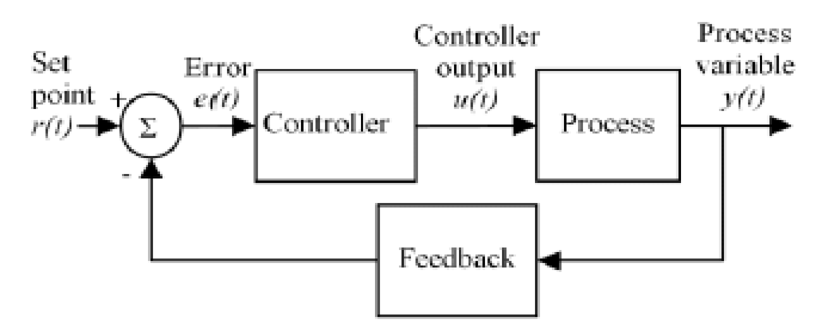



جَامعة
المَنارة

# Lecture (4)

# Block Diagrams and Signal Flow

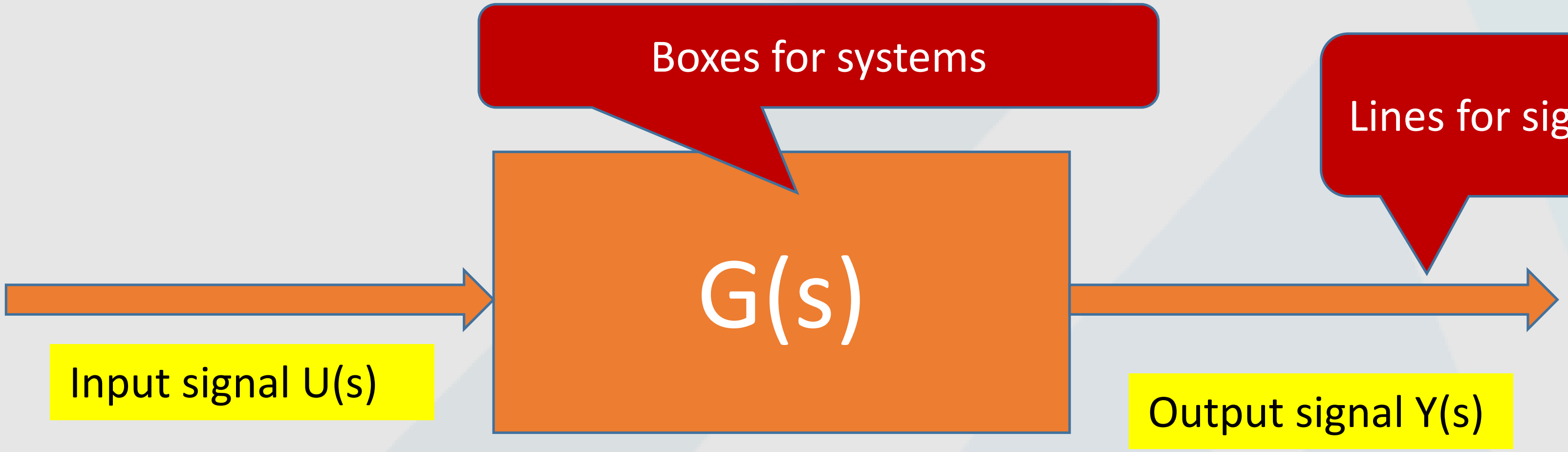
Mechatronics Engineering Department
Assistant Professor Isam Asaad

# References


- Gopal, M. - Control Systems_ Principles and Design 3rd edition- Tata McGraw Hill Publishing Co. Ltd. (2008)
- Modern Control Systems, Richard C. Dorf and Robert H. Bishop, Prentice Hall, 12th edition, 2010, ISBN-10: 0-13-602458-0
- Modelling, Dynamics and Control, University of Sheffield, John Anthony Rossiter.
- Control Systems Course, professor Aniket Khandekar, Zeal college of engineering and Research, Pune.

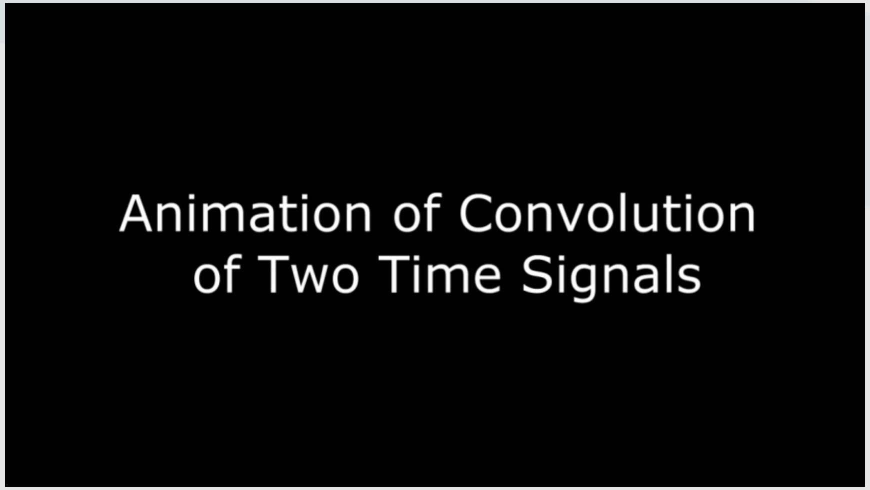
# Block diagram representations

A block diagram represents dependencies between signals.

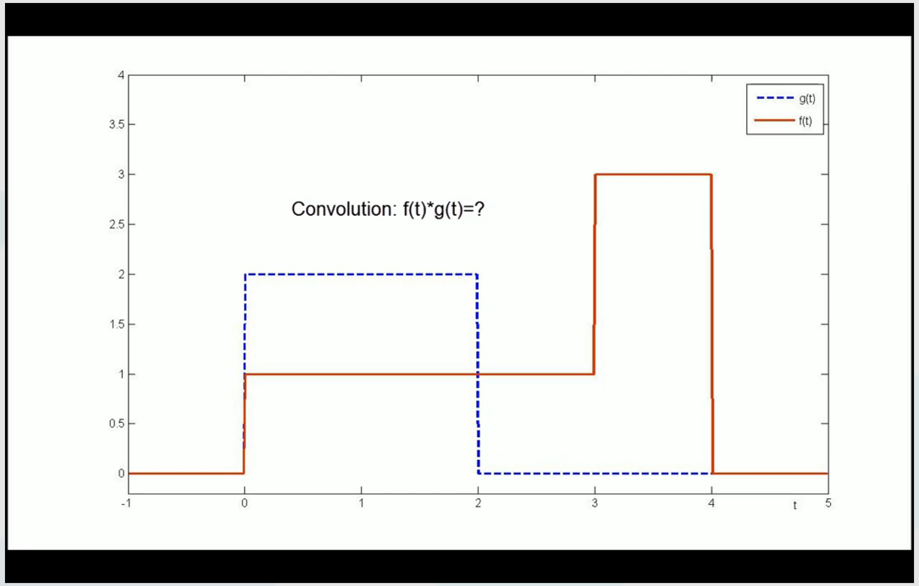



$$Y(s) = G(s)U(s)$$

# Block diagram representations (Convolution)




https://www.youtube.com/watch?v=C1N55M1VD2o




https://www.youtube.com/watch?v=jwlfSIBNqP8

$$Y(s) = G(s)U(s)$$

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\,d\tau$$

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)\,d\tau$$

https://www.youtube.com/watch?v=kkm0DXSMtPI

# Systems in series

This part deals with scenarios where systems are arranged in series, so for example the output of system 1 is the input to system 2.

System 1 with input u and output x.

$$u = 5\frac{dx}{dt} + 7x$$

$$2x = 6\frac{dy}{dt} + 10y$$

System 2 with input x and output y.

The proposal is to use Laplace to reduce the computation of y to simple multiplications.

# Systems in series 2

First, take Laplace of each model in turn:

$$u = 5\frac{dx}{dt} + 7x \quad \Rightarrow \quad X(s) = \left[\frac{1}{5s+7}\right]U(s) = G_1(s)U(s)$$

$$2x = 6$$

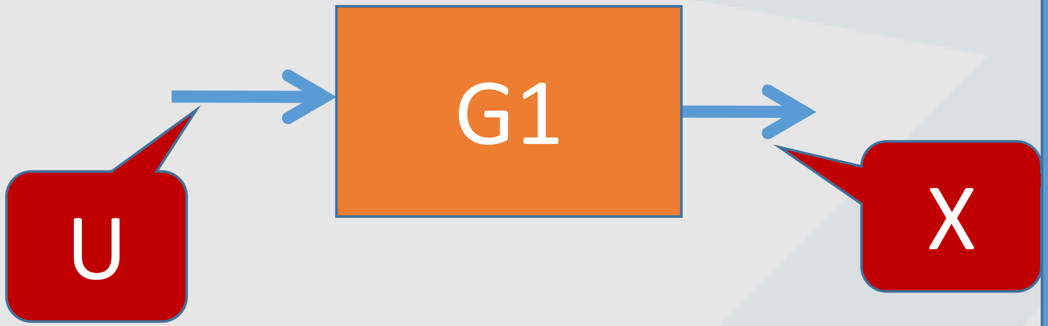
Next, elimina

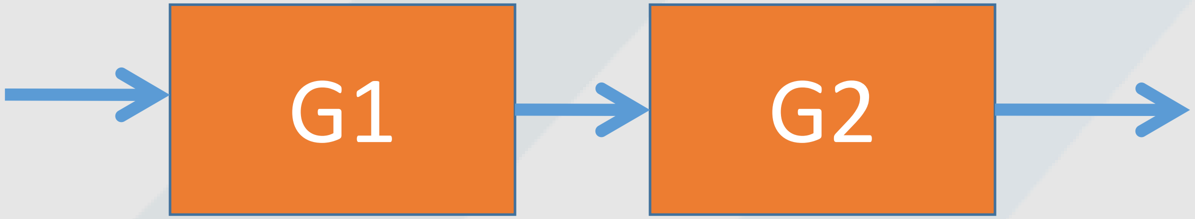
$$Y(s) =$$

# Systems in series 2

Finally, this can be represented in a single block diagram.

$$X(s) = G_1(s)U(s);$$



# Systems in series example 2

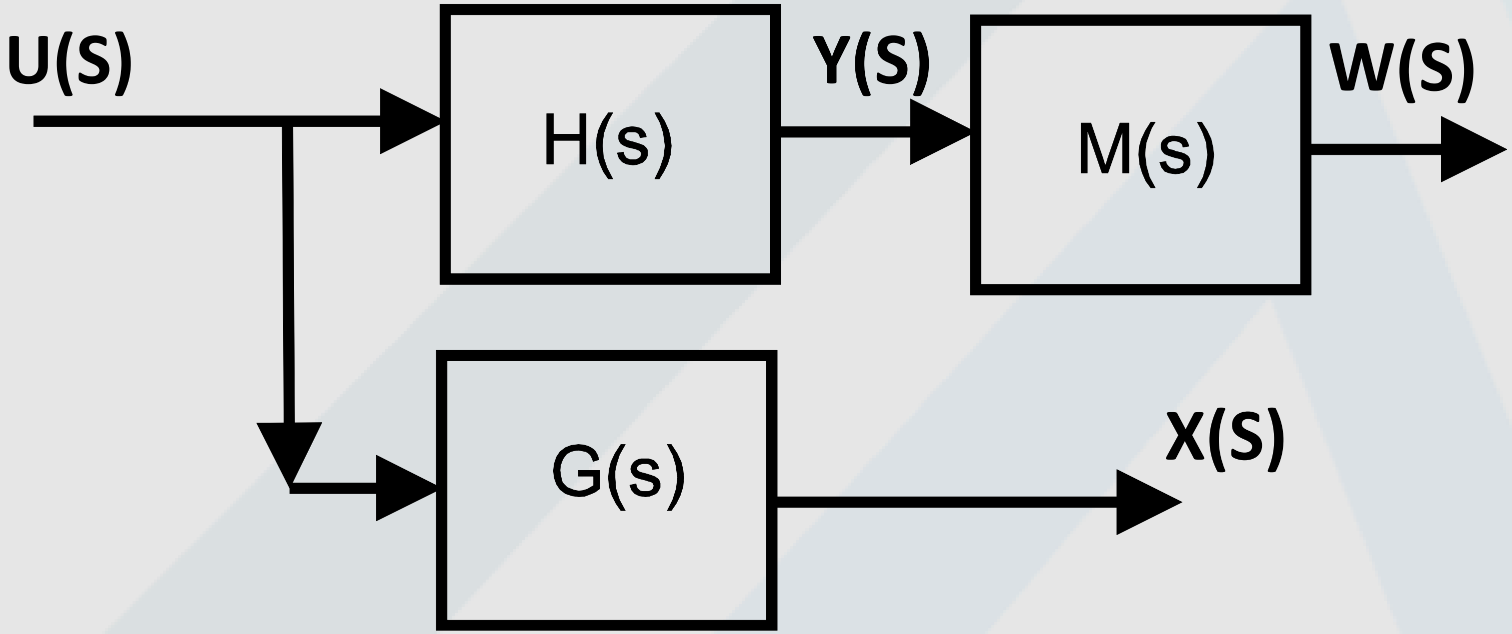
A system's behaviour is governed by the following ODEs. Find a single model to represent the input to output relationship (that is u ➔ y).

$$\frac{dw}{dt}+4w=2u; \quad \frac{dz}{dt}+z=w; \quad \frac{dy}{dt}+3y=4z;$$

# Sharing an input signal

Taking the definitions provided on the previous slide.

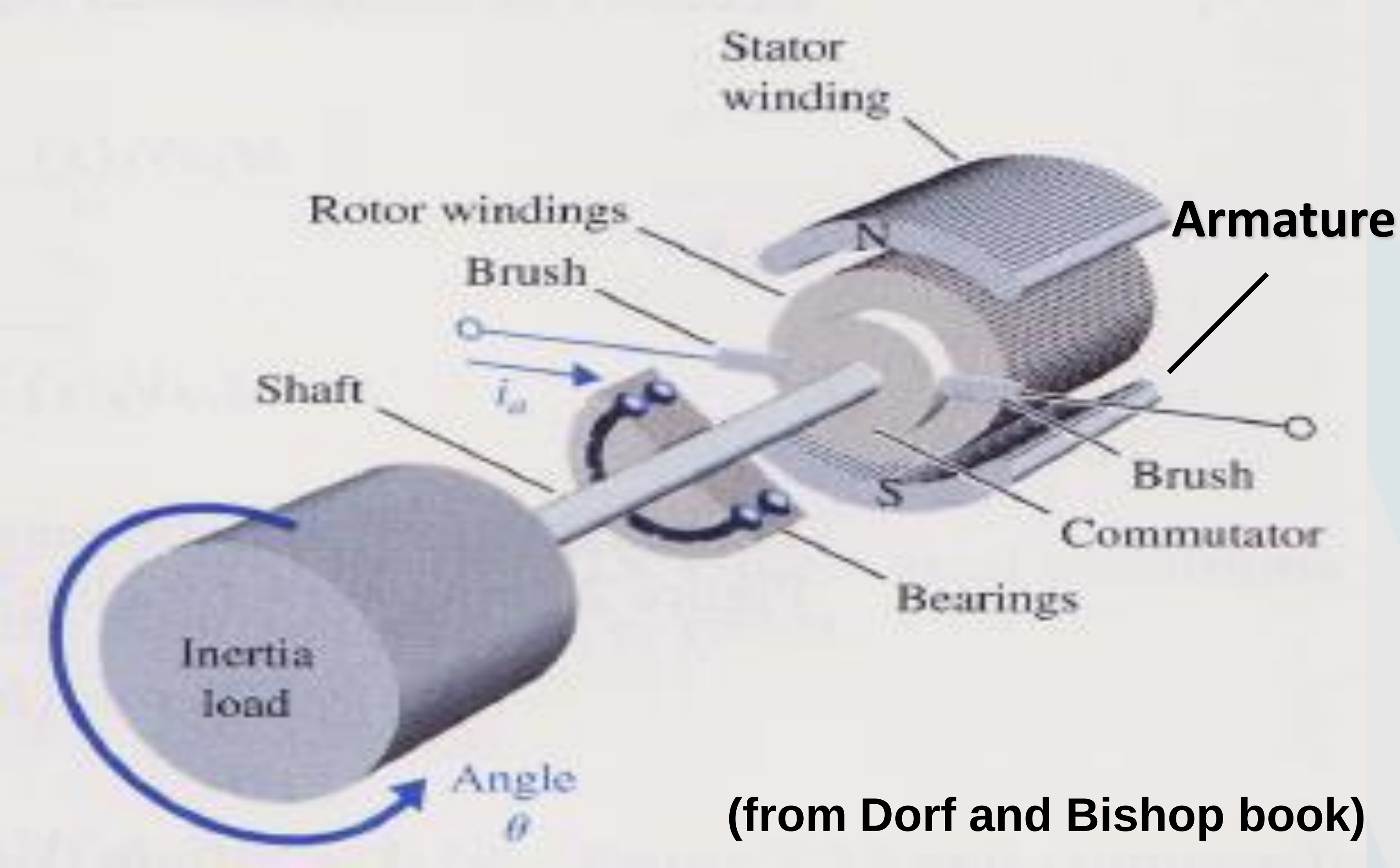
$$X(s) = G(s)U(s); \quad Y(s) = H(s)U(s); \quad W(s) = M(s)Y(s)$$



# What is DC motor?

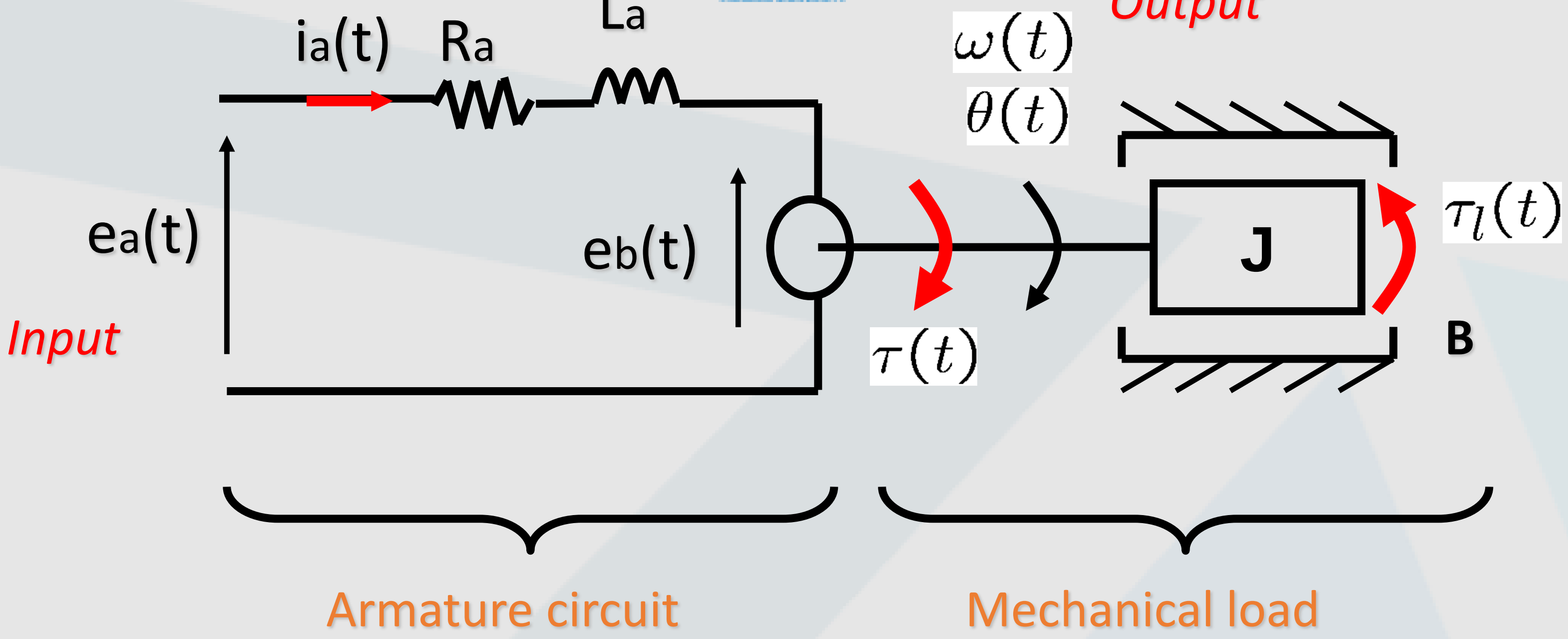
*An actuator, converting electrical energy into rotational mechanical energy*

# How does DC motor work?




(from Dorf and Bishop book)

# Model of DC motor




"$a$" :armature

- $e_a$ :applied voltage
- $i_a$ :armature current

"$b$" :back EMF

mechanical

- $\theta$ :angular position
- $\omega$ :angular velocity
- $J$ : rotor inertia
- $B$ : viscous friction

# Modeling of DC motor: time domain

- Armature circuit $$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

- Connection between mechanical/electrical parts
  - Motor torque $$\tau(t) = K_\tau i_a(t)$$
  - Back EMF $$e_b(t) = K_b \omega(t)$$

- Mechanical load

Load torque

$$J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t) - \tau_l(t)$$

- Angular position $$\omega(t) = \dot{\theta}(t)$$

# Modeling of DC motor: s-domain

- Armature circuit

$$I_a(s) = \frac{1}{R_a + L_a s}(E_a(s) - E_b(s))$$

- Connection between mechanical/electrical parts
  - Motor torque
  - Back EMF

$$T(s) = K_\tau I_a(s)$$

$$E_b(s) = K_b \Omega(s)$$

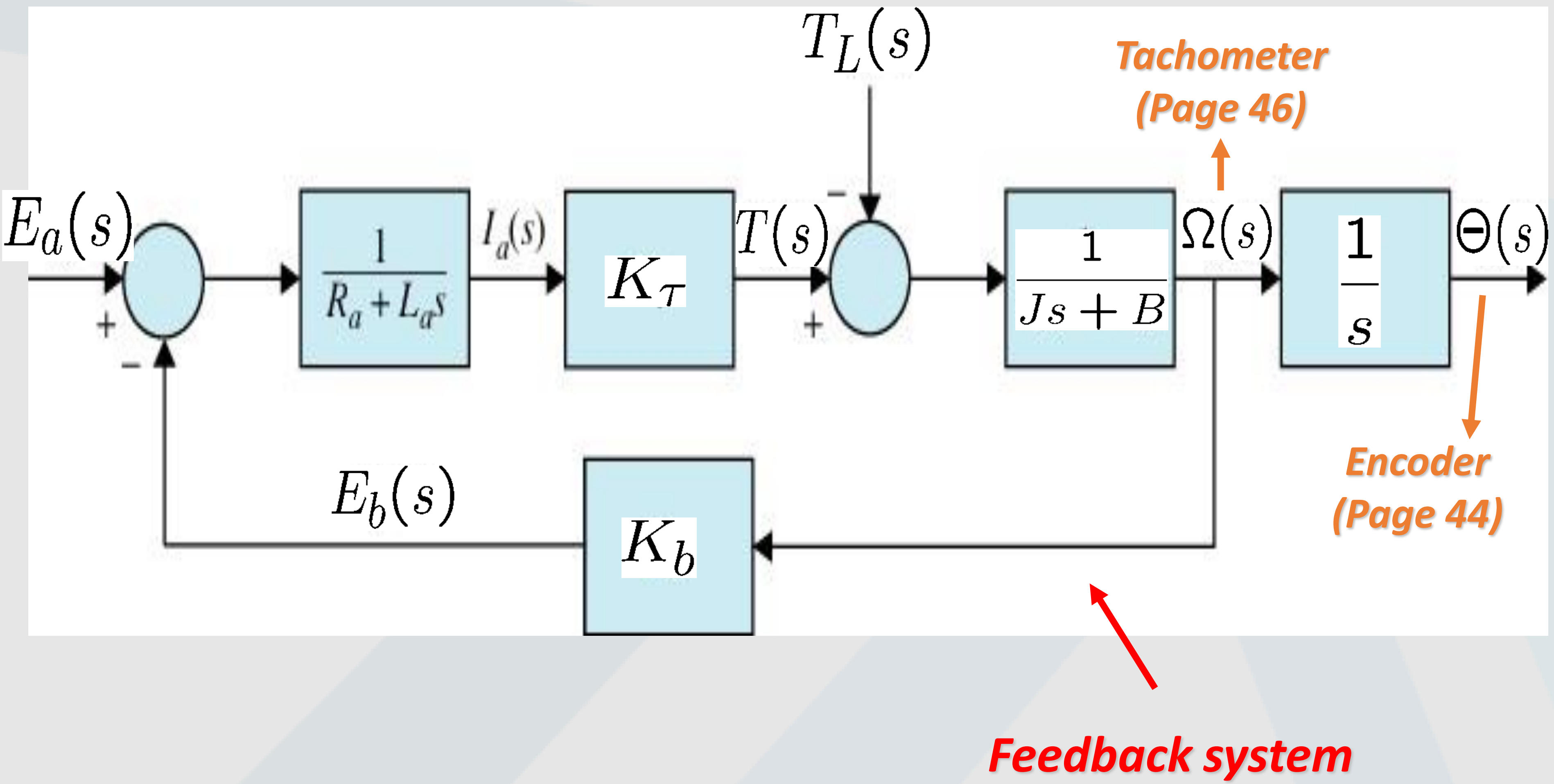
- Mechanical load

$$\Omega(s) = \frac{1}{Js + B}(T(s) - T_L(s))$$

- Angular position

$$\Theta(s) = \frac{1}{s}\Omega(s)$$

# DC motor: Block diagram



# Block Diagram Models

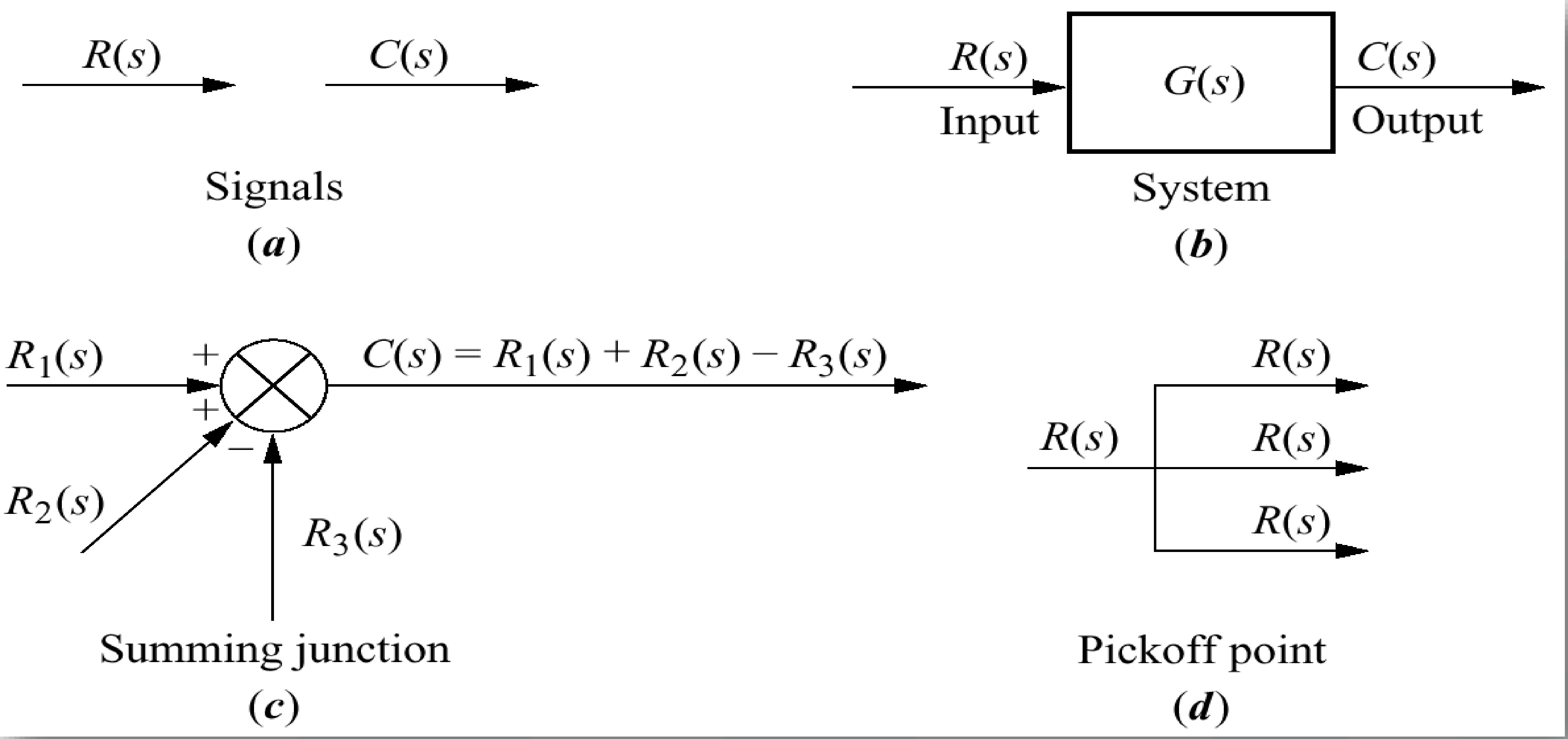
- A block diagram of a system is a pictorial representation of the functions performed by each component and of the flow of signals.

- Such diagram depicts the interrelationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating more realistically the signal flows of the actual system.

- Transfer function can be represented as a block diagram:

$$R(s) \longrightarrow \boxed{\dfrac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0}} \longrightarrow C(s)$$

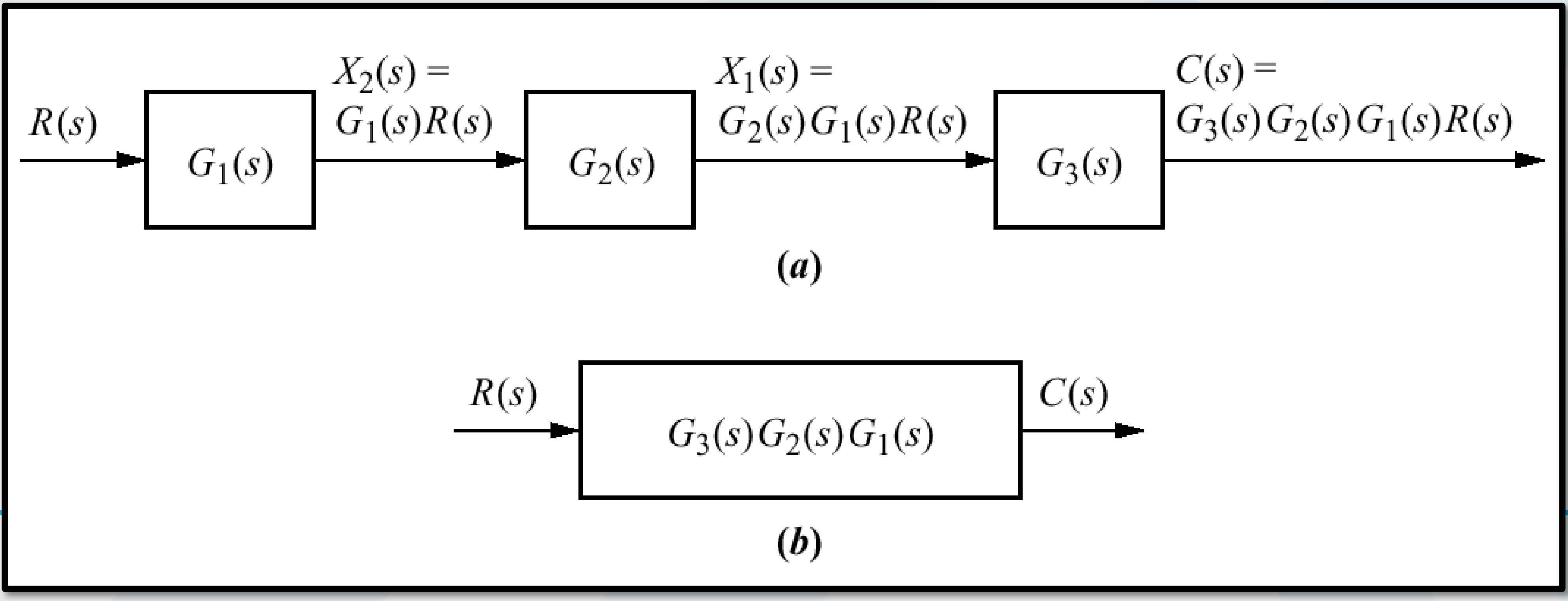
# Components Of a block diagram for a LTI system



# Block Diagram Reduction

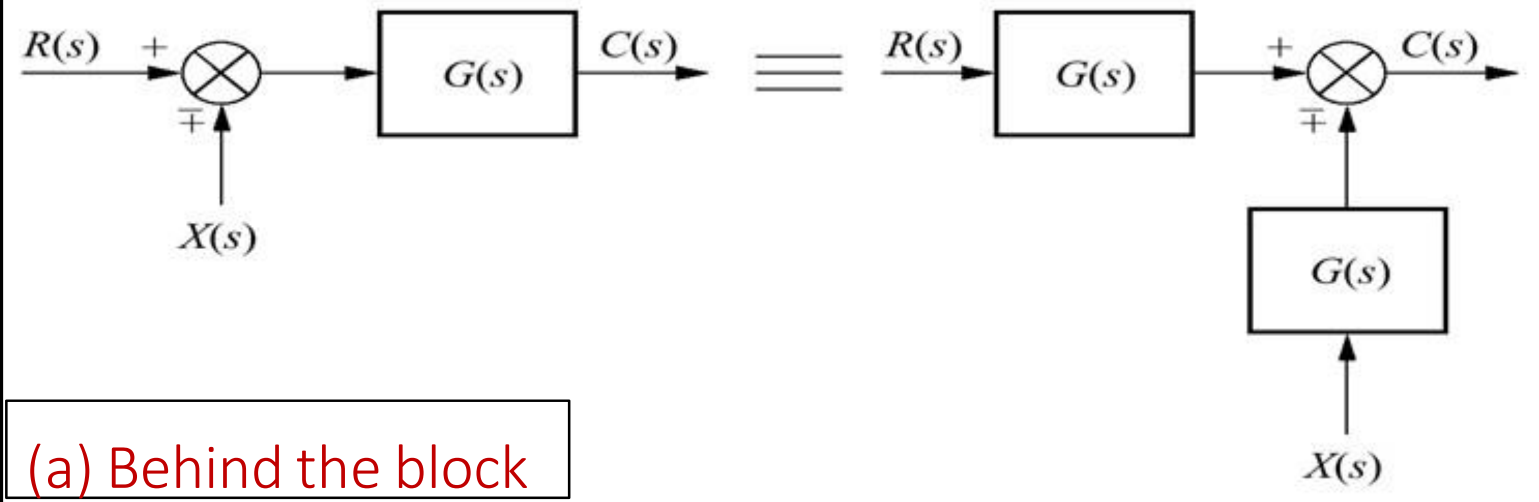
- **Rules for reduction of the block diagram:**

1. Any number of cascaded blocks can be reduced by a single block representing transfer function being a product of transfer functions of all cascaded blocks.

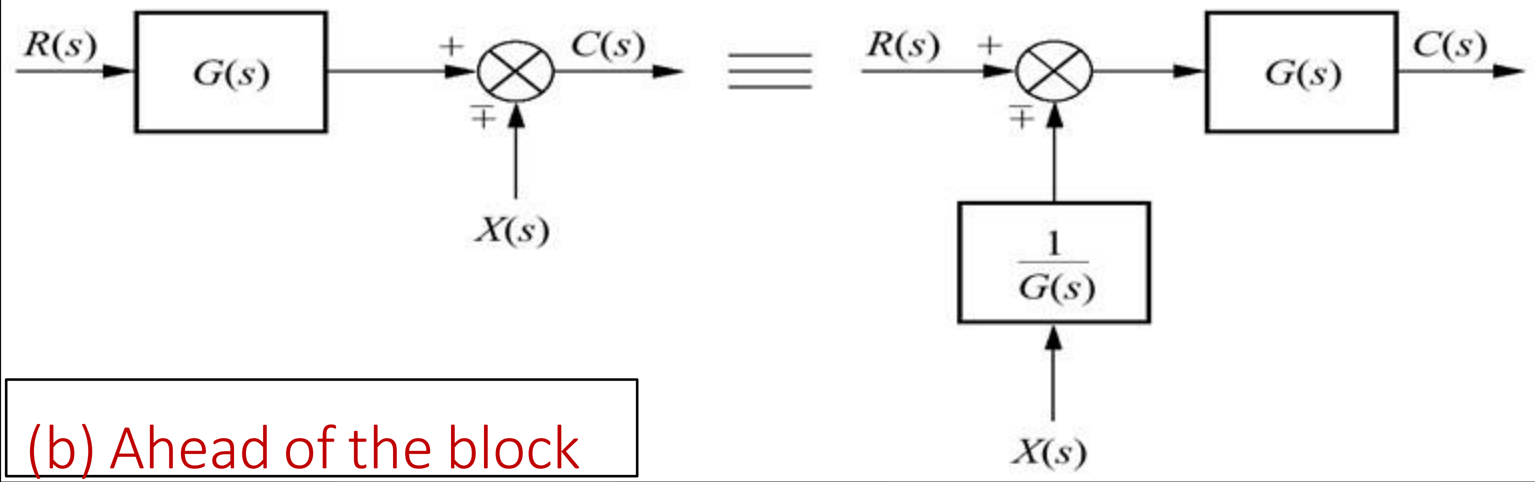


# Block Diagram Reduction

## 2. Moving a summing point



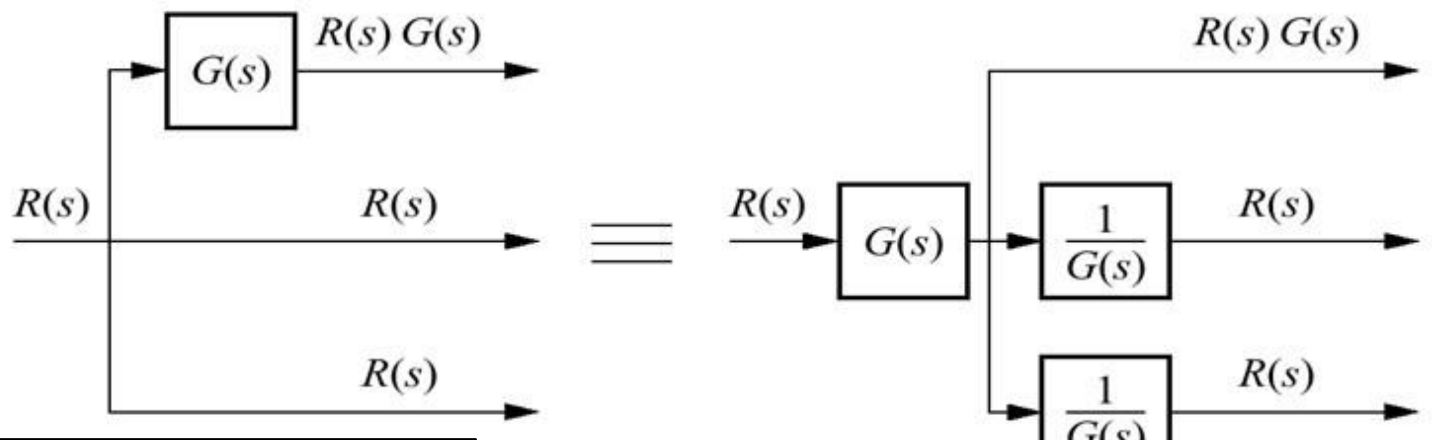

(a) Behind the block



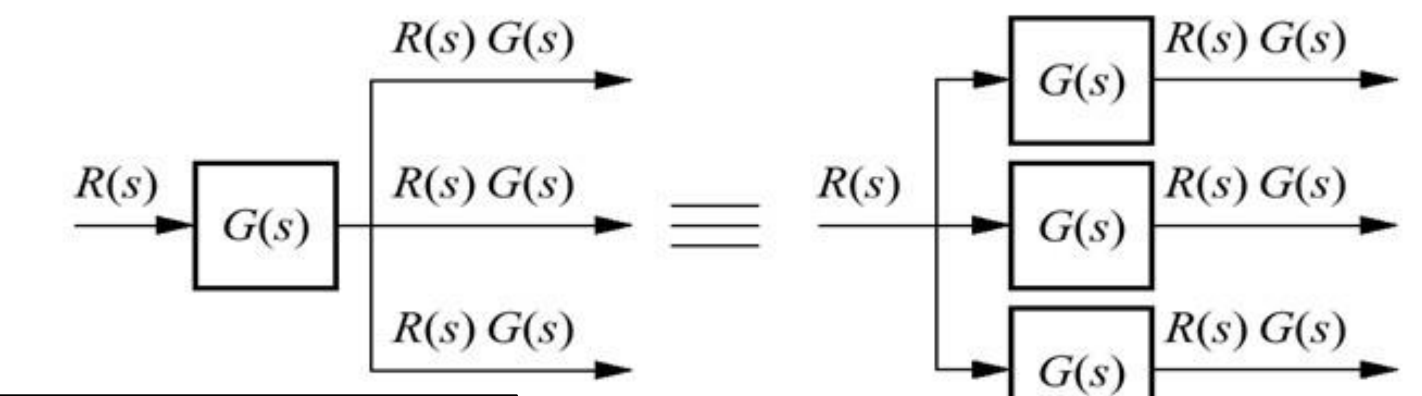

(b) Ahead of the block

# Block Diagram Reduction

## 3. Moving a pickoff point



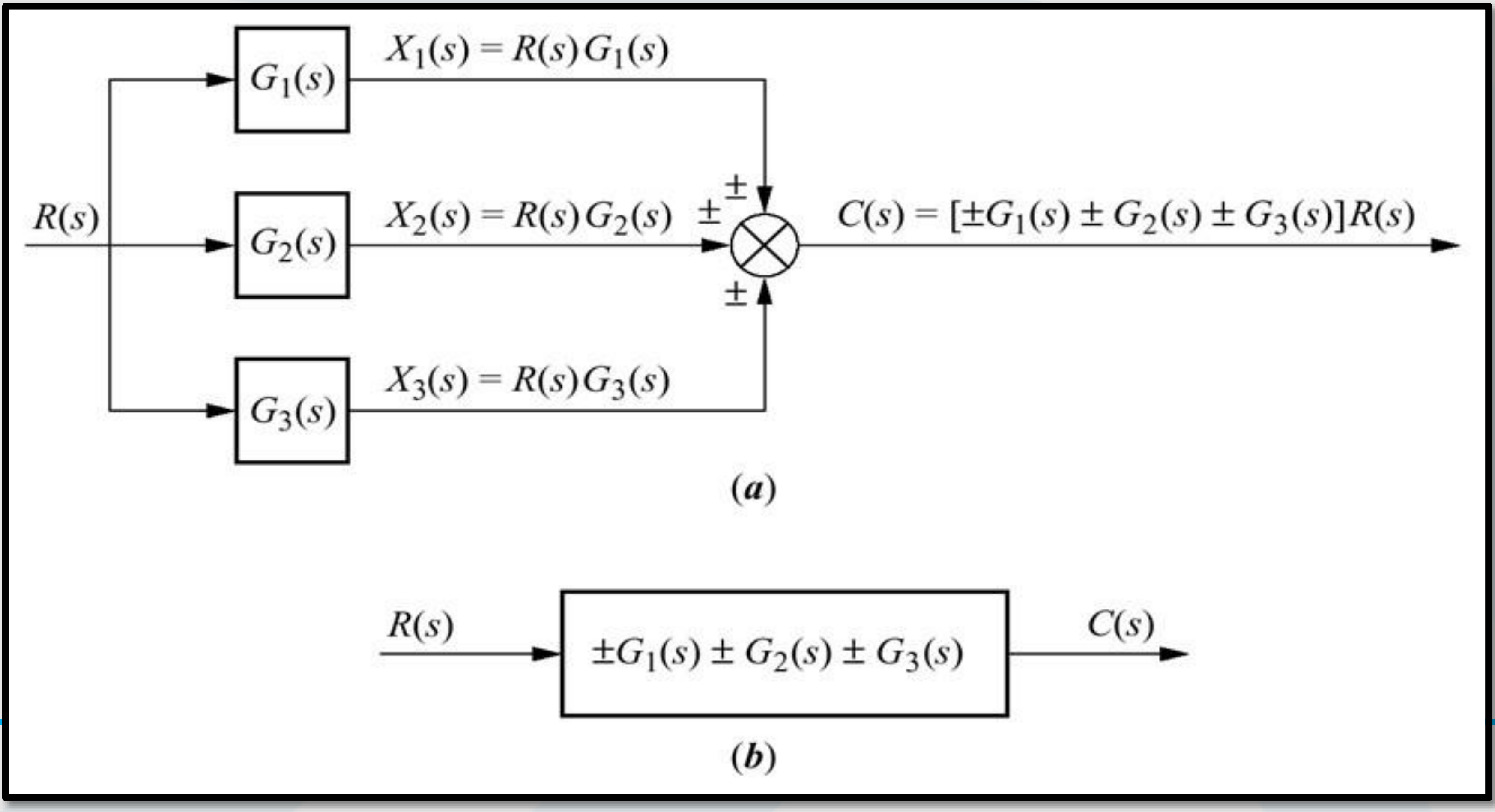

(a) Behind the block




(b) Ahead of the block

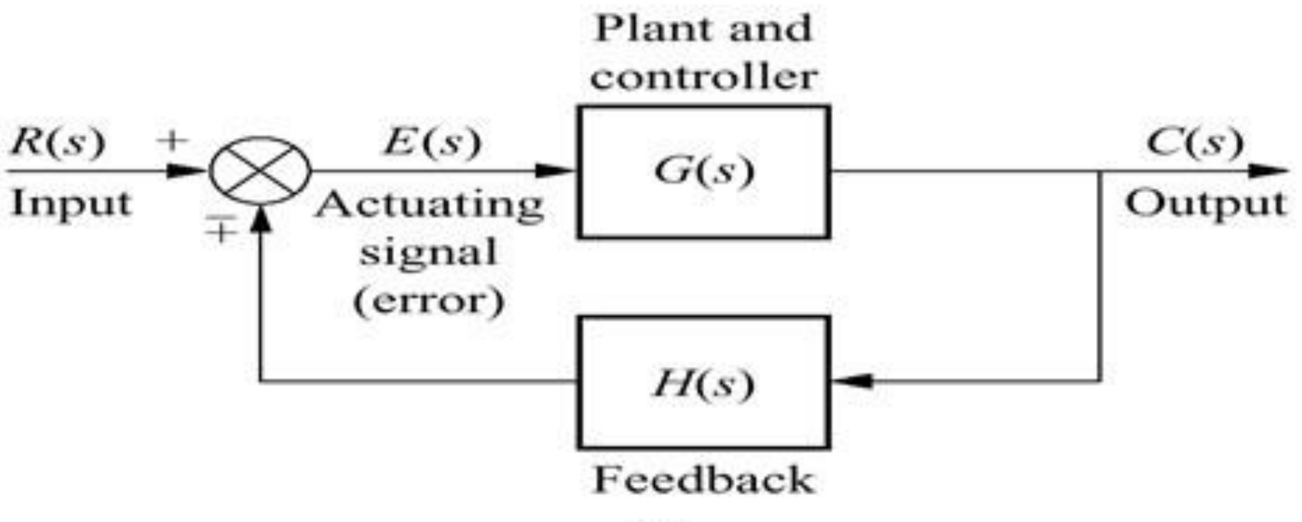
# Block Diagram Reduction

4. Equivalent transfer function for parallel subsystems is the sum of their transfer functions



# Block Diagram Reduction

5. Feedback control system



(a)

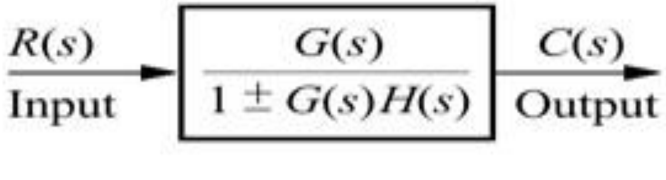
$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)C(s)$$

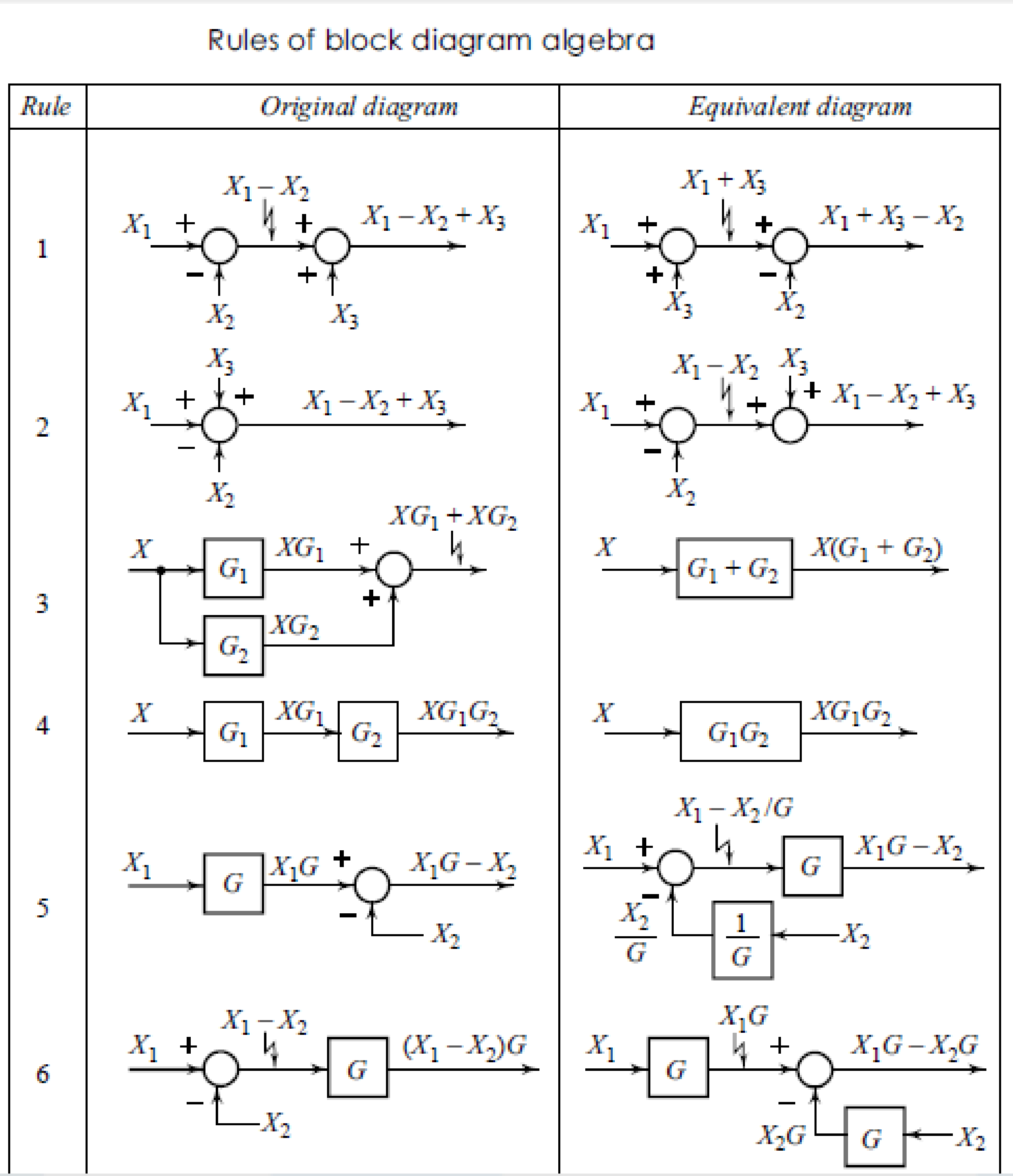
$$C(s) = G(s)[R(s) - H(s)C(s)]$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

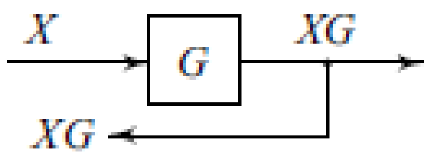
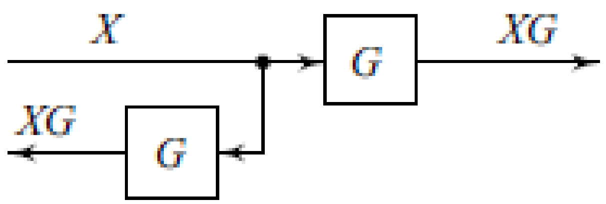
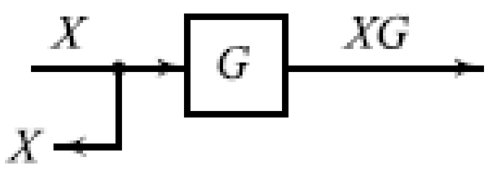
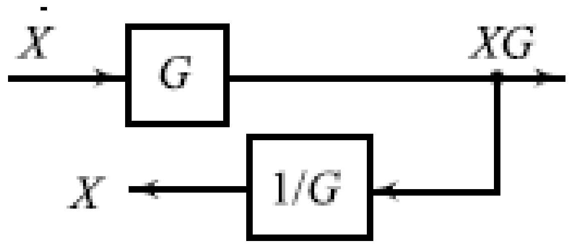
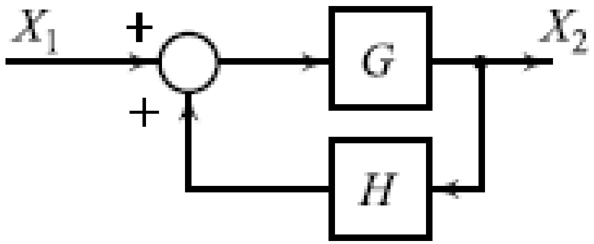
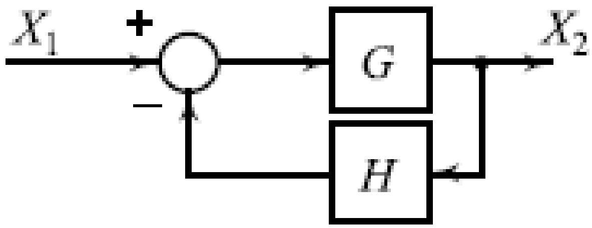
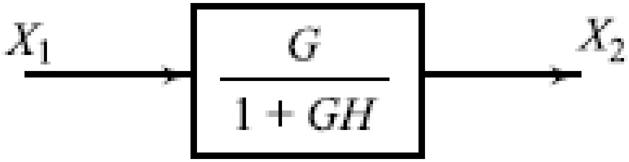


(b)

# Block Diagra



# Block Diagram Reduction

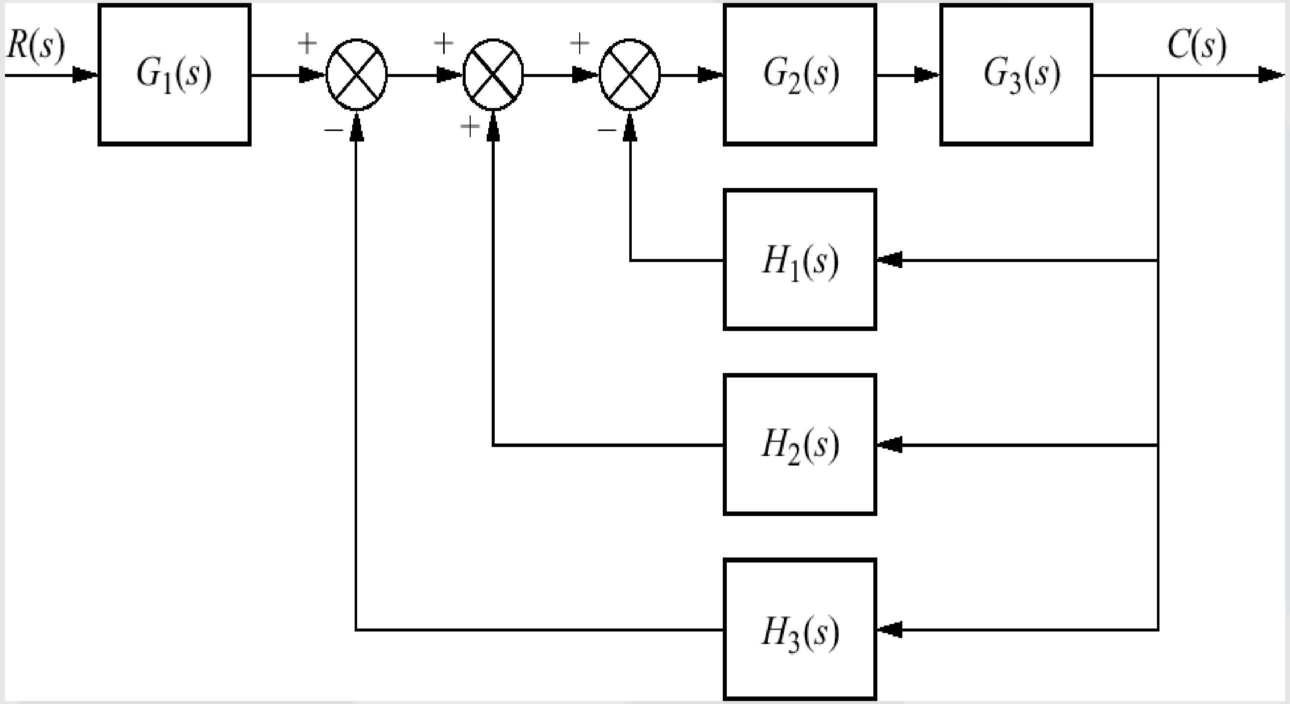
| Rule | Original diagram | Equivalent diagram |
|---|---|---|
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |

# Block Diagram Reduction

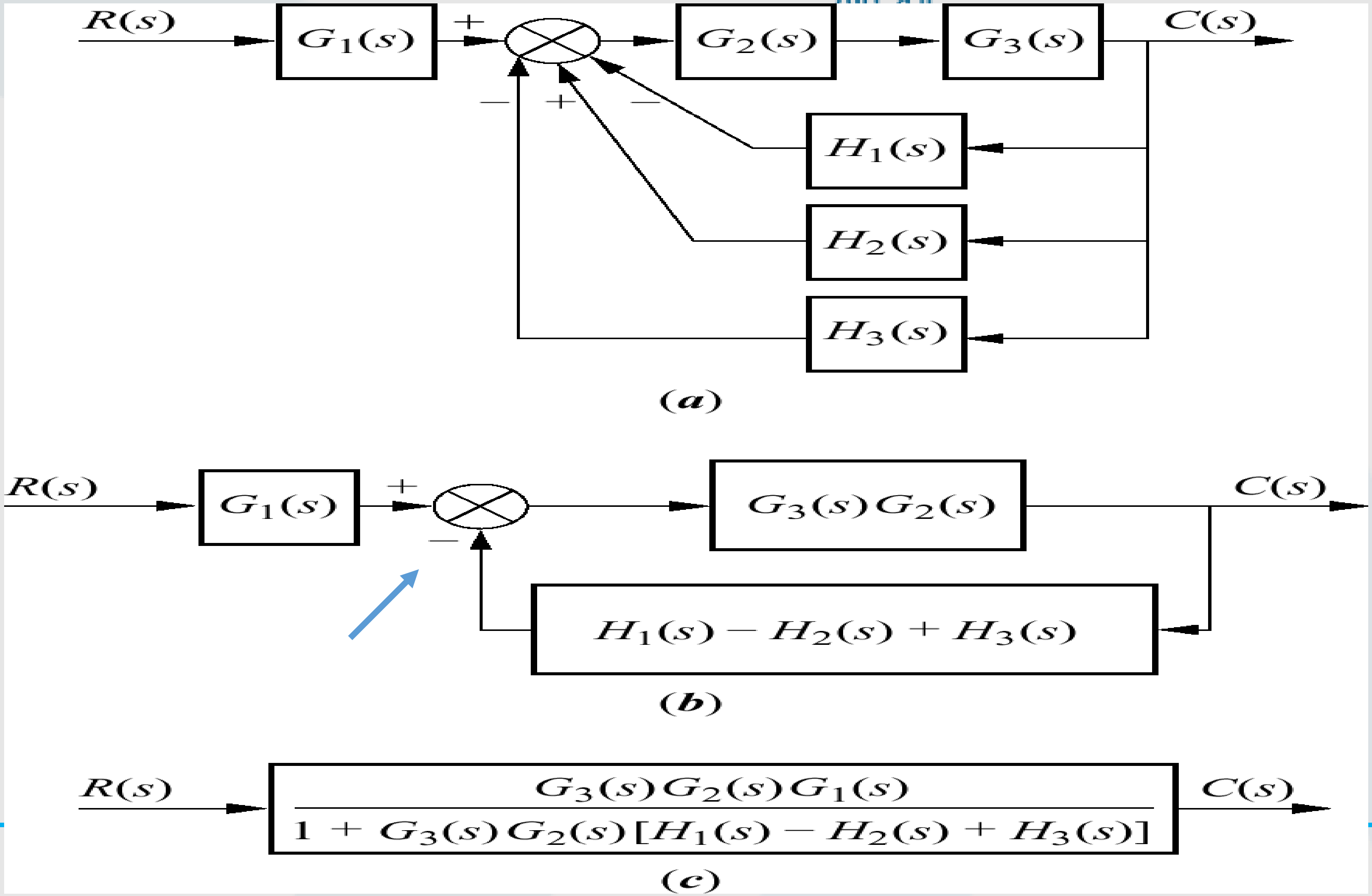
- Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.
  - **Rule 1** − Check for the blocks connected in series and simplify.
  - **Rule 2** − Check for the blocks connected in parallel and simplify.
  - **Rule 3** − Check for the blocks connected in feedback loop and simplify.
  - **Rule 4** − If there is difficulty with take-off point while simplifying, shift it towards right.
  - **Rule 5** − If there is difficulty with summing point while simplifying, shift it towards left.
  - **Rule 6** − Repeat the above steps till you get the simplified form, i.e., single block.

# Block diagram reduction via familiar forms for Example1

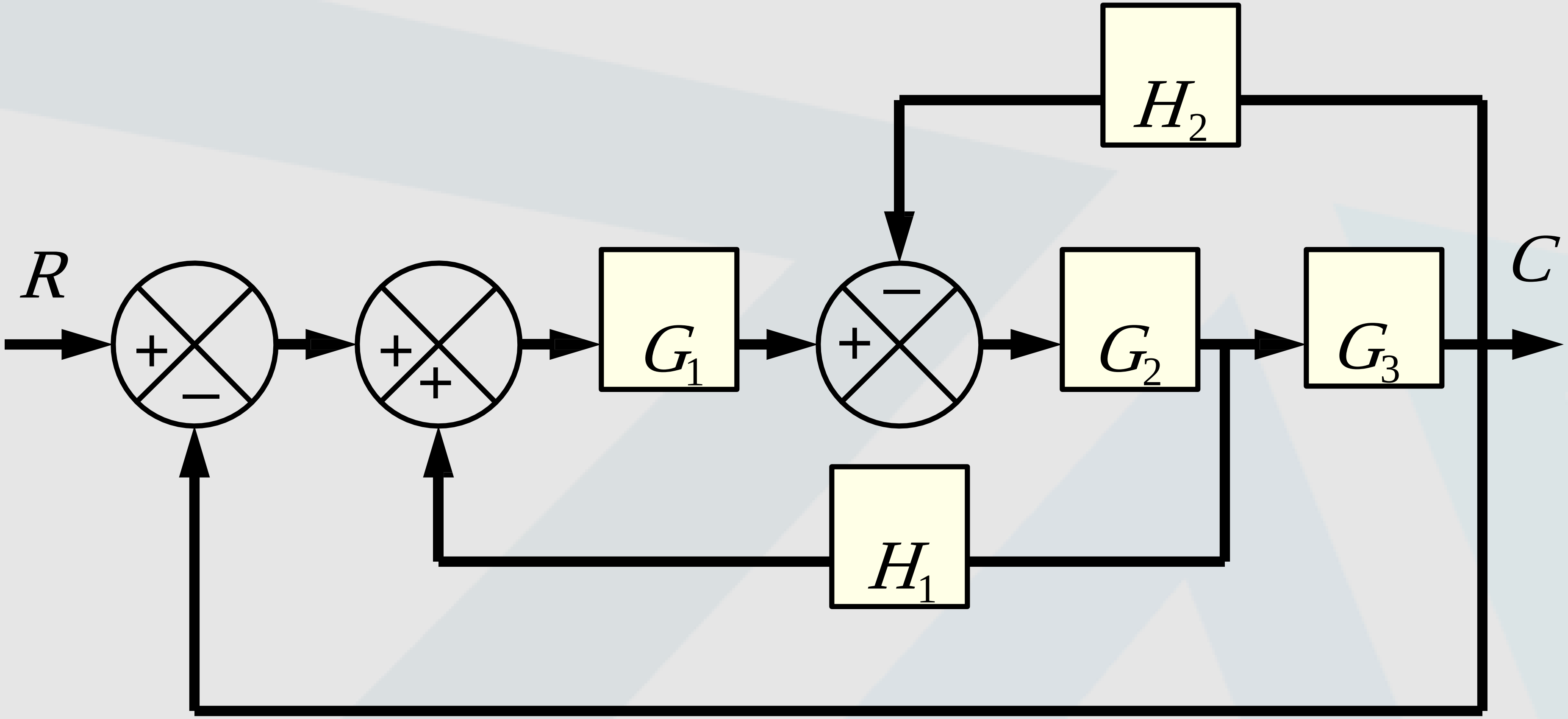
**Problem:** Reduce the block diagram shown in figure to a single transfer function



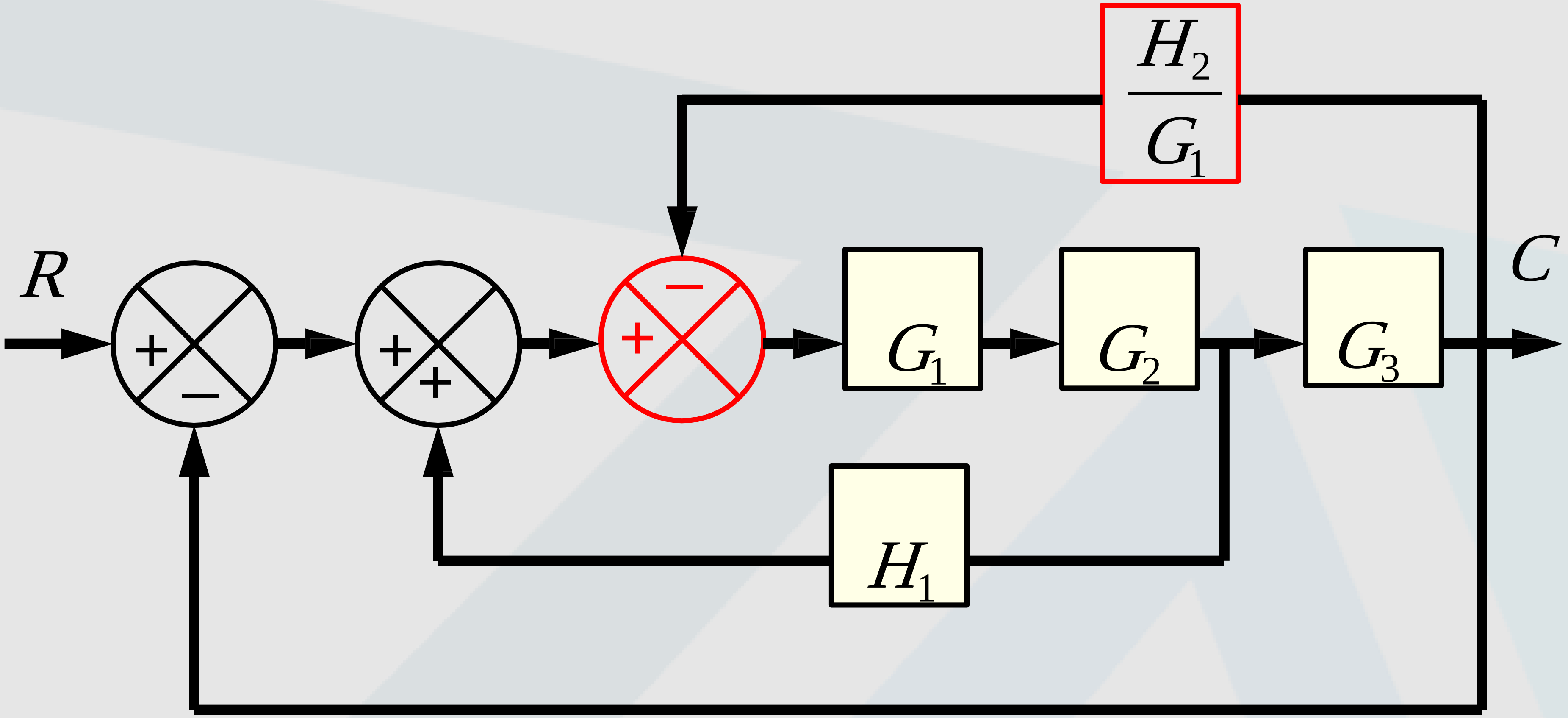
# Block diagram reduction via familiar forms for Example Cont.



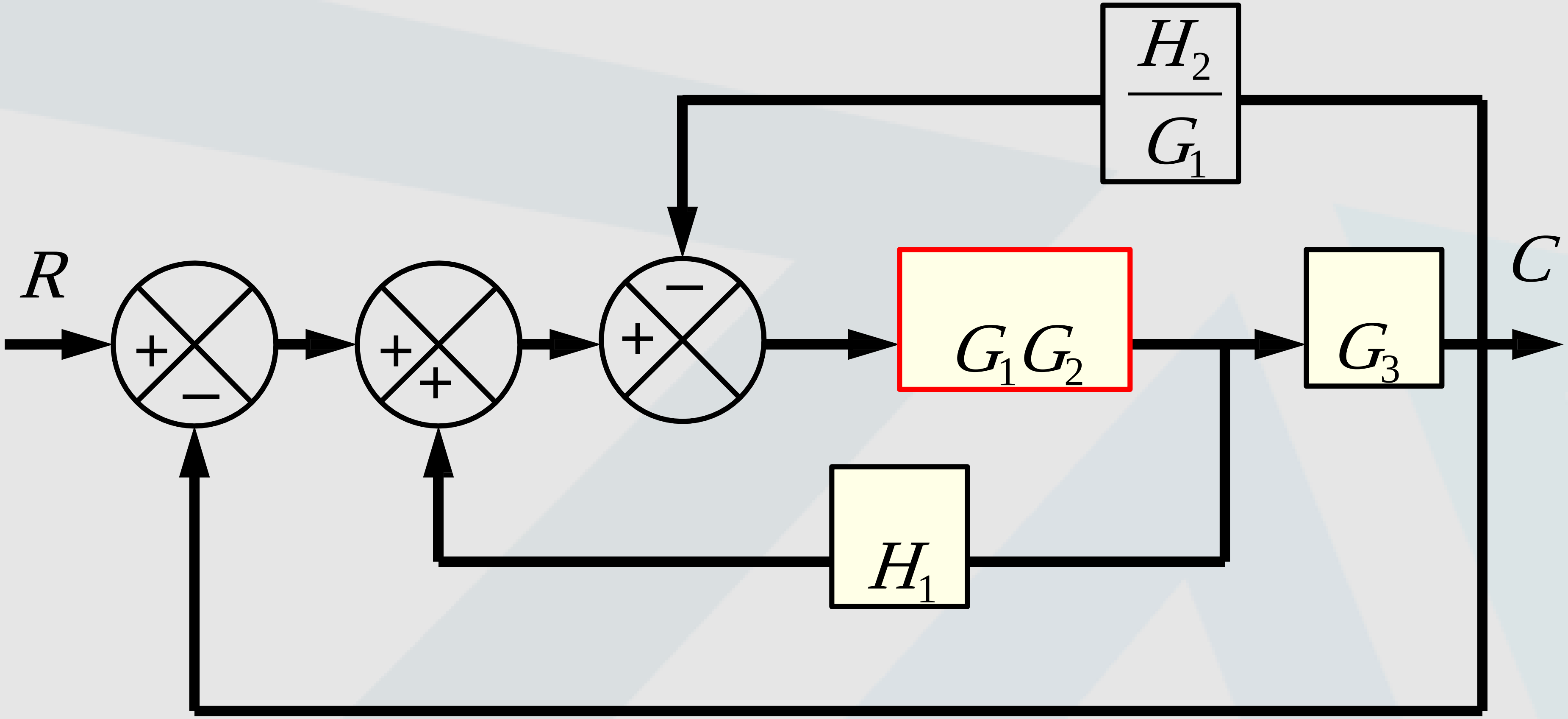
# Example 2



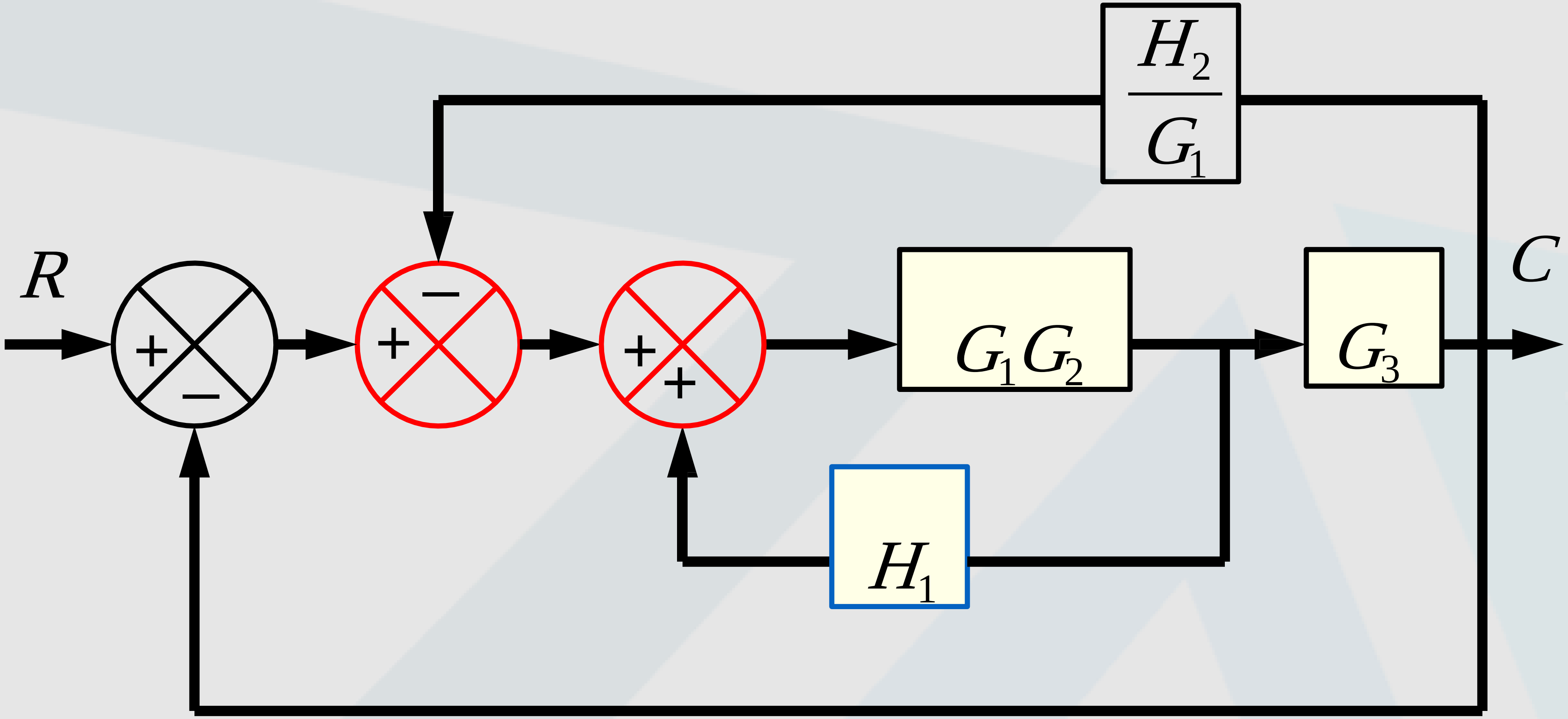
# Moving the summing point ahead of G1, we have:



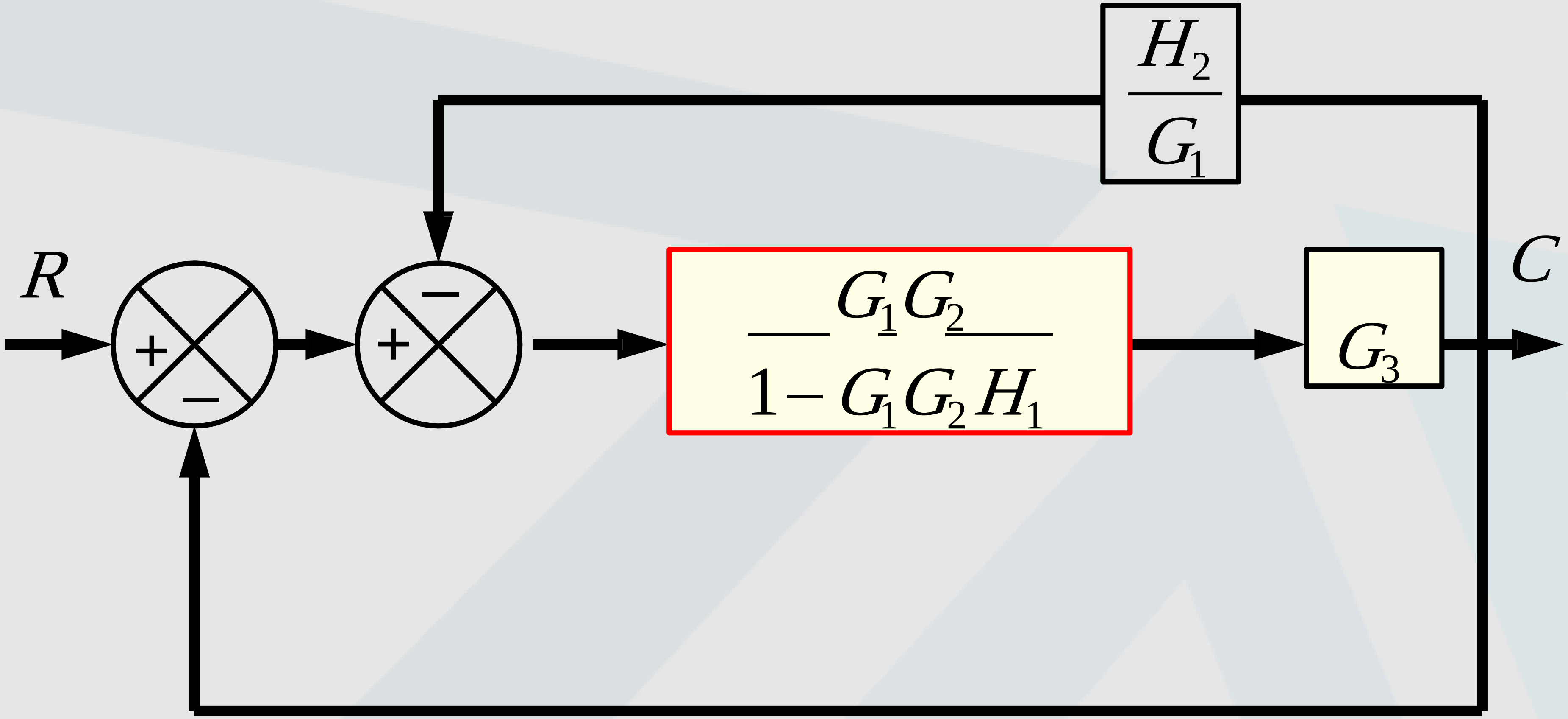
# Combing G1 and G2 in Cascade, we get:



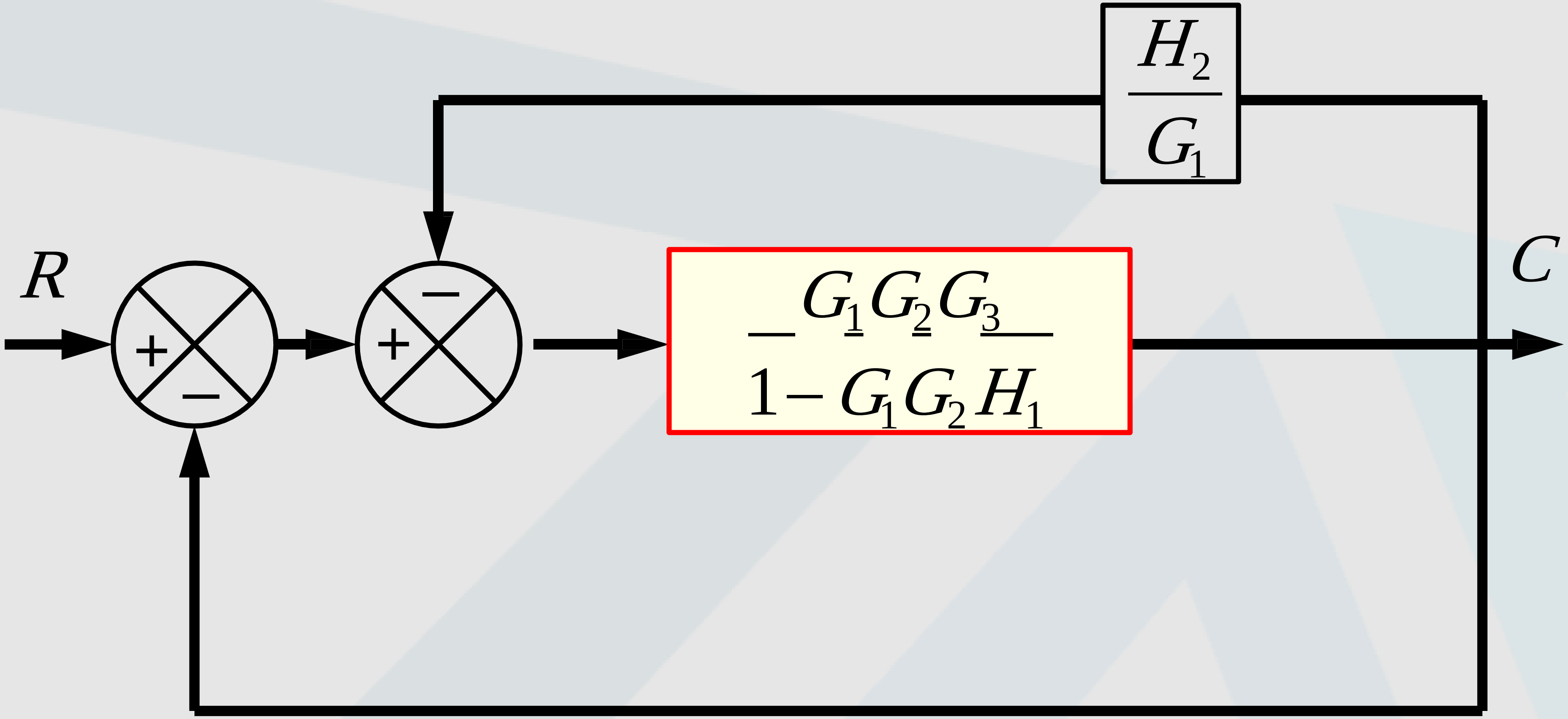
# Eliminating the feedback loop G1, G2 and H1 we get:



Combing the two blocks in Cascade, we get



Similarly eliminating the second feedback loop we get:

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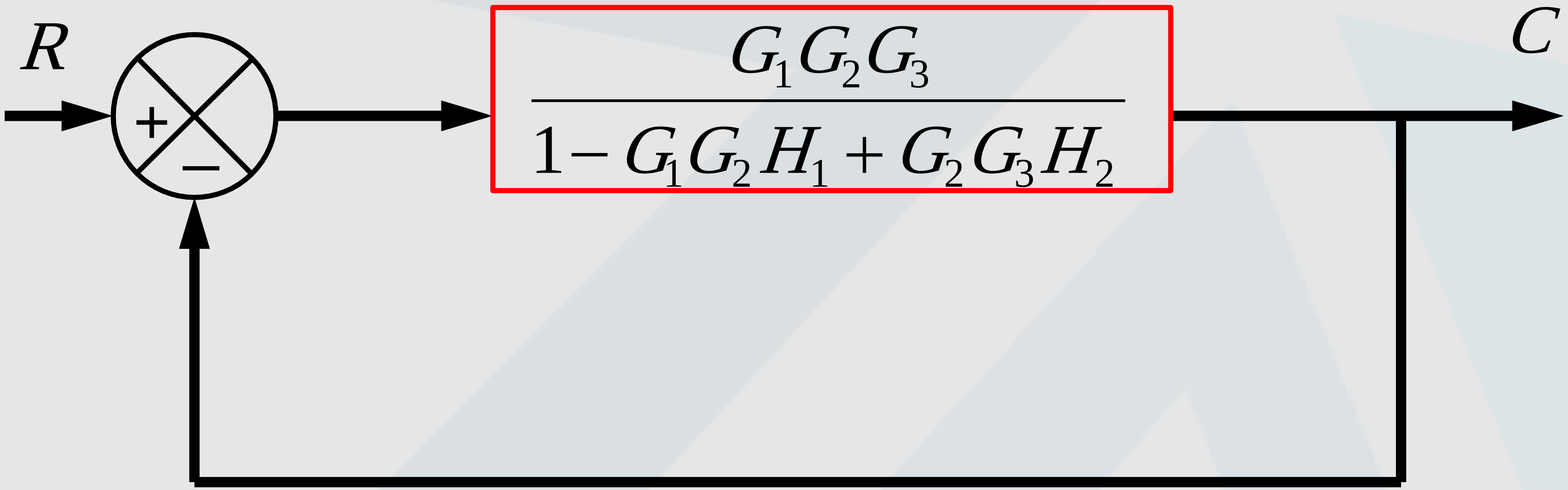
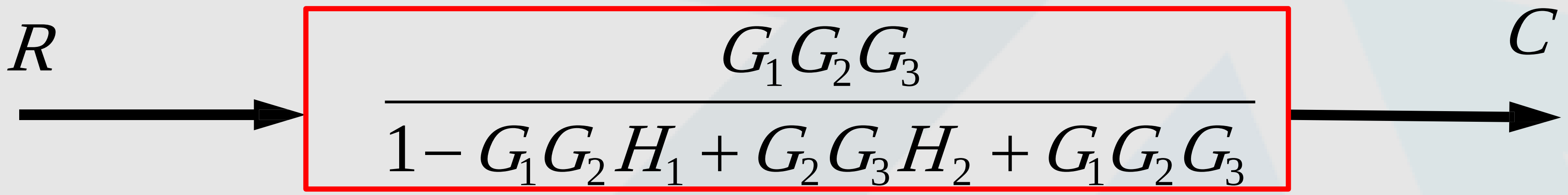
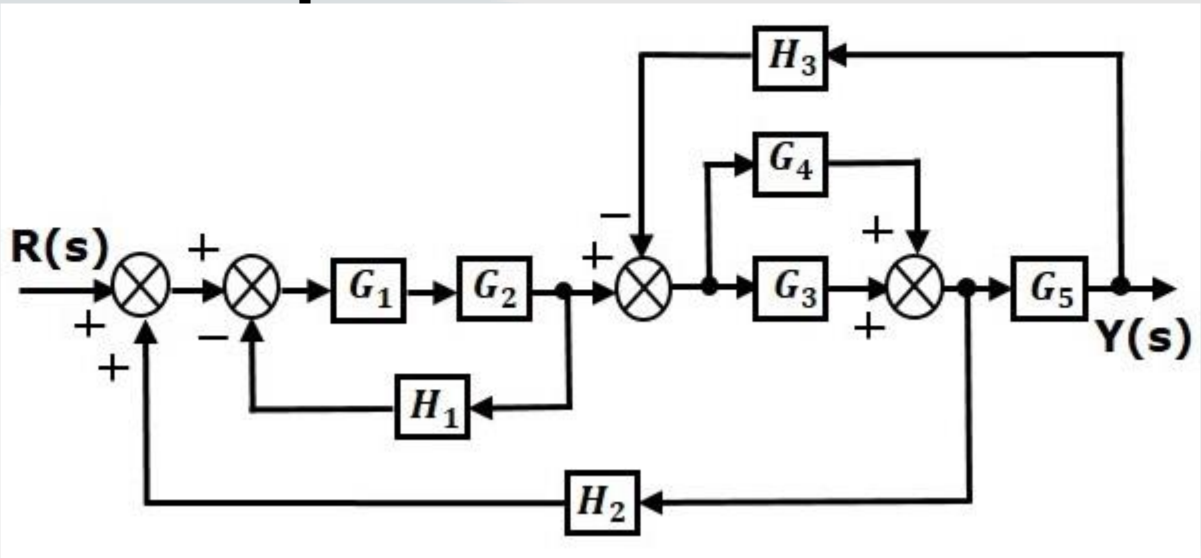
Similarly eliminating the third feedback loop we get:

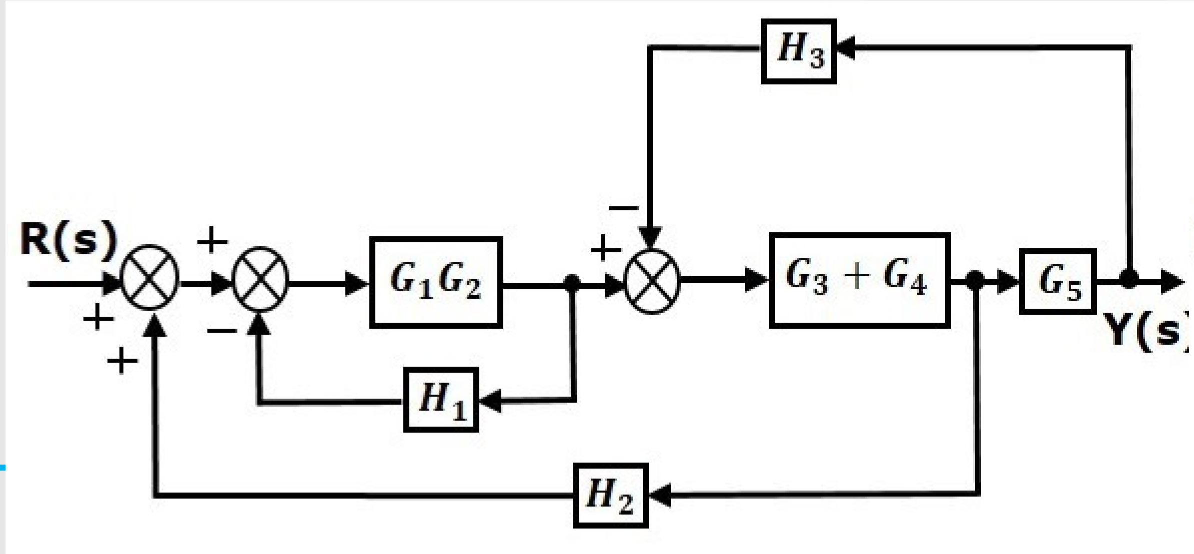


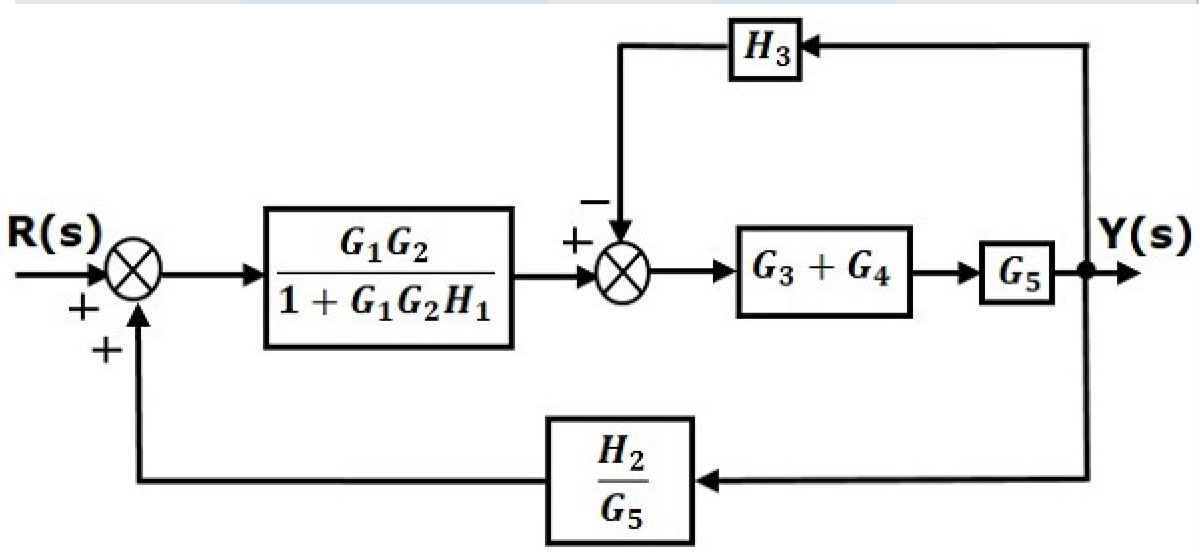
# The system is reduced to the following block diagram:



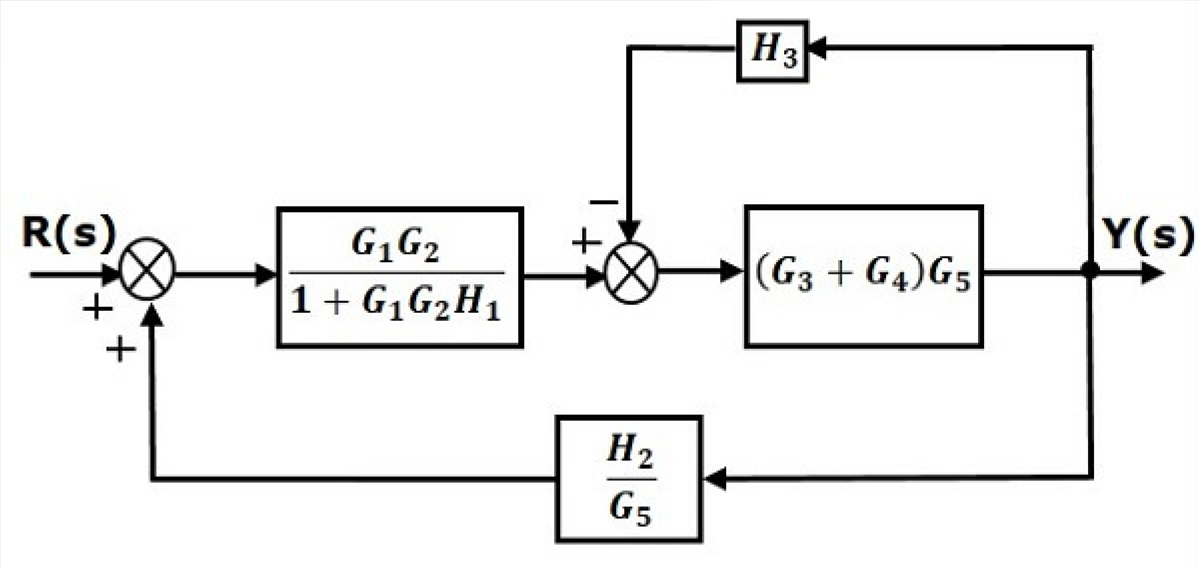
# Example3

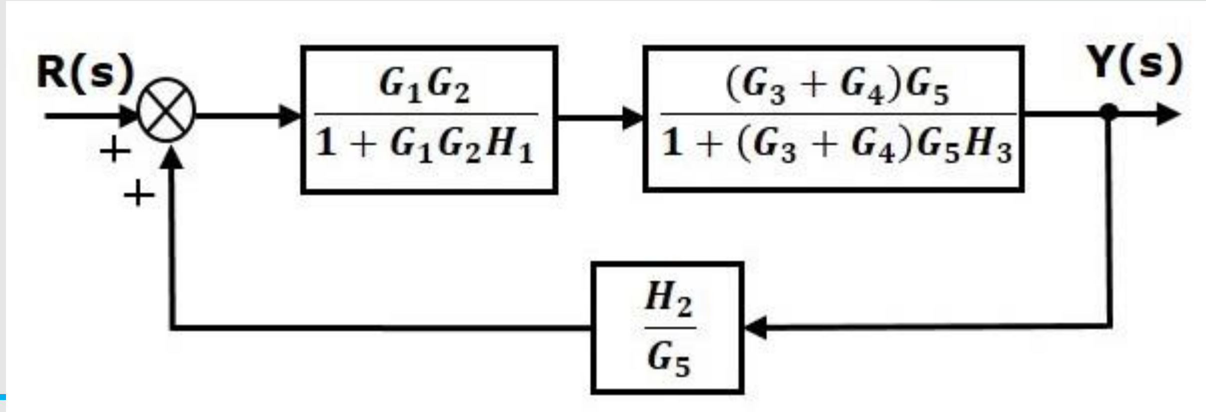

R(s)
Y(s)
$G_1$
$G_2$
$G_3$
$G_4$
$G_5$
$H_1$
$H_2$
$H_3$

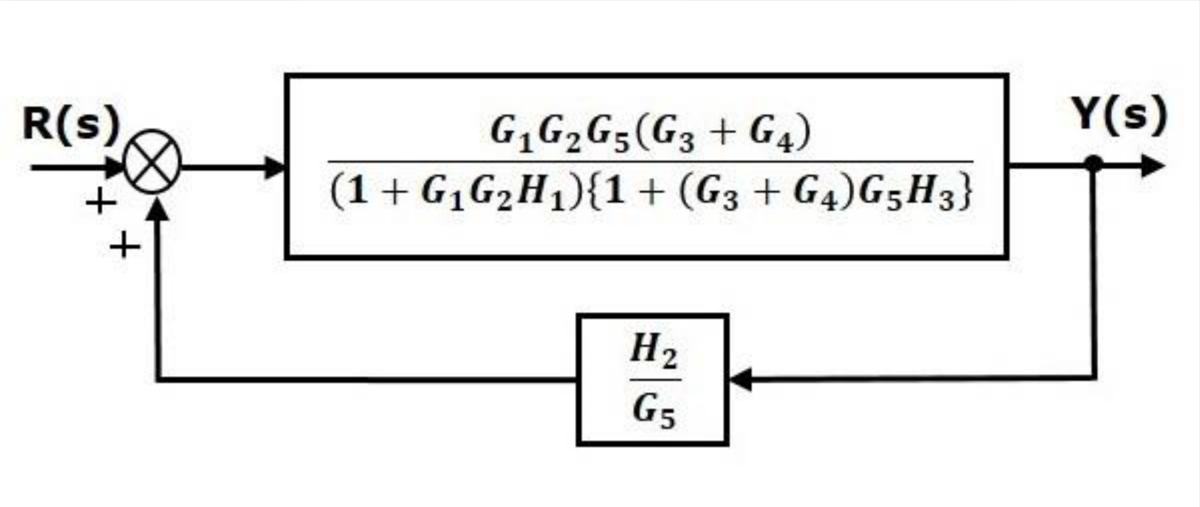

R(s)
$G_1G_2$
$G_3 + G_4$
$G_5$
$H_1$
$H_2$
$H_3$

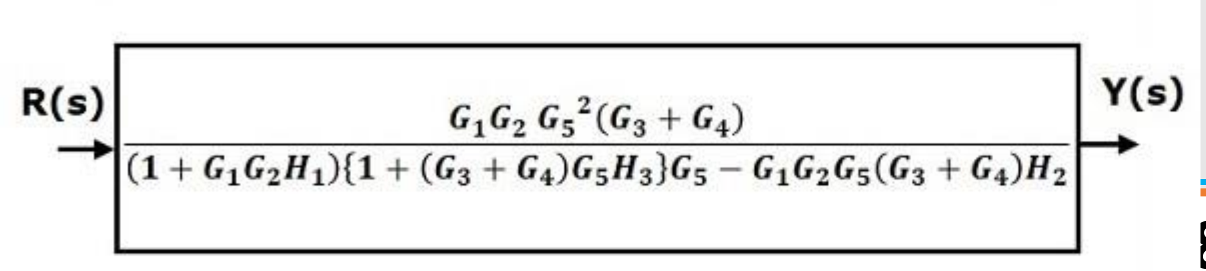

R(s)
Y(s)
$\frac{G_1G_2}{1 + G_1G_2H_1}$
$G_3 + G_4$
$G_5$
$H_3$
$\frac{H_2}{G_5}$

# Example3

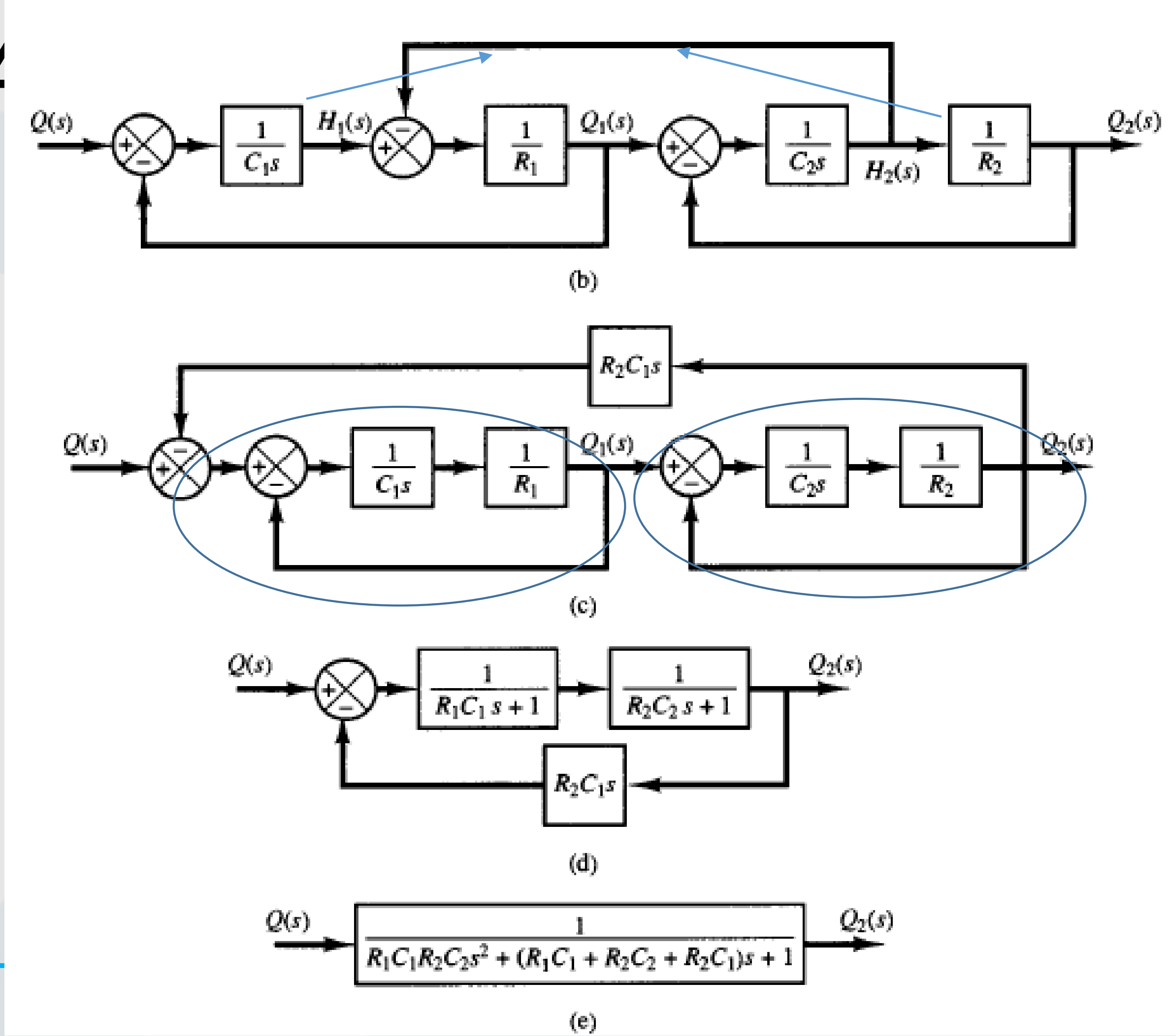

R(s)
Y(s)
$H_3$
$\frac{G_1G_2}{1+G_1G_2H_1}$
$(G_3+G_4)G_5$
$\frac{H_2}{G_5}$
+
+
+
−


R(s)
Y(s)
$\frac{G_1G_2}{1+G_1G_2H_1}$
$\frac{(G_3+G_4)G_5}{1+(G_3+G_4)G_5H_3}$
$\frac{H_2}{G_5}$
+
+


R(s)
Y(s)
$\frac{G_1G_2G_5(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}}$
$\frac{H_2}{G_5}$
+
+

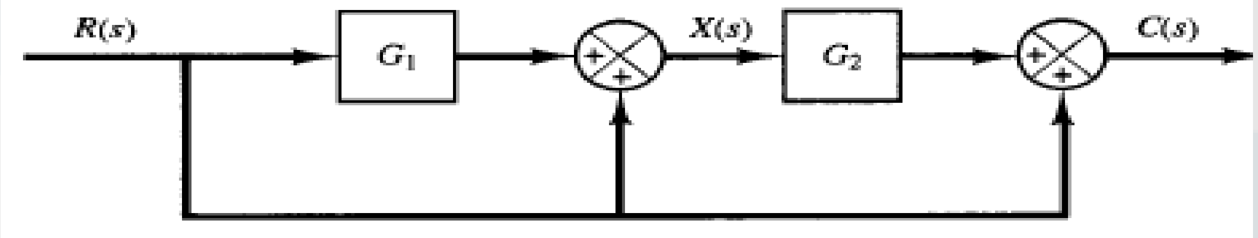

R(s)
Y(s)
$\frac{G_1G_2\,{G_5}^2(G_3+G_4)}{(1+G_1G_2H_1)\{1+(G_3+G_4)G_5H_3\}G_5-G_1G_2G_5(G_3+G_4)H_2}$

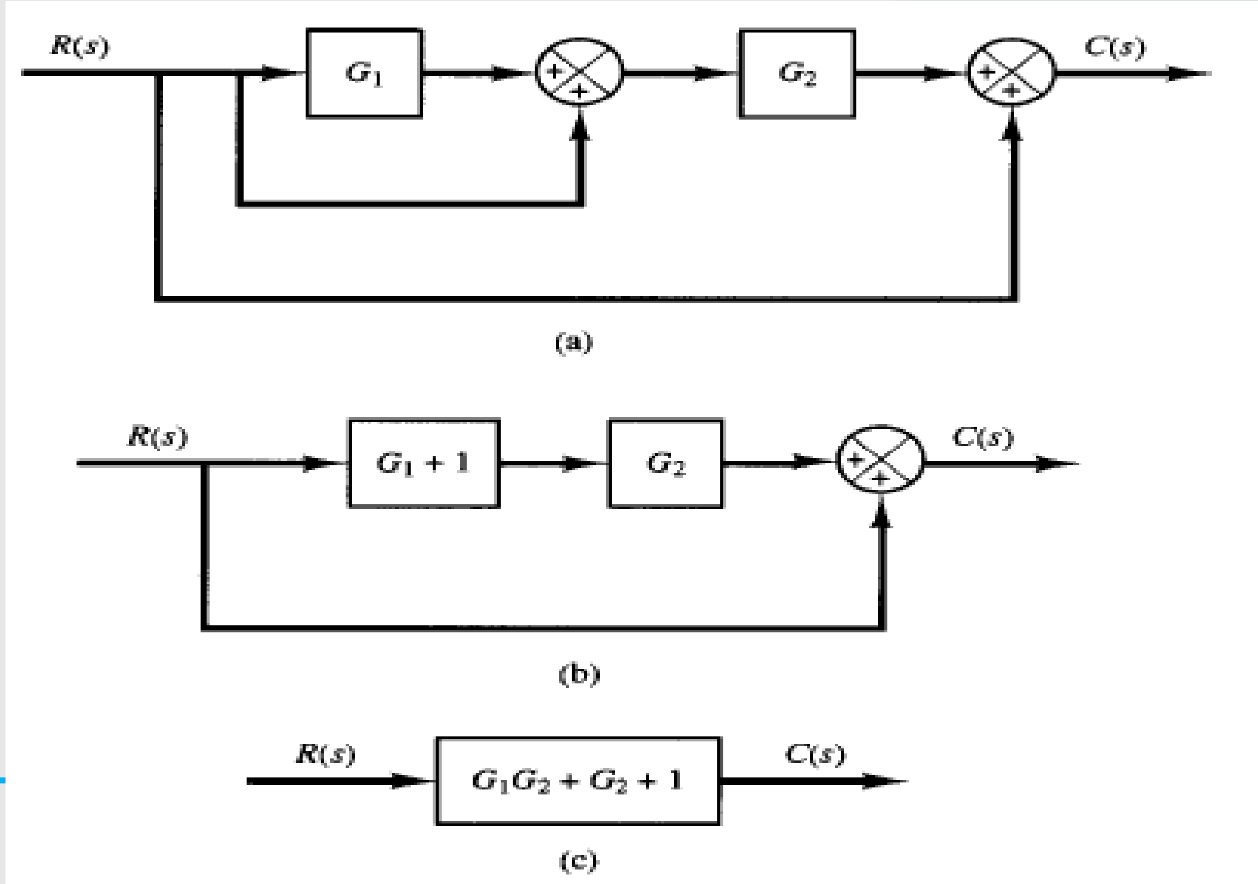
# Example4



Q(s)
1/(C1s)
H1(s)
1/R1
Q1(s)
1/(C2s)
H2(s)
1/R2
Q2(s)
(b)
R2C1s
Q(s)
1/(C1s)
1/R1
Q1(s)
1/(C2s)
1/R2
Q2(s)
(c)
Q(s)
1/(R1C1s+1)
1/(R2C2s+1)
Q2(s)
R2C1s
(d)
Q(s)
1/(R1C1R2C2s^2 + (R1C1 + R2C2 + R2C1)s + 1)
Q2(s)
(e)

# Example5




$$X(s) = G_1R(s) + R(s)$$

The output signal $C(s)$ is the sum of $G_2X(s)$ and $R(s)$. Hence

$$C(s) = G_2X(s) + R(s) = G_2[G_1R(s) + R(s)] + R(s)$$

And so we have the same result as before:

$$\frac{C(s)}{R(s)} = G_1G_2 + G_2 + 1$$