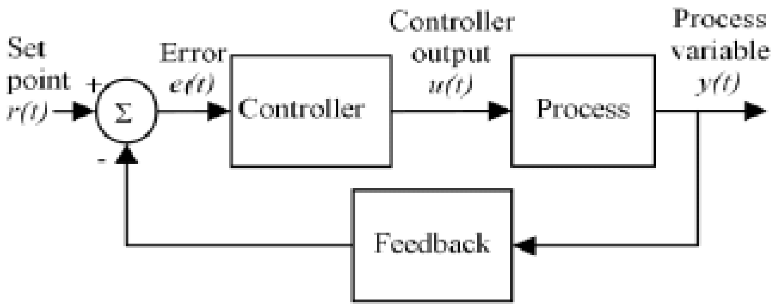

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Lecture (6)
The Concept of Stability
Stability Analysis Using Routh-Hurwitz Method

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References

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- The concept of dominant poles
- Role of a pole (location) in system response
- Stability analysis using Routh-Hurwitz method

The Concept of Stability

- A stable system is a dynamic system with a bounded response to a bounded input.
- Consider the **closed loop** transfer function of a system as :

$$T(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0} = \frac{N(s)}{\Delta(s)}$$

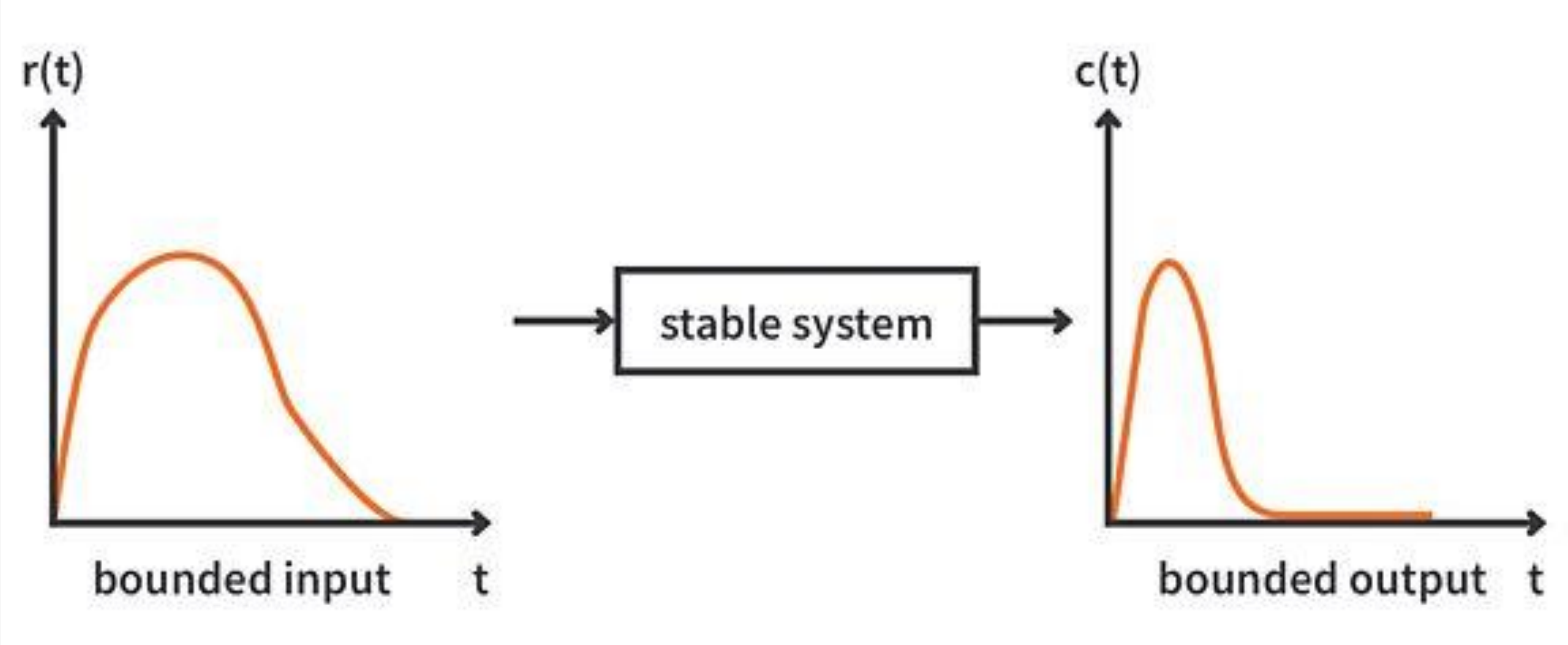
- The characteristic equation or polynomial of the system which is given by:

$$\Delta(s) = Q(s) = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0$$

- For the system described by T(s) to be stable, the root of the characteristic equation must lie in the left half plane.
- The Routh-Hurwitz criteria or test is a numerical procedure for determining the number of right half-plane (RHP) and imaginary axis roots of the characteristic polynomial.

The Concept of Stability

- This is commonly called as BIBO Stability meaning – Bounded Input Bounded Output Stability.



The concept of dominant poles

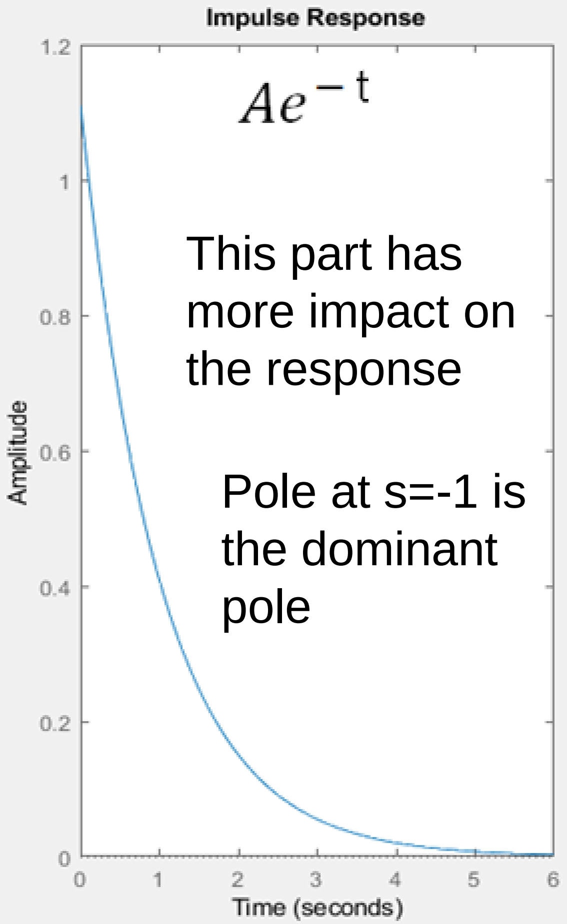
- The concept of dominant poles is important in the analysis and design of control systems.
- In the context of linear time-invariant (LTI) systems, a pole refers to a point in the complex plane where the denominator of the transfer function becomes zero.
- The poles of a system play a crucial role in determining its dynamic behavior and stability.
- In a control system, a dominant pole is a pole that significantly influences the system's response, particularly in the part of the transient response.

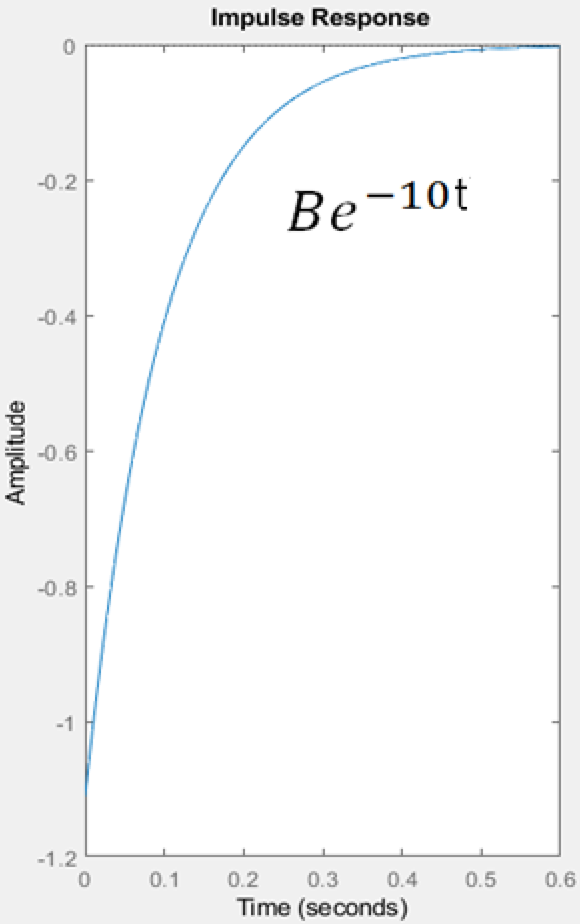
The concept of dominant poles

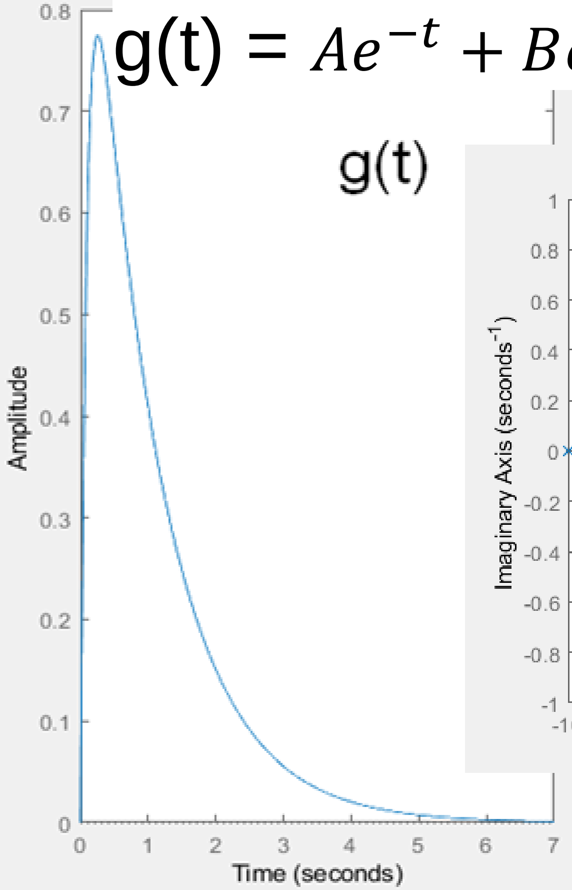
- A pole/complex conjugate pole pair closest to the imaginary axis in the s-plane is called as dominant pole/pole pair.

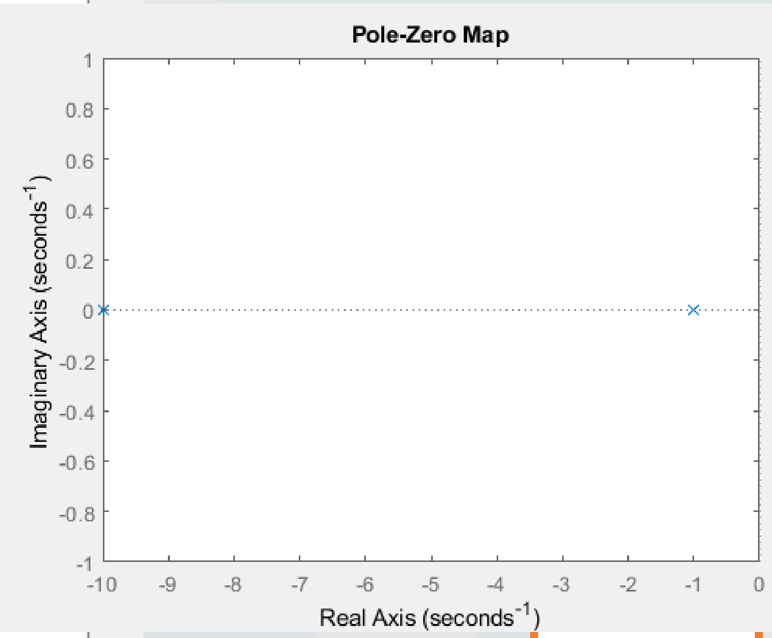
Let $G(s) = \frac{10}{(s+1)(s+10)} = \frac{A}{s+1} + \frac{B}{s+10}$

$g(t) = Ae^{-t} + Be^{-10t}$;A=10/9,B=-10/9









The concept of dominant poles

- Let $S_d = -\xi w_n \pm j w_d$
- The non dominant poles shall be selected at least 5 times away from the dominant poles.
- If $S_a = -2 \pm j3$ then other poles shall have $|re\{pole\}| > 10$.
- Those poles shall be in the left of s=-10.

Stability, nature of response for various pole locations and stability from pole locations

- Stability- A system is said to be stable if it produces bounded output for every bounded input.
- If $\int_{-\infty}^{+\infty} |g(t)|dt < \infty$ then system is stable

  where g(t) is the impulse response of the system.

Relative stability- A system that has less settling time is relatively more stable. In other words, the system that has its most dominant pole away from the imaginary axis in the left half of s-plane.

Conditional stability - A system is said to be conditionally stable if its stability depends on some condition

Ex. Let $G(s) = \dfrac{k}{s(s+2)(s+5)}$

If range of k for stability is finite then system is conditionally stable.

If range of k is 0<k<∞ then system is stable for all finite values of k & is unconditionally stable (**absolutely stable system**).

Stability, nature of response for various pole locations and stability from pole locations

Marginally Stable System

- If the system is stable by producing an output signal with constant amplitude for bounded input, then it is known as **marginally stable system**.
  - i.e. a unit step response for a unit impulse input.
- The closed loop control system is marginally stable if one (only one) pole exists in the origin of s-plane.

Stability, nature of response for various pole locations and stability from pole locations

Marginally Stable System

Notice:

- Some references says:

  If the system is stable by producing an output signal with constant amplitude and constant frequency of oscillations for bounded input, then it is known as **marginally stable system**.

Role of a pole (location) in system response

Let $H(s) = \dfrac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$ $m \leq n$

$$H(s) = K \frac{(s - z_1)(s - z_2) \ldots (s - z_{m-1})(s - z_m)}{(s - p_1)(s - p_2) \ldots (s - p_{n-1})(s - p_n)}$$

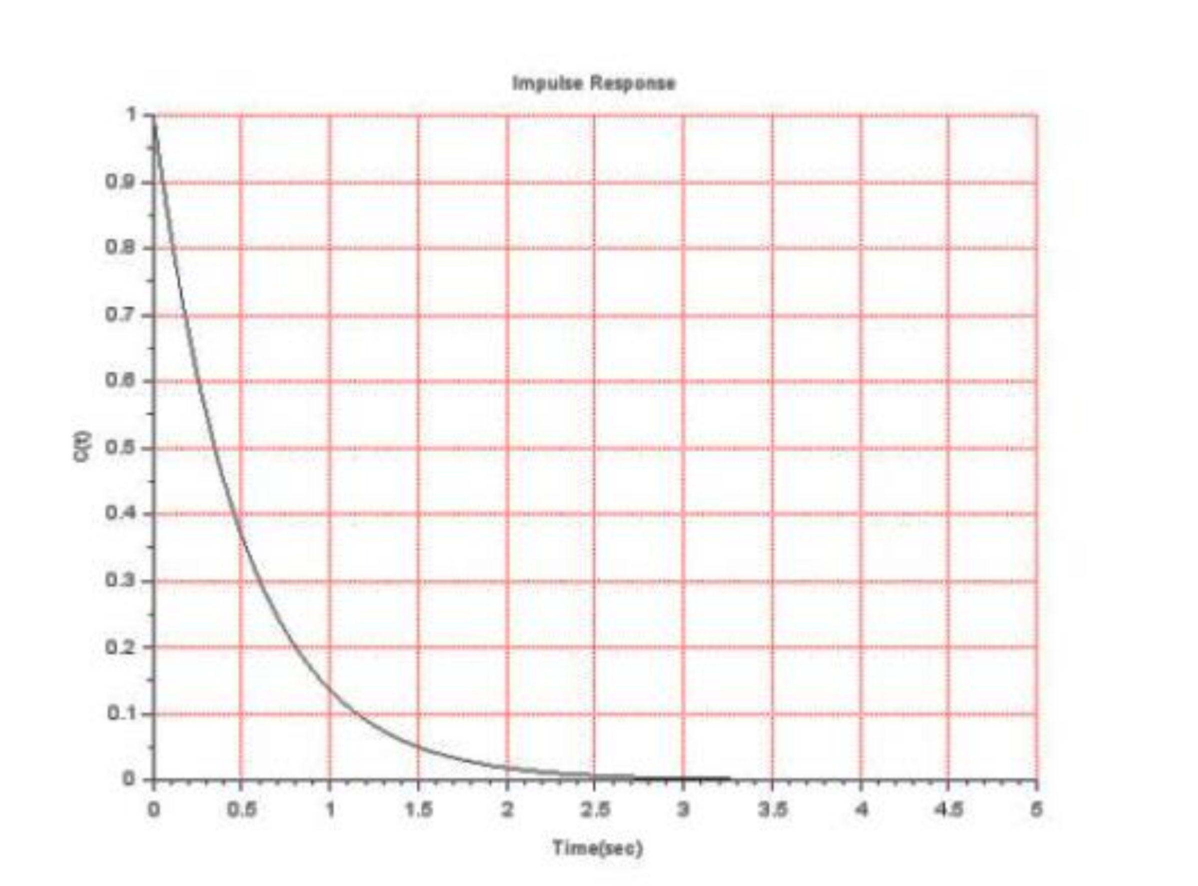
Different pole locations can be:

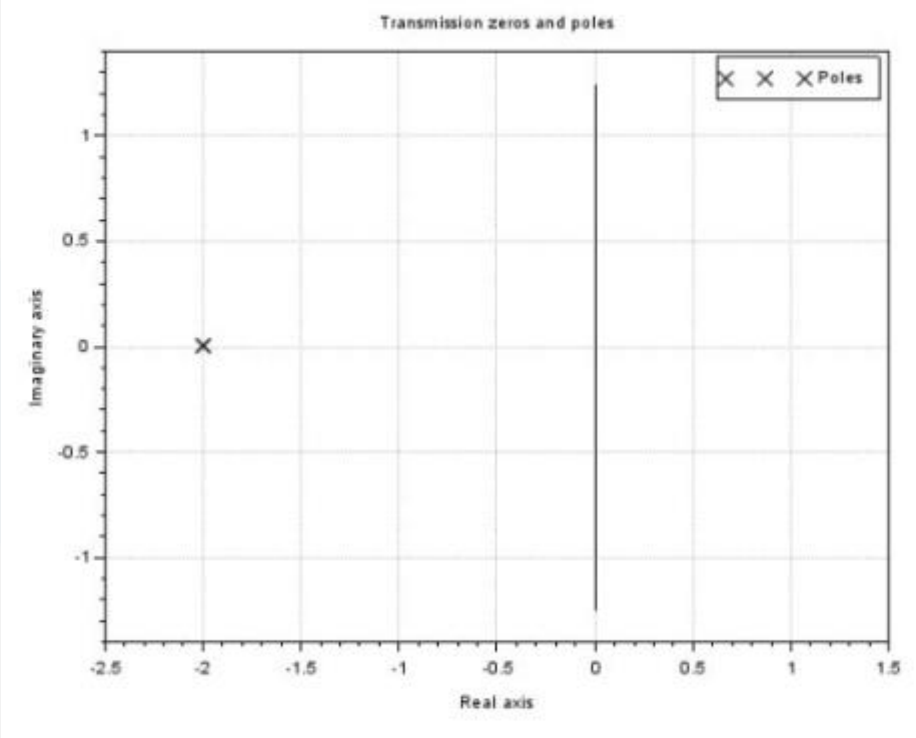
1. Single real pole
2. Complex conjugate pole pair
3. Repeated real poles
4. Repeated complex conjugate pole pairs

Case 1 – Poles on the negative real axis

- Consider a simple pole at s = -2
- This means the transfer function would be $G(s) = \frac{1}{s+2}$

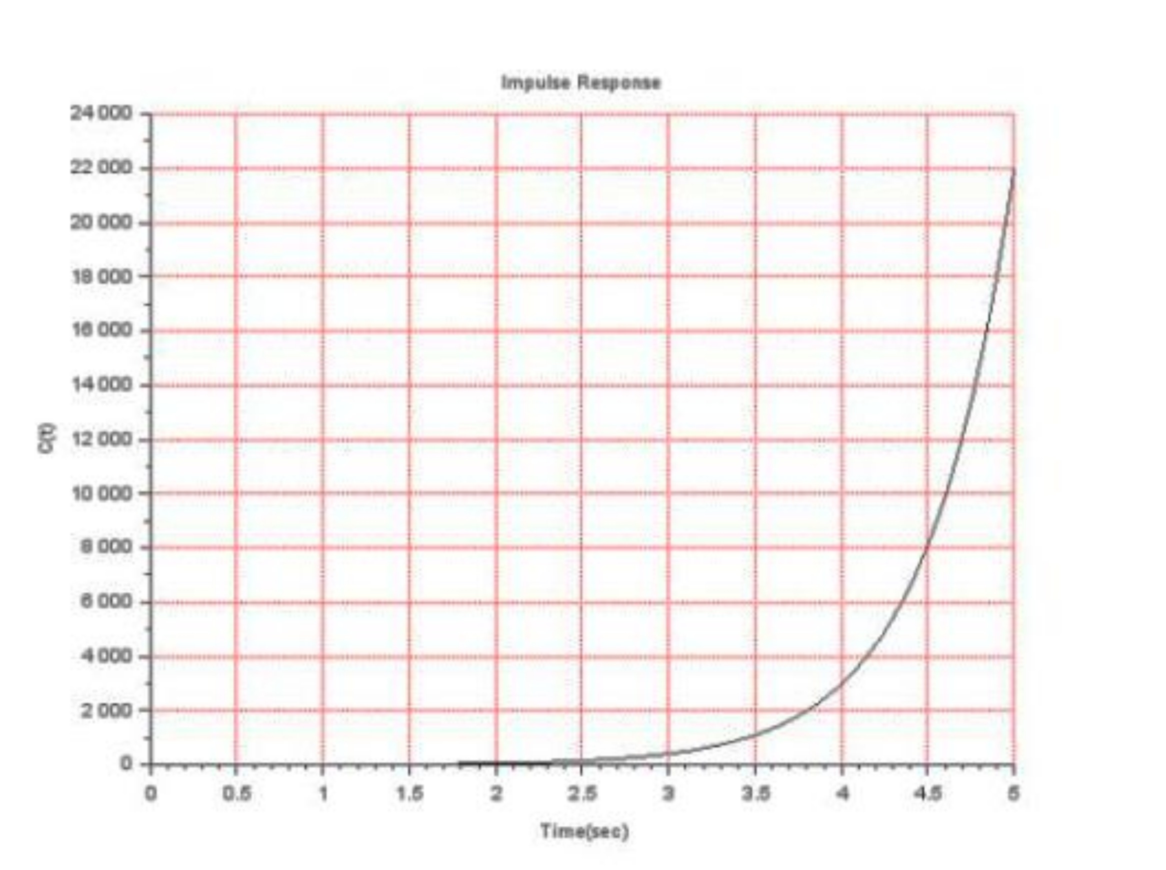
$g(t)=e^{-2t}$

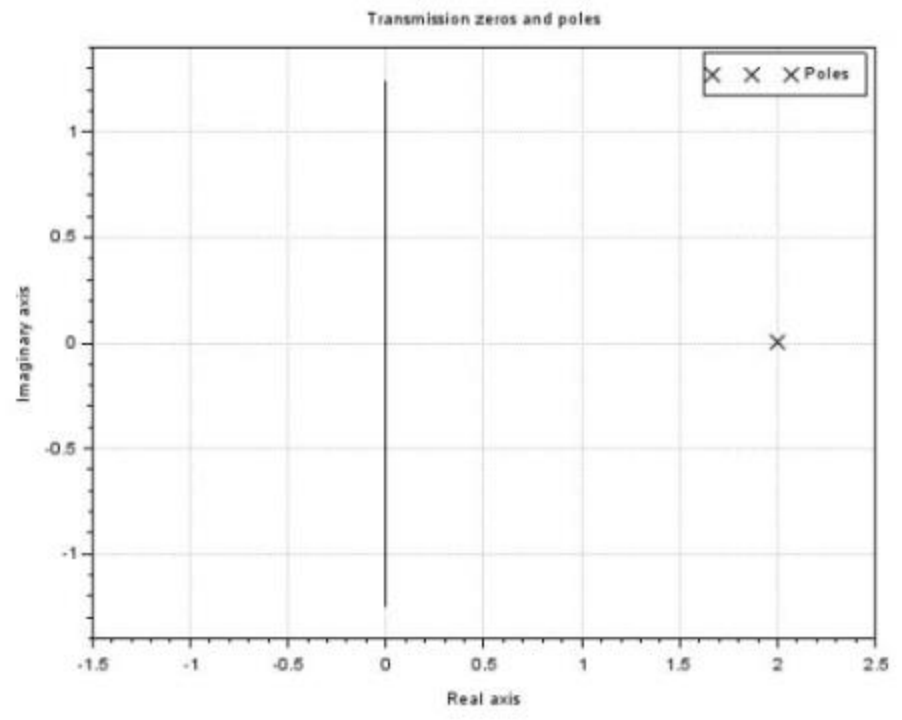




Case 2 – Poles on positive real axis

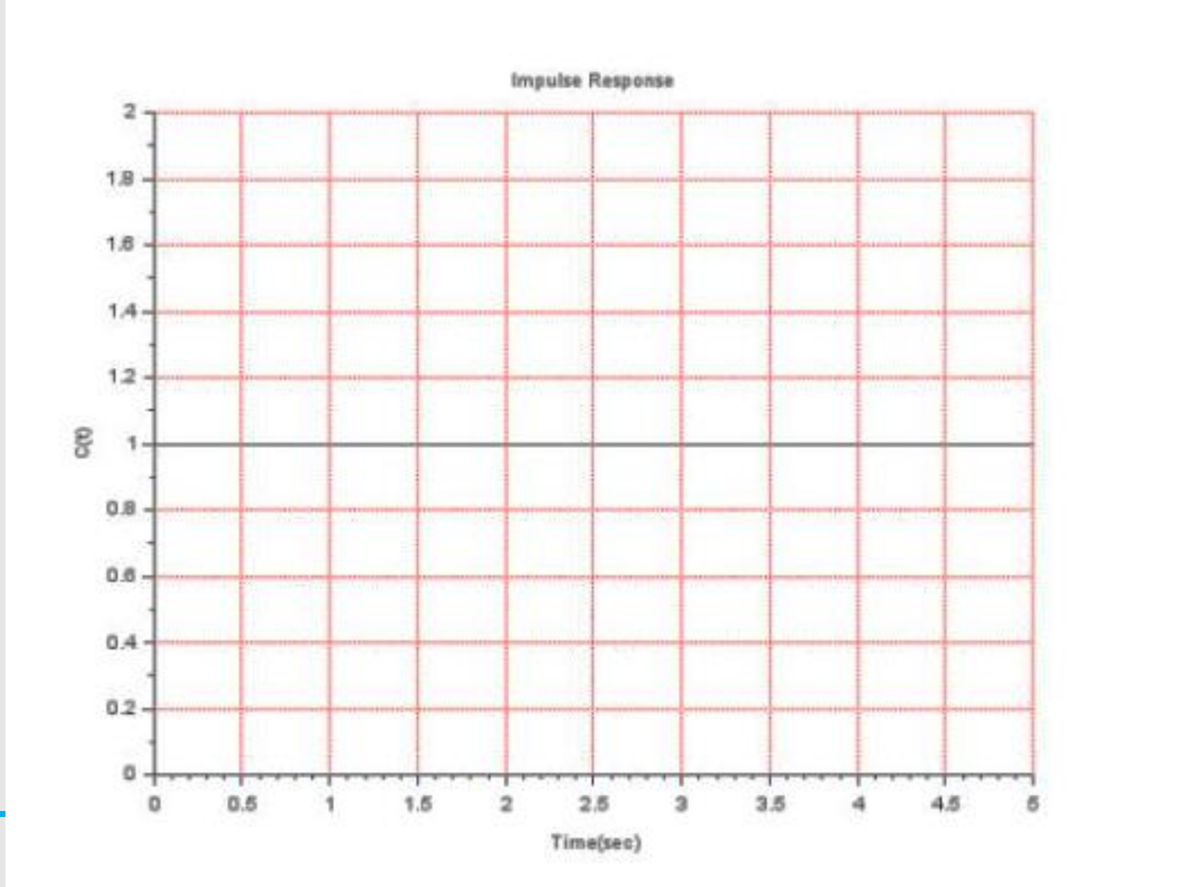
- Consider a simple pole at s = +2
- This means the transfer function would be $G(s) = \frac{1}{s-2}$ $g(t)=e^{2t}$

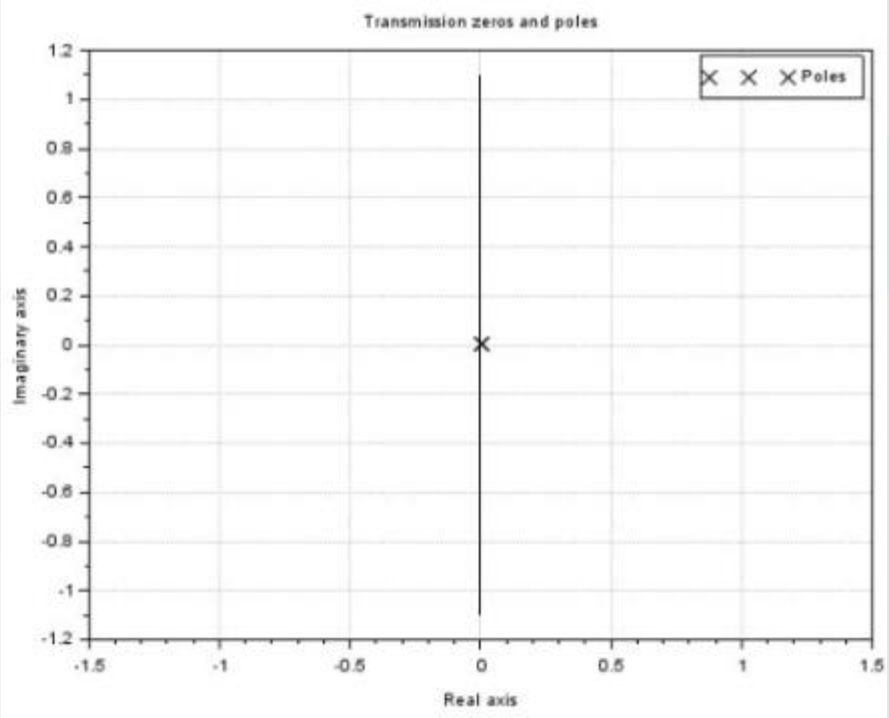




Case 3 – One poles at origin

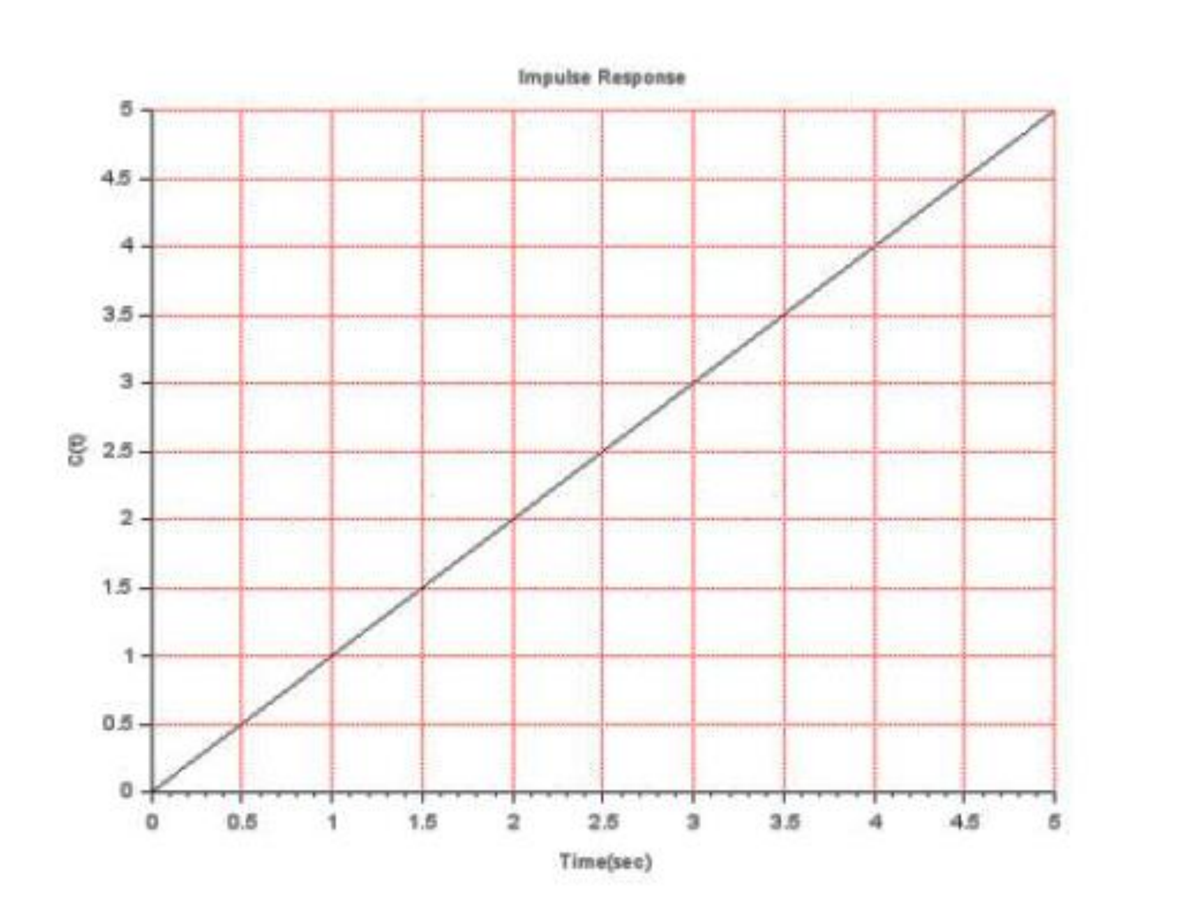
- Consider a simple pole at s = 0
- This means the transfer function would be $G(s) = \frac{1}{s}$ $g(t)=e^{0t}$

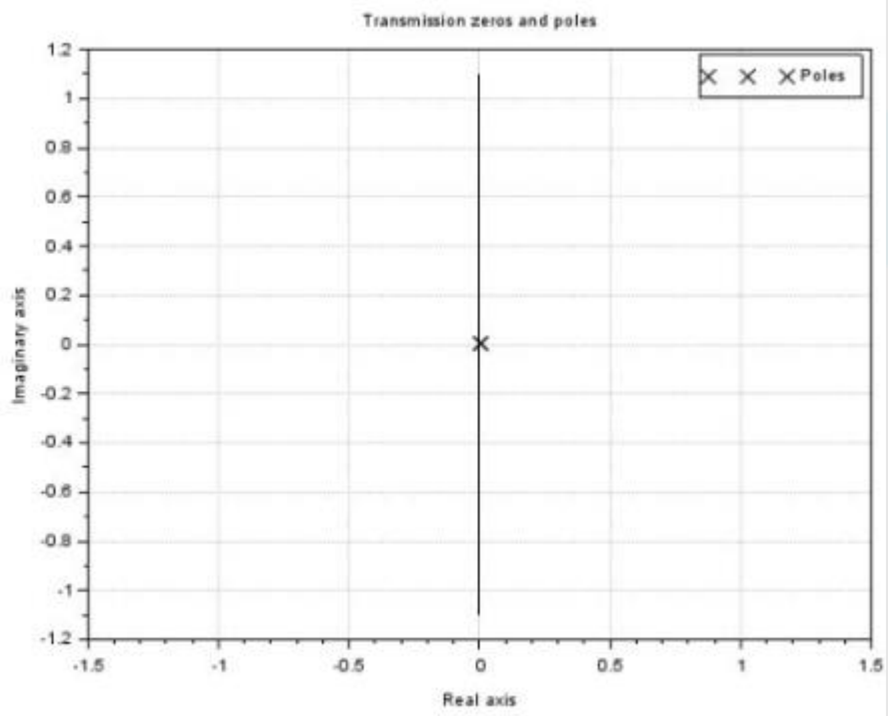




Case 4 – Two poles at origin

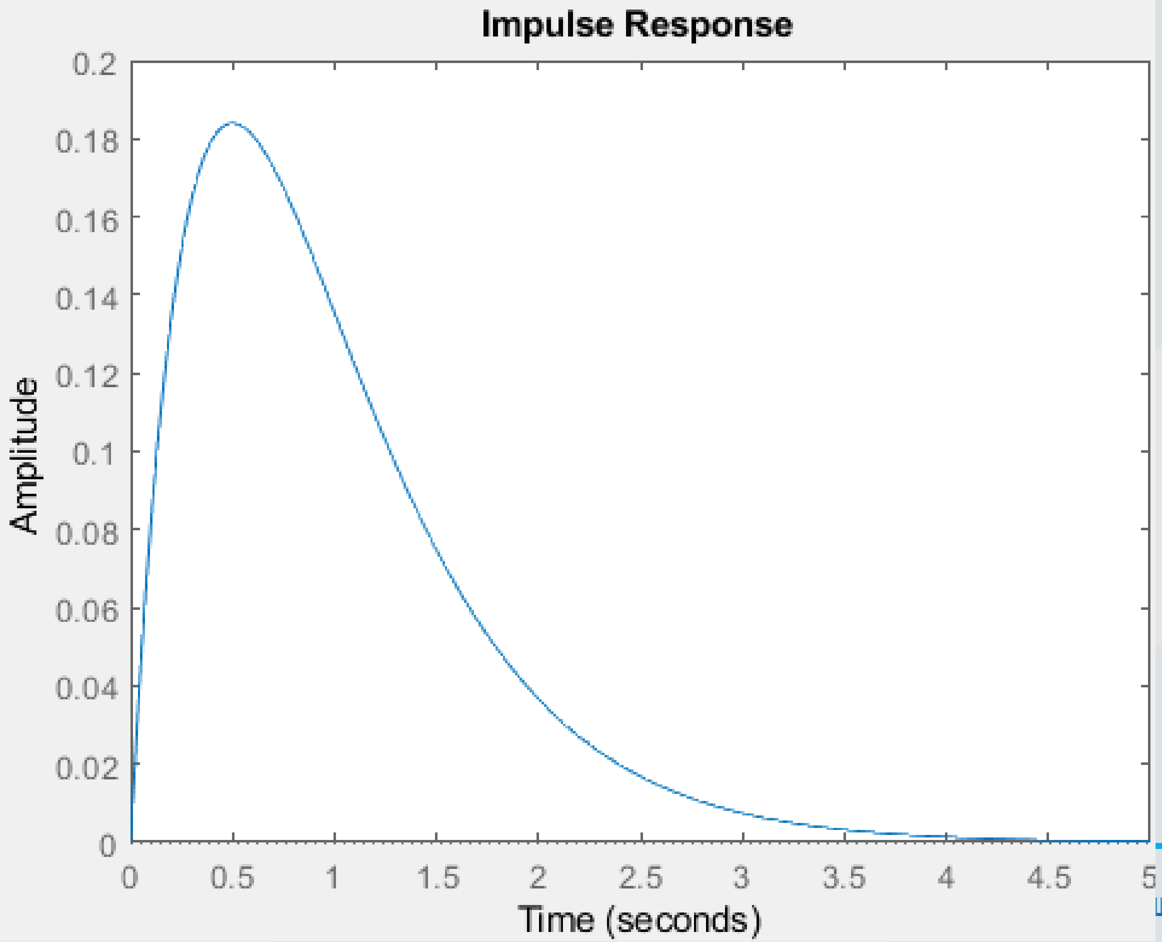
- Consider a repeated pole at s = 0 (repeated 2 times)(It is a pair of complex poles with jw=0)
- This means the transfer function would be $G(s) = \frac{1}{s^2}$ g(t)=t

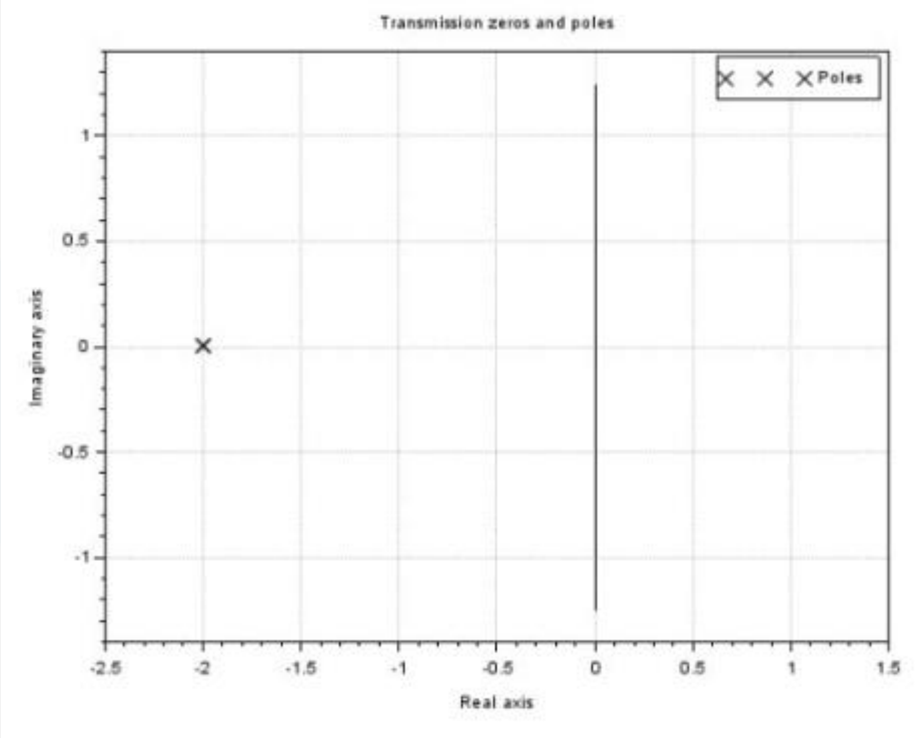




Case 5 – Repeated real poles in LHP

- Consider a repeated pole at s = -2 (repeated 2 times)
- This means the transfer function would be $G(s) = \frac{1}{(s+2)^2}$ $g(t)=te^{-2t}$

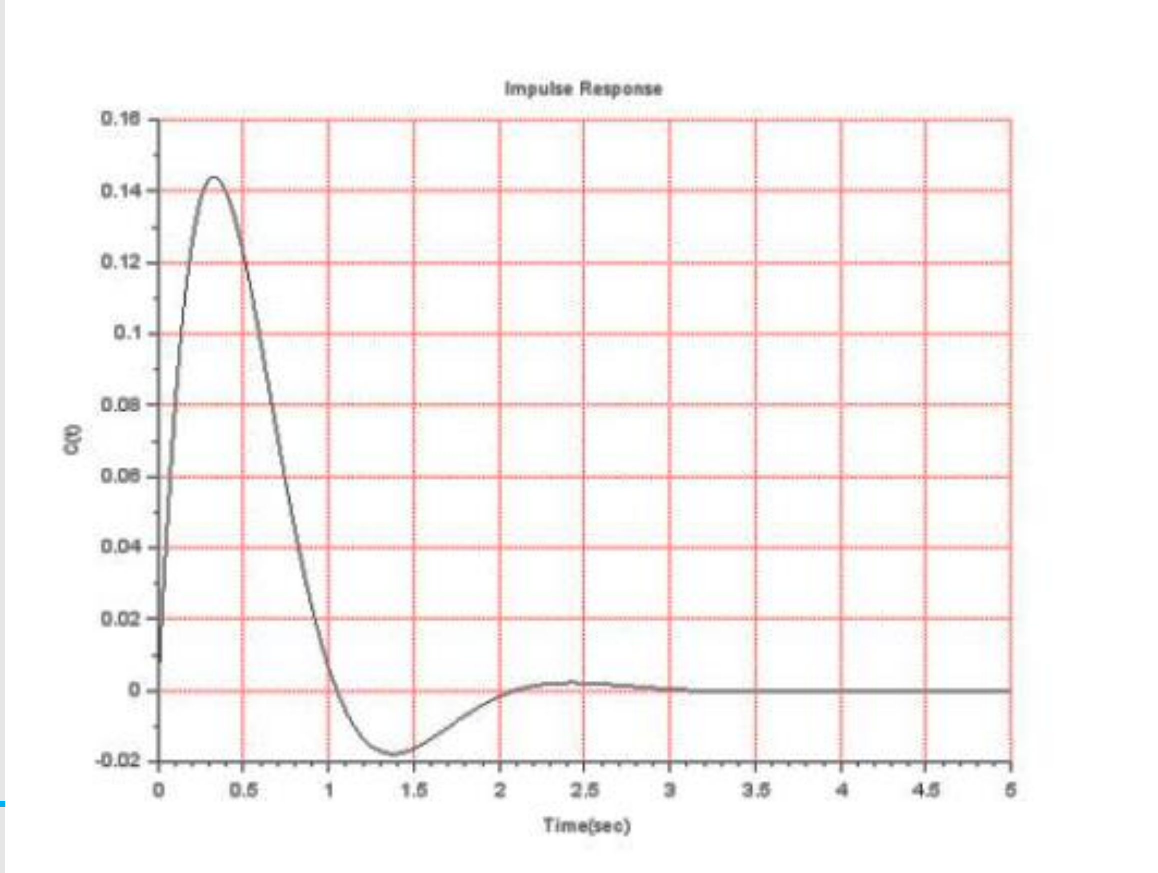


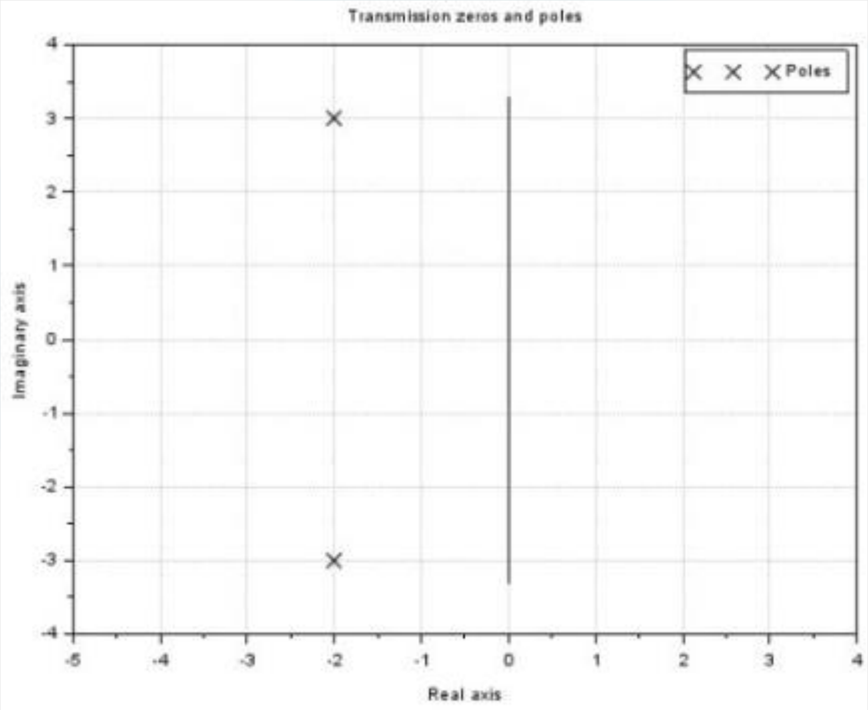


Case 6 – Complex pole in the left half of S-plane

- complex pole pair at s = -2+3*j* and at *s* = -2-3*j*
- This means the transfer function would be $G(s) = \frac{1}{(s+2-3j)(s+2+3j)} = \frac{1}{(s+2)^2+3^2} = \frac{1}{s^2+4s+13}$

$g(t)=0.33\sin(3t)*e^{-2t}$



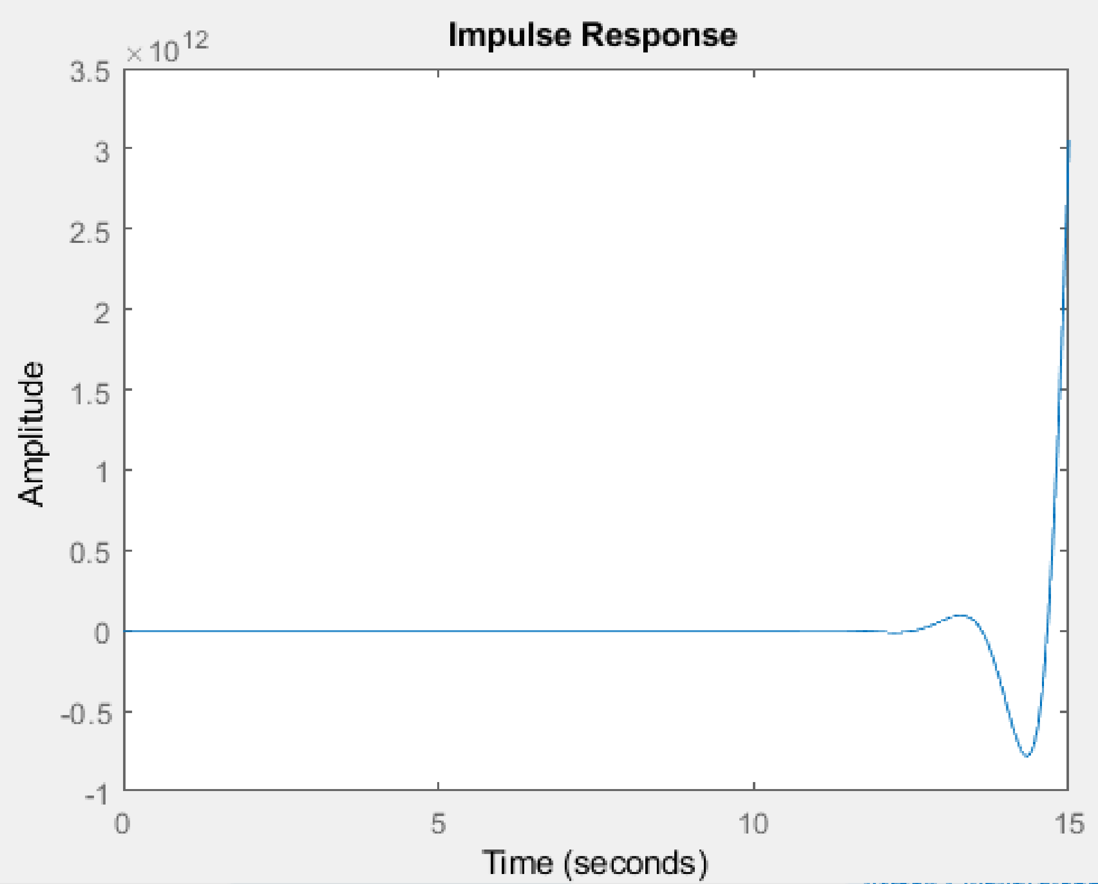


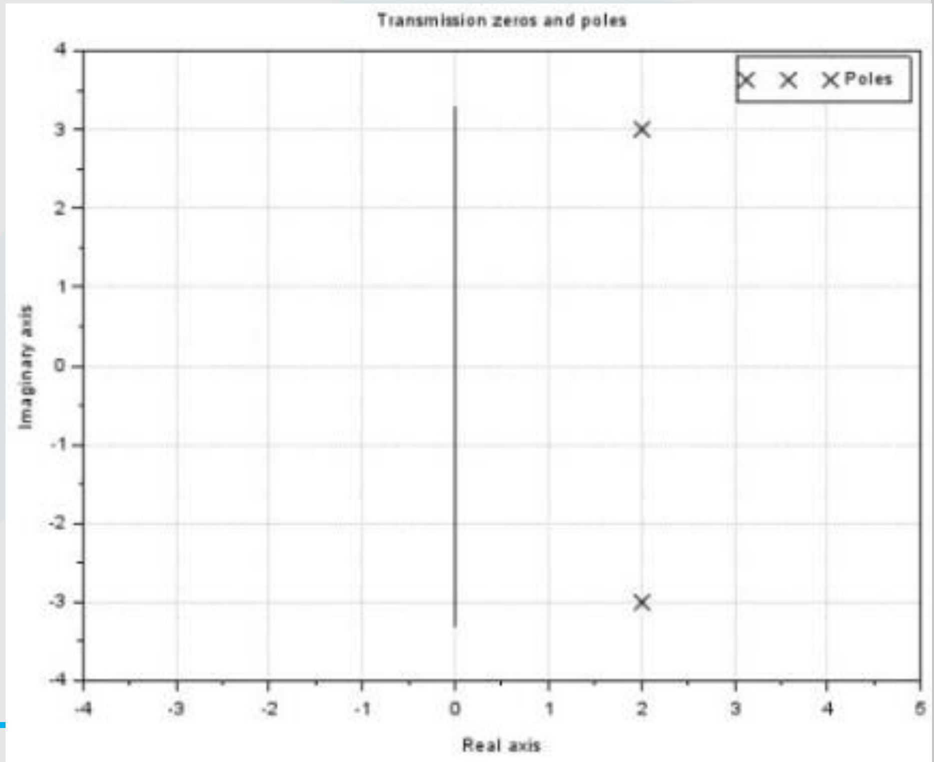
Case 7 – Complex poles in the right half of the S-plane

- complex pole pair at s = +2+3*j* and at *s* = +2-3*j*
- This means the transfer function would be

$$G(s) = \frac{1}{(s-2-3j)(s-2+3j)} = \frac{1}{(s-2)^2+3^2} = \frac{1}{s^2-4s+13}$$

g(t)=0.33sin(3t)*e^{+2t}



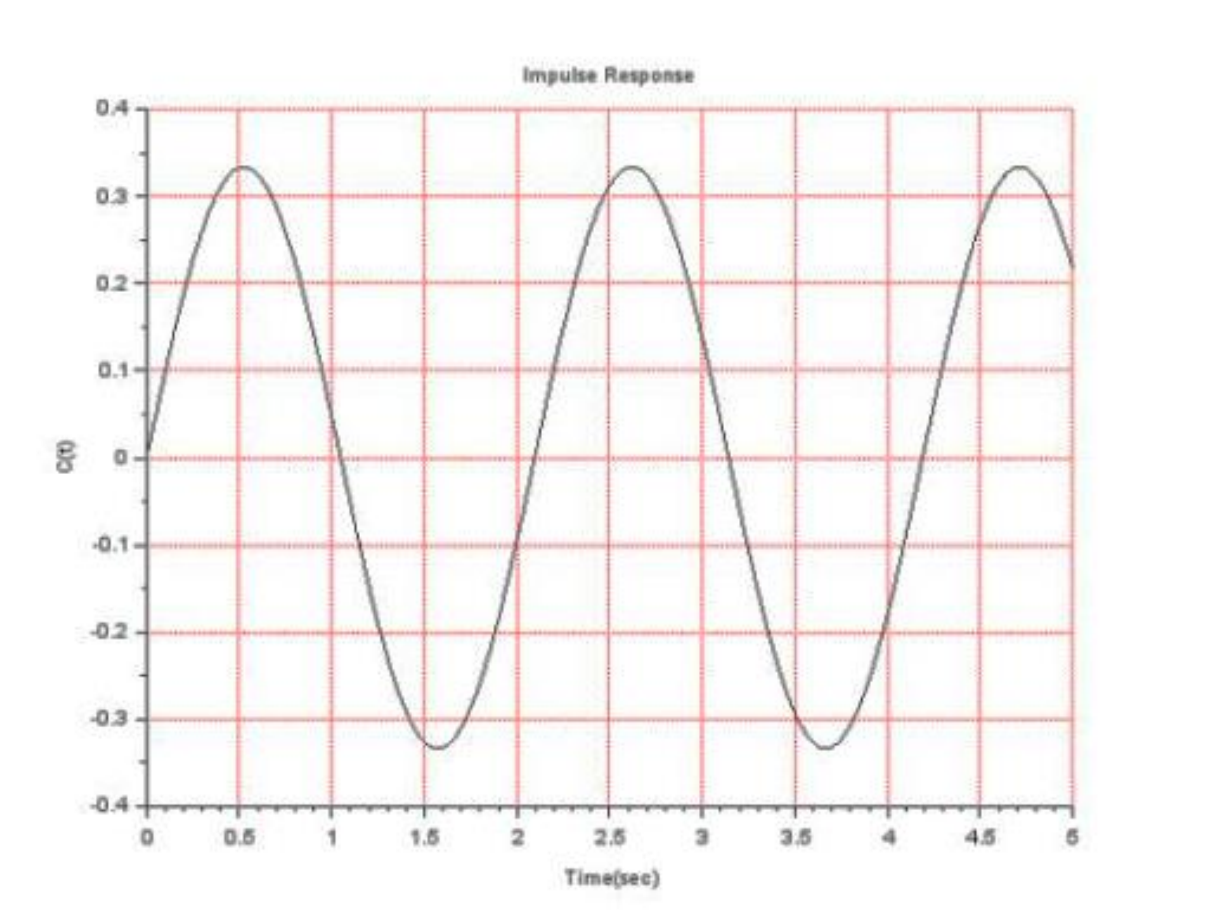


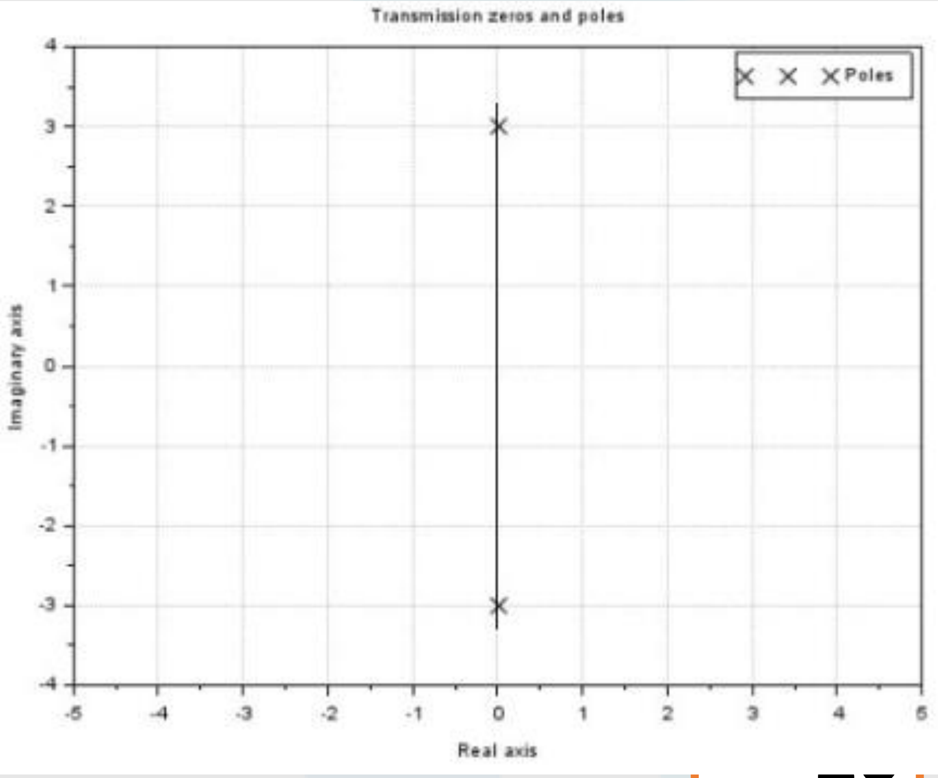
Case 8 – Poles on the imaginary axis (One pair)

- complex pole pair at s =+3*j* and at *s* = -3*j*
- This means the transfer function would be

$$G(s) = \frac{1}{(s-3j)(s+3j)} = \frac{1}{s^2+3^2} = \frac{1}{s^2+9}$$

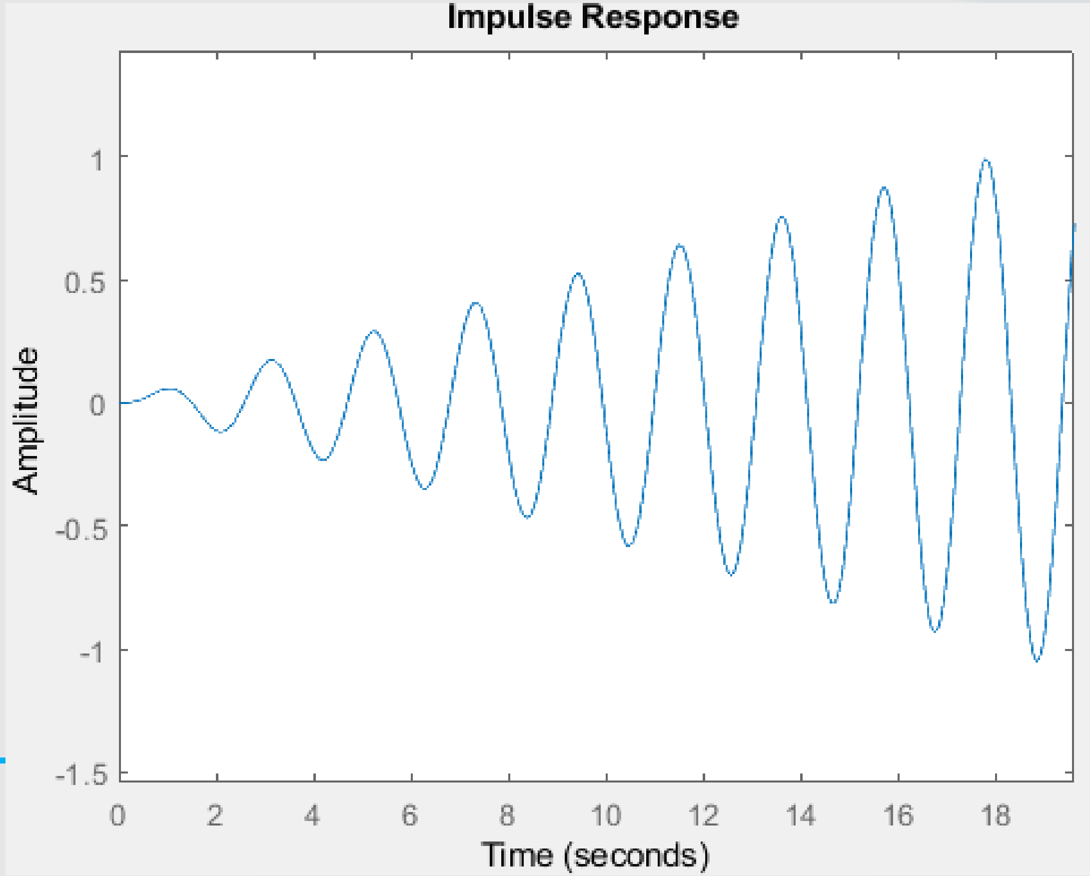
g(t)=0.33sin(3t)



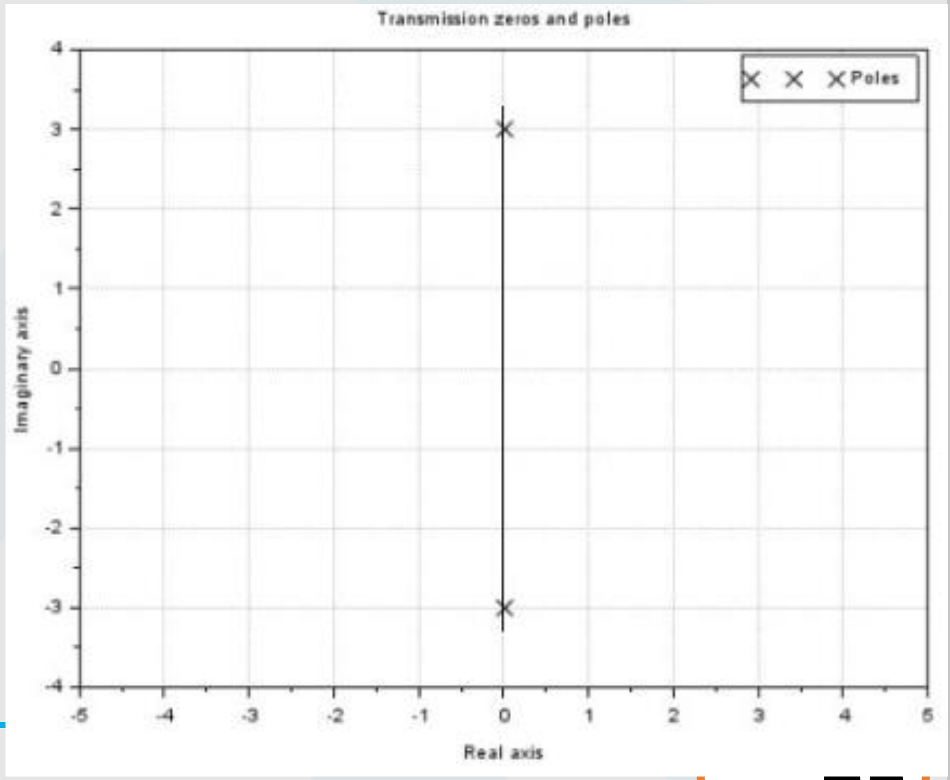


Case 9 – Poles on the imaginary axis (multiple pairs)

- Consider two complex pole pairs at s =+3*j* and at *s* = -3*j*
- This means the transfer function would be $G(s) = \frac{1}{((s-3j)(s+3j))^2} = \frac{1}{(s^2+3^2)^2} = \frac{1}{(s^2+9)^2}$

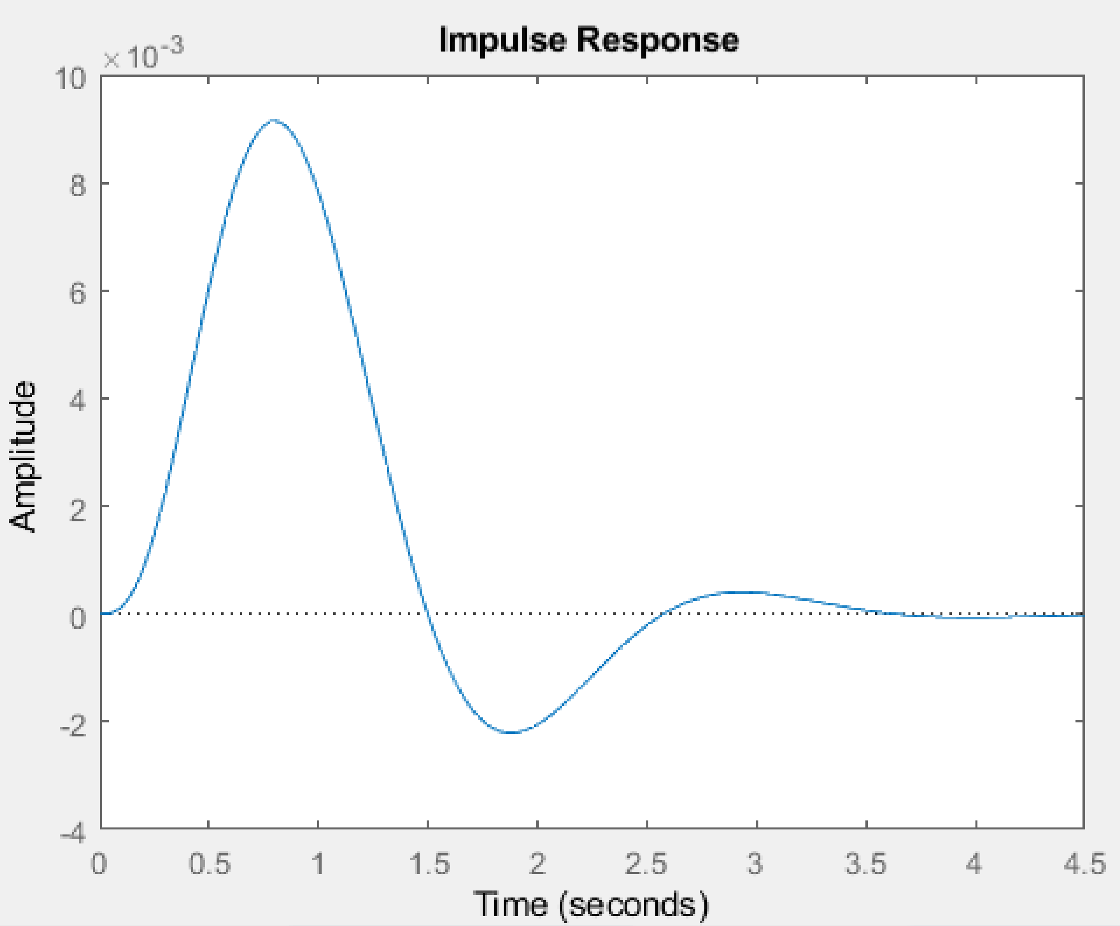


g(t)=sin(3*t)/54 - (t*cos(3*t))/18



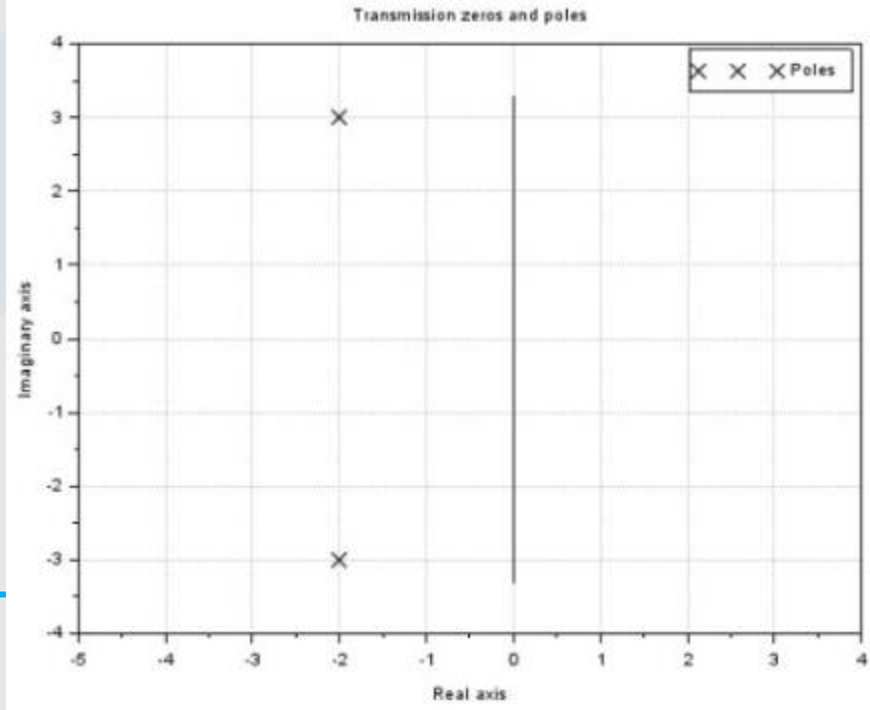
Case 10 – Repeated pairs of poles in LHP

- Consider a repeated complex pole pair at s = -2+3*j* and at *s* = -2-3*j* (repeated 2 times)
- This means the transfer function would be $G(s) = \dfrac{1}{((s+2-3j)(s+2+3j))^2} = \dfrac{1}{((s+2)^2+3^2)^2} = \dfrac{1}{(s^2+4s+13)^2}$



g(t)=(sin(3*t)*exp(-2*t))/54 - (t*(cos(3*t)*exp(-2*t) - sin(3*t)*exp(-2*t)*1i))/36 - (t*(cos(3*t)*exp(-2*t) + sin(3*t)*exp(-2*t)*1i))/36

$$g(t) = \frac{\sin(3t)e^{-2t}}{54} - \frac{t(\cos(3t)e^{-2t} - \sin(3t)e^{-2t}i)}{36} - \frac{t(\cos(3t)e^{-2t} + \sin(3t)e^{-2t}i)}{36}$$

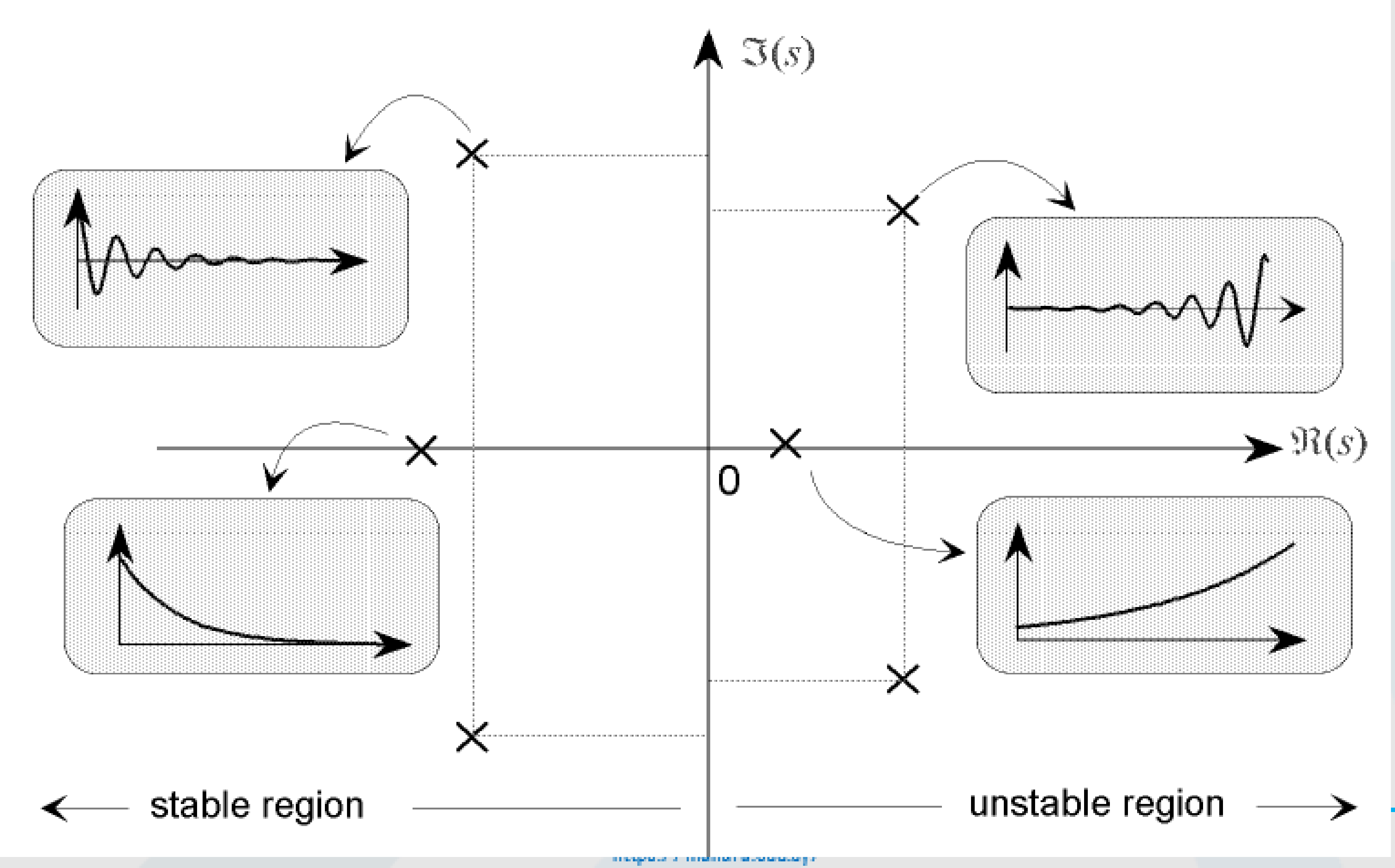


Role of a pole (location) in system response

Let's conclude a few points based on the cases discussed above.

- If **all** the poles lie in the left half of the s-plane, then the system is stable.
- If the system has one non-repeated pole on the imaginary axis (in the origin of s-plane), then the system is marginally stable.
- If **any** pole lies in the right half of the s-plane, then the system is unstable
- If the system has any number of pair of poles on the imaginary axis, then the system is unstable.

Role of a pole (location) in system response



Role of a pole (location) in system response

Notice:

Some references say:

- If the system has two or more poles in the same location on the imaginary axis, then the system is unstable.
- If the system has one pole or non-repeated pair of poles on the imaginary axis, then the system is marginally stable.

Routh-Hurwitz criterion

The Routh-Hurwitz Method Stability Criteria

- **The method requires two steps**:
- Step #1: Generate a table called a Routh table (Routh array) as follows:
  - Consider the characteristic equation which is given by:

$$\Delta(s) = Q(s) = a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

Rules for Routh table creation

- Any row of the Routh table can be multiplied by a positive constant without changing the values of the rows below.
- To avoid the division by zero, an epsilon is assigned to replace the zero in the first column.

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2			
s^1			
s^0			

Initial layout for Routh table

The Routh-Hurwitz Method Stability Criteria

- Further rows of the schedule are then completed as follows:

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

Completed Routh table

The Routh-Hurwitz Method Stability Criteria

- **Step #2**: Interpret the Routh table to tell how many closed-loop system poles are in the:
  - left half-plane
  - right half-plane.
- The number of roots of the polynomial that are in the right half-plane is equal to the number of sign changes in the first column.
- **Notes**:
  - 1- If the coefficients of the characteristic equation have differing algebraic, there is at least one RHP root. For Example:

$$\Delta(s) = Q(s) = 7s^5 + 5s^4 - 3s^3 - 2s^2 + s + 10$$

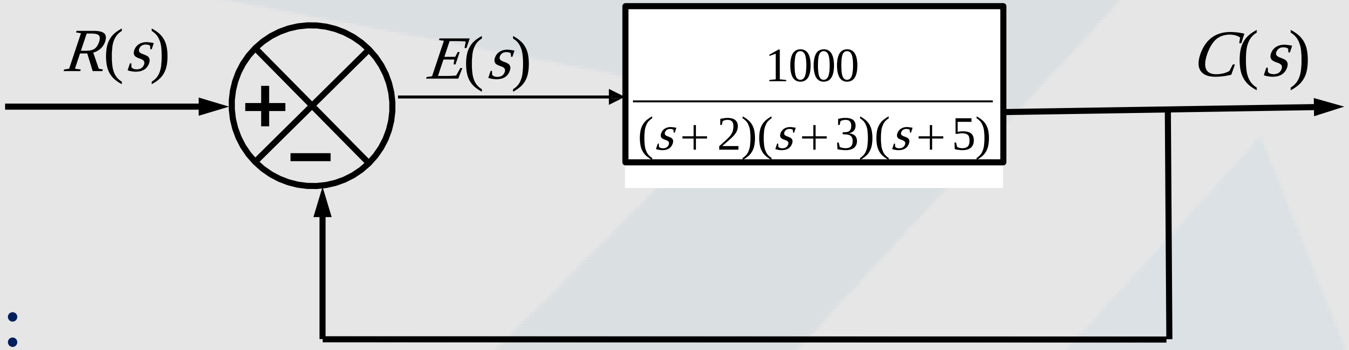
    - Has definitely one or more RHP roots.
  - 2- If one or more of the coefficients of the characteristic equation have zero value, there are imaginary or RHP roots or both. For Example:

$$\Delta(s) = Q(s) = s^6 + 3s^5 + 2s^4 + 8s^2 + 3s + 17$$

    - has imaginary axis roots indicated by missing s^3 term.

Example1

- For the system shown in Figure below, determine closed loop transfer function T(s) and then apply the Routh-Hurwitz Method to check its Stability .



- **Solution**: :
  - **Step #1**: Find the system closed loop transfer function T(s)

$$T(s)=\frac{G(s)}{1+G(s)H(s)}=\frac{\dfrac{1000}{(s+2)(s+3)(s+5)}}{1+\dfrac{1000}{(s+2)(s+3)(s+5)}}=\frac{1000}{s^3+10s^2+31s+1030}$$

Example1 (Cont.)

- Therefore, the characteristic equation is given by:

$$\Delta(s) = s^3 + 10s^2 + 31s + 1030$$

- Step #2: Generate the Routh table as follows:

Divide by 10

s^3	1	31	0
s^2	~~10~~ 1	~~1030~~ 103	0
s^1	$\frac{-\begin{vmatrix}1 & 31\\1 & 103\end{vmatrix}}{1} = -72$	$\frac{-\begin{vmatrix}1 & 0\\1 & 0\end{vmatrix}}{1} = 0$	$\frac{-\begin{vmatrix}1 & 0\\1 & 0\end{vmatrix}}{1} = 0$
s^0	$\frac{-\begin{vmatrix}1 & 103\\-72 & 0\end{vmatrix}}{-72} = 103$	$\frac{-\begin{vmatrix}1 & 0\\-72 & 0\end{vmatrix}}{-72} = 0$	$\frac{-\begin{vmatrix}1 & 0\\-72 & 0\end{vmatrix}}{-72} = 0$

- Step #3: Since. There are two sign changes in the left column.
  - therefore, the system is unstable and has two roots in the RHP.

Example2

- Apply the Routh-Hurwitz Method to determine the stability of a the closed loop system whose transfer function is given by:

$$T(s) = \frac{10}{3s^5 + 5s^4 + 6s^3 + 3s^2 + 2s + 1}$$

- **Solution**: Generate the Routh table as follows:

s^5	3	6	2
s^4	5	3	1
s^3	4.2	1.4	
s^2	1.33	1	
s^1	-1.75		
s^0	1		

There are two sign changes in the left column, therefore, the system has two RHP roots and hence it is unstable

Example3

Example – consider a system with closed loop char. eqn
$Q(s) = s^4 + 4s^3 + 6s^2 + 2s + 3 = 0$ and investigate stability.

Soln we have, $Q(s) = s^4 + 4s^3 + 6s^2 + 2s + 3 = 0$
Then Routh array is as under

s^4	1	6	3
s^3	4	2	
s^2	5.5	3	
s^1	−0.1818		
s^0	3		

s^4	+
s^3	+
s^2	+
s^1	−
s^0	+

There are two sign changes in the first column of Routh array. Hence system is unstable with 2 poles in R.H.P of s-plane.

Example4

Example- Let $Q(s) = s^4 + 4s^3 + 6s^2 + 2s + 1 = 0$

Investigate stability.

sol^n- we have, $Q(s) = s^4 + 4s^3 + 6s^2 + 2s + 1 = 0$

Then Routh array is as under.

s^4	1	6	1
s^3	4	2	
s^2	5.5	1	
s^1	1.2727		
s^0	1		

As there is no sign change in the Routh array (first column) the system is stable.

Example5

Let us find the stability of the control system having characteristic equation,

$$s^4 + 3s^3 + 3s^2 + 2s + 1 = 0$$

s^4	1	3	1
s^3	3	2	
s^2	$\frac{(3\times3)-(2\times1)}{3} = \frac{7}{3}$	$\frac{(3\times1)-(0\times1)}{3} = \frac{3}{3} = 1$	
s^1	$\frac{\left(\frac{7}{3}\times2\right)-(1\times3)}{\frac{7}{3}} = \frac{5}{7}$		
s^0	1		

All the elements of the first column of the Routh array are positive. There is no sign change in the first column of the Routh array. So, the control system is stable.

Special cases of Routh stability test and their remedy

- **Special case-1** when a zero (0) appears in the first column of Routh array, further calculation is not possible as every element will be infinity and thus the formation of Routh array terminates.

Let $Q(s) = s^3 + a_1 s^2 + a_2 s + a_3 = 0$

Then the Routh array will be

$$\begin{array}{c|cc} s^3 & 1 & a_2 \\ s^2 & a_1 & a_3 \\ s^1 & \frac{a_1 a_2 - a_3}{a_1} = 0 & \end{array}$$

if $a_1 a_2 - a_3 = 0$

- **Remedy 1**- Replace 0 with small positive constant $\mathcal{E} \rightarrow 0$ and then determine the remaining elements in terms of $\mathcal{E}$ to Complete the Routh array and then replace $\mathcal{E}$ with 0 to check for sign change in the first column of Routh array.

Example1

Example- Let $Q(s) = s^4+3s^3+2s^2+6s+4=0$

Investigate stability.

sol- we have $Q(s) = s^4+3s^3+2s^2+6s+4=0$

Then Routh array is

s^4	1	2	4
s^3	3	6	
s^2	$0 \to \varepsilon$	4	

Compute Routh array

s^4	1	2	4
s^3	3	6	
s^2	ε	4	
s^1	$\frac{6\varepsilon-12}{\varepsilon}$		
s^0	4		

In row s^1

$$\lim_{\varepsilon \to 0} \frac{6\varepsilon-12}{\varepsilon} = -\infty$$

The sign profile of R.A. is

s^4	+
s^3	+
s^2	+
s^1	−
s^0	+

As there are two sign changes the system is unstable.

Example2

Let us find the stability of the control system having characteristic equation,

$$s^4 + 2s^3 + s^2 + 2s + 1 = 0$$

s^4	1	1	1
s^3	~~2~~ 1	~~2~~ 1	
s^2	$\frac{(1\times1)-(1\times1)}{1} = 0$	$\frac{(1\times1)-(0\times1)}{1} = 1$	
s^1			
s^0			

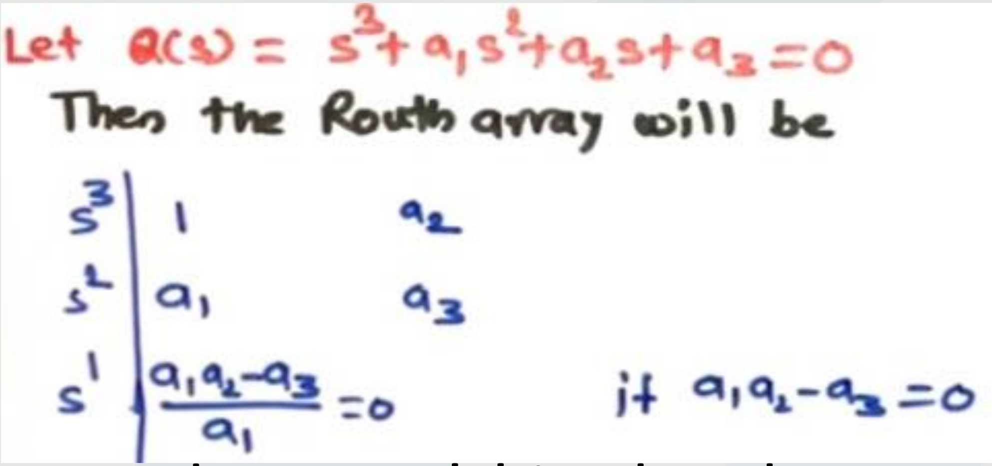
s^4	1	1	1
s^3	1	1	
s^2	ϵ	1	
s^1	$\frac{(\epsilon\times1)-(1\times1)}{\epsilon} = \frac{\epsilon-1}{\epsilon}$		
s^0	1		

s^4	1	1	1
s^3	1	1	
s^2	0	1	
s^1	$-\infty$		
s^0	1		

There are two sign changes in the first column of Routh table. Hence, the control system is unstable.

Special cases of Routh stability test and their remedy

- Special case-1 when a zero (0) appears in the first column of Routh array, further calculation is not possible as every element will be infinity and thus the formation of Routh array terminates.



- **Remedy 2**- In this remedy every 's' in the characteristic equation is replaced by (1/z) to get new characteristic equation. Then form Routh array for this new characteristic equation and check for sign changes.

Example

Example- Let $Q(s) = s^4 + 2s^3 + 2s^2 + 4s + 3 = 0$

$\underline{sol^n}$ $Q(s) = s^4 + 2s^3 + 2s^2 + 4s + 3 = 0$

Routh array is

s^4	1	2	3
s^3	2	4	
s^2	0		

Replace s with $\frac{1}{z}$ to get

$$Q(z) = \frac{1}{z^4} + \frac{2}{z^3} + \frac{2}{z^2} + \frac{4}{z} + 3 = 0$$

$$= 3z^4 + 4z^3 + 2z^2 + 2z + 1 = 0$$

Routh array for $Q(z)$ is

z^4	3	2	1
z^3	4	2	
z^2	0.5	1	
z^1	−6		
z^0	1		

∴ As there are two sign changes in the first column of Routh array, the system is unstable.

Special cases of Routh stability test and their remedy

- **Special case-2** If all elements in a row of Routh array have all zero elements then computation of Routh away terminates.
- **Remedy** - In this case, the row just above the row having all zeros is considered, and its auxiliary equation is obtained. This auxiliary equation is differentiated with respect to 's' to get a new auxiliary equation for the row having all zeros. Then use this for further computations to form a Routh array.
- **In this case, if there is a sign change in the first column of Routh array system is unstable. Otherwise, the system is marginally stable.**

Example1

Let us find the stability of the control system having characteristic equation,

$$s^5 + 3s^4 + s^3 + 3s^2 + s + 3 = 0$$

s^5	1	1	1
s^4	~~3~~ 1	~~3~~ 1	~~3~~ 1
s^3	$\frac{(1\times1)-(1\times1)}{1} = 0$	$\frac{(1\times1)-(1\times1)}{1} = 0$	
s^2			
s^1			
s^0			

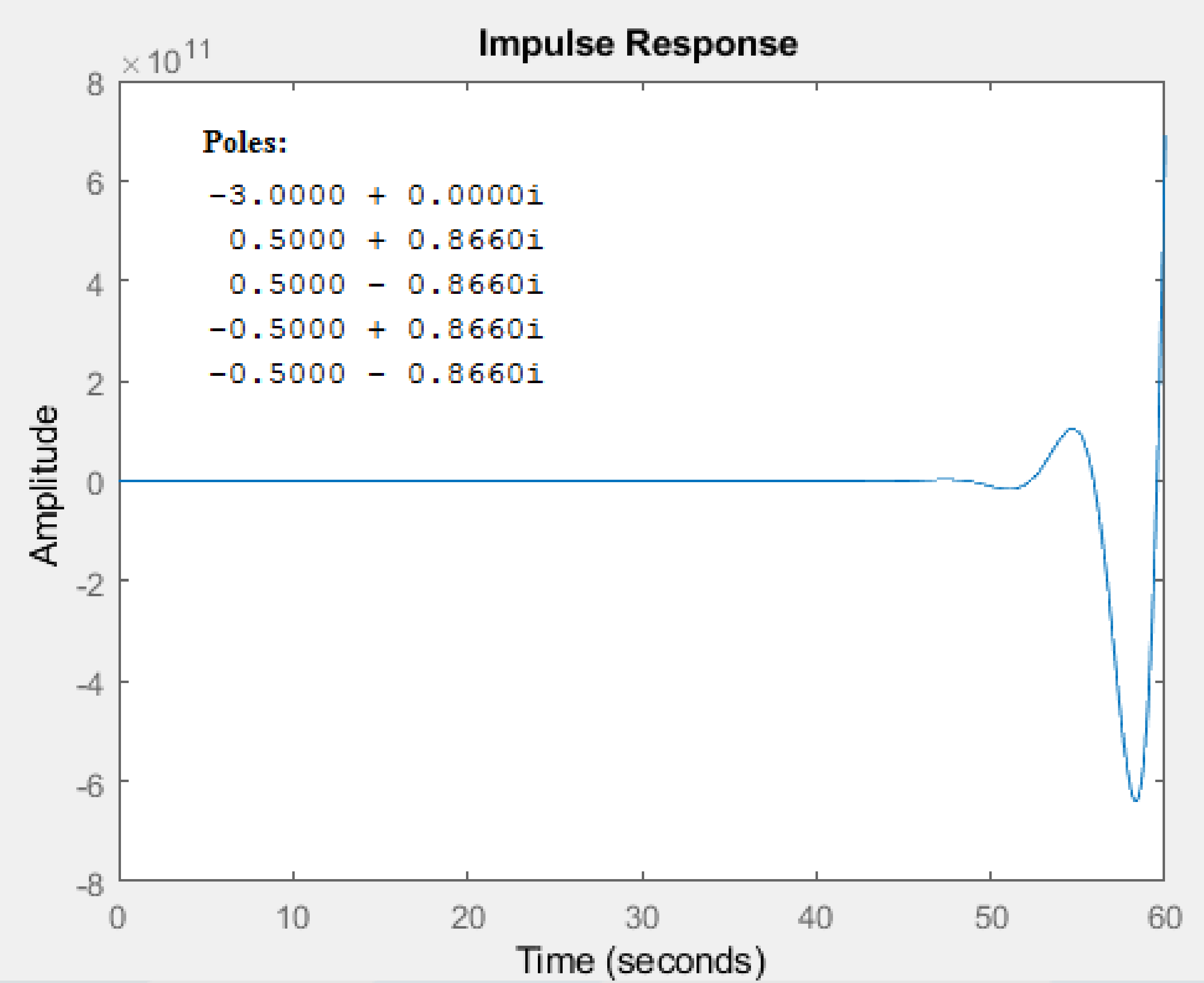
$$A(s) = s^4 + s^2 + 1$$

$$\frac{dA(s)}{ds} = 4s^3 + 2s$$

s^5	1	1	1
s^4	1	1	1
s^3	~~4~~ 2	~~2~~ 1	
s^2	$\frac{(2\times1)-(1\times1)}{2} = 0.5$	$\frac{(2\times1)-(0\times1)}{2} = 1$	
s^1	$\frac{(0.5\times1)-(1\times2)}{0.5} = \frac{-1.5}{0.5} = -3$		
s^0	1		

There are two sign changes in the first column of Routh table. Hence, the control system is unstable.

Example1



Impulse Response
×10^11
8
6
4
2
0
-2
-4
-6
-8
Amplitude
0
10
20
30
40
50
60
Time (seconds)
Poles:
-3.0000 + 0.0000i
0.5000 + 0.8660i
0.5000 - 0.8660i
-0.5000 + 0.8660i
-0.5000 - 0.8660i

Example2

Example- consider a system with char. eqn $Q(s)=s^6+3s^5+5s^4+9s^3+8s^2+6s+4=0$

Investigate stability using Routh stability test.

sol^n we have, $Q(s)=s^6+3s^5+5s^4+9s^3+8s^2+6s+4=0$

The Routh array is as follows

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	0	0		

consider row s^4 that has A.E.

$$A(s)=2s^4+6s^2+4$$

$$\frac{dA(s)}{ds}=8s^3+12s$$

s^6	1	5	8	4
s^5	3	9	6	
s^4	2	6	4	
s^3	8	12		
s^2	3	4		
s^1	1.3333			
s^0	4			

As there is no sign change in the first column of Routh array, the system is marginally stable.

Application of Routh stability test in system analysis (conditional stability)

Example1

Example-1 The unity feed back system has open loop transfer function $G(s) = \frac{k}{s(s+2)(s+5)(s+10)}$. Determine range of k for stability, values of k and frequency of oscillations at marginal stability.

Example1

Sol^n We have, $G(s) = \frac{K}{s(s+2)(s+5)(s+10)}$, $H(s) = 1$

The closed loop char. eq^n is $1 + G(s)H(s) = 0$

$$\therefore 1 + \frac{K}{s(s+2)(s+5)(s+10)} = 0$$

$$\therefore s^4 + 17s^3 + 80s^2 + 100s + K = 0$$

Routh array for C.L.C.E. is,

s^4	1	80	K
s^3	17	100	
s^2	74.1176	K	
s^1	$\frac{7411.76 - 17K}{74.1176}$		
s^0	K		

For stability,

1. $K > 0$
2. $\frac{7411.76 - 17K}{74.1176} > 0$

$\therefore K < \frac{7411.76}{17}$

$K < 435.9862$

$\therefore$ The range of K for stability is

$0 < K < 435.9862$

At marginal stability,

$K_{mar} = 435.9862$

From row s^2 we get,

$74.1176 s^2 + K = 0$

$\therefore 74.1176 s^2 + 435.9862 = 0$

$s^2 = -5.8824$

$\therefore s = \pm j\,2.4253$

Also on imaginary axis

$s = \pm j\omega$

$\therefore \omega = 2.4253$ rad/sec

↳ Freq. of oscillation

Example3

Example-3 For the system with char. eqn $Q(s)= s^3+2ks^2+(k+2)s+4=0$
Determine the range of k for stability.

soln: we have $Q(s)= s^3+2ks^2+(k+2)s+4=0$

Routh array is as follows

s^3	1	$k+2$
s^2	$2k$	4
s^1	$\frac{(k+2)2k-4}{2k}$	
s^0	4	

For stability

1. $k>0$

2. $(k+2)2k-4>0$

$\therefore 2k^2+4k-4>0$

$\therefore k^2+2k-2>0$

$\therefore (k-0.732)(k+2.732)>0$

$\therefore k>0.732$

$k>-2.732$

$\therefore$ For stability

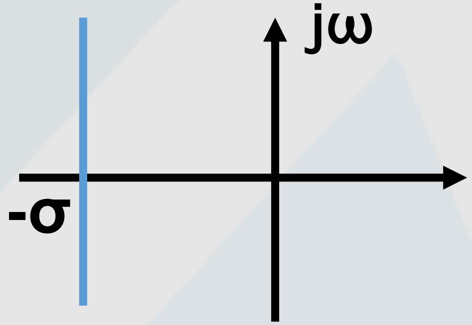
$k>0.732$

$\therefore$ Range of k for stability is

$0.732<k<\infty$

Application of Routh stability test for system analysis (relative stability)

- For the stable system all poles are towards left of imaginary axis in s-plane.
- Our interest is to check where all poles are towards left of $s=-\sigma$.



- Routh stability test can be used to check this by substituting $S=Z-\sigma$ in characteristic equation Q(S) to get new characteristic equation Q(Z).
- Then apply Routh stability test to Q(Z) and if there is no sign change in the first column of Routh array then all poles are towards the left of $s=-\sigma$.

Example1

Example-1 check whether all poles of system with closed loop C.E.
$Q(s) = s^3 + 10s^2 + 31s + 30$ are towards left of $s = -1$

Sol^n we have $Q(s) = s^3 + 10s^2 + 31s + 30 = 0$

Then $Q(z) = (z-1)^3 + 10(z-1)^2 + 31(z-1) + 30 = 0$

$\therefore z^3 - 3z^2 + 3z - 1 + 10z^2 - 20z + 10 + 31z - 31 + 30 = 0$

$\therefore Q(z) = z^3 + 7z^2 + 14z + 8 = 0$

Routh array for Q(z) is,

z^3	1	14
z^2	7	8
z^1	12.8571	
z^0	8	

As there is no sign change in the first column of Routh array, all poles of Q(s) are in the left of $s = -1$.

Example2

Example-2 For the system with closed loop char. eqn $Q(s) = s^3 + 4s^2 + 6s + k = 0$. determine range of k for stability. Also determine range of k such that all closed loop poles are left of $s = -1$.

Sol^n^ we have $Q(s) = s^3 + 4s^2 + 6s + k = 0$

Routh array is as follows

s^3	1	6
s^2	4	k
s^1	$\frac{24-k}{4}$	
s^0	k	

For stability

1. $k > 0$
2. $k < 24$

$\therefore$ Range of k for stability is

$0 < k < 24$

Example2

$$Q(z) = (z-1)^3 + 4(z-1)^2 + 6(z-1) + k = 0$$

$$Q(z) = z^3 - 3z^2 + 3z - 1 + 4z^2 - 8z + 4 + 6z - 6 + k = 0$$

$$\therefore Q(z) = z^3 + z^2 + z + k - 3$$

Routh array is as follows

z^3	1	1
z^2	1	$k-3$
z^1	$\frac{1-k+3}{1}$	
z^0	$k-3$	

For stability

1. $k - 3 > 0$

   $\therefore k > 3$

2. $4 - k > 0$

   $\therefore k < 4$

$\therefore$ The range of k for having all poles in the left of $s = -1$ is,

$$3 < k < 4$$