Calculus 1

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Calculus 1

Lecture 9

Integrals

Chapter 5
Integrals

Antiderivatives

More generally, starting with a function f, we want to find a function F whose derivative is f. If such a function F exists, it is called an *antiderivative* of f. Antiderivatives are the link connecting the two major elements of calculus: derivatives and definite integrals.

Finding Antiderivatives

DEFINITION A function F is an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Antiderivatives

EXAMPLE 1 Find an antiderivative for each of the following functions.

(a) $f(x) = 2x$ **(b)** $g(x) = \cos x$ **(c)** $h(x) = \frac{1}{x} + 2e^{2x}$

Solution We need to think backward here: What function do we know has a derivative equal to the given function?

(a) $F(x) = x^2$ **(b)** $G(x) = \sin x$ **(c)** $H(x) = \ln|x| + e^{2x}$

Each answer can be checked by differentiating. The derivative of $F(x) = x^2$ is $2x$. The derivative of $G(x) = \sin x$ is $\cos x$, and the derivative of $H(x) = \ln|x| + e^{2x}$ is $(1/x) + 2e^{2x}$. ■

The function $F(x) = x^2$ is not the only function whose derivative is $2x$. The function $x^2 + 1$ has the same derivative. So does $x^2 + C$ for any constant C. Are there others?

Antiderivatives

THEOREM 8 If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Antiderivatives

EXAMPLE 2 Find an antiderivative of $f(x) = 3x^2$ that satisfies $F(1) = -1$.

Solution Since the derivative of x^3 is $3x^2$, the general antiderivative

$$F(x) = x^3 + C$$

gives all the antiderivatives of $f(x)$. The condition $F(1) = -1$ determines a specific value for C. Substituting $x = 1$ into $F(x) = x^3 + C$ gives

$$F(1) = (1)^3 + C = 1 + C.$$

Since $F(1) = -1$, solving $1 + C = -1$ for C gives $C = -2$. So

$$F(x) = x^3 - 2$$

Antiderivatives

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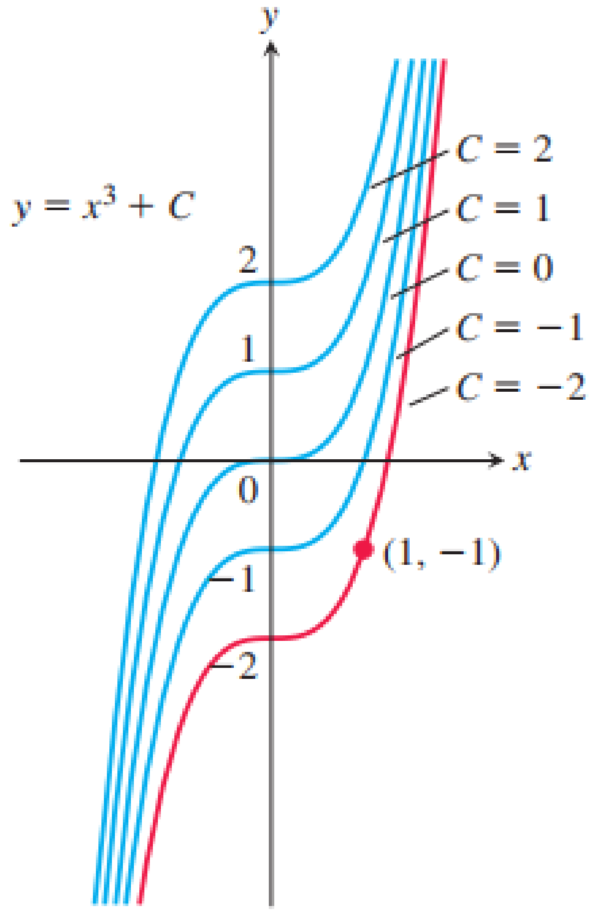

Antiderivatives

TABLE 4.2 Antiderivative formulas, k a nonzero constant

Function	General antiderivative	Function	General antiderivative
1. x^n	$\frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$	8. e^{kx}	$\frac{1}{k}e^{kx} + C$
2. $\sin kx$	$-\frac{1}{k}\cos kx + C$	9. $\frac{1}{x}$	$\ln\lvert x\rvert + C, \quad x \neq 0$
3. $\cos kx$	$\frac{1}{k}\sin kx + C$	10. $\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\sin^{-1} kx + C$
4. $\sec^2 kx$	$\frac{1}{k}\tan kx + C$	11. $\frac{1}{1+k^2x^2}$	$\frac{1}{k}\tan^{-1} kx + C$
5. $\csc^2 kx$	$-\frac{1}{k}\cot kx + C$	12. $\frac{1}{x\sqrt{k^2x^2-1}}$	$\sec^{-1} kx + C, kx > 1$
6. $\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$	13. a^{kx}	$\left(\frac{1}{k \ln a}\right)a^{kx} + C, a > 0, a \neq 1$
7. $\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$		

Antiderivatives

EXAMPLE 3 Find the general antiderivative of each of the following functions.

(a) $f(x) = x^5$ (b) $g(x) = \frac{1}{\sqrt{x}}$ (c) $h(x) = \sin 2x$

(d) $i(x) = \cos \frac{x}{2}$ (e) $j(x) = e^{-3x}$ (f) $k(x) = 2^x$

Antiderivatives

Solution In each case, we can use one of the formulas listed in Table 4.2.

(a) $F(x) = \dfrac{x^6}{6} + C$ — Formula 1 with $n = 5$

(b) $g(x) = x^{-1/2}$, so

$$G(x) = \frac{x^{1/2}}{1/2} + C = 2\sqrt{x} + C$$

Formula 1 with $n = -1/2$

(c) $H(x) = \dfrac{-\cos 2x}{2} + C$ — Formula 2 with $k = 2$

(d) $I(x) = \dfrac{\sin(x/2)}{1/2} + C = 2\sin\dfrac{x}{2} + C$ — Formula 3 with $k = 1/2$

(e) $J(x) = -\dfrac{1}{3}e^{-3x} + C$ — Formula 8 with $k = -3$

(f) $K(x) = \left(\dfrac{1}{\ln 2}\right)2^x + C$ — Formula 13 with $a = 2$, $k = 1$ ■

Antiderivatives

TABLE 4.3 Antiderivative linearity rules

		Function	General antiderivative
1.	*Constant Multiple Rule*:	$kf(x)$	$kF(x) + C$, k a constant
2.	*Negative Rule*:	$-f(x)$	$-F(x) + C$
3.	*Sum or Difference Rule*:	$f(x) \pm g(x)$	$F(x) \pm G(x) + C$

EXAMPLE 4 Find the general antiderivative of

$$f(x) = \frac{3}{\sqrt{x}} + \sin 2x.$$

Antiderivatives

$$F(x) = 3G(x) + H(x) + C$$
$$= 6\sqrt{x} - \frac{1}{2}\cos 2x + C$$

Indefinite Integrals

A special symbol is used to denote the collection of all antiderivatives of a function f.

DEFINITION The collection of all antiderivatives of f is called the **indefinite integral** of f with respect to x, and is denoted by

$$\int f(x)\,dx.$$

The symbol $\int$ is an **integral sign**. The function f is the **integrand** of the integral, and x is the **variable of integration**.

Indefinite Integrals

$$\int 2x\,dx = x^2 + C,$$

$$\int \cos x\,dx = \sin x + C,$$

$$\int \left(\sec^2 x + \frac{1}{2\sqrt{x}}\right) dx = \tan x + \sqrt{x} + C$$

EXAMPLE 6 Evaluate

$$\int (x^2 - 2x + 5)\,dx.$$

Indefinite Integrals

Solution If we recognize that $(x^3/3) - x^2 + 5x$ is an antiderivative of $x^2 - 2x + 5$, we can evaluate the integral as

$$\int (x^2 - 2x + 5)\,dx = \overbrace{\frac{x^3}{3} - x^2 + 5x}^{\text{antiderivative}} + \underbrace{C}_{\text{arbitrary constant}}.$$

If we do not recognize the antiderivative right away, we can generate it term-by-term with the Sum, Difference, and Constant Multiple Rules:

$$\begin{aligned}\int (x^2 - 2x + 5)\,dx &= \int x^2\,dx - \int 2x\,dx + \int 5\,dx \\ &= \int x^2\,dx - 2\int x\,dx + 5\int 1\,dx \\ &= \left(\frac{x^3}{3} + C_1\right) - 2\left(\frac{x^2}{2} + C_2\right) + 5(x + C_3) \\ &= \frac{x^3}{3} + C_1 - x^2 - 2C_2 + 5x + 5C_3.\end{aligned}$$

Indefinite Integrals

$$\int (x^2 - 2x + 5)\,dx = \int x^2\,dx - \int 2x\,dx + \int 5\,dx$$

$$= \frac{x^3}{3} - x^2 + 5x + C.$$

Integration by Parts

Integration by Parts Formula

$$\int u\,dv = uv - \int v\,du \qquad (2)$$

EXAMPLE 1 Find

$$\int x \cos x\,dx.$$

Solution We use the formula $\int u\,dv = uv - \int v\,du$ with

$$u = x, \qquad dv = \cos x\,dx,$$
$$du = dx, \qquad v = \sin x. \qquad \text{Simplest antiderivative of } \cos x$$

Then

$$\int x \cos x\,dx = x \sin x - \int \sin x\,dx = x \sin x + \cos x + C. \qquad \blacksquare$$

Integration by Parts

EXAMPLE 2 Find $\int \ln x \, dx$.

Solution

$$u(x) = \ln x \quad \text{and} \quad v'(x) = 1.$$

We differentiate $u(x)$ and find an antiderivative of $v'(x)$,

$$u'(x) = \frac{1}{x} \quad \text{and} \quad v(x) = x.$$

Then

$$\int \underbrace{\ln x}_{u(x)} \cdot \underbrace{1}_{v'(x)} \, dx = \underbrace{(\ln x)}_{u(x)} \underbrace{x}_{v(x)} - \int \underbrace{x}_{v(x)} \underbrace{\frac{1}{x}}_{u'(x)} \, dx$$ Integration by parts formula

$$= x \ln x - x + C$$ Simplify and integrate. ■

Integration by Parts

EXAMPLE 3 Evaluate

$$\int x^2 e^x \, dx.$$

Solution We use the integration by parts formula Equation (1) with

$$u(x) = x^2 \quad \text{and} \quad v'(x) = e^x.$$

We differentiate $u(x)$ and find an antiderivative of $v'(x)$,

$$u'(x) = 2x \quad \text{and} \quad v(x) = e^x.$$

We summarize this choice by setting $du = u'(x)\,dx$ and $dv = v'(x)\,dx$, so

$$du = 2x\,dx \quad \text{and} \quad dv = e^x\,dx.$$

We then have

$$\int \underbrace{x^2}_{u}\,\underbrace{e^x\,dx}_{dv} = \underbrace{x^2}_{u}\,\underbrace{e^x}_{v} - \int \underbrace{e^x}_{v}\,\underbrace{2x\,dx}_{du}.$$

Integration by parts formula

Integration by Parts

$u = x,\ dv = e^x\,dx$. Then $du = dx,\ v = e^x$, and

$$\int \underbrace{x}_{u}\underbrace{e^x\,dx}_{dv} = \underbrace{x}_{u}\underbrace{e^x}_{v} - \int \underbrace{e^x}_{v}\,\underbrace{dx}_{du} = xe^x - e^x + C.$$

Integration by parts Equation (2)
$u = x,\ dv = e^x\,dx$
$v = e^x, \quad du = dx$

Using this last evaluation, we then obtain

$$\int x^2 e^x\,dx = x^2 e^x - 2\int xe^x\,dx$$
$$= x^2 e^x - 2xe^x + 2e^x + C,$$

where the constant of integration is renamed after substituting for the integral on the right. ∎

Integration by Parts

EXAMPLE 4 Evaluate

$$\int e^x \cos x \, dx.$$

Solution Let $u = e^x$ and $dv = \cos x \, dx$. Then $du = e^x \, dx$, $v = \sin x$, and

$$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x \, dx. \qquad u(x) = e^x, \quad v(x) = \sin x$$

The second integral is like the first except that it has $\sin x$ in place of $\cos x$. To evaluate it, we use integration by parts with

$$u = e^x, \qquad dv = \sin x \, dx, \qquad v = -\cos x, \qquad du = e^x \, dx.$$

Integration by Parts

Then

$$\int e^x \cos x\, dx = e^x \sin x - \left(-e^x \cos x - \int (-\cos x)(e^x\, dx)\right) \qquad u(x) = e^x, \quad v(x) = -\cos x$$

$$= e^x \sin x + e^x \cos x - \int e^x \cos x\, dx.$$

The unknown integral now appears on both sides of the equation, but with opposite signs. Adding the integral to both sides and adding the constant of integration give

$$2\int e^x \cos x\, dx = e^x \sin x + e^x \cos x + C_1.$$

Dividing by 2 and renaming the constant of integration give

$$\int e^x \cos x\, dx = \frac{e^x \sin x + e^x \cos x}{2} + C.$$

Integration by Parts

EXAMPLE 5 Obtain a formula that expresses the integral

$$\int \cos^n x\,dx$$

Solution We may think of $\cos^n x$ as $\cos^{n-1} x \cdot \cos x$. Then we let

$$u = \cos^{n-1} x \qquad \text{and} \qquad dv = \cos x\,dx,$$

so that

$$du = (n-1)\cos^{n-2} x\,(-\sin x\,dx) \qquad \text{and} \qquad v = \sin x.$$

Integration by parts then gives

$$\begin{aligned}\int \cos^n x\,dx &= \cos^{n-1} x \sin x + (n-1)\int \sin^2 x \cos^{n-2} x\,dx \\ &= \cos^{n-1} x \sin x + (n-1)\int (1-\cos^2 x)\cos^{n-2} x\,dx \\ &= \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx.\end{aligned}$$

Integration by Parts

If we add

$$(n-1)\int \cos^n x\, dx$$

to both sides of this equation, we obtain

$$n\int \cos^n x\, dx = \cos^{n-1} x \sin x + (n-1)\int \cos^{n-2} x\, dx.$$

We then divide through by n, and the final result is

$$\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\, dx.$$

$$\int \cos^3 x\, dx = \frac{\cos^2 x \sin x}{3} + \frac{2}{3}\int \cos x\, dx$$

$$= \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x + C.$$

Finding Antiderivatives

In Exercises 1–24, find an antiderivative for each function. Do as many as you can mentally. Check your answers by differentiation.

1. **a.** $2x$ **b.** x^2 **c.** $x^2 - 2x + 1$
2. **a.** $6x$ **b.** x^7 **c.** $x^7 - 6x + 8$
3. **a.** $-3x^{-4}$ **b.** x^{-4} **c.** $x^{-4} + 2x + 3$
4. **a.** $2x^{-3}$ **b.** $\frac{x^{-3}}{2} + x^2$ **c.** $-x^{-3} + x - 1$
5. **a.** $\frac{1}{x^2}$ **b.** $\frac{5}{x^2}$ **c.** $2 - \frac{5}{x^2}$
6. **a.** $-\frac{2}{x^3}$ **b.** $\frac{1}{2x^3}$ **c.** $x^3 - \frac{1}{x^3}$
7. **a.** $\frac{3}{2}\sqrt{x}$ **b.** $\frac{1}{2\sqrt{x}}$ **c.** $\sqrt{x} + \frac{1}{\sqrt{x}}$
8. **a.** $\frac{4}{3}\sqrt[3]{x}$ **b.** $\frac{1}{3\sqrt[3]{x}}$ **c.** $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$
9. **a.** $\frac{2}{3}x^{-1/3}$ **b.** $\frac{1}{3}x^{-2/3}$ **c.** $-\frac{1}{3}x^{-4/3}$

1. (a) x^2 (b) $\frac{x^3}{3}$ (c) $\frac{x^3}{3} - x^2 + x$

2. (a) $3x^2$ (b) $\frac{x^8}{8}$ (c) $\frac{x^8}{8} - 3x^2 + 8x$

3. (a) x^{-3} (b) $-\frac{x^{-3}}{3}$ (c) $-\frac{x^{-3}}{3} + x^2 + 3x$

4. (a) $-x^{-2}$ (b) $-\frac{x^{-2}}{4} + \frac{x^3}{3}$ (c) $\frac{x^{-2}}{2} + \frac{x^2}{2} - x$

5. (a) $\frac{-1}{x}$ (b) $\frac{-5}{x}$ (c) $2x + \frac{5}{x}$

6. (a) $\frac{1}{x^2}$ (b) $\frac{-1}{4x^2}$ (c) $\frac{x^4}{4} + \frac{1}{2x^2}$

7. (a) $\sqrt{x^3}$ (b) $\sqrt{x}$ (c) $\frac{2}{3}\sqrt{x^3} + 2\sqrt{x}$

8. (a) $x^{4/3}$ (b) $\frac{1}{2}x^{2/3}$ (c) $\frac{3}{4}x^{4/3} + \frac{3}{2}x^{2/3}$

9. (a) $x^{2/3}$ (b) $x^{1/3}$ (c) $x^{-1/3}$

11. a. $\frac{1}{x}$ b. $\frac{7}{x}$ c. $1-\frac{5}{x}$

12. a. $\frac{1}{3x}$ b. $\frac{2}{5x}$ c. $1+\frac{4}{3x}-\frac{1}{x^2}$

13. a. $-\pi \sin \pi x$ b. $3 \sin x$ c. $\sin \pi x - 3 \sin 3x$

14. a. $\pi \cos \pi x$ b. $\frac{\pi}{2} \cos \frac{\pi x}{2}$ c. $\cos \frac{\pi x}{2} + \pi \cos x$

15. a. $\sec^2 x$ b. $\frac{2}{3} \sec^2 \frac{x}{3}$ c. $-\sec^2 \frac{3x}{2}$

16. a. $\csc^2 x$ b. $-\frac{3}{2} \csc^2 \frac{3x}{2}$ c. $1 - 8 \csc^2 2x$

17. a. $\csc x \cot x$ b. $-\csc 5x \cot 5x$ c. $-\pi \csc \frac{\pi x}{2} \cot \frac{\pi x}{2}$

18. a. $\sec x \tan x$ b. $4 \sec 3x \tan 3x$ c. $\sec \frac{\pi x}{2} \tan \frac{\pi x}{2}$

19. a. e^{3x} b. e^{-x} c. $e^{x/2}$

20. a. e^{-2x} b. $e^{4x/3}$ c. $e^{-x/5}$

11. (a) $\ln|x|$ (b) $7\ln|x|$ (c) $x-5\ln|x|$

12. (a) $\frac{1}{3}\ln|x|$ (b) $\frac{2}{5}\ln|x|$ (c) $x+\frac{4}{3}\ln|x|+\frac{1}{x}$

13. (a) $\cos(\pi x)$ (b) $-3\cos x$ (c) $\frac{-\cos(\pi x)}{\pi}+\cos(3x)$

14. (a) $\sin(\pi x)$ (b) $\sin\left(\frac{\pi x}{2}\right)$ (c) $\left(\frac{2}{\pi}\right)\sin\left(\frac{\pi x}{2}\right)+\pi\sin x$

15. (a) $\tan x$ (b) $2\tan\left(\frac{x}{3}\right)$ (c) $-\frac{2}{3}\tan\left(\frac{3x}{2}\right)$

16. (a) $-\cot x$ (b) $\cot\left(\frac{3x}{2}\right)$ (c) $x+4\cot(2x)$

17. (a) $-\csc x$ (b) $\frac{1}{5}\csc(5x)$ (c) $2\csc\left(\frac{\pi x}{2}\right)$

18. (a) $\sec x$ (b) $\frac{4}{3}\sec(3x)$ (c) $\frac{2}{\pi}\sec\left(\frac{\pi x}{2}\right)$

19. (a) $\frac{1}{3}e^{3x}$ (b) $-e^{-x}$ (c) $2e^{x/2}$

20. (a) $-\frac{1}{2}e^{-2x}$ (b) $\frac{3}{4}e^{4x/3}$ (c) $-5e^{-x/5}$

21. **a.** 3^x **b.** 2^{-x} **c.** $\left(\frac{5}{3}\right)^x$

22. **a.** $x^{\sqrt{3}}$ **b.** x^{π} **c.** $x^{\sqrt{2}-1}$

21. (a) $\frac{1}{\ln 3}\cdot 3^x$ (b) $\frac{-1}{\ln 2}\cdot 2^{-x}$ (c) $\frac{1}{\ln(5/3)}\cdot\left(\frac{5}{3}\right)^x$

22. (a) $\frac{1}{\sqrt{3}+1}x^{\sqrt{3}+1}$ (b) $\frac{1}{\pi+1}x^{\pi+1}$ (c) $\frac{1}{\sqrt{2}}x^{\sqrt{2}}$

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33. $\int x^{-1/3}\,dx$

34. $\int x^{-5/4}\,dx$

35. $\int (\sqrt{x} + \sqrt[3]{x})\,dx$

36. $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$

37. $\int \left(8y - \frac{2}{y^{1/4}}\right) dy$

38. $\int \left(\frac{1}{7} - \frac{1}{y^{5/4}}\right) dy$

39. $\int 2x(1 - x^{-3})\,dx$

40. $\int x^{-3}(x + 1)\,dx$

41. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\,dt$

42. $\int \frac{4 + \sqrt{t}}{t^3}\,dt$

43. $\int (-2\cos t)\,dt$

44. $\int (-5\sin t)\,dt$

45. $\int 7\sin\frac{\theta}{3}\,d\theta$

46. $\int 3\cos 5\theta\,d\theta$

47. $\int (-3\csc^2 x)\,dx$

48. $\int \left(-\frac{\sec^2 x}{3}\right) dx$

49. $\int \frac{\csc\theta\cot\theta}{2}\,d\theta$

50. $\int \frac{2}{5}\sec\theta\tan\theta\,d\theta$

33. $\int x^{-1/3}dx = \frac{x^{2/3}}{\frac{2}{3}} + C = \frac{3}{2}x^{2/3} + C$

34. $\int x^{-5/4}dx = \frac{x^{-1/4}}{-\frac{1}{4}} + C = \frac{-4}{\sqrt[4]{x}} + C$

35. $\int\left(\sqrt{x} + \sqrt[3]{x}\right)dx = \int\left(x^{1/2} + x^{1/3}\right)dx = \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{4/3}}{\frac{4}{3}} + C = \frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$

36. $\int\left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right)dx = \int\left(\frac{1}{2}x^{1/2} + 2x^{-1/2}\right)dx = \frac{1}{2}\left(\frac{x^{3/2}}{\frac{3}{2}}\right) + 2\left(\frac{x^{1/2}}{\frac{1}{2}}\right) + C = \frac{1}{3}x^{3/2} + 4x^{1/2} + C$

37. $\int\left(8y - \frac{2}{y^{1/4}}\right)dy = \int\left(8y - 2y^{-1/4}\right)dy = \frac{8y^2}{2} - 2\left(\frac{y^{3/4}}{\frac{3}{4}}\right) + C = 4y^2 - \frac{8}{3}y^{3/4} + C$

38. $\int\left(\frac{1}{7} - \frac{1}{y^{5/4}}\right)dy = \int\left(\frac{1}{7} - y^{-5/4}\right)dy = \frac{1}{7}y - \left(\frac{y^{-1/4}}{-\frac{1}{4}}\right) + C = \frac{y}{7} + \frac{4}{y^{1/4}} + C$

39. $\int 2x\left(1 - x^{-3}\right)dx = \int\left(2x - 2x^{-2}\right)dx = \frac{2x^2}{2} - 2\left(\frac{x^{-1}}{-1}\right) + C = x^2 + \frac{2}{x} + C$

40. $\int x^{-3}(x+1)\,dx = \int\left(x^{-2} + x^{-3}\right)dx = \frac{x^{-1}}{-1} + \left(\frac{x^{-2}}{-2}\right) + C = -\frac{1}{x} - \frac{1}{2x^2} + C$

41. $\int \frac{t\sqrt{t} + \sqrt{t}}{t^2}\,dt = \int\left(\frac{t^{3/2}}{t^2} + \frac{t^{1/2}}{t^2}\right)dt = \int\left(t^{-1/2} + t^{-3/2}\right)dt = \frac{t^{1/2}}{\frac{1}{2}} + \left(\frac{t^{-1/2}}{-\frac{1}{2}}\right) + C = 2\sqrt{t} - \frac{2}{\sqrt{t}} + C$

42. $\int \frac{4+\sqrt{t}}{t^3}\,dt = \int\left(\frac{4}{t^3}+\frac{t^{1/2}}{t^3}\right)dt = \int\left(4t^{-3}+t^{-5/2}\right)dt = 4\left(\frac{t^{-2}}{-2}\right)+\left(\frac{t^{-3/2}}{-\frac{3}{2}}\right)+C = -\frac{2}{t^2}-\frac{2}{3t^{3/2}}+C$

43. $\int -2\cos t\; dt = -2\sin t + C$

44. $\int -5\sin t\; dt = 5\cos t + C$

45. $\int 7\sin\frac{\theta}{3}\,d\theta = -21\cos\frac{\theta}{3}+C$

46. $\int 3\cos 5\theta\, d\theta = \frac{3}{5}\sin 5\theta + C$

47. $\int -3\csc^2 x\, dx = 3\cot x + C$

48. $\int -\frac{\sec^2 x}{3}\,dx = -\frac{\tan x}{3}+C$

49. $\int \frac{\csc\theta\cot\theta}{2}\,d\theta = -\frac{1}{2}\csc\theta + C$

50. $\int \frac{2}{5}\sec\theta\tan\theta\, d\theta = \frac{2}{5}\sec\theta + C$

51. $\int (e^{3x} + 5e^{-x})\, dx$

52. $\int (2e^{x} - 3e^{-2x})\, dx$

53. $\int (e^{-x} + 4^{x})\, dx$

54. $\int (1.3)^{x}\, dx$

55. $\int (4 \sec x \tan x - 2 \sec^2 x)\, dx$

56. $\int \frac{1}{2}(\csc^2 x - \csc x \cot x)\, dx$

57. $\int (\sin 2x - \csc^2 x)\, dx$

58. $\int (2 \cos 2x - 3 \sin 3x)\, dx$

59. $\int \frac{1 + \cos 4t}{2}\, dt$

60. $\int \frac{1 - \cos 6t}{2}\, dt$

61. $\int \left(\frac{1}{x} - \frac{5}{x^2 + 1}\right) dx$

63. $\int 3x^{\sqrt{3}}\, dx$

64. $\int x^{\sqrt{2}-1}\, dx$

51. $\int (e^{3x} + 5e^{-x})dx = \frac{e^{3x}}{3} - 5e^{-x} + C$

52. $\int (2e^{x} - 3e^{-2x})dx = 2e^{x} + \frac{3}{2}e^{-2x} + C$

53. $\int (e^{-x} + 4^{x})dx = -e^{-x} + \frac{4^{x}}{\ln 4} + C$

54. $\int (1.3)^{x}\,dx = \frac{(1.3)^{x}}{\ln(1.3)} + C$

55. $\int (4\sec x\tan x - 2\sec^{2} x)\,dx = 4\sec x - 2\tan x + C$

56. $\int \frac{1}{2}(\csc^{2} x - \csc x\cot x)\,dx = -\frac{1}{2}\cot x + \frac{1}{2}\csc x + C$

57. $\int (\sin 2x - \csc^{2} x)\,dx = -\frac{1}{2}\cos 2x + \cot x + C$

58. $\int (2\cos 2x - 3\sin 3x)\,dx = \sin 2x + \cos 3x + C$

59. $\int \frac{1+\cos 4t}{2}\,dt = \int \left(\frac{1}{2} + \frac{1}{2}\cos 4t\right)dt = \frac{1}{2}t + \frac{1}{2}\left(\frac{\sin 4t}{4}\right) + C = \frac{t}{2} + \frac{\sin 4t}{8} + C$

60. $\int \frac{1-\cos 6t}{2}\,dt = \int \left(\frac{1}{2} - \frac{1}{2}\cos 6t\right)dt = \frac{1}{2}t - \frac{1}{2}\left(\frac{\sin 6t}{6}\right) + C = \frac{t}{2} - \frac{\sin 6t}{12} + C$

61. $\int\left(\frac{1}{x}-\frac{5}{x^2+1}\right)dx=\ln|x|-5\tan^{-1}x+C$

63. $\int 3x^{\sqrt{3}}dx=\frac{3x^{(\sqrt{3}+1)}}{\sqrt{3}+1}+C$

64. $\int x^{(\sqrt{2}-1)}dx=\frac{x^{\sqrt{2}}}{\sqrt{2}}+C$

65. $\int (1 + \tan^2 \theta)\, d\theta$

(*Hint:* $1 + \tan^2 \theta = \sec^2 \theta$)

66. $\int (2 + \tan^2 \theta)\, d\theta$

67. $\int \cot^2 x\, dx$

(*Hint:* $1 + \cot^2 x = \csc^2 x$)

68. $\int (1 - \cot^2 x)\, dx$

69. $\int \cos \theta (\tan \theta + \sec \theta)\, d\theta$

70. $\int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta$

65. $\int (1+\tan^2\theta)\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C$

66. $\int (2+\tan^2\theta)\,d\theta = \int (1+1+\tan^2\theta)\,d\theta = \int (1+\sec^2\theta)\,d\theta = \theta + \tan\theta + C$

67. $\int \cot^2 x\,dx = \int (\csc^2 x - 1)\,dx = -\cot x - x + C$

68. $\int (1-\cot^2 x)\,dx = \int (1-(\csc^2 x - 1))\,dx = \int (2-\csc^2 x)\,dx = 2x + \cot x + C$

69. $\int \cos\theta(\tan\theta + \sec\theta)\,d\theta = \int (\sin\theta + 1)\,d\theta = -\cos\theta + \theta + C$

70. $\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta = \int \left(\frac{\csc\theta}{\csc\theta - \sin\theta}\right)\left(\frac{\sin\theta}{\sin\theta}\right)\,d\theta = \int \frac{1}{1-\sin^2\theta}\,d\theta = \int \frac{1}{\cos^2\theta}\,d\theta = \int \sec^2\theta\,d\theta = \tan\theta + C$

1. $\int x \sin \frac{x}{2}\, dx$

3. $\int t^2 \cos t\, dt$

10. $\int (x^2 - 2x + 1)e^{2x}\, dx$

4. $\int x^2 \sin x\, dx$

6. $\int_1^e x^3 \ln x\, dx$

22. $\int e^{-y} \cos y\, dy$

1. $u = x,\ du = dx;\ dv = \sin\frac{x}{2}\,dx,\ v = -2\cos\frac{x}{2};$

$$\int x\sin\frac{x}{2}\,dx = -2x\cos\frac{x}{2} - \int\left(-2\cos\frac{x}{2}\right)dx = -2x\cos\left(\frac{x}{2}\right) + 4\sin\left(\frac{x}{2}\right) + C$$

$$\int t^2\cos t\,dt = t^2\sin t + 2t\cos t - 2\sin t + C$$

$$\int x^2\sin x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + C$$

6. $u = \ln x,\ du = \frac{dx}{x};\ dv = x^3\,dx,\ v = \frac{x^4}{4};$

$$\int_1^e x^3\ln x\,dx = \left[\frac{x^4}{4}\ln x\right]_1^e - \int_1^e \frac{x^4}{4}\frac{dx}{x} = \frac{e^4}{4} - \left[\frac{x^4}{16}\right]_1^e = \frac{3e^4+1}{16}$$

$$\int\left(x^2-2x+1\right)e^{2x}dx=\tfrac{1}{2}\left(x^2-2x+1\right)e^{2x}-\tfrac{1}{4}(2x-2)e^{2x}+\tfrac{1}{4}e^{2x}+C$$
$$=\left(\tfrac{1}{2}x^2-\tfrac{3}{2}x+\tfrac{5}{4}\right)e^{2x}+C$$

22. $I=\int e^{-y}\cos y\,dy;[u=\cos y,\,du=-\sin y\,dy;dv=e^{-y}\,dy,\,v=-e^{-y}]$

$\Rightarrow I=-e^{-y}\cos y-\int\left(-e^{-y}\right)(-\sin y)\,dy=-e^{-y}\cos y-\int e^{-y}\sin y\,dy;$

$[u=\sin y,\,du=\cos y\,dy;\,dv=e^{-y}dy,\,v=-e^{-y}]\Rightarrow I=-e^{-y}\cos y-\left(-e^{-y}\sin y-\int\left(-e^{y}\right)\cos y\,dy\right)$

$=-e^{-y}\cos y+e^{-y}\sin y-I+C'\Rightarrow 2I=e^{-y}\left(\sin y-\cos y\right)+C'\Rightarrow I=\tfrac{1}{2}\left(e^{-y}\sin y-e^{-y}\cos y\right)+C,$ where

$C=\frac{C'}{2}$ is another arbitrary constant

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Thank you for your attention