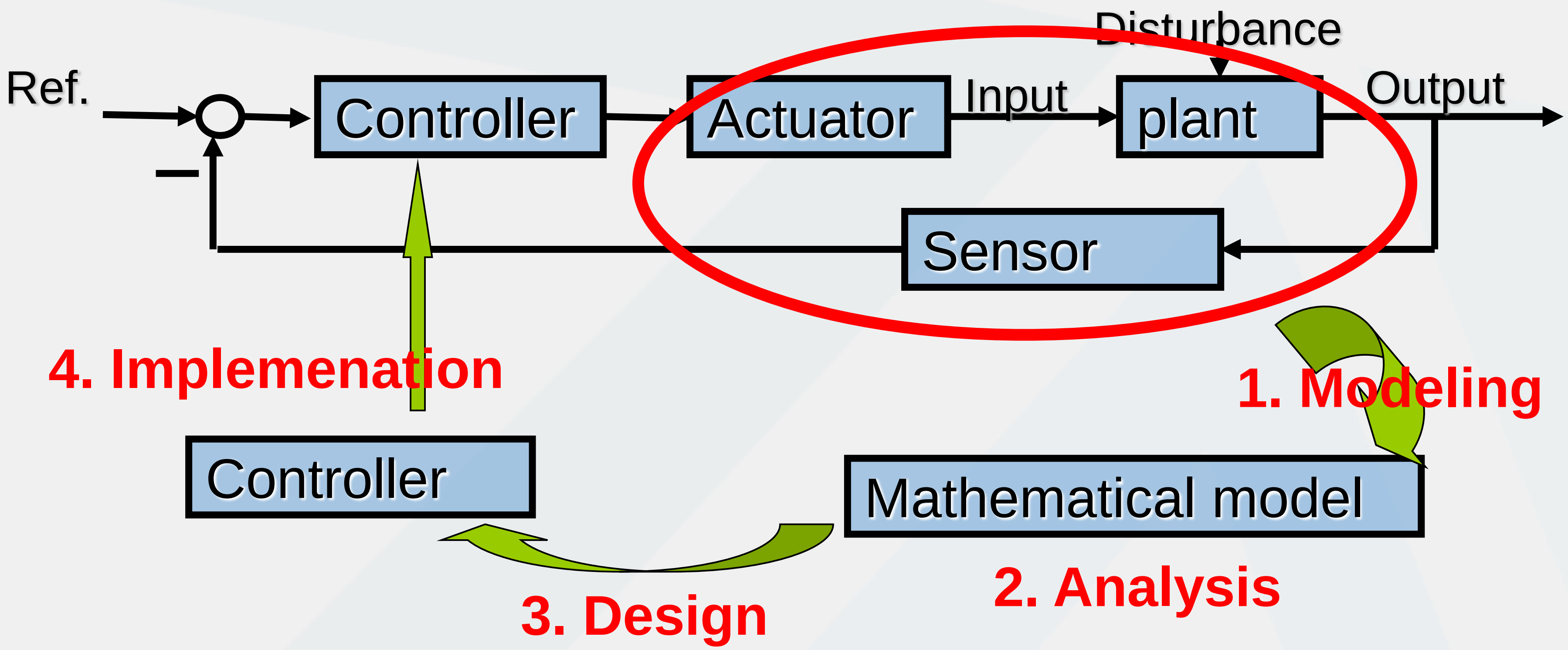
Mathematical Models of Systems Objectives:

- Control systems rely on quantitative mathematical models of physical systems.
- These models describe dynamic behavior with ordinary differential equations, covering various systems such as mechanical, hydraulic, and electrical.
- To handle the inherent nonlinearity of most physical systems, we may use linearization approximations, then we will use Laplace transform methods to get a transfer function.
- Transfer functions are introduced to represent the input-output relationships of components and subsystems and are typically organized into block diagrams or signal-flow graphs for graphical analysis.

Mathematical Model in Control Engineering:

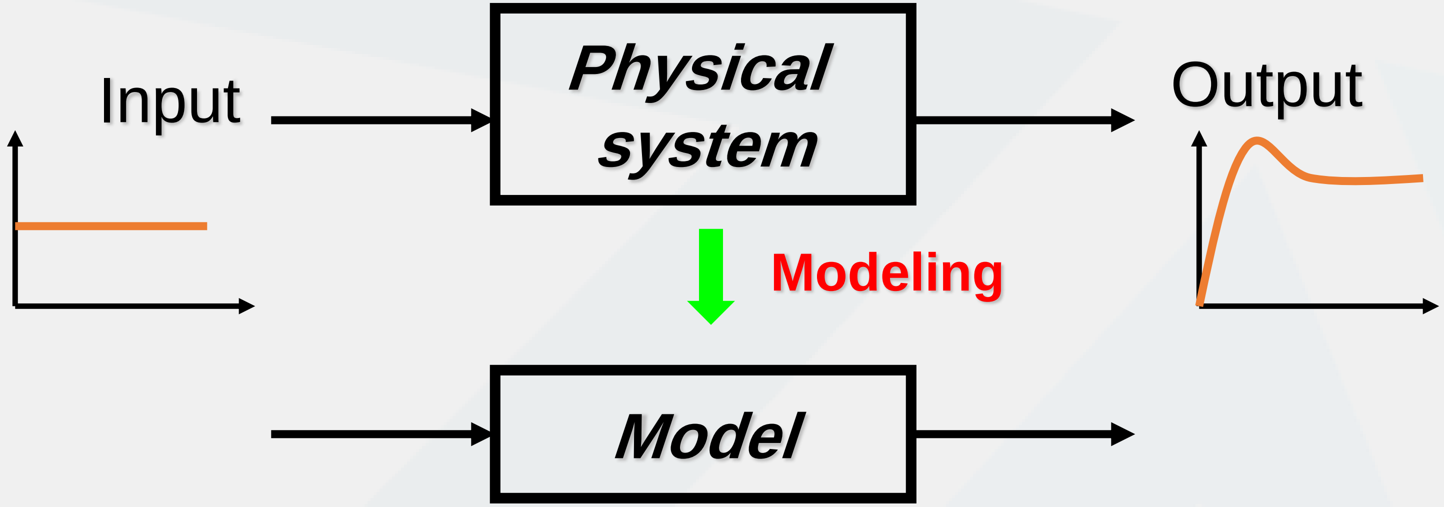
- The creation of a mathematical model is a fundamental task in control engineering for analysis and design.
- A mathematical model is a set of differential equations that accurately represents a dynamic system's behavior.
- Note that there can be multiple mathematical models for the same system, depending on the context.

Controller design procedure (preview)



Mathematical model

- Representation of the input-output (signal) relation of a physical system



Important remarks on models

- Modeling is the most important and difficult task in control system design.
- No mathematical model exactly represents a physical system.

$$\text{Math model} \neq \text{Physical system}$$
$$\text{Math model} \approx \text{Physical system}$$

- Do not confuse models with physical systems!
- In this lecture, we may use the term "system" to mean a mathematical model.

Laplace Transform Properties

TABLE 2.2 Laplace transform theorems

Item no.	Theorem	Name
1.	$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$	Definition
2.	$\mathscr{L}[kf(t)] = kF(s)$	Linearity theorem
3.	$\mathscr{L}[f_1(t)+f_2(t)] = F_1(s)+F_2(s)$	Linearity theorem
4.	$\mathscr{L}[e^{-at}f(t)] = F(s+a)$	Frequency shift theorem
5.	$\mathscr{L}[f(t-T)] = e^{-sT}F(s)$	Time shift theorem
6.	$\mathscr{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right)$	Scaling theorem
7.	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0-)$	Differentiation theorem
8.	$\mathscr{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$	Differentiation theorem
9.	$\mathscr{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$	Differentiation theorem
10.	$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right] = \frac{F(s)}{s}$	Integration theorem
11.	$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem[1]
12.	$f(0+) = \lim_{s\to\infty} sF(s)$	Initial value theorem[2]

[1]For this theorem to yield correct finite results, all roots of the denominator of $F(s)$ must have negative real parts, and no more than one can be at the origin.

[2]For this theorem to be valid, $f(t)$ must be continuous or have a step discontinuity at $t = 0$ (that is, no impulses or their derivatives at $t = 0$).

Table 2.2

https://www.youtube.com/watch?v=HyKNko7yoHo&list=PL6vQCK4n4KX_2J5WaopJWt8P3Fz_qUYZi

TRANSFER FUNCTION

- Transfer functions are commonly used to characterize the input—output relationships of components or systems that can be described by linear, time-invariant, differential equations.

- The *transfer function* of a linear, time-invariant, differential equation system is defined as "**the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero**".

TRANSFER FUNCTION

- The general form of the differential equation for LTI-System is given by

$$\boldsymbol{a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \dots + a_{n-1}\dot{y} + a_n\, y = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \dots + b_{m-1}\dot{x} + b_m\, x}$$

Time domain model

- where y is the system output and x is the input of the System
- The transfer function of this system is obtained by taking the Laplace transforms of both sides of Equation (under the assumption that all initial conditions are zero).

$$a_0 S^n Y(s) + \cdots + a_{n-1} S^1 Y(s) + a_n\, Y(s) = b_0 S^{\boldsymbol{m}} \boldsymbol{X}(s) + \cdots + b_{m-1} S^{\boldsymbol{1}} \boldsymbol{X}(s) + b_m\, \boldsymbol{X}(\boldsymbol{s})$$

Then: $[a_0 S^n + \cdots + a_{n-1} S^1 + a_n]\,Y(s) = [b_0 S^{\boldsymbol{m}} + \cdots + b_{m-1} S^{\boldsymbol{1}} + b_m]\, \boldsymbol{X}(\boldsymbol{s})$

TRANSFER FUNCTION

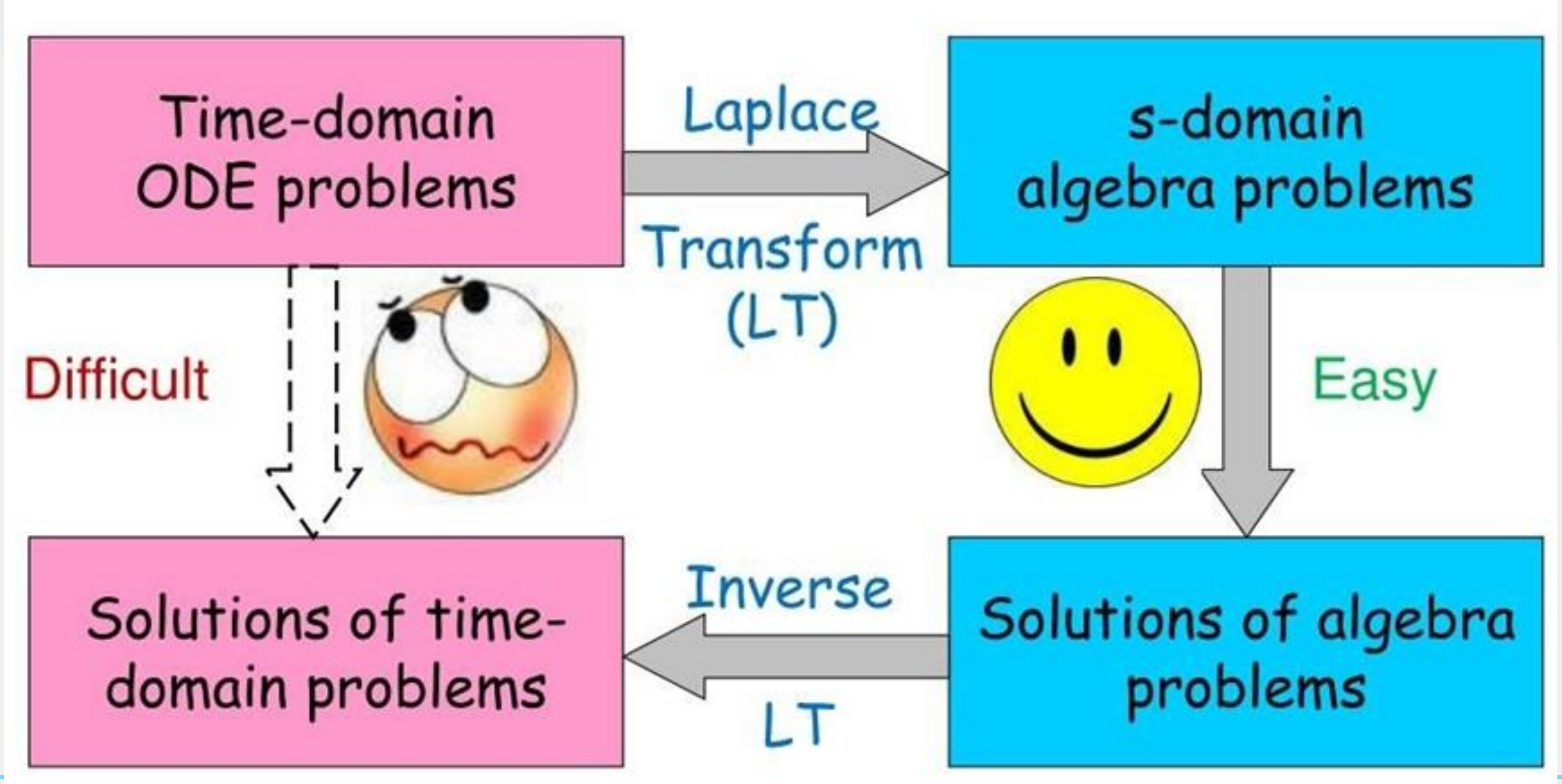
- Then the transfer function is

$$\boldsymbol{Transfer\ Function = G(s) = \left[\frac{Laplace\ of\ Output}{Laplace\ of\ Input}\right]_{Assuming\ Zero\ initial\ Condition}}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_0 S^{\boldsymbol{m}} + \cdots + b_{m-1} S^{\boldsymbol{1}} + b_m}{a_0 S^n + a_1 S^{n-1} + \cdots + a_{n-1} S^1 + a_n}$$

- Poles: are roots of the denominator (Values of s such that transfer function becomes infinite)
- Zeros: are roots of the numerator (Values of s such that transfer function becomes 0)

Why do we need LAPLACE transform?



Transfer function (Conclusion):

1. The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.
2. The transfer function is a property of a system itself, independent of the magnitude and nature of the input or driving function.
3. The transfer function includes the units necessary to relate the input to the output; however, it does not provide any information concerning the physical structure of the system (The transfer functions of many physically different systems can be identical).
4. If the transfer function of a system is known, the output or response can be studied for various forms of inputs with a view toward understanding the nature of the system.
5. If the transfer function of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system. Once established, a transfer function gives a full description of the dynamic characteristics of the system, as distinct from its physical description.

Modeling methods

Analytic method

- According to
  - A. Newton's Law of Motion
  - B. Law of Kirchhoff
  - C. System structure and parameters

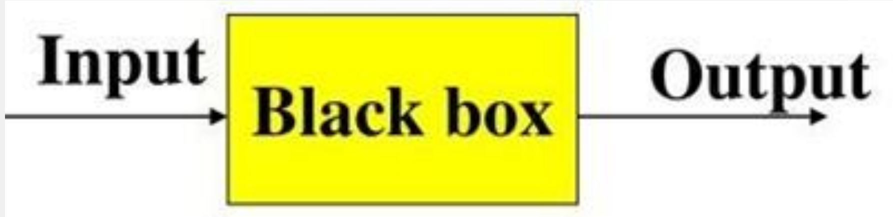
The mathematical expression of system input and output can be derived.

- Thus, we build the mathematical model (suitable for simple systems).

Modeling methods

• System identification method

- Building the system model based on the system input-output signal.
- This method is usually applied when there is little information available for the system.



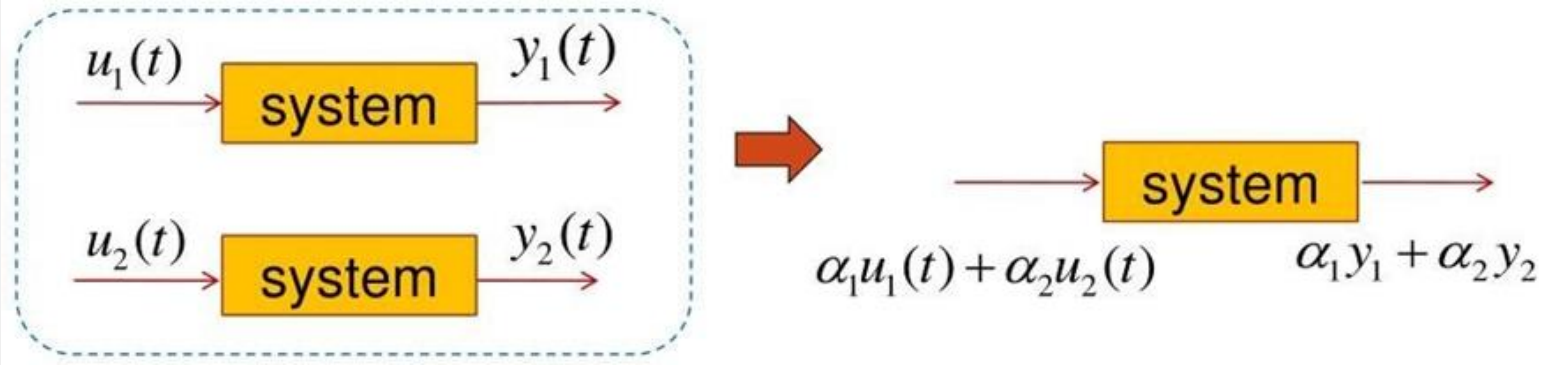
Neural Networks,
Fuzzy Systems

- Black box: the system is totally unknown.
- Grey box: the system is partially known.

Why Focus on Linear Time-Invariant (LTI) System

What is linear system?

-A system is called linear if the principle of superposition applies.



Why Focus on Linear Time-Invariant (LTI) System

Advantages of linear systems:

The overall response of a linear system can be obtained by

-- decomposing the input into a sum of elementary signals

-- figuring out each response to the respective elementary signal

-- adding all these responses together.

Models of system components

Subsystem types found in controls systems

Subsystem	Components
Electrical	Motors, Solenoids, Transducers, Control Electronics
Mechanical	Control Valves, Gear Boxes, Linkages
Liquid Flow	Piping , Tanks, Pumps, Compressors, Filters
Gas Flow	Piping , Tanks, Pumps, Compressors, Filters
Thermal	Heating Elements, Heat Exchangers, Insulation,

Models of system components

Systems' behavior defined by component characteristics

Example: electrical components

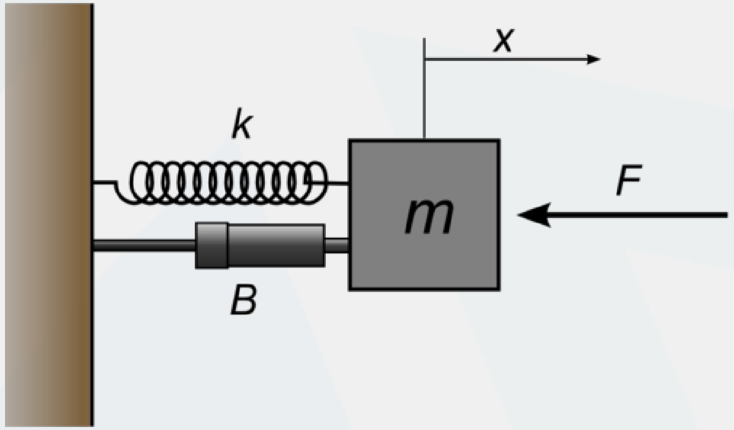
- Resistance
- Capacitance
- Inductance
- Delay
- Voltage
- Current
- Charge

Models of system components

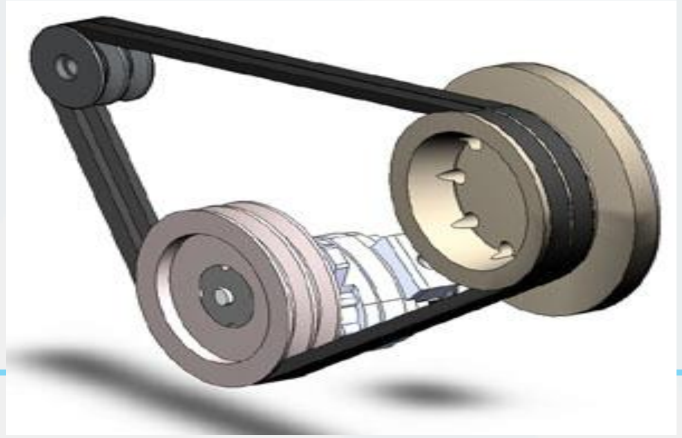
Systems' behavior defined by component characteristics

Example: Mechanical Systems

- Translational
  - Linear Motion

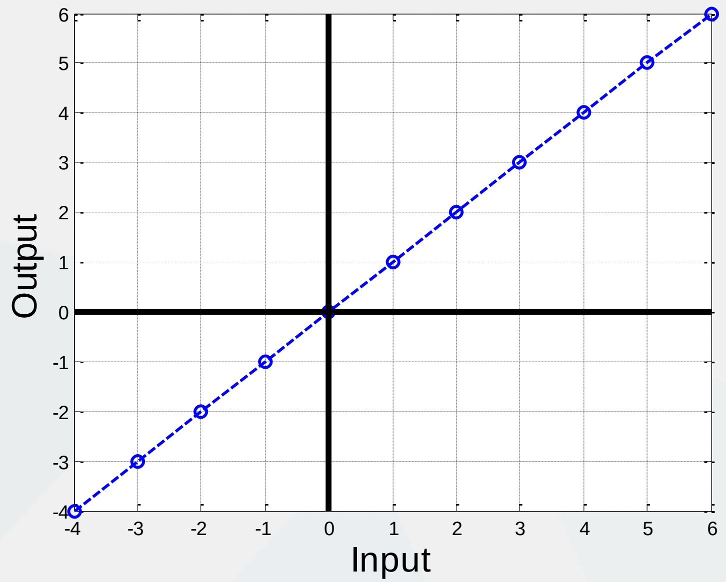


- Rotational
  - Rotational Motion



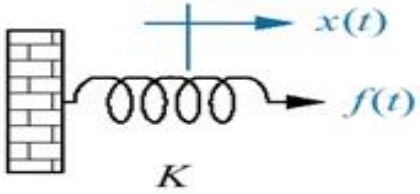
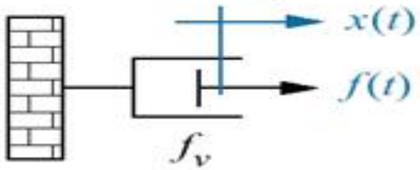
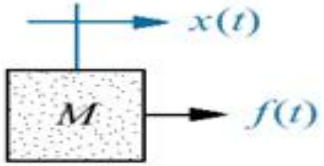
Modelling basics:

1. Awareness that many components can be modelled by simple linear equations of the same structure.
2. Same mathematical structure implies analogous behaviour so understanding one means understanding all!
3. Learn governing equations and models for some key engineering components.



Describing Differential Equations for Translation Mechanical Elements

Using Laplace Transform

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = Bv(t)$	$f(t) = B\frac{dx(t)}{dt}$	Bs
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), B = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

Common Uses of Dashpots

Door Stoppers

Vehicle Suspension

Heavy Duty Mechanical Scale with Dashpot

Bridge Suspension

Flyover Suspension

Shocks in a vehicle

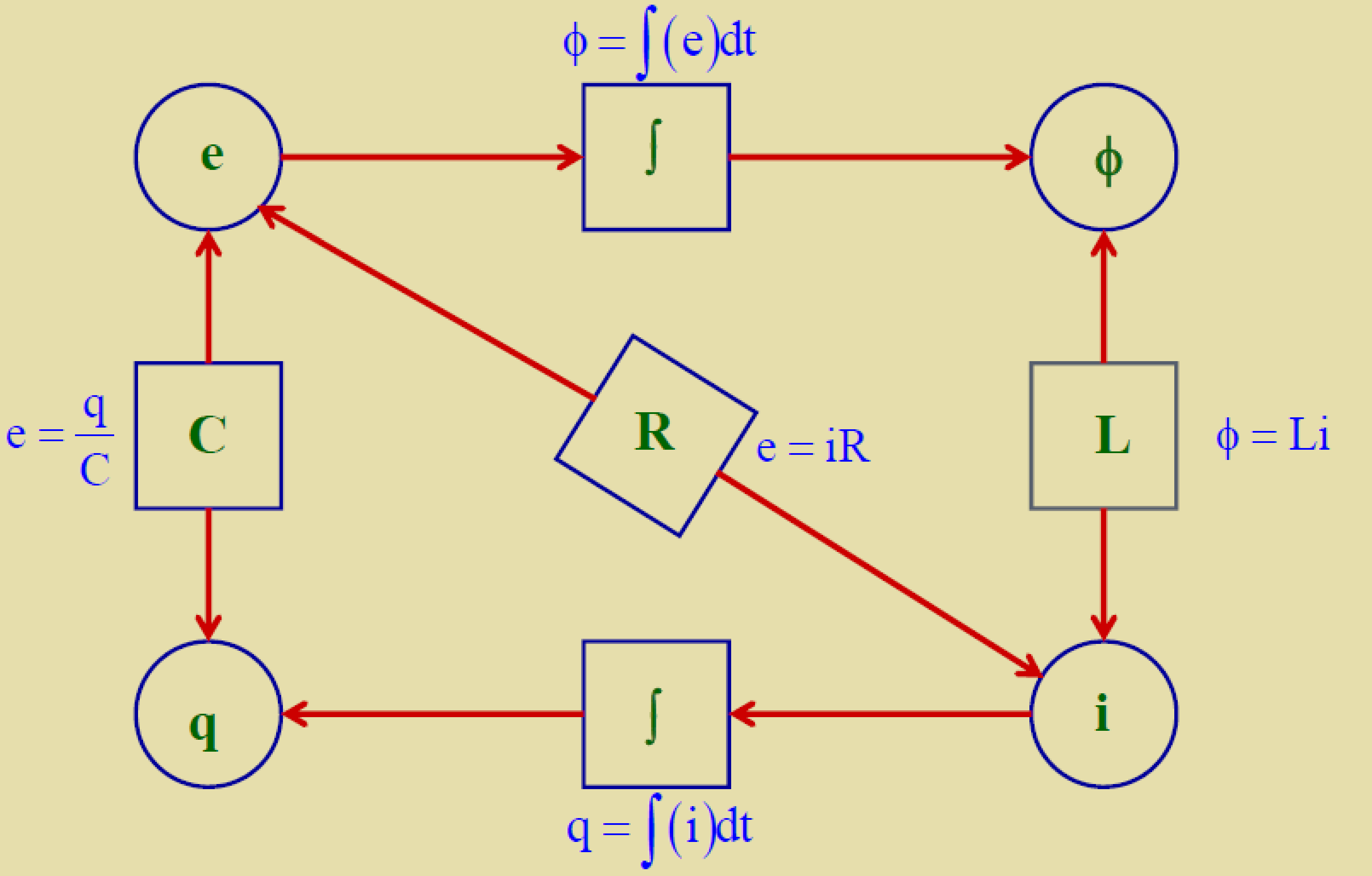
Describing Differential Equations for Electrical and Electronic Elements

TABLE 2.3 Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

Table 2.3

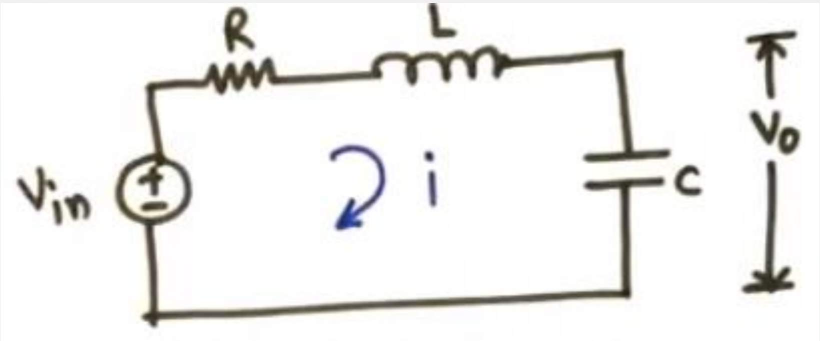


General Model Structure for Electrical Systems

Numerical Examples

Example1

Determine the differential equation and transfer function



Applying KVL we get

$$V_{in}(t) = R\,i(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(\tau)\,d\tau \quad -(I)$$

$$V_o(t) = \frac{1}{C}\int i(\tau)\,d\tau \quad -(II)$$

Taking Laplace transform on both sides of (I)

$$V_{in}(s) = R\,I(s) + Ls\,I(s) + \frac{1}{Cs}I(s)$$

$$= \left[Ls + R + \frac{1}{Cs}\right]I(s)$$

$$= \left[\frac{LCs^2 + RCs + 1}{Cs}\right]I(s)$$

Taking L.T. on both sides of (II)

$$V_o(s) = \frac{1}{Cs}I(s)$$

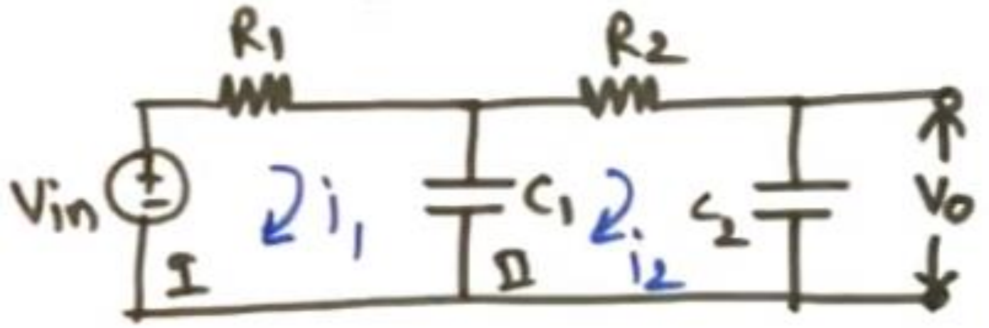
∴ T.F. is

$$\frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{Cs}I(s)}{\left[\frac{LCs^2+RCs+1}{Cs}\right]I(s)} = \frac{1}{LCs^2+RCs+1}$$

$$= \frac{1/LC}{s^2 + \frac{R}{L}s + 1/LC}$$

Example2

Determine the differential equation and transfer function



Applying KVL to loop I we get,

$$v_{in}(t) = R_1 i_1(t) + \frac{1}{C_1}\int (i_1 - i_2)\,dt \quad \text{— (I)}$$

Applying KVL to loop II we get,

$$\frac{1}{C_1}\int (i_1 - i_2)\,dt = R_2 i_2 + \frac{1}{C_2}\int i_2\,dt \quad \text{— (II)}$$

Also, $V_o = \frac{1}{C_2}\int i_2\,dt$ — (III)

Taking L.T. of (I) - (III) we get

$$V_{in}(s) = R_1 I_1(s) + \frac{1}{C_1 s}[I_1(s) - I_2(s)]$$

$$= \left[R_1 + \frac{1}{C_1 s}\right] I_1(s) - \frac{1}{C_1 s} I_2(s)$$

$$= \left[\frac{R_1 C_1 s + 1}{C_1 s}\right] I_1(s) - \frac{1}{C_1 s} I_2(s) \quad \text{— (IV)}$$

$$\frac{1}{C_1 s}[I_1(s) - I_2(s)] = R_2 I_2(s) + \frac{1}{C_2 s} I_2(s)$$

$$\frac{1}{C_1 s} I_1(s) = \left[\frac{1}{C_1 s} + R_2 + \frac{1}{C_2 s}\right] I_2(s)$$

$$I_1(s) = \left[1 + R_2 C_1 s + \frac{C_1}{C_2}\right] I_2(s) \quad \text{— (V)}$$

$$V_o(s) = \frac{1}{C_2 s} I_2(s) \quad \text{— (VI)}$$

From (IV) & (V) we get,

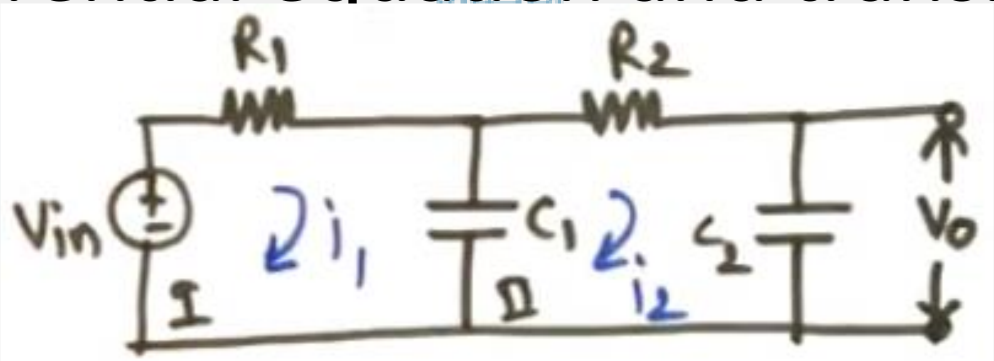
$$V_{in}(s) = \left[\frac{R_1 C_1 s + 1}{C_1 s}\right]\left[1 + R_2 C_1 s + \frac{C_1}{C_2}\right] I_2(s) - \frac{1}{C_1 s} I_2(s)$$

$$= \frac{(1 + R_1 C_1 s)\left(1 + R_2 C_1 s + \frac{C_1}{C_2}\right) - 1}{C_1 s} I_2(s)$$

$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{C_2 s}}{\frac{(1 + R_1 C_1 s)\left(1 + R_2 C_1 s + \frac{C_1}{C_2}\right) - 1}{C_1 s}}$$

Example2 (continued)

Determine the differential equation and transfer function



$$\therefore \frac{V_0(s)}{V_{in}(s)} = \frac{\frac{1}{C_2 s}}{\frac{\left[1 + R_2C_1s + \frac{C_1}{C_2} + R_1C_1s + R_1R_2C_1^2s^2 + \frac{R_1C_1^2s}{C_2} - 1\right]}{C_1 s}}$$

$$= \frac{\frac{1}{C_2 s}}{\frac{C_2R_2C_1s + C_1 + R_1C_1C_2s + R_1R_2C_1^2C_2s^2 + R_1C_1^2s}{C_1C_2s}}$$

$$= \frac{\frac{1}{C_2 s}}{\frac{R_2C_2s + 1 + R_1C_2s + R_1R_2C_1C_2s^2 + R_1C_1s}{C_2 s}}$$

$$= \frac{1}{R_1R_2C_1C_2s^2 + R_1C_1s + R_1C_2s + R_2C_2s + 1}$$

Example3 (Lead Compensator)

Determine the D.E. and T.F.

Applying KCL at node A we get

$$i_1 + i_2 = i_3$$

$$\therefore \frac{V_{in} - V_o}{R_1} + C\frac{d}{dt}(V_{in} - V_o) = \frac{V_o}{R_2}$$

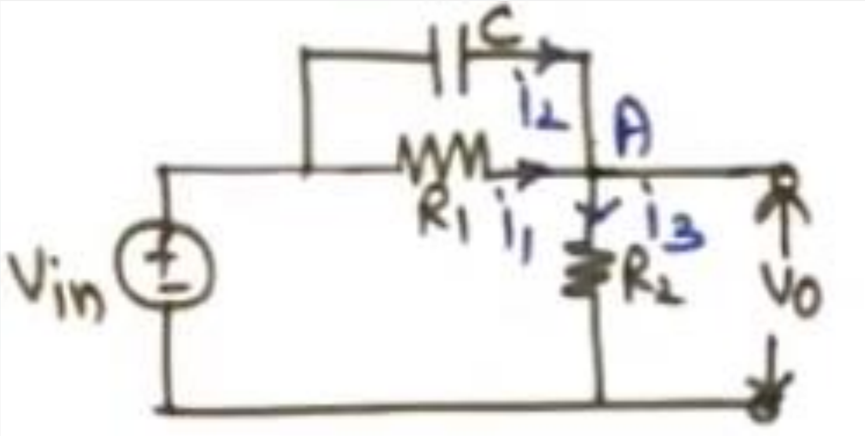
Applying L.T. we get

$$\frac{V_{in}(s) - V_o(s)}{R_1} + cs\left[V_{in}(s) - V_o(s)\right] = \frac{V_o(s)}{R_2}$$

$$V_{in}(s)\left[\frac{1}{R_1} + cs\right] = V_o(s)\left[\frac{1}{R_1} + cs + \frac{1}{R_2}\right]$$

$$V_{in}(s)\left[\frac{1 + R_1 cs}{R_1}\right] = V_o(s)\left[\frac{R_2 + R_1 R_2 cs + R_1}{R_1 R_2}\right]$$

$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{R_2(1 + R_1 cs)}{R_1 + R_2 + R_1 R_2 cs}$$



Example4 (Lag Compensator)

Determine the D.E. and T.F.

Applying KVL we get

$$V_{in} = R_1 i + R_2 i + \frac{1}{C}\int i\,dt \quad \text{— (I)}$$

$$V_o = R_2 i + \frac{1}{C}\int i\,dt \quad \text{— (II)}$$

Applying L.T. to (I) & (II) we get

$$V_{in}(s) = R_1 I(s) + R_2 I(s) + \frac{1}{Cs} I(s)$$

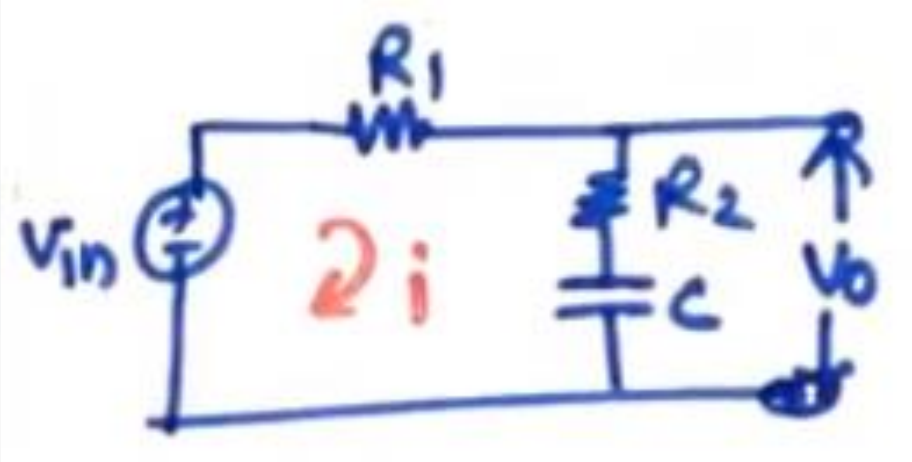
$$= \left[R_1 + R_2 + \frac{1}{Cs}\right] I(s)$$

$$= \frac{(R_1+R_2)Cs + 1}{Cs} I(s)$$

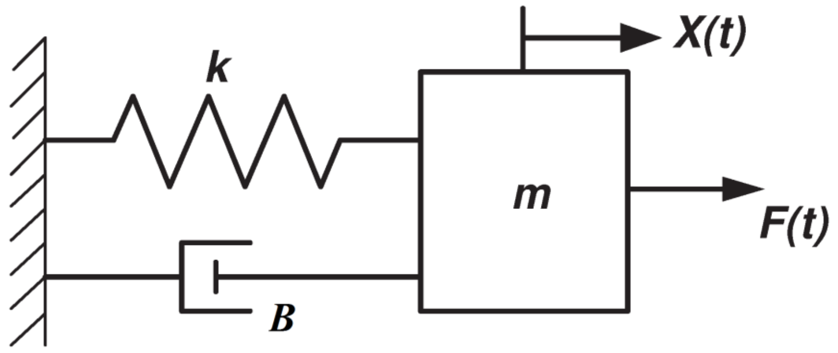
$$V_o(s) = R_2 I(s) + \frac{1}{Cs} I(s)$$

$$= \left[\frac{R_2 Cs + 1}{Cs}\right] I(s)$$

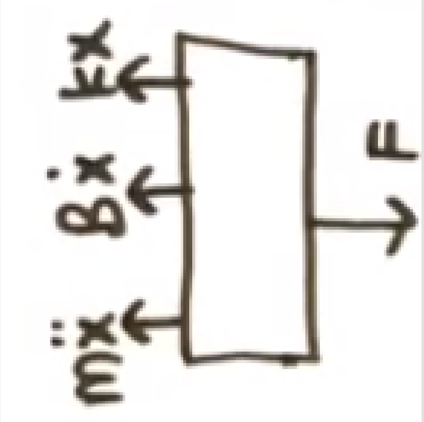
$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{1 + R_2 Cs}{1 + (R_1+R_2)Cs}$$



Free Block Diagram (FBD):



- The free body diagram helps in <u>visualizing</u> the <u>forces</u> <u>acting</u> on the <u>mass</u>.
- Case study: spring-mass-damper system.

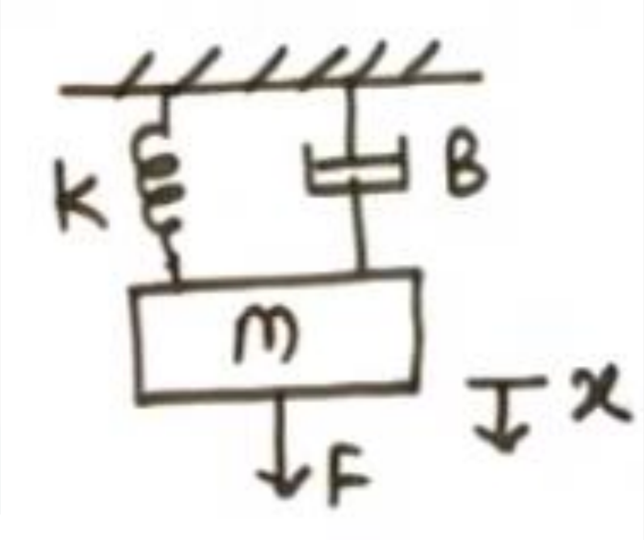


1. External Force (f):
   - **The <u>external</u> <u>force</u> f <u>acts</u> <u>directly</u> on the <u>mass</u>. It's the "driving force" that initiates the <u>system's</u> <u>motion</u>.**
   - **This force is <u>responsible</u> for the mass's <u>acceleration</u>.**

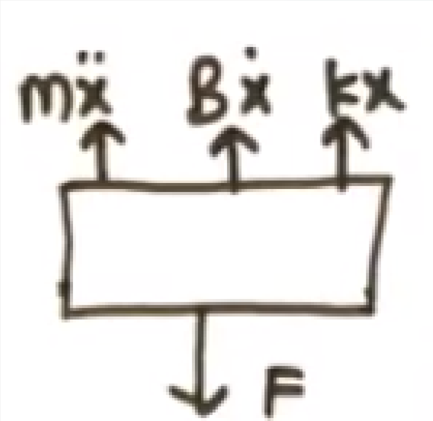
2. Internal Forces:
   - **Inertia (m*$\ddot{x}$): This is the force <u>due</u> to the <u>mass's</u> <u>inertia</u> (<u>resistance</u> to <u>change</u> in motion). The <u>mass</u> <u>resists</u> <u>acceleration</u>.**
   - **Spring Force (k*x): The spring exerts a <u>force</u> <u>proportional</u> to its <u>deformation</u> (the <u>change</u> in <u>length</u>). This force always acts <u>opposite</u> the <u>direction</u> of <u>deformation</u>.**
   - **Damping Force (b$\dot{x}$): The damper <u>opposes</u> the <u>mass's</u> <u>velocity</u> ($\dot{x}$). This force is <u>proportional</u> to the <u>velocity</u> and acts in the <u>opposite</u> <u>direction</u>.**

Example5

Determine the D.E. and T.F.



F.B.D. of the given system is



$$F = m\ddot{x} + B\dot{x} + kx$$

Applying L.T. we get

$$F(s) = ms^2X(s) + BsX(s) + kX(s)$$

$$F(s) = [ms^2 + Bs + k]X(s)$$

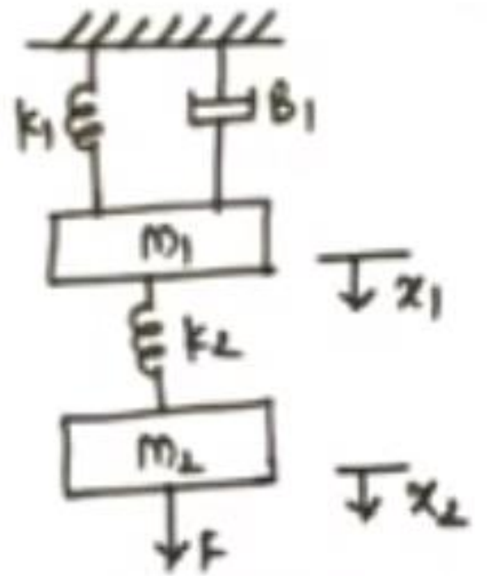
$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + Bs + k}$$

$$= \frac{1/m}{s^2 + \frac{B}{m}s + \frac{k}{m}}$$

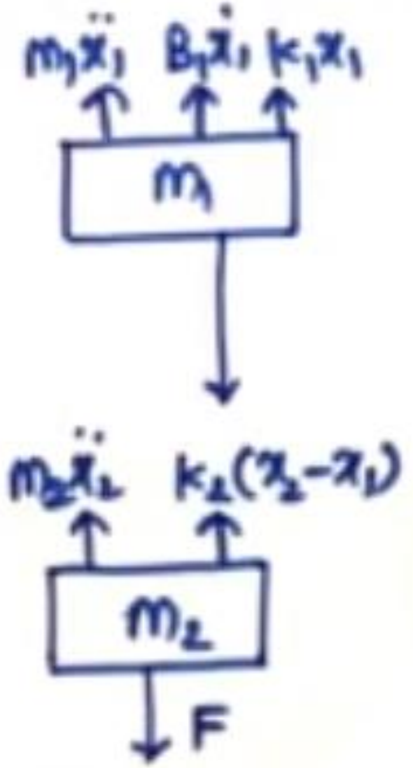
Example6

Determine the D.E. and T.F.

Determine T.F. $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$ for the system



F.B.D. of given system is



Force balance of m_1 gives

$$m_1\ddot{x}_1 + B_1\dot{x}_1 + k_1x_1 = k_2(x_2-x_1)$$

Applying L.T. we get

$$m_1 s^2 X_1(s) + B_1 s X_1(s) + k_1 X_1(s) = k_2[X_2(s) - X_1(s)]$$

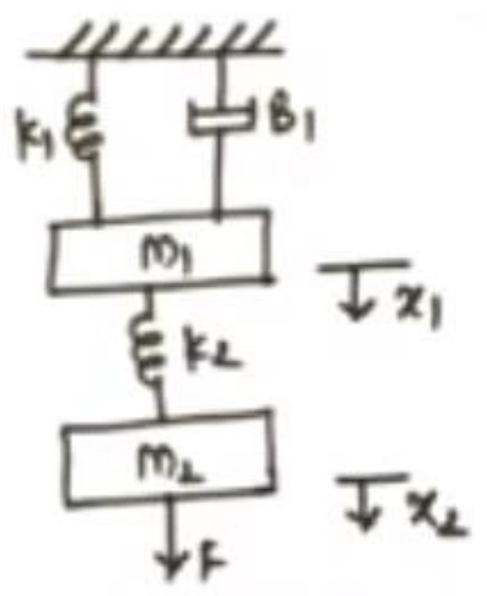
$$\therefore [m_1s^2+B_1s+k_1+k_2]\,X_1(s) = k_2X_2(s)$$

$$\therefore X_1(s) = \frac{k_2}{m_1s^2+B_1s+k_1+k_2}X_2(s)$$

$$X_2(s) = \frac{m_1s^2+B_1s+k_1+k_2}{k_2}X_1(s)$$

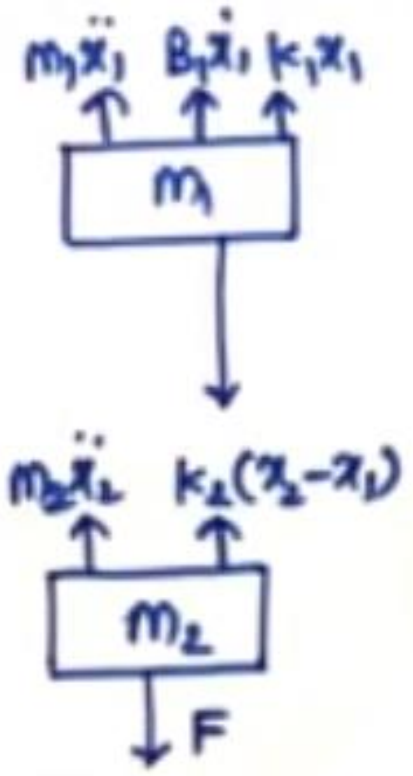
Example6

Determine the D.E. and T.F.



Determine T.F. $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$ for the system

F.B.D. of given system is



Force balance of m_2 gives

$$F = m_2\ddot{x}_2 + k_2(x_2 - x_1)$$

Applying L.T. we get

$$F(s) = m_2 s^2 X_2(s) + k_2 X_2(s) - k_2 X_1(s)$$

$$= [m_2 s^2 + k_2] X_2(s) - k_2 X_1(s)$$

$$\therefore F(s) = \frac{(m_2 s^2 + k_2)(m_1 s^2 + B_1 s + k_1 + k_2) X_1(s)}{k_2} - k_2 X_1(s)$$

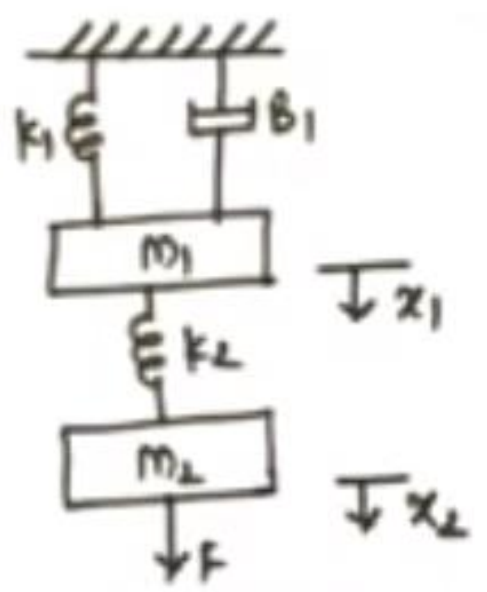
$$= \frac{(m_2 s^2 + k_2)(m_1 s^2 + B_1 s + k_1 + k_2) - k_2^2}{k_2} X_1(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{k_2}{(m_2 s^2 + k_2)(m_1 s^2 + B_1 s + k_1 + k_2) - k_2^2}$$

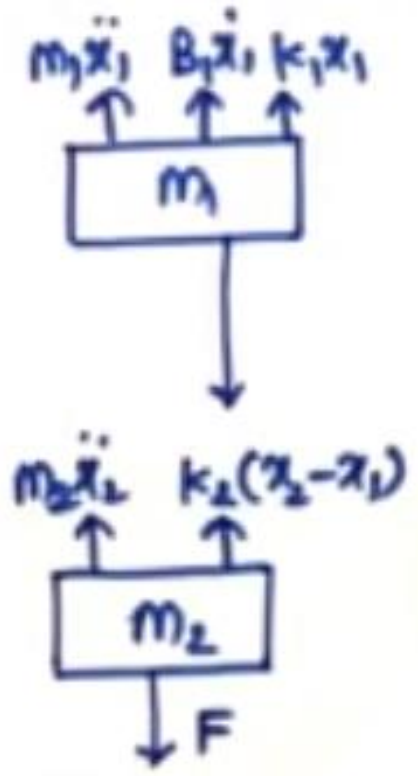
Example6

Determine the D.E. and T.F.

Determine T.F. $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$ for the system



F.B.D. of given system is



$$F(s) = [m_2s^2+k_1]X_2(s) - k_2X_1(s)$$

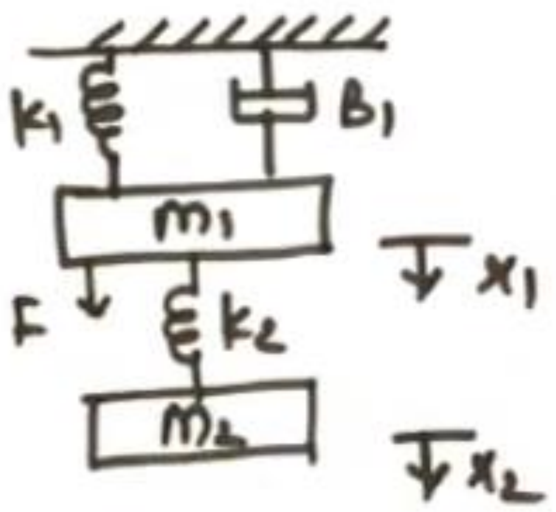
$$= [m_2s^2+k_1]X_2(s) - k_2\left[\frac{k_2}{m_1s^2+B_1s+k_1+k_2}\right]X_2(s)$$

$$= \frac{(m_2s^2+k_1)(m_1s^2+B_1s+k_1+k_2)-k_2^2}{m_1s^2+B_1s+k_1+k_2}X_2(s)$$

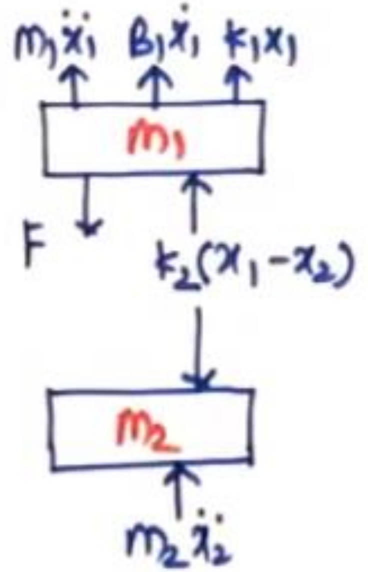
$$\therefore \frac{X_2(s)}{F(s)} = \frac{m_1s^2+B_1s+k_1+k_2}{(m_2s^2+k_1)(m_1s^2+B_1s+k_1+k_2)-k_2^2}$$

Example7

Determine the D.E. and T.F.



The F.B.D. of given system is



Applying Force balance to m_1 we get

$$F = m_1\ddot{x}_1 + B_1\dot{x}_1 + k_1x_1 + k_2(x_1 - x_2)$$

Applying L.T. we get

$$F(s) = m_1s^2X_1(s) + B_1sX_1(s) + k_1X_1(s) + k_2(X_1(s) - X_2(s)]$$

$$= [m_1s^2 + B_1s + k_1 + k_2]X_1(s) - k_2X_2(s)$$

Applying force balance to m_2 we get

$$m_2\ddot{x}_2 = k_2(x_1 - x_2)$$

Applying L.T. we get

$$m_2s^2X_2(s) = k_2[X_1(s) - X_2(s)]$$

$$[m_2s^2 + k_2]X_2(s) = k_2X_1(s)$$

$$X_1(s) = \frac{m_2s^2 + k_2}{k_2}X_2(s)$$

$$X_2(s) = \frac{k_2}{m_2s^2 + k_2}X_1(s)$$

$$F(s) = [m_1s^2 + B_1s + k_1 + k_2]X_1(s) - \frac{k_2^2}{m_2s^2 + k_2}X_1(s)$$

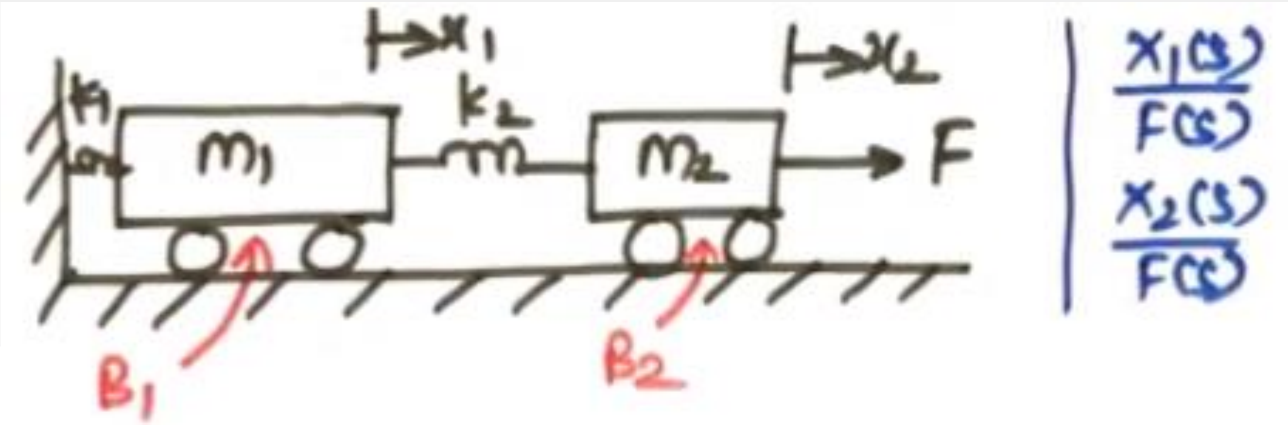
$$= \frac{(m_1s^2 + B_1s + k_1 + k_2)(m_2s^2 + k_2) - k_2^2}{m_2s^2 + k_2}X_1(s)$$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{m_2s^2 + k_2}{(m_1s^2 + B_1s + k_1 + k_2)(m_2s^2 + k_2) - k_2^2}$$

Find $\frac{X_2(s)}{F(s)}$ by replacing $X_1(s)$ with $X_2(s)$

Example8

Determine the D.E. and T.F.



The F.B.D is as shown below

Applying force balance to m_1 we get

$$m_1\ddot{x}_1 + B_1\dot{x}_1 + k_1x_1 = k_2(x_2 - x_1)$$

Applying L.T. we get

$$[m_1s^2 + B_1s + k_1 + k_2]X_1(s) = k_2X_2(s)$$

$$X_1(s) = \frac{k_2}{m_1s^2 + B_1s + k_1 + k_2}X_2(s)$$

$$X_2(s) = \frac{m_1s^2 + B_1s + k_1 + k_2}{k_2}X_1(s)$$

Applying force balance to m_2 we get

$$F = m_2\ddot{x}_2 + B_2\dot{x}_2 + k_2(x_2 - x_1)$$

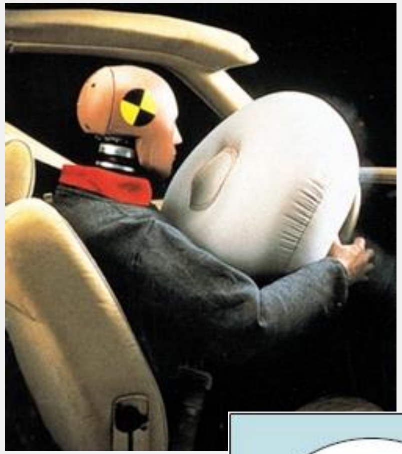
Applying L.T. we get

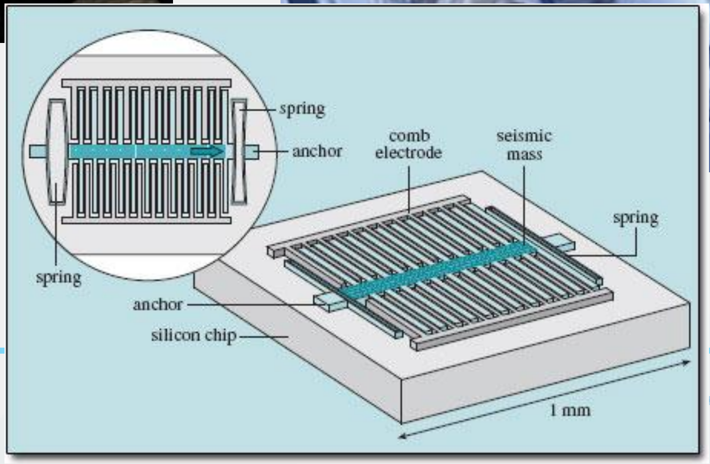
$$F(s) = [m_2s^2 + B_2s + k_2]X_2(s) - k_2X_1(s)$$

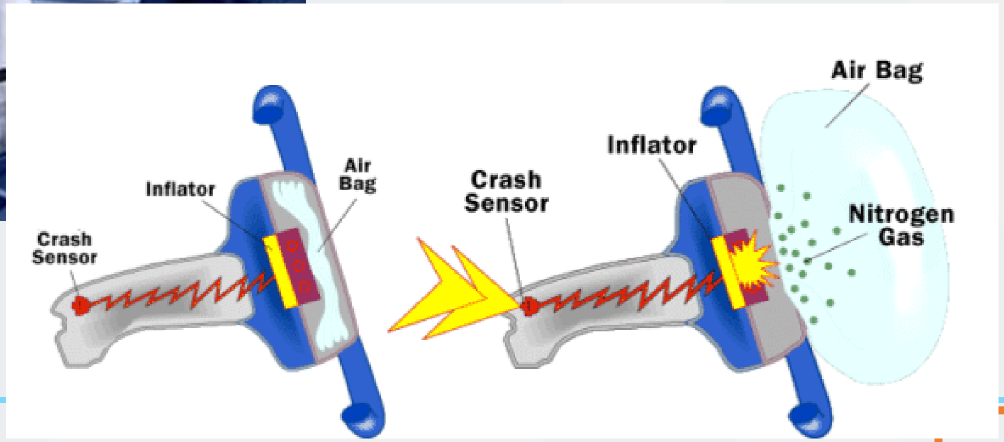
Determine $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$

EX. Air bag and accelerometer

- Tiny MEMS accelerometer
  - Microelectromechanical systems (MEMS)



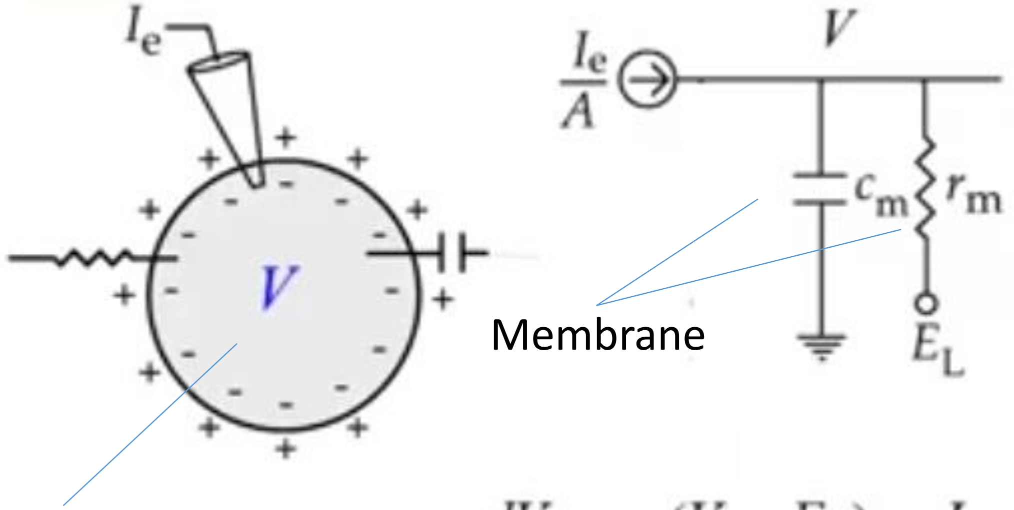




(Pictures from various websites)

EX. RC Circuit Model of the Membrane

- Basic model of membrane of a neuron:



$c_m \approx 10\ \mathrm{nF/mm^2}$

$r_m \approx 1\ \mathrm{M\Omega\ mm^2}$

$C_m = c_m A$

$R_m = r_m / A$

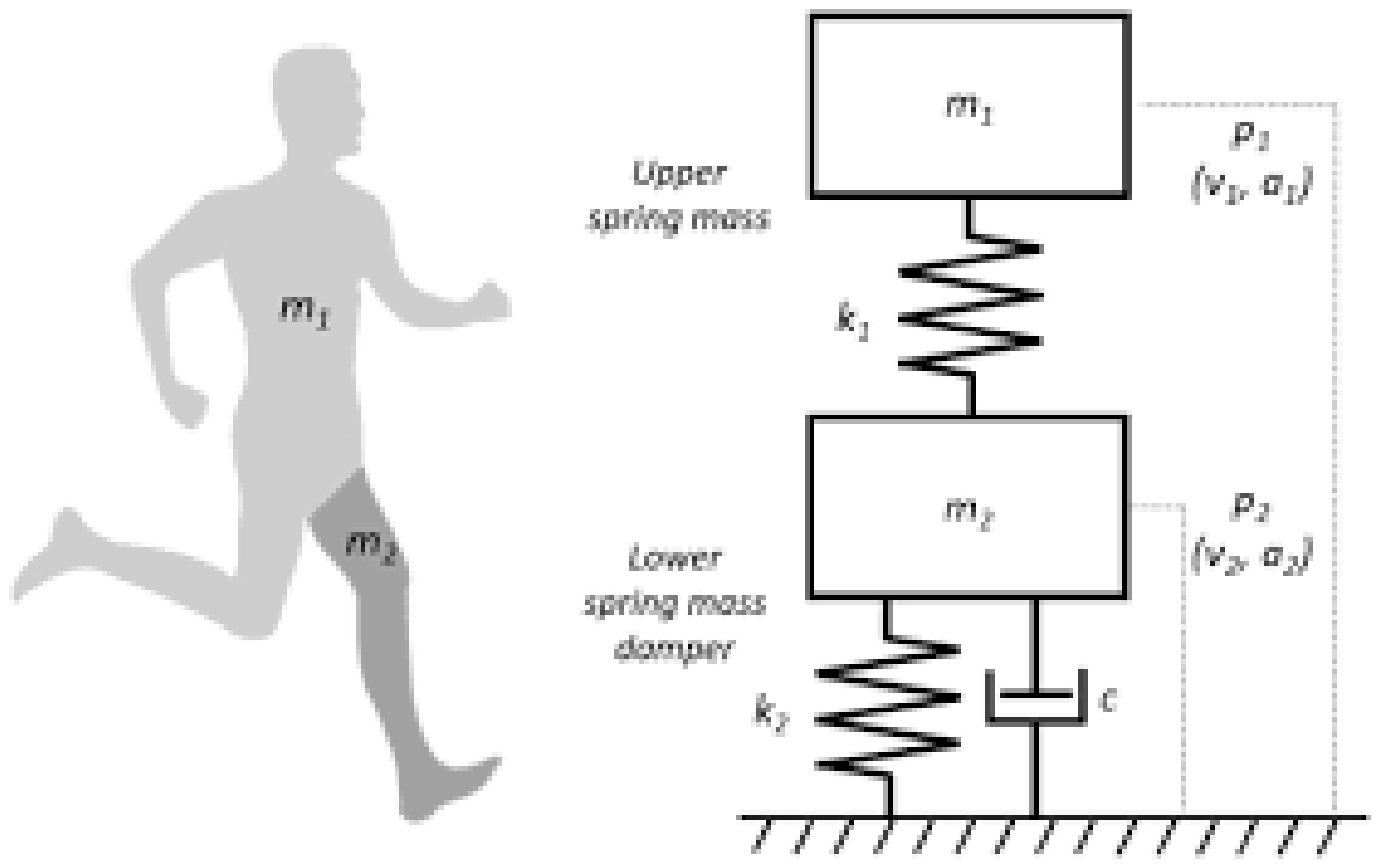
$$c_m \frac{dV}{dt} = -\frac{(V - E_L)}{r_m} + \frac{I_e}{A}$$

Or Equivalently:

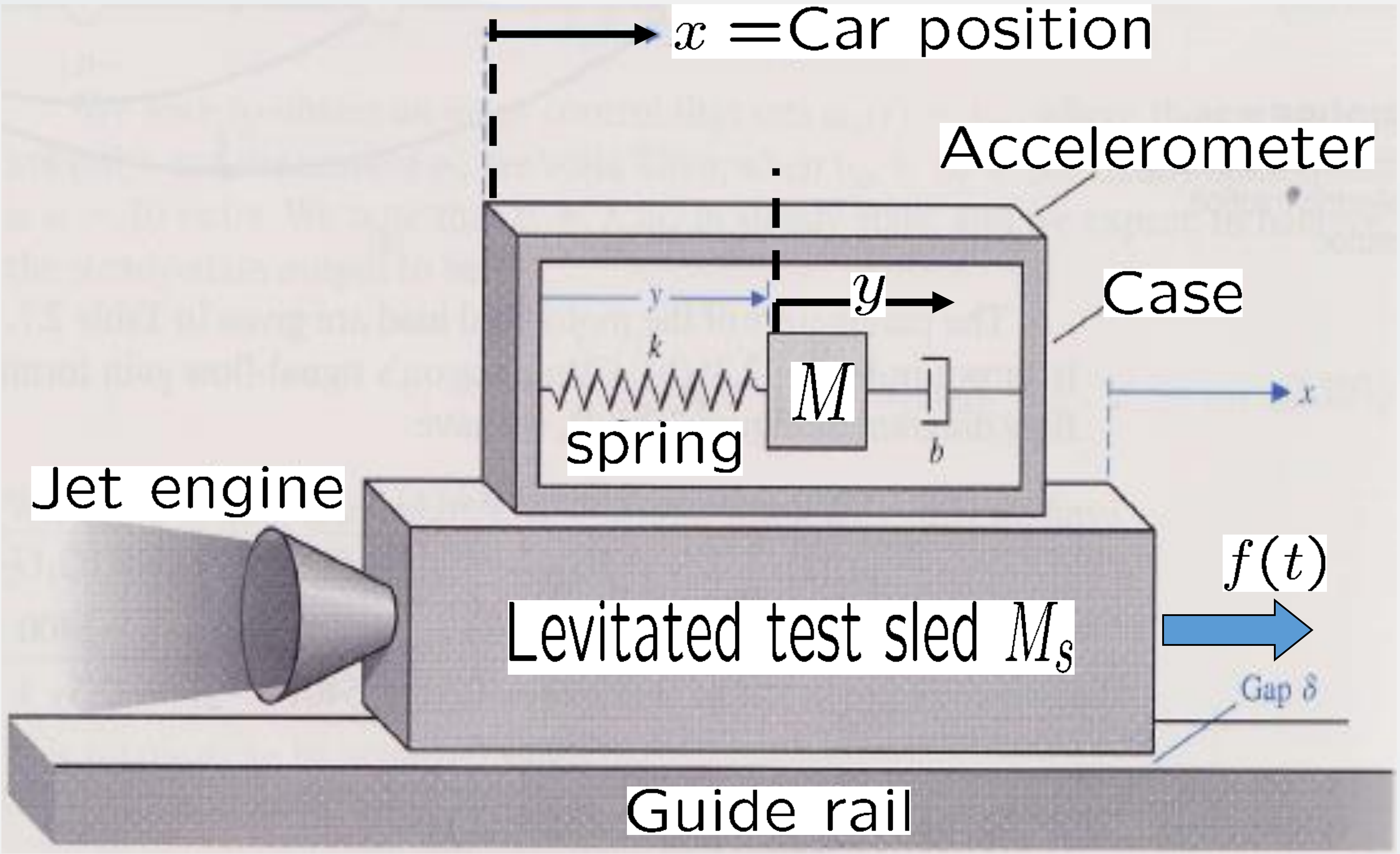
$\tau_m = r_m c_m = R_m C_m$ is the membrane time constant

$$\tau_m \frac{dV}{dt} = -(V - E_L) + I_e R_m$$

EX. Human Body

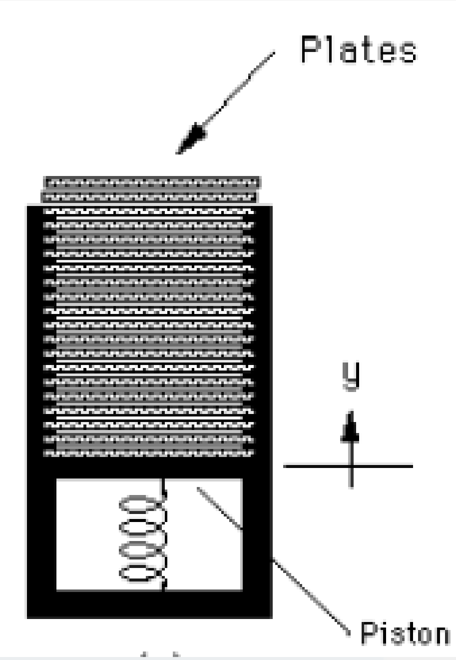


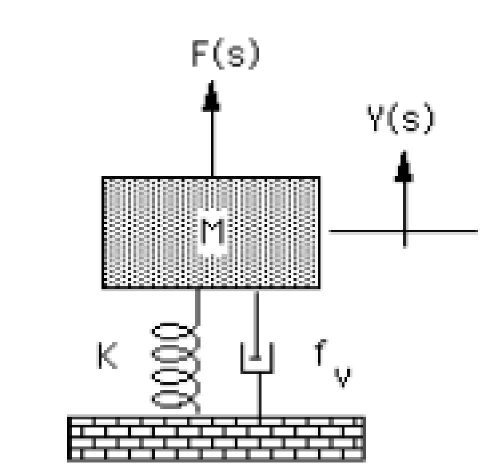
Ex: Mechanical accelerometer



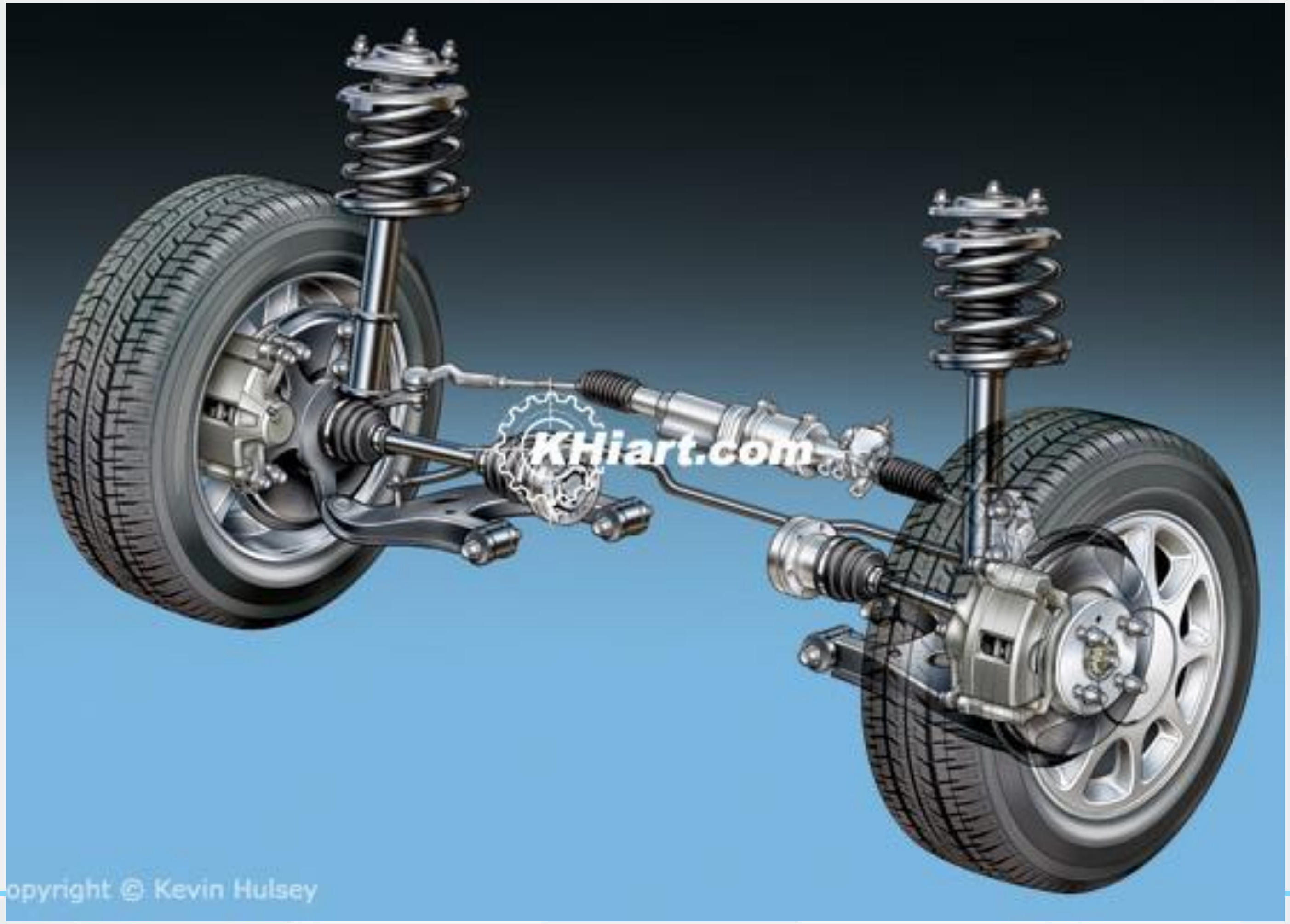
Example

- Restaurant plate dispenser

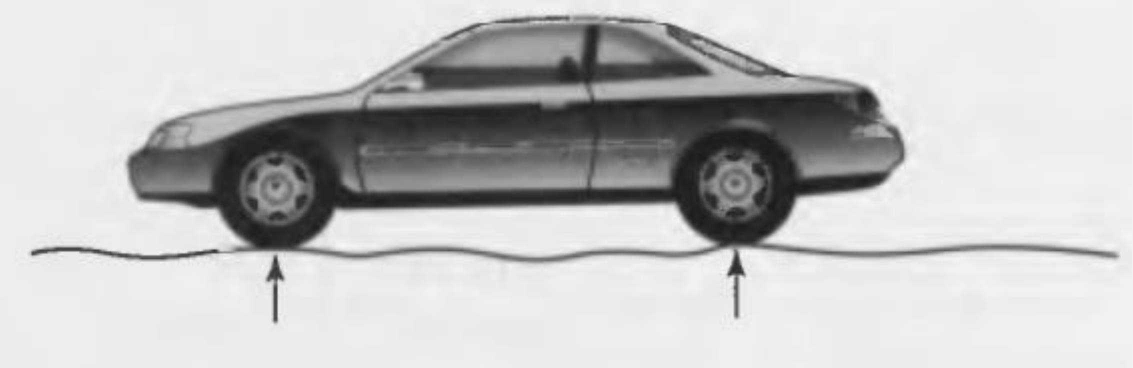


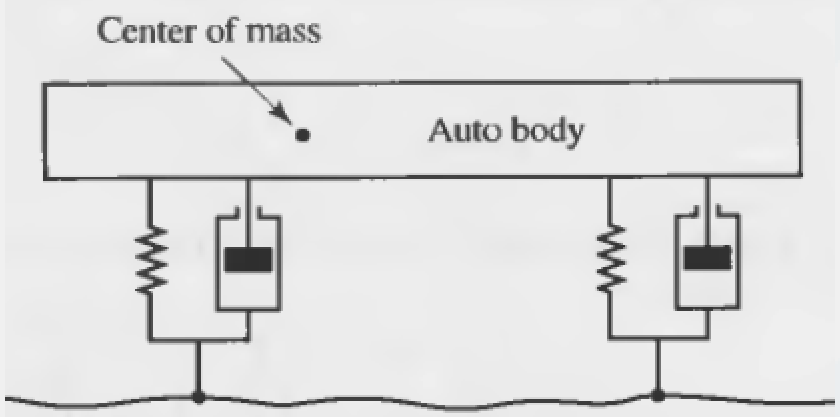


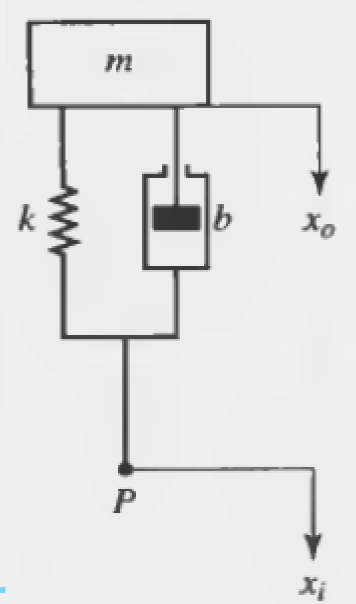
Example: Automobile Suspension



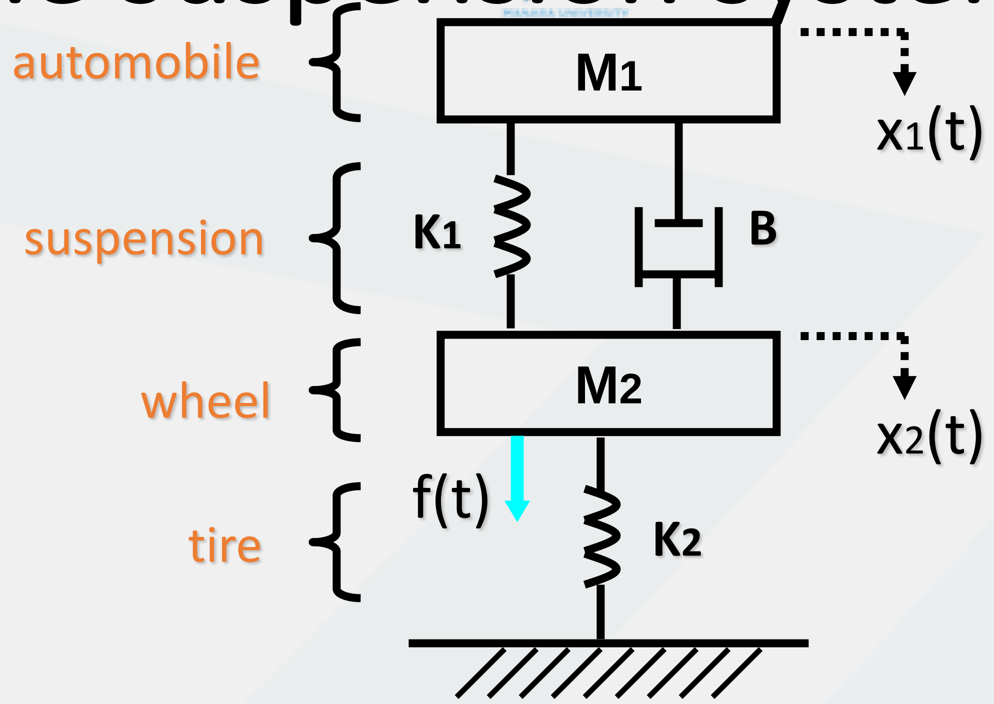
Automobile Suspension







Automobile suspension system



Example: Train Suspension