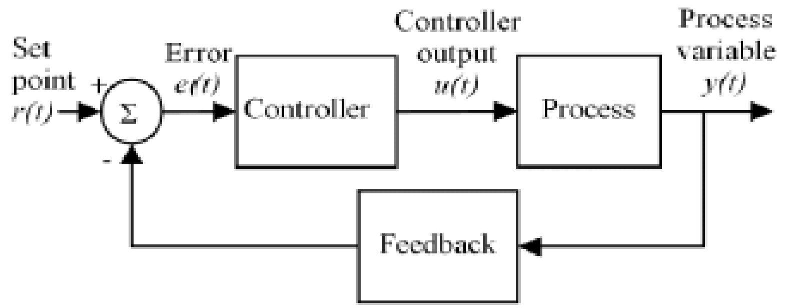

Lecture (4)

Time Domain Analysis of Feedback Control Systems Performance and Characteristics –Part1

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References

- Control Systems Course, professor Aniket Khandekar, Zeal college of engineering and Research, Pune.
- Gopal, M. - Control Systems_ Principles and Design 3rd edition-Tata McGraw Hill Publishing Co. Ltd. (2008)
- Modern Control Systems, Richard C. Dorf and Robert H. Bishop, Prentice Hall, 12th edition, 2010, ISBN-10: 0-13-602458-0
- Modelling, Dynamics and Control, University of Sheffield, John Anthony Rossiter.
- https://www.tutorialspoint.com/control_systems/

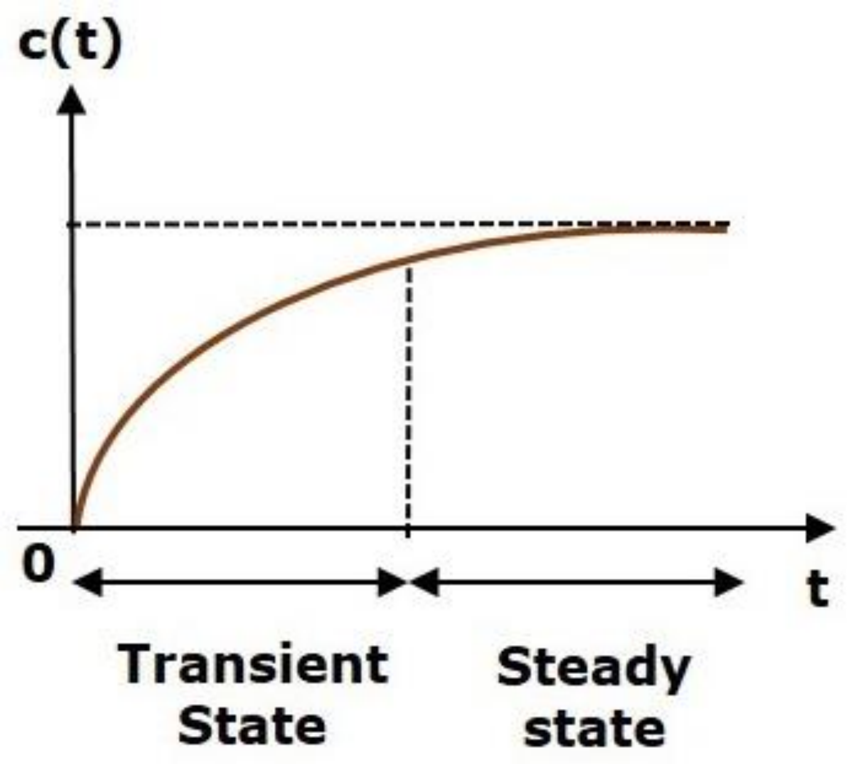
Contents

What is Time Response?

- If the output of control system for an input varies with respect to time, then it is called the **time response** of the control system.
- The time response consists of two parts.
  - Transient response
  - Steady state response

Mathematically, we can write the time response c(t) as

$$c(t)=c_{tr}(t)+c_{ss}(t)$$



Where,

- $c_{tr}(t)$ is the transient response
- $c_{ss}(t)$ is the steady state response

Order and Type of a system

- **Order of the system:**
  - Consider a system defined by the transfer function:

$$T(s) = \frac{C(s)}{R(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$

  - The order of this system is **n** which is defined by the highest power for s in the denominator.

- **Examples:**

1st order system

$$\frac{C(s)}{R(s)} = \frac{5}{4s+1}$$

2nd order system

$$\frac{C(s)}{R(s)} = \frac{10s}{s^2 + 4s + 4}$$

4th order system

$$\frac{C(s)}{R(s)} = \frac{10s^2}{3s^4 + 2s^3 + s^2 + 4s + 3}$$

Order and Type of a system

- **The system type Number:**
  - It is defined as the number of poles at the origin of the open loop transfer function G(s)H(s).
  - Consider the open loop transfer function of a system as :

$$G(s)H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{s^c (a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0)}$$

  - The system of type **c** and has an order of **n+c**

- **Examples:**

$$G(s)H(s) = \frac{50}{(s+1)(s+4)}$$

➔ **System of type 0**

$$G(s)H(s) = \frac{10s^2 + 3}{s^2(3s^4 + 2s^3 + s^2 + 4s + 3)}$$

➔ **System of type 2**

Order and Type of a system (Examples)

Determine order and type of the following systems

1. $G(s) = \dfrac{s^3+4s^2+7s+3}{s^4+4s^3+2s^2+5s+4}$ Order = 4, Type = 0

2. $G(s) = \dfrac{s^2+5s+7}{s^3+4s^2+5s}$ order = 3, Type = 1

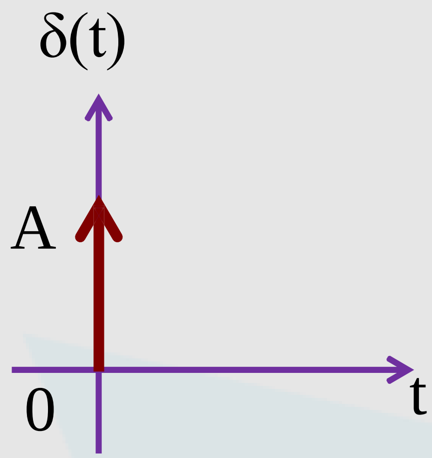
3. $G(s) = \dfrac{s+3}{s(s+3)(s+7)+10}$ order = 3, Type = 0

Standard Test Signals

❑ Impulse-Function

- The impulse signal imitates the <u>sudden</u> <u>shock</u> characteristic of actual input signal.

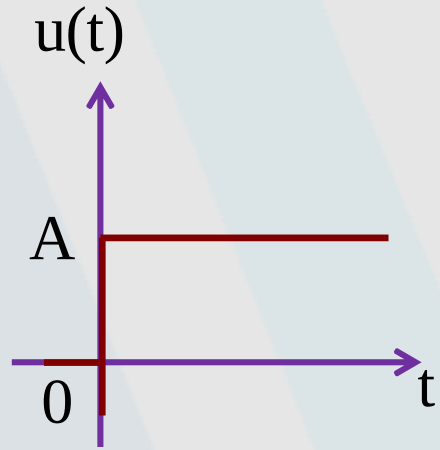
$$u(t) = \delta(t) = \begin{cases} A & t = 0 \\ 0 & t \neq 0 \end{cases} \quad ==> \quad U(s) = A$$



❑ Step-function

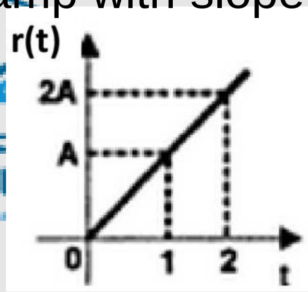
- The step signal imitates the <u>sudden</u> <u>change</u> characteristic of actual input signal.

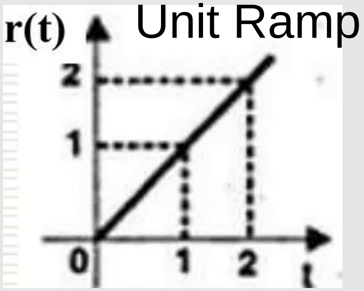
$$u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases} \quad ==> \quad U(s) = \frac{A}{s}$$

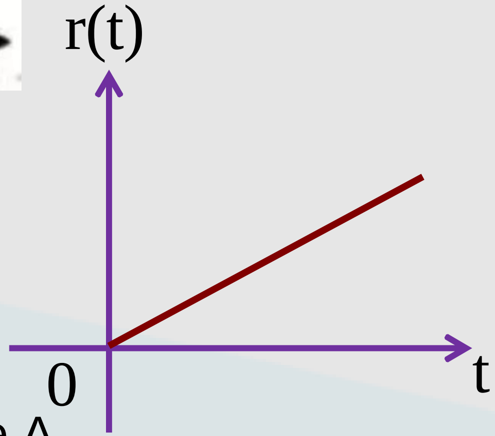


Standard Test Signals

Ramp with slope A



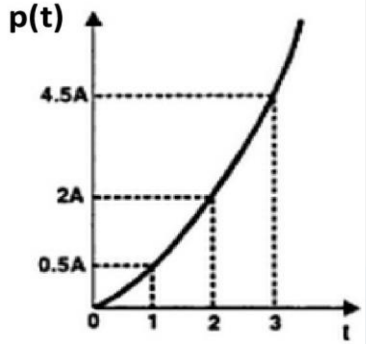




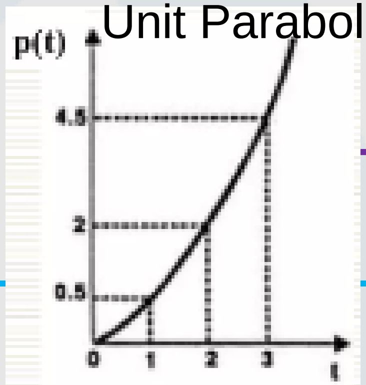
- **Ramp-function**
  - The ramp signal imitates the <u>constant</u> <u>velocity</u> characteristic of actual input signal.

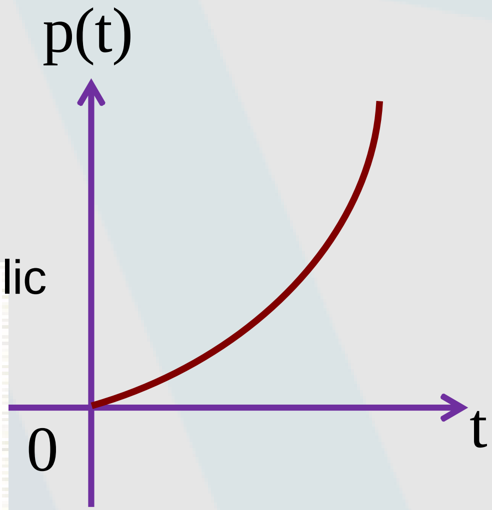
$$r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases} \quad ==> \quad U(s) = \frac{A}{s^2}$$

Parabolic with slope A



Unit Parabolic



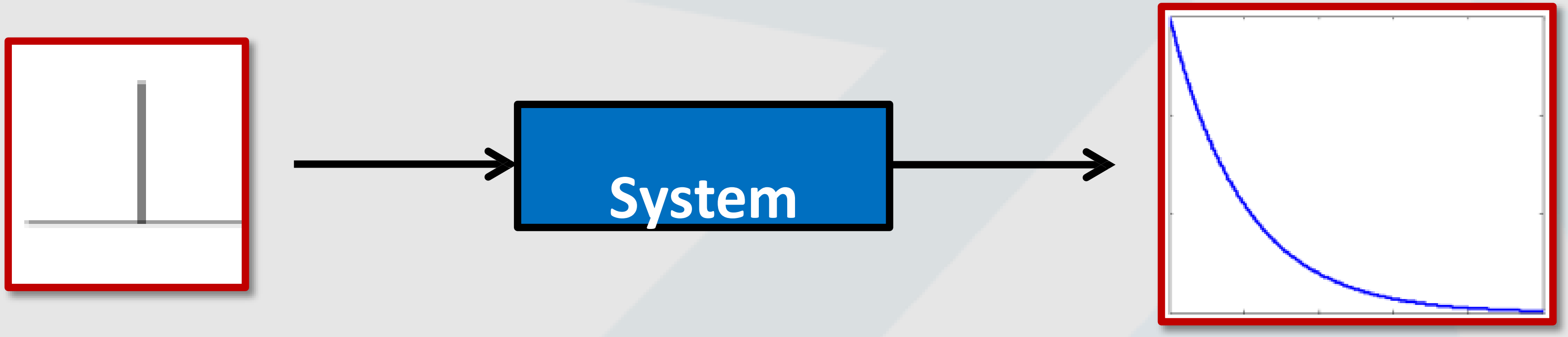


- **Parabolic-function**
  - The parabolic signal imitates the <u>constant</u> <u>acceleration</u> characteristic of actual input signal.

$$p(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases} \quad ==> \quad U(s) = \frac{A}{s^3}$$

Time Response of Control Systems

- Time response of a dynamic system is response to an input expressed as a function of time.

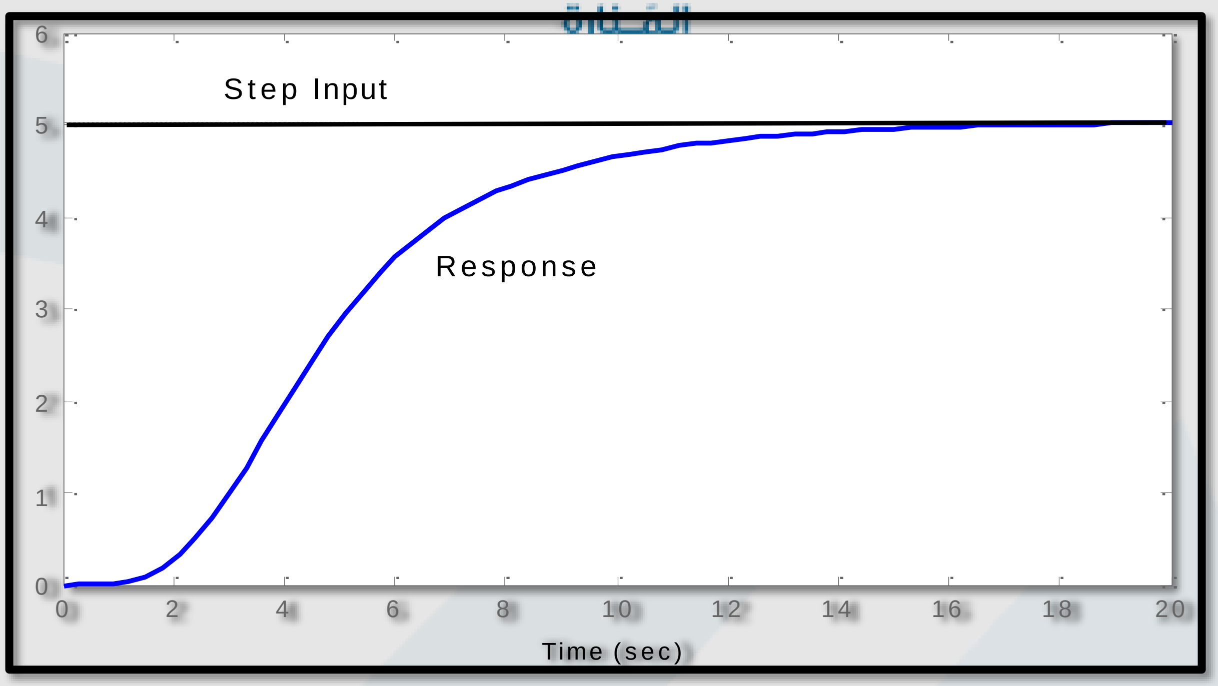


- The time response of any system has two components:
  - Transient response
  - Steady-state response.

$$\lim_{t \to \infty} c_{tr}(t) = 0$$

$$c(t) = c_{tr}(t) + c_{ss}(t)$$

Time Response of Control Systems



- When the <u>response</u> of the <u>system</u> is <u>changed</u> from rest or equilibrium it <u>takes</u> some <u>time</u> to <u>settle</u> <u>down</u>.
- **<u>Transient response</u>** is the <u>response</u> of a system <u>from</u> <u>rest</u> or equilibrium <u>to</u> <u>steady</u> <u>state</u>.
- The <u>response</u> of the system <u>after</u> the <u>transient</u> <u>response</u> is called **<u>steady state response</u>.**

Time Response of Control Systems

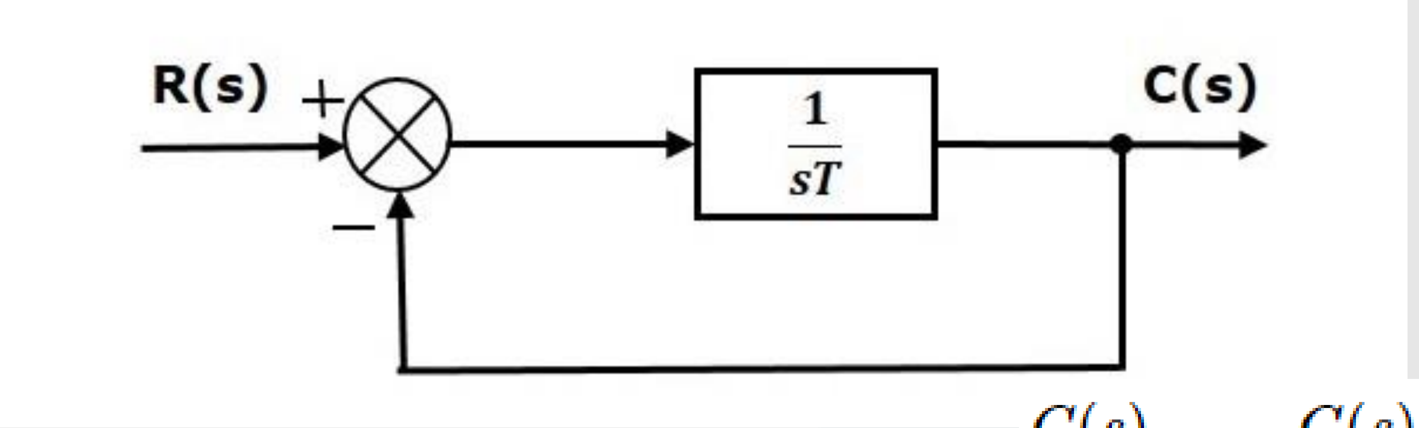
- Transient response depends upon the system poles only and not on the type of input.

- It is therefore sufficient to analyze the transient response using a step input.

- The steady-state response depends on system dynamics, system type, and the input quantity.

- It is then examined using different test signals by the final value theorem.

Response of First Order System



We know that the transfer function of the closed loop control system has unity negative feedback is,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{1+\frac{1}{sT}} = \frac{1}{sT+1}$$

$$C(s) = \left(\frac{1}{sT+1}\right)R(s)$$

First order system, T or τ is the **time constant**

Response of First Order System

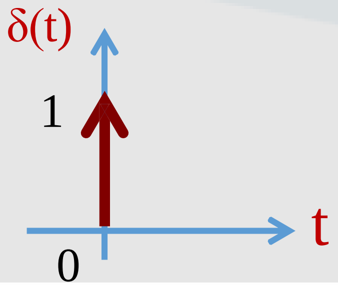
- The Standard form system transfer function G(s) is given by:

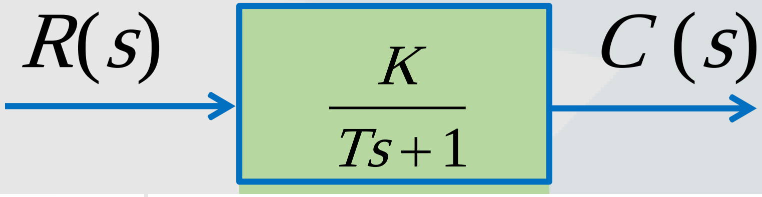
$$G(S)=\frac{C(s)}{R(s)}=\frac{k}{Ts+1}$$

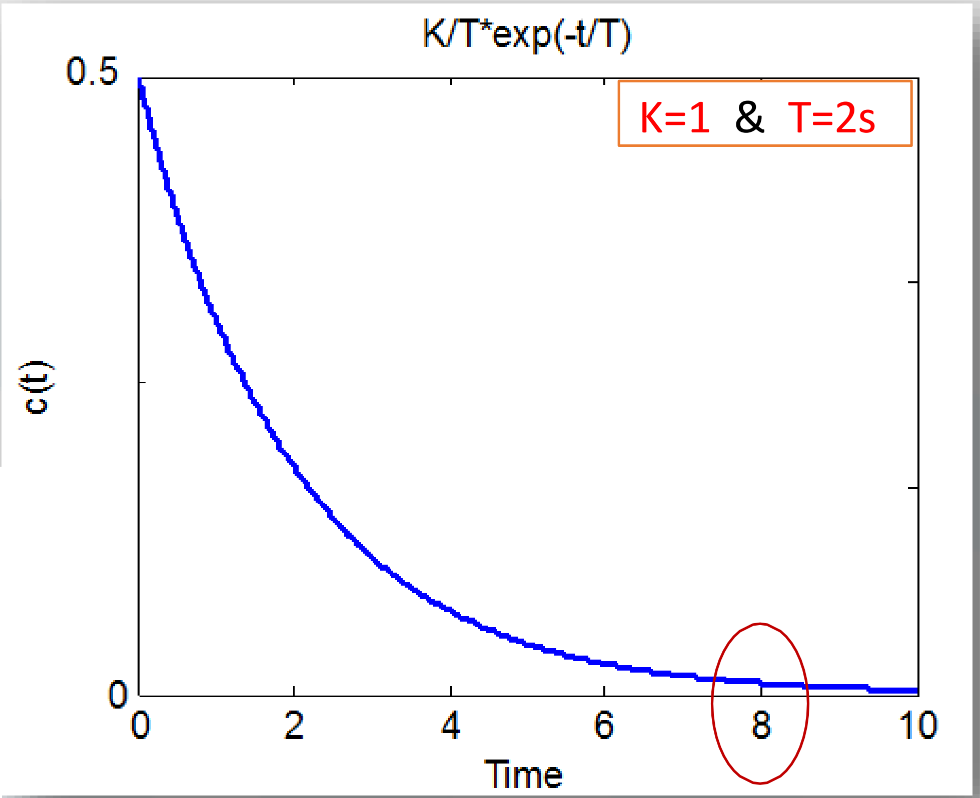
- Where ***K*** is the DC gain and ***T or τ*** is the time constant of the system.
  - Time constant is a measure of how quickly a 1st order system responds to a unit step input.
  - DC Gain of the system is ratio between the input signal and the steady state value of output.
- The first order system has only one pole at ***1/T***

Impulse Response of First Order System

- Consider the following 1st order system and unit impulse signal is applied as an input:







$r(t) = \delta(t)$

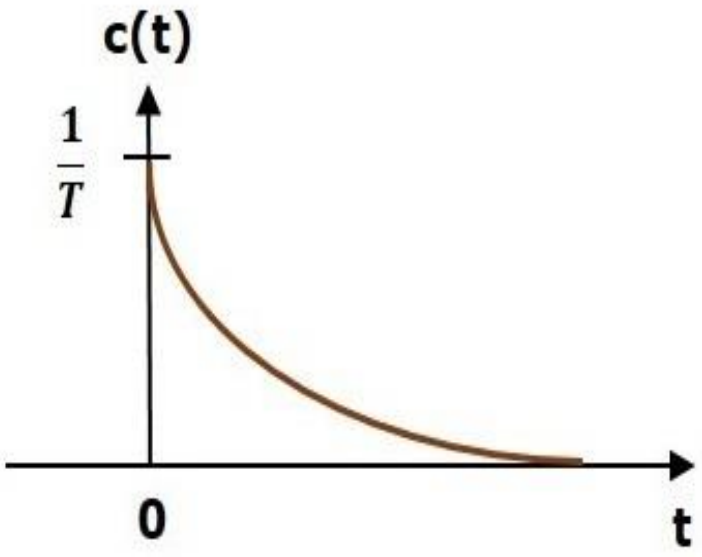
$R(s) = 1$

$C(s) = \left(\frac{1}{sT+1}\right) R(s)$

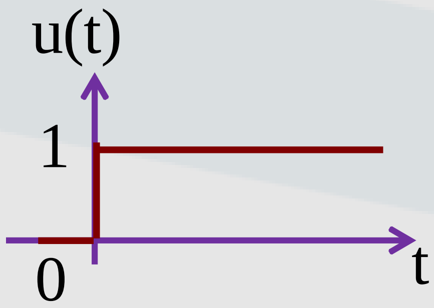
$C(s) = \left(\frac{1}{sT+1}\right)(1) = \frac{1}{sT+1}$

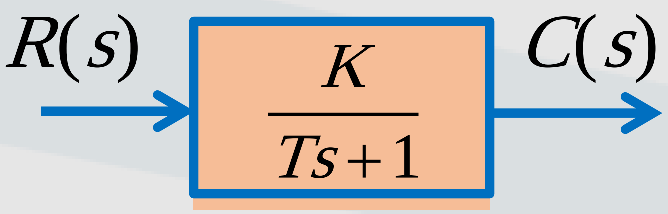
$C(s) = \frac{1}{T\left(s+\frac{1}{T}\right)} \Rightarrow C(s) = \frac{1}{T}\left(\frac{1}{s+\frac{1}{T}}\right)$

$c(t) = \frac{1}{T} e^{\left(-\frac{t}{T}\right)} u(t)$



Step Response of First Order System





$$r(t) = u(t)$$

$$R(s) = \frac{1}{s}$$

$$C(s) = \left(\frac{1}{sT+1}\right) R(s)$$

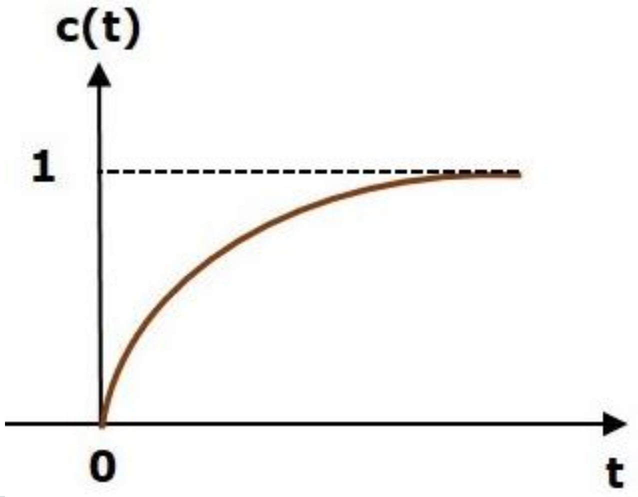
$$C(s) = \left(\frac{1}{sT+1}\right)\left(\frac{1}{s}\right) = \frac{1}{s\,(sT+1)}$$

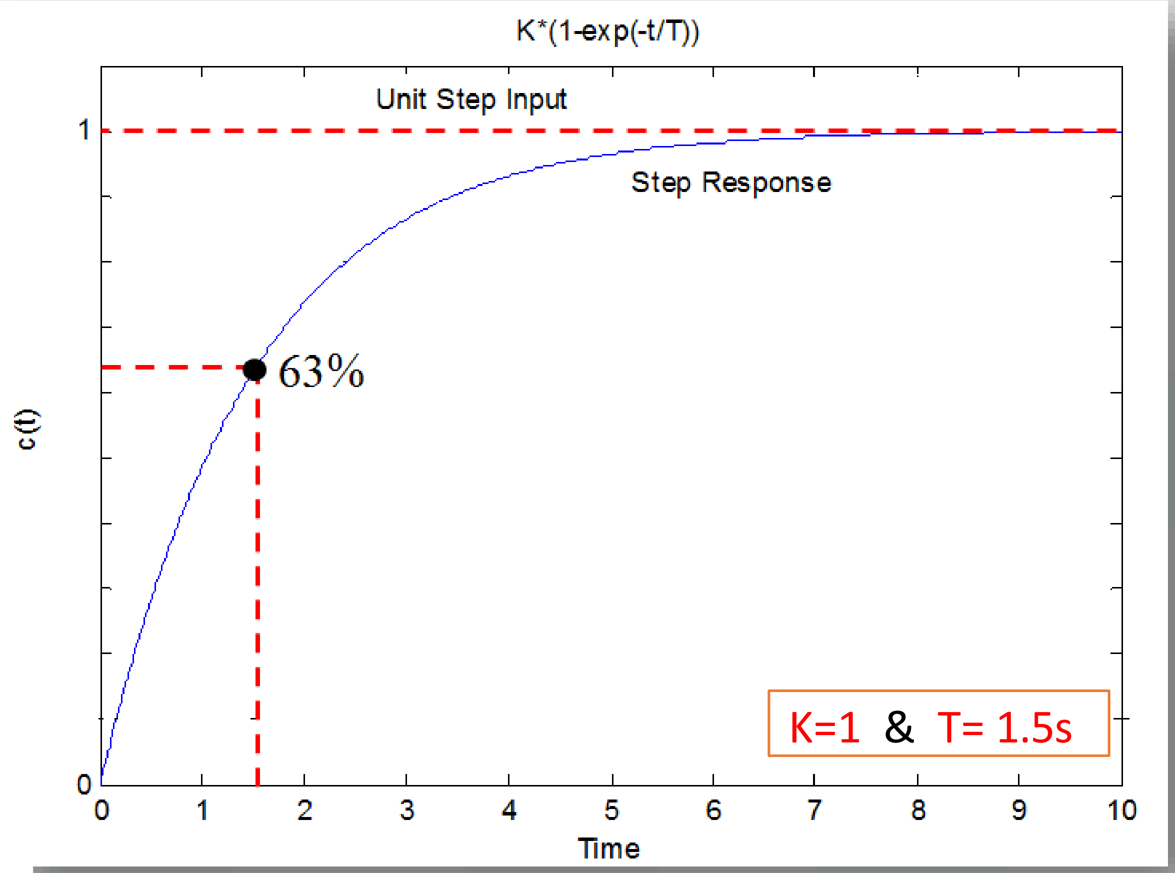
$$\Rightarrow C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

$$c(t) = \left(1 - e^{-\left(\frac{t}{T}\right)}\right) u(t)$$

$$c_{tr}(t) = -e^{-\left(\frac{t}{T}\right)} u(t)$$

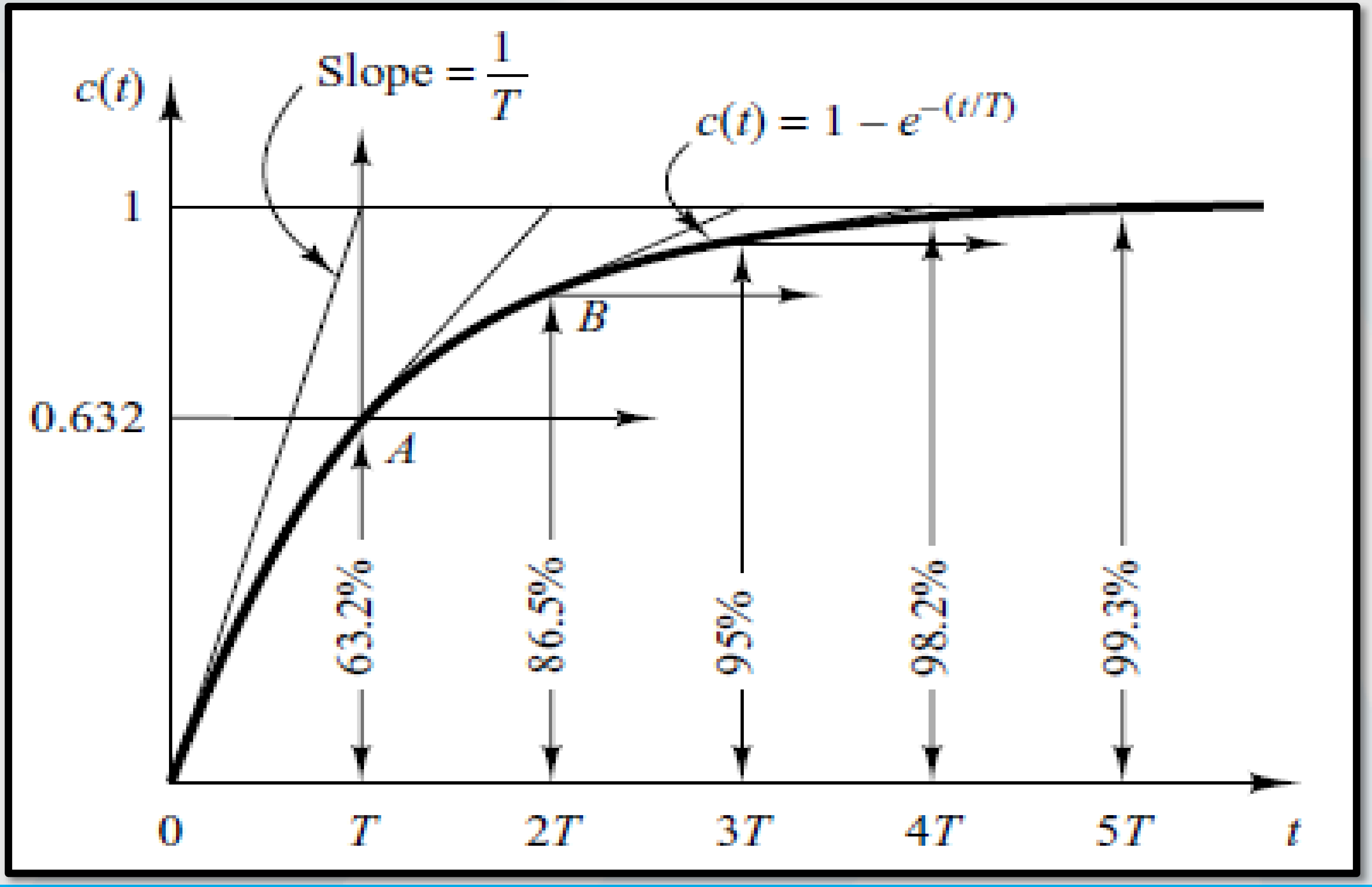
$$c_{ss}(t) = u(t)$$





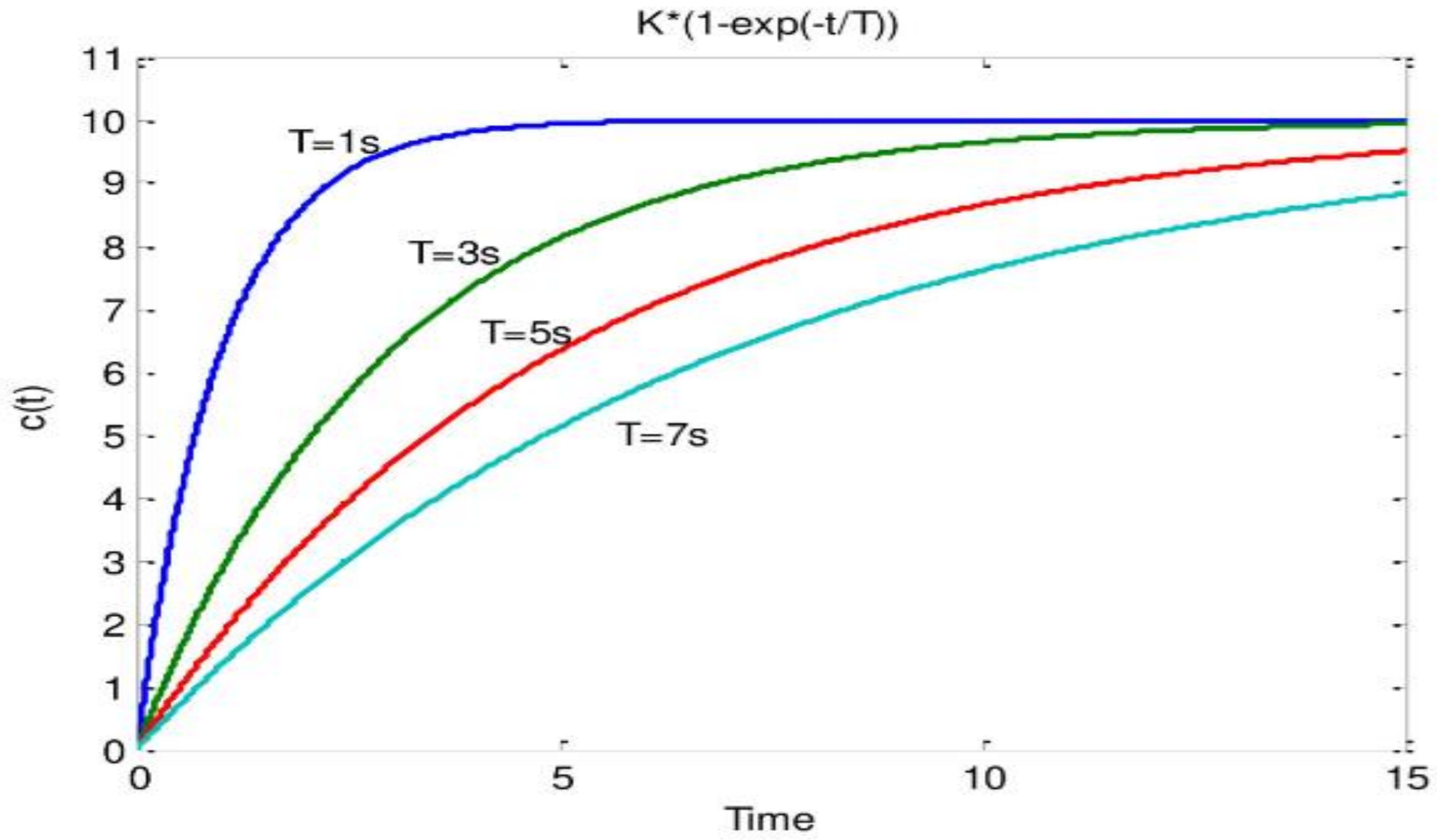
Step Response of First Order System

- System takes five time constants to reach its final value.



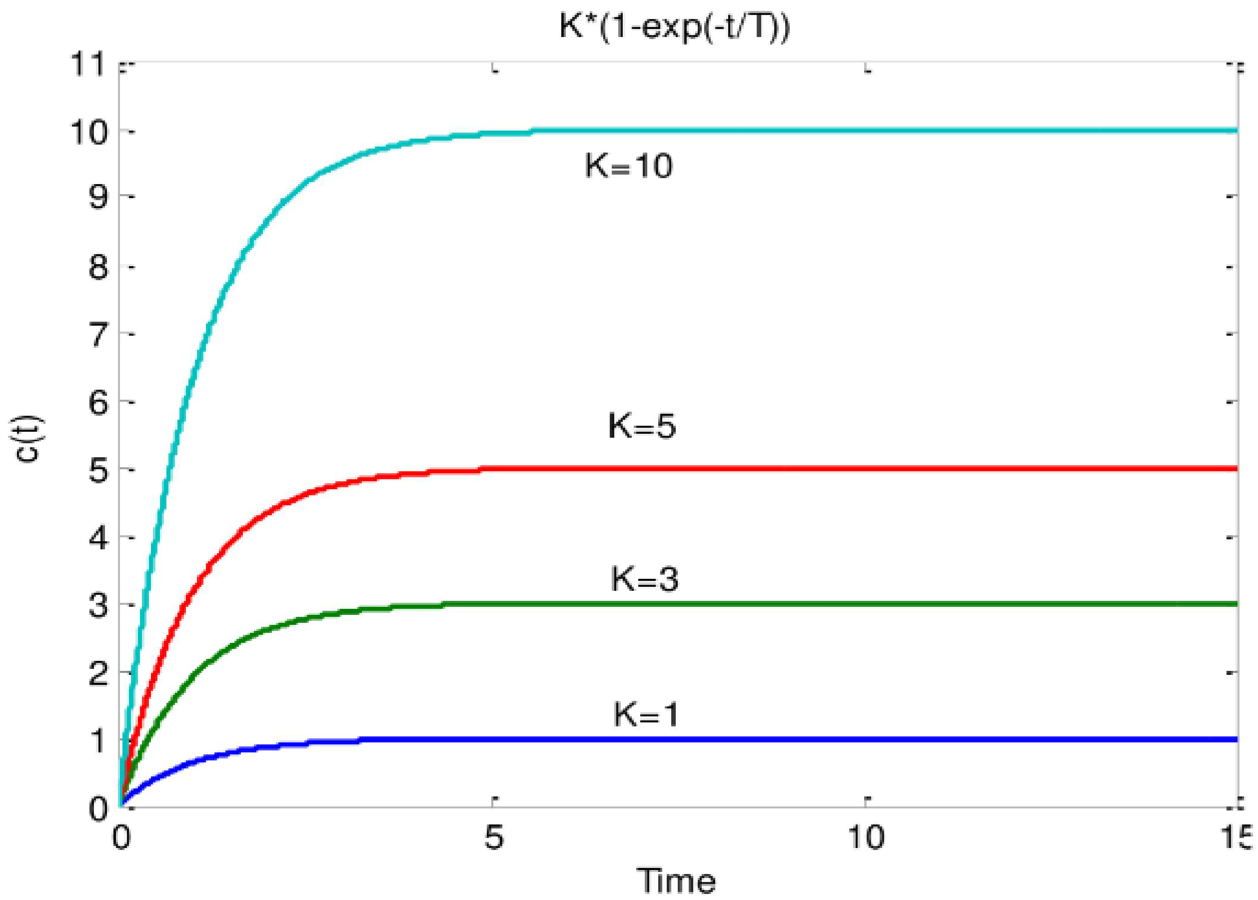
Step Response of First Order System

If K=10 and T=1, 3, 5, 7 $c(t) = K\left(1 - e^{-t/T}\right)$



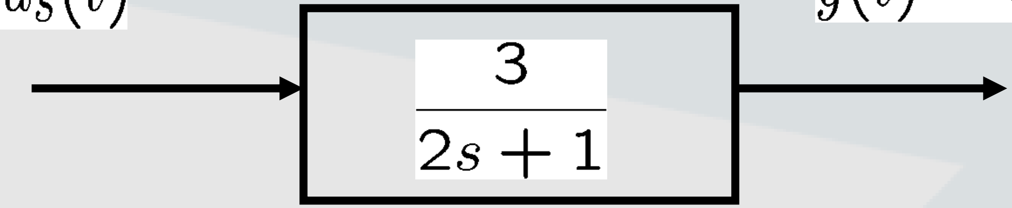
Step Response of First Order System

If K=1, 3, 5, 10 and T=1 $$c(t) = K\left(1 - e^{-t/T}\right)$$



Example of transient & steady-state responses

$$r(t) = u_s(t)$$



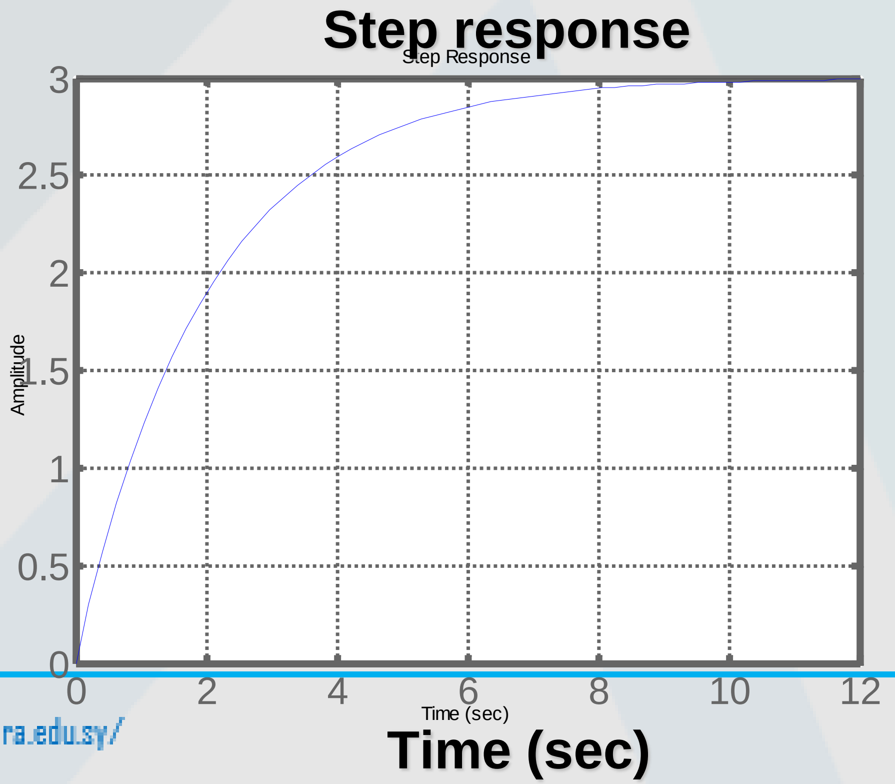
$$y(t) = 3 - 3e^{-\frac{t}{2}}$$

- Transient response

$$y_t(t) = -3e^{-\frac{t}{2}}$$

- Steady-state response

$$y_{ss}(t) = 3$$



Step Response of First Order System (Numerical example1)

Consider a system with T.F. $G(s) = \frac{1}{1+10s}$. Determine the time taken by the system to reach 90% of the steady state when subjected unit step input.

<u>Soln</u> We have $G(s) = \frac{1}{1+10s}$

$\therefore \tau = 10$

Also for first order system subjected to step i/p

$$y(t) = 1 - e^{-t/\tau}$$

$$0.9 = 1 - e^{-t/10}$$

$$e^{-t/10} = 0.1$$

$$-\frac{t}{10} = -2.3026$$

$$\therefore t = 23.026 \text{ sec.}$$

Step Response of First Order System (Numerical example2)

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For the same system when subjected to unit impulse

1. determine output at $t = 15$ sec
2. determine the time at which output is 0.05

<u>sol</u>

Sol^n: For impulse input

$$y(t) = \frac{1}{\tau} e^{-t/\tau}$$

At $t = 15$

$$y(t) = \frac{1}{10} e^{-15/10} = 0.0223$$

$$y(t) = 0.05$$

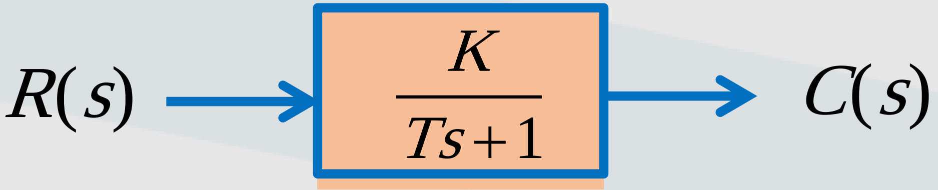
$$\therefore 0.05 = \frac{1}{10} e^{-t/10}$$

$$e^{-t/10} = 0.5$$

$$-\frac{t}{10} = -0.693$$

$$\therefore t = 6.93 \text{ sec}$$

Ramp Response of First Order System



$$r(t) = tu(t)$$

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \left(\frac{1}{sT+1}\right) R(s)$$

$$C(s) = \left(\frac{1}{sT+1}\right)\left(\frac{1}{s^2}\right) = \frac{1}{s^2(sT+1)}$$

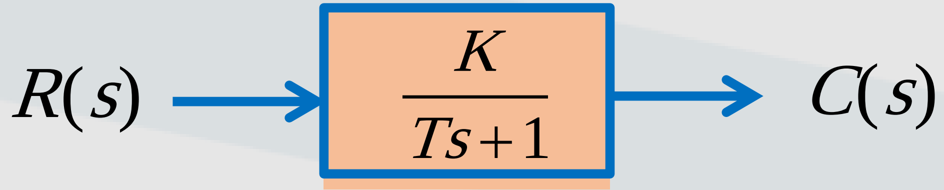
$$\Rightarrow C(s) = \frac{1}{s^2} - \frac{T}{s} + \frac{T}{s+\frac{1}{T}}$$

$$c(t) = \left(t - T + Te^{-\left(\frac{t}{T}\right)}\right) u(t)$$

$$c_{tr}(t) = Te^{-\left(\frac{t}{T}\right)} u(t)$$

$$c_{ss}(t) = (t-T)u(t)$$

Parabolic Response of First Order System



$$r(t) = \frac{t^2}{2} u(t)$$

$$R(s) = \frac{1}{s^3}$$

$$C(s) = \left(\frac{1}{sT+1}\right) R(s)$$

$$C(s) = \left(\frac{1}{sT+1}\right)\left(\frac{1}{s^3}\right) = \frac{1}{s^3(sT+1)}$$

$$c(t) = \left(\frac{t^2}{2} - Tt + T^2 - T^2 e^{-\left(\frac{t}{T}\right)}\right) u(t)$$

$$C_{tr}(t) = -T^2 e^{-\left(\frac{t}{T}\right)} u(t)$$

$$C_{ss}(t) = \left(\frac{t^2}{2} - Tt + T^2\right) u(t)$$

Summary of response of a LTI first order system

	input:	output:
unit ramp	$r(t) = t$	$c(t) = t - \tau\left(1 - e^{-t/\tau}\right)$
	$\downarrow \frac{d}{dt}$	$\downarrow \frac{d}{dt}$
unit step	$r(t) = u(t)$	$c(t) = 1 - e^{-(t/\tau)}$
	$\downarrow \frac{d}{dt}$	$\downarrow \frac{d}{dt}$
unit impulse	$r(t) = \delta(t)$	$c(t) = \frac{1}{\tau} e^{-(t/\tau)}$