Lecture Outline

- Condition for the Equilibrium of a Particle. • شرط توازن النقطة المادية.
- Concurrent Coplanar Systems. • القوى المتلاقية المستوية.
- The Free-Body Diagram. • مخطط الجسم الحر.
- Pulley, cables and Spring. • الحبل، البكرة، النابض.
- Examples • أمثلة.

1. Condition for the Equilibrium of a Particle

1.شرط توازن النقطة المادية:

تكون النقطة المادية في حالة توازن إذا كانت في سكون تام أو تتحرك بسرعة منتظمة،

أي إذا كان تسارعها معدوماً: $\vec{a} = 0$.

- **A Particle is at *equilibrium* if:**

- At rest, or
- Moving at constant velocity.

Acceleration must vanish: $\vec{a} = 0$

- **Newton's second law of motion:** $\sum \vec{F} = m\vec{a}$,

where $\sum \vec{F}$ is the vector sum of all the forces acting on the particle

Condition for the Equilibrium of a Particle: $\sum \vec{F}$.

وحسب قانون نيوتن الثاني يكون شرط توازن نقطة مادية مادية، هو انعدام محصلة القوى المؤثرة عليها

$$\sum \vec{F} = 0 .$$

2. Concurrent Coplanar Systems 2.جملة القوى المستوية المتلاقية

A particle is subjected to coplanar forces in the *x*-*y* plane.

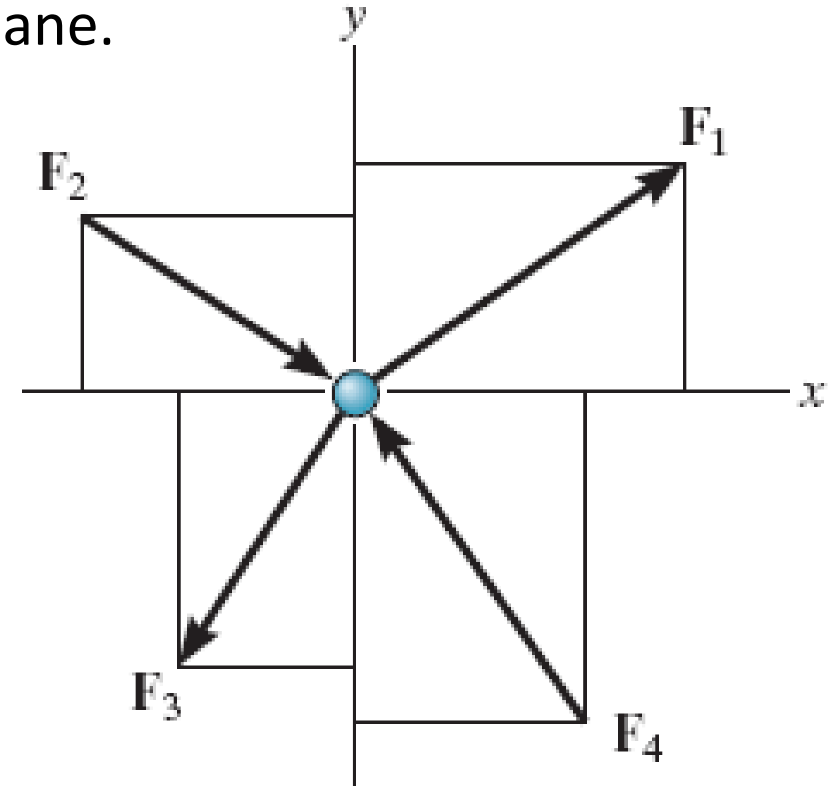
The vector equilibrium condition is:

$$\sum \vec{F} = 0.: \quad \vec{F} = F_x \hat{\imath} + F_y \hat{\jmath}$$

Resolve into *x* and *y* components for equilibrium

$$\sum F_x = 0 \;\; \& \;\; \sum F_y = 0$$

Scalar equations of equilibrium require that the algebraic sum of the *x* & *y* components to equal to zero.



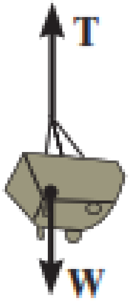
معادلتا التوازن:

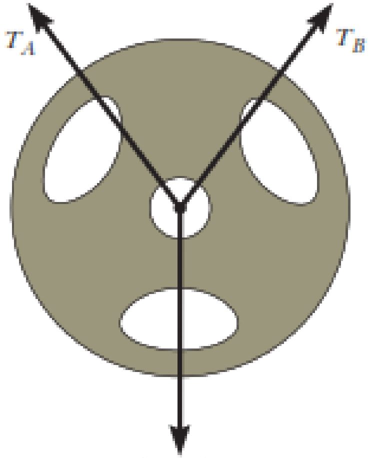
المجموع الجبري لمساقط القوى المؤثرة على كل من محوري الإحداثيات (الاختياريين) معدوم.

$$\sum F_x = 0 \;\; \& \;\; \sum F_y = 0$$

3. The Free-Body Diagram

- Best representation of all the forces ($\sum F$) which acts on a body (particle).
- A sketch showing the particle "free" from the surroundings with all the forces acting on it.
- Consider two common connections in this subject:
  - – Cables with or without Pulley.
  - – Spring or rubber cord.



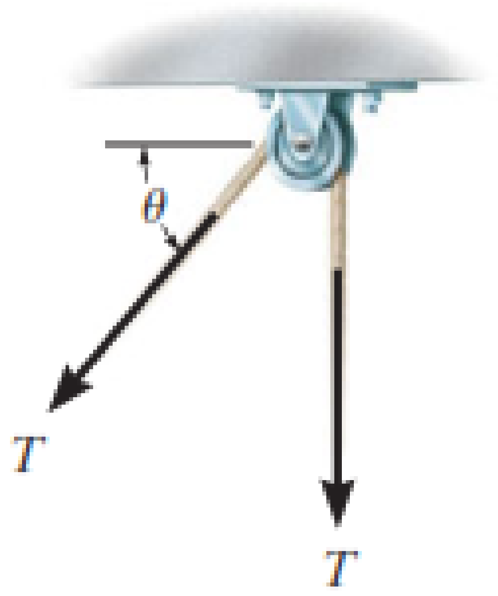


مخطط الجسم الحر: عزل الجسم أو النقطة المادية عن محيطها واستبدال علاقتها بهذا المحيط بقوى مناسبة

3. The Free-Body Diagram … common connections

Cables and Pulley

- Cables (or cords) are assumed of negligible weight & cannot stretch.
- Tension always acts in the direction of the cable.
- Tension force must have a constant magnitude for equilibrium
- For any angle θ, the cable is subjected to a constant tension *T*



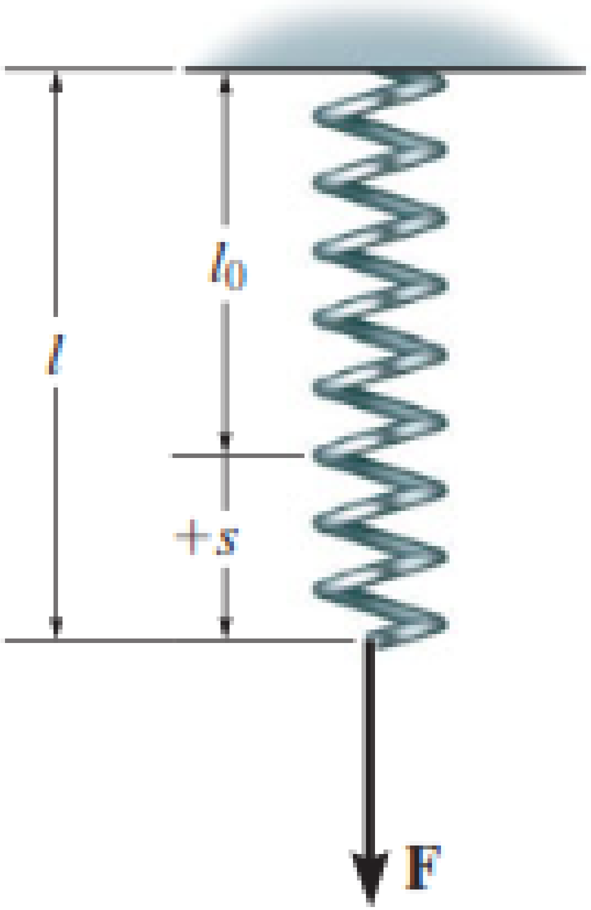
Cable is in tension

قوتا الشد في طرفي حبل مار على بكرة مثالية، متساويتان في الشدة، أما اتجاه كل منهما فهو استقامة الحبل من طرفها.

3. The Free-Body Diagram … common connections

Spring or Rubber Cord.

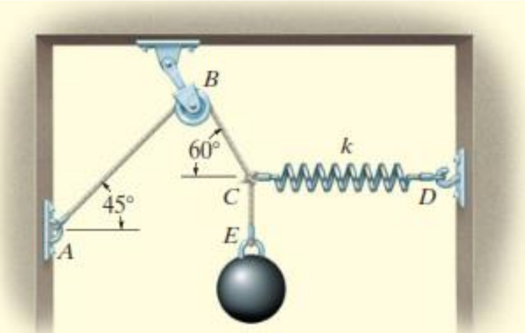
- Linear elastic spring: change in length is directly proportional to the force acting on it.
- Spring constant or *stiffness* k : defines the elasticity of the spring.
- Magnitude of force when spring is elongated or compressed, is: $F = k\,s = k(l - l_0)$



القوة في نابض مرن مثالي تتناسب مع الاستطالة. وعامل التناسب هو معامل صلابة النابض k

$$F = k\,s = k(l - l_0)$$

Example 1. A sphere has a mass of 6kg and is supported. Draw a free-body diagram of the sphere, cord CE and the knot at C.

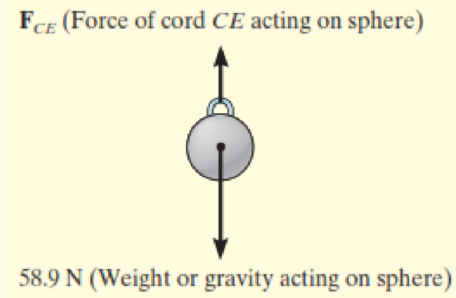


(a)

FBD of Sphere. Fig. b.

Two forces acting, weight and the force on cord CE.

Weight of 6kg ($9.81 m/s^2$) = 58.9N



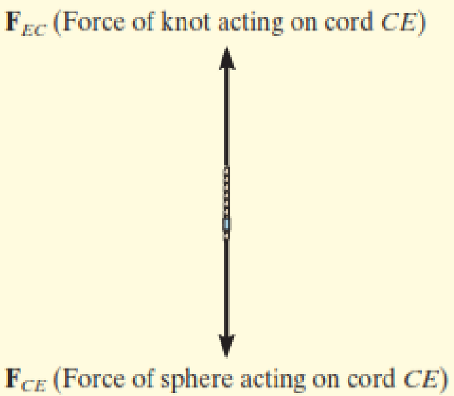
(b)

FBD of Cord CE. Fig. c.

Two forces acting: sphere and knot. Newton's 3rd Law:

$\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull the cord in tension

For equilibrium, $\mathbf{F}_{CE} = \mathbf{F}_{EC}$

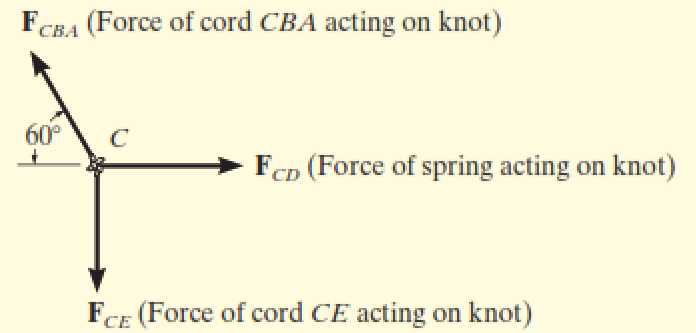


(c)

FBD at Knot. Fig. d.

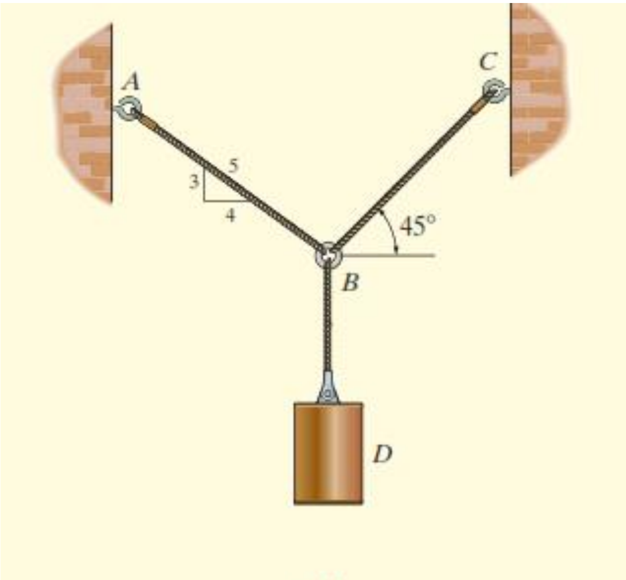
3 forces acting: cord CBA, cord CE and spring CD.

Important to know that the weight of the sphere does not act directly on the knot but subjected to by the cord CE.



(d)

Example 2 . Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig.a.



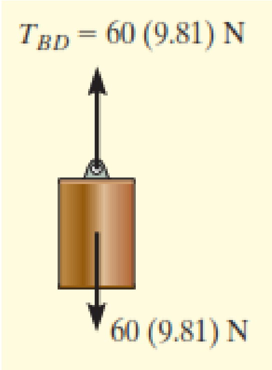
(a)

Solution

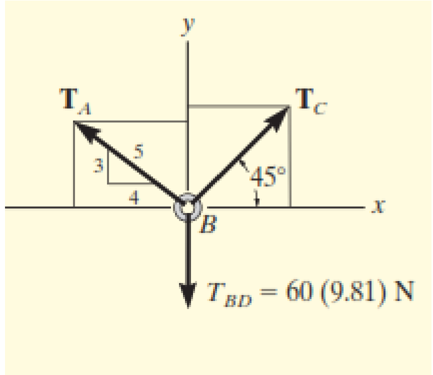
Free-Body Diagrams.

The tension in cable BD, acting on the cylinder in addition to its weight, is shown in the free-body diagram (Fig.b.). Due to equilibrium, this tension is T_{BD}=60(9.81)N= 589 N.

The forces in cables BA, BC can be determined by investigating the equilibrium of ring B. Its free-body diagram is shown in Fig.c.



(b)



(c)

The magnitudes of T_A and T_C are unknown, but their directions are known. From the Eq. Eqs.

Equilibrium Equations (Eq. Eqs.).

$$\sum F_x = 0: -T_A(4/5)+T_C\cos 45° = 0$$

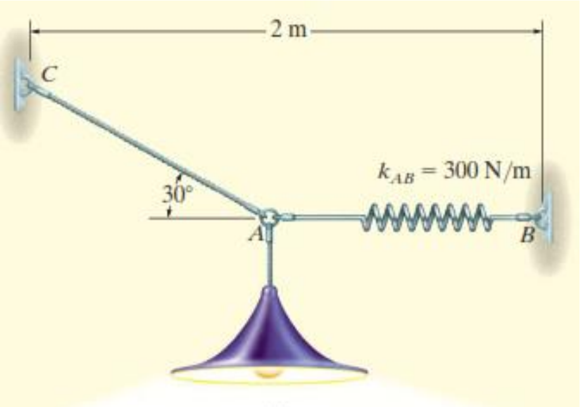
$$\sum F_y = 0: T_A(3/5)+T_C\sin 45° -589 = 0$$

Solving to get T_A =420N & T_C =476N

Example 3.

1. Determine the two forces T_{AB} & T_{AC} that the spring AB & the cord AC apply to the ring *A*, if the spring *AB* stays horizontal and the mass of the suspended lamp is 8 kg.

2. Determine the length of the cord *AC* if the undeformed length of the spring *AB* is l_{AB}^{0}= 0.4 m, and the its stiffness is k = 300N/m.



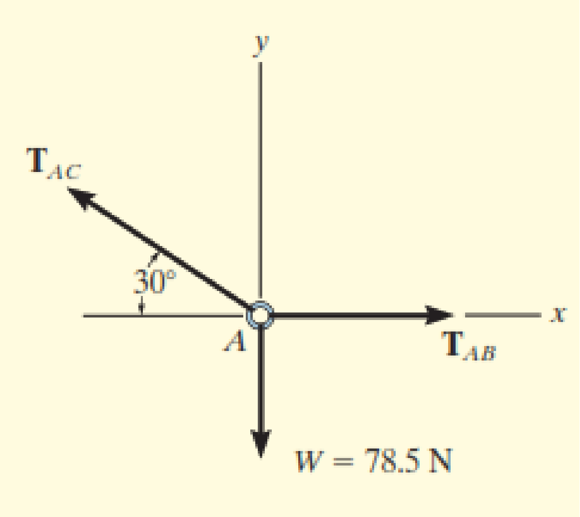
(a)

Solution 1.

FBD of Ring A

Three forces acting, force by cable AC, force in spring AB and weight of the lamp:

$W = gm = (9.81)(8) \approx 78.5\text{N}$



(b)

Eq. Eqs.

$$\left.\begin{aligned}\sum F_x = 0;\ T_{AB} - T_{AC}\cos 30^\circ = 0\\ \sum F_y = 0;\ T_{AC}\sin 30^\circ - 78.5N = 0\end{aligned}\right\}$$ Solving to get $T_{AC} = 157\text{N}$ and $T_{AB} = 136\text{N}$

Solution 2.

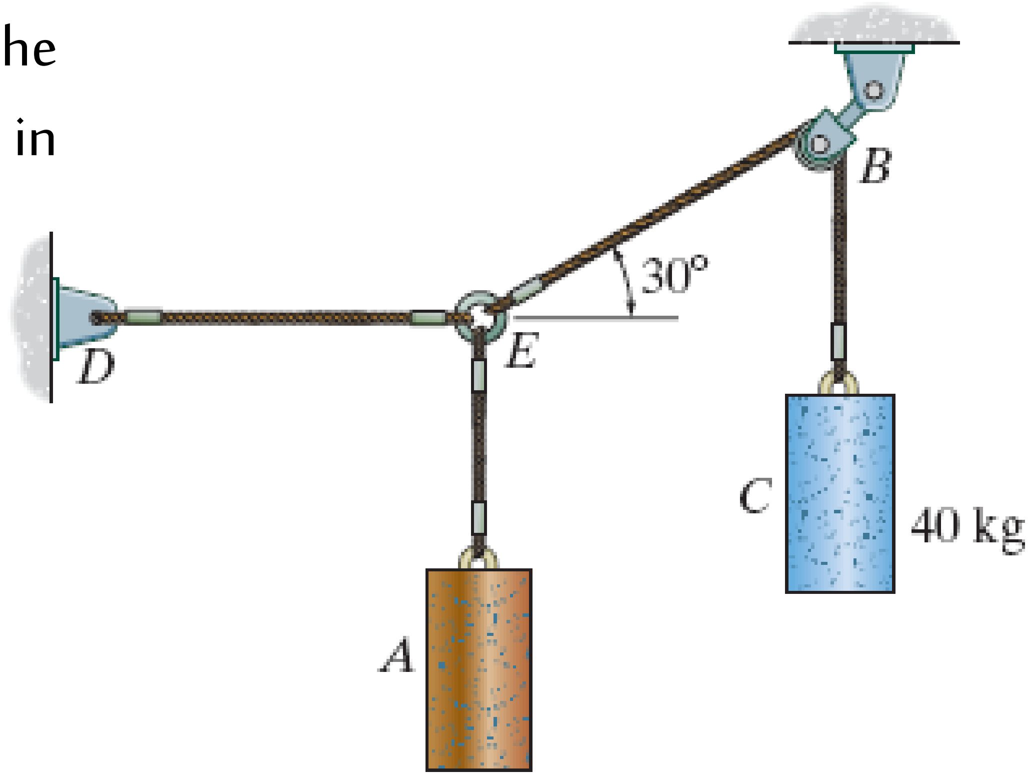
If force on cable AB is known, stretch of the spring is found by:

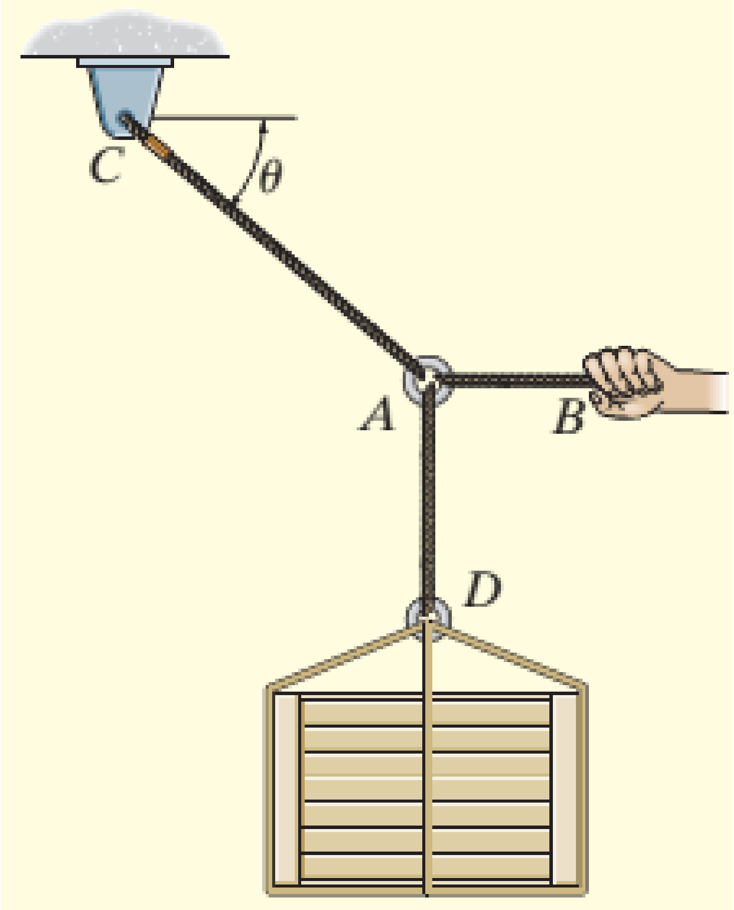
$T_{AB} = k_{AB}s_{AB} \Rightarrow 136 = 300s_{AB} \Rightarrow s_{AB} = 0.453\text{m}$

For stretched length, $l_{AB} = l^{o}_{AB} + s_{AB} \Rightarrow l_{AB} = 0.4\text{m} + 0.453\text{m} = 0.853\text{m}$ For horizontal distance BC,

$2\text{m} = l_{AC}\cos 30^\circ + 0.853\text{m} \Rightarrow l_{AC} = 1.32\text{m}$

Ex.1. If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.

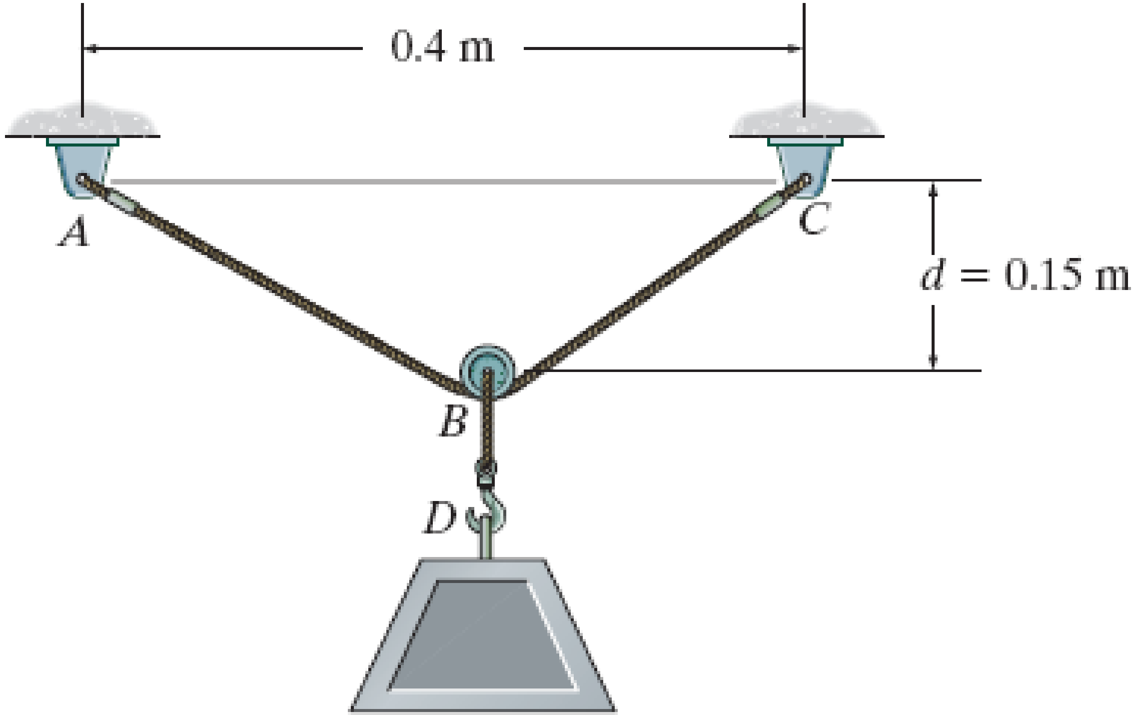


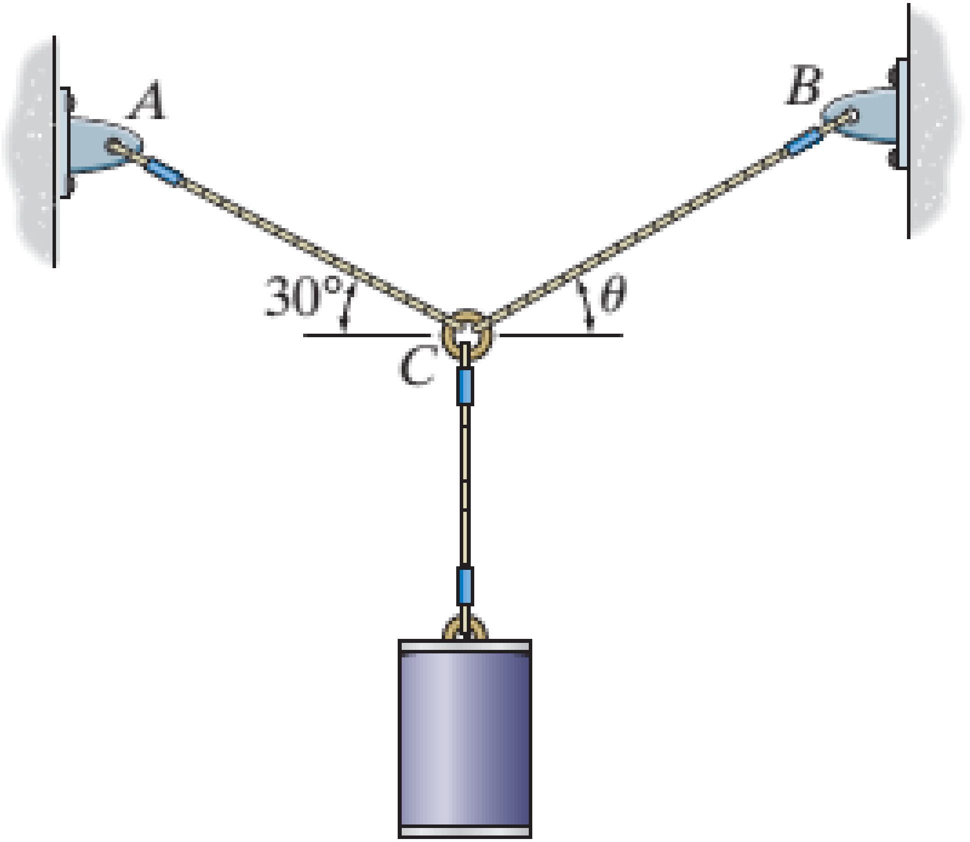


Ex.2. The 200-kg crate is suspended using the ropes AB, AC and AD. Each rope can withstand a maximum force of 10kN before it breaks.

If AB remains horizontal, determine the smallest angle θ to which the crate can be suspended before one of the ropes breaks.

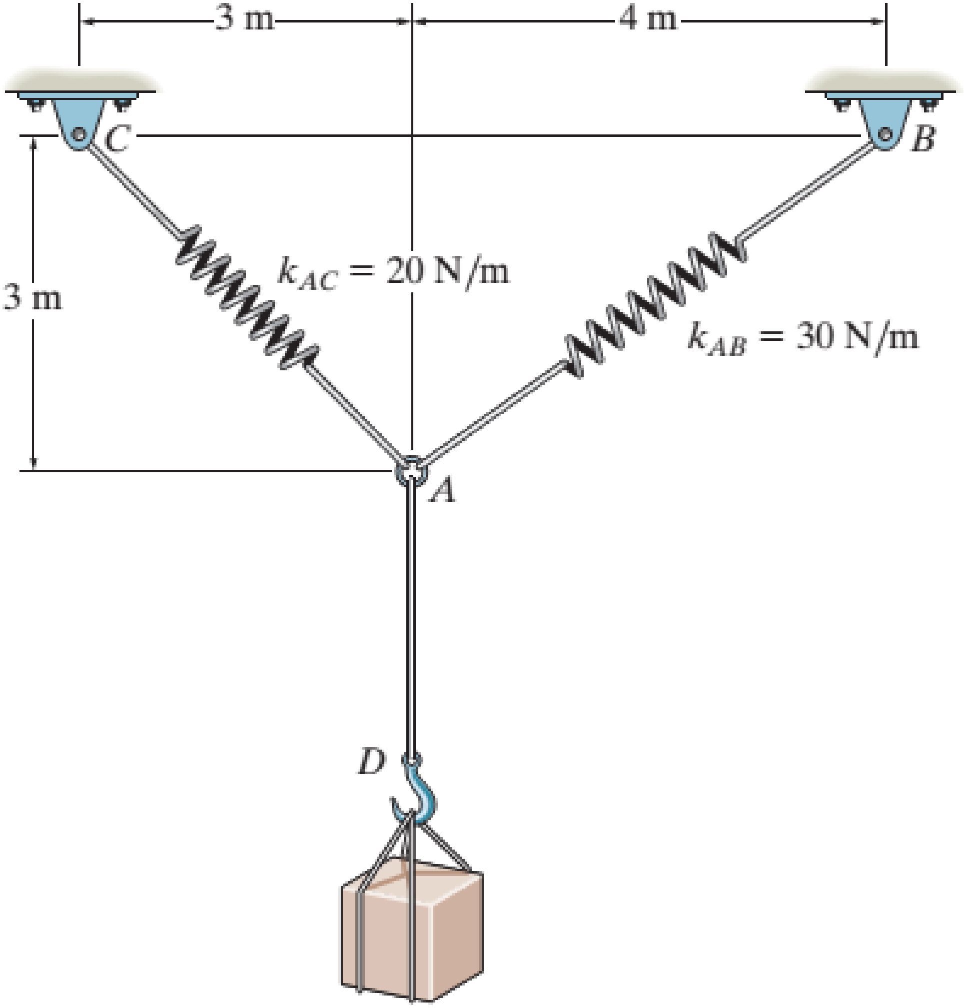
Ex.3. If the 5-kg block is suspended from the pulley B and the sag of the cord is $d = 0.15$ m, determine the force in cord ABC. Neglect the size of the pulley.

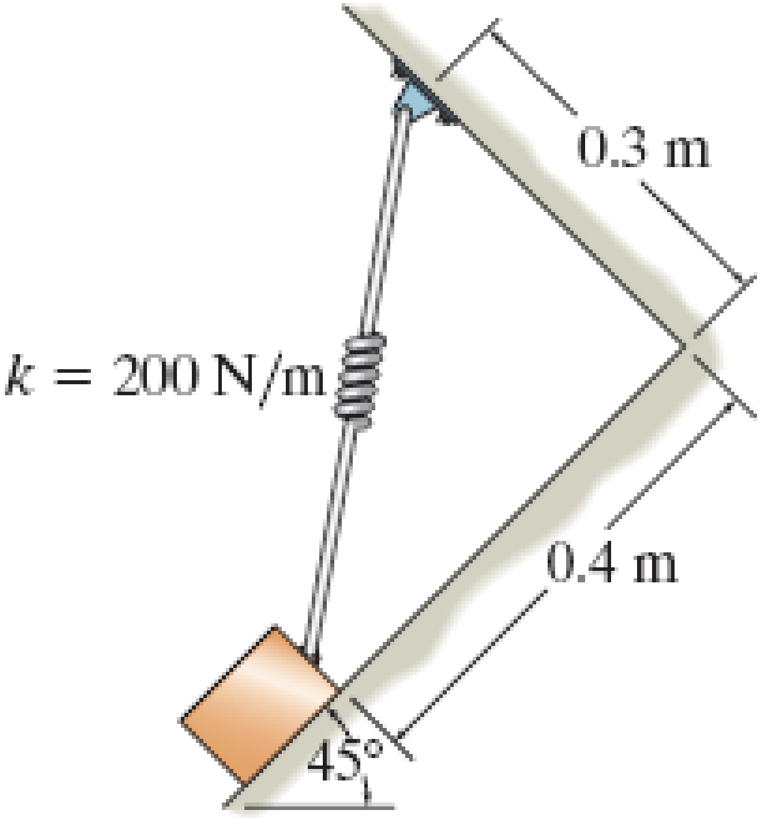




Ex.4. If cable CB is subjected to a tension that is twice that of cable CA, determine the angle for equilibrium of the 10-kg cylinder. Also, what are the tensions in wires CA and CB ?

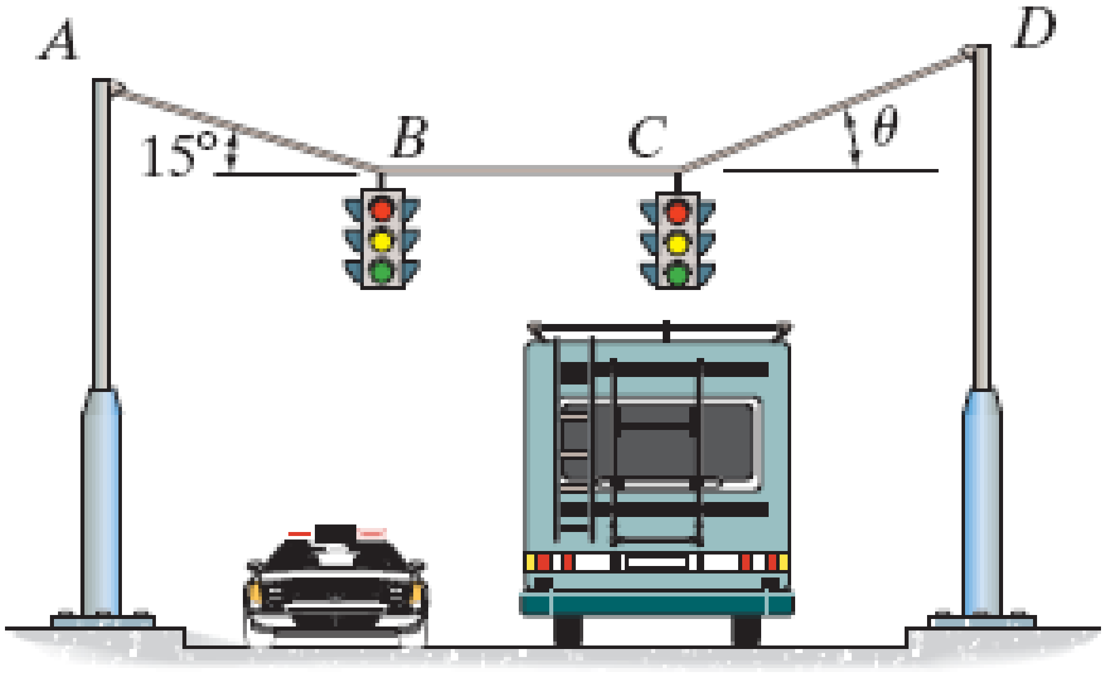
Ex.5. The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D and the unstretched length of spring AC.

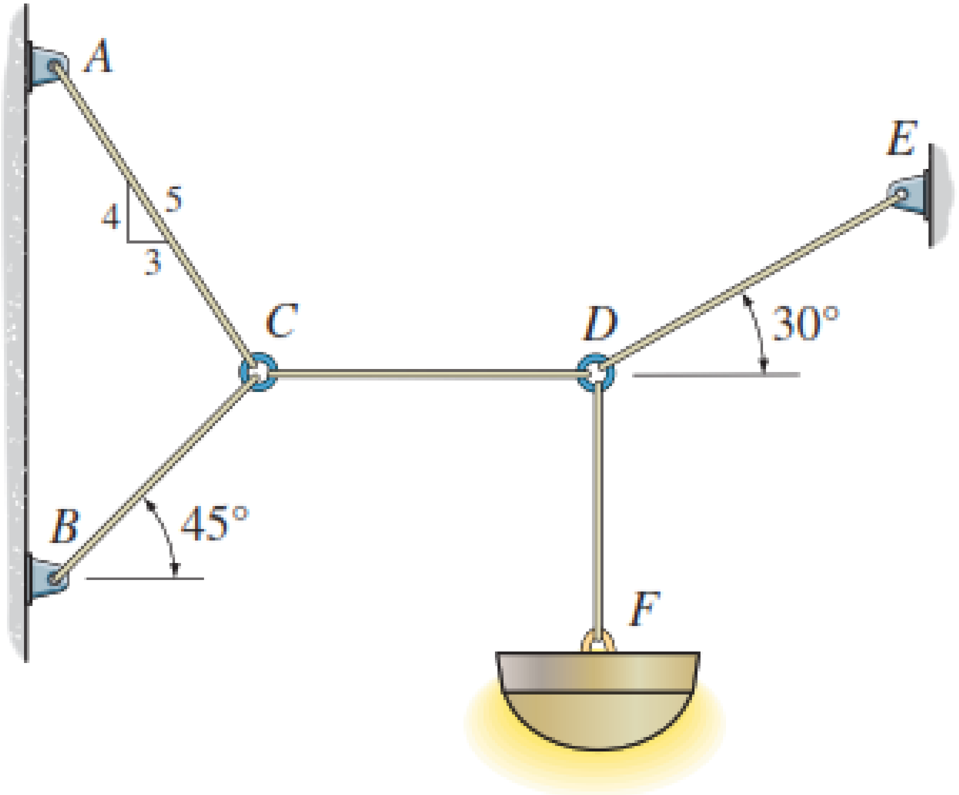




Ex.6. The block has a mass of 5 kg and rests on the smooth plane. Determine the unstretched length of the spring.

Ex.7. Determine the tension in cables AB, BC, and CD, necessary to support the 10-kg and 15-kg traffic lights at B and C, respectively. Also, find the angle θ.

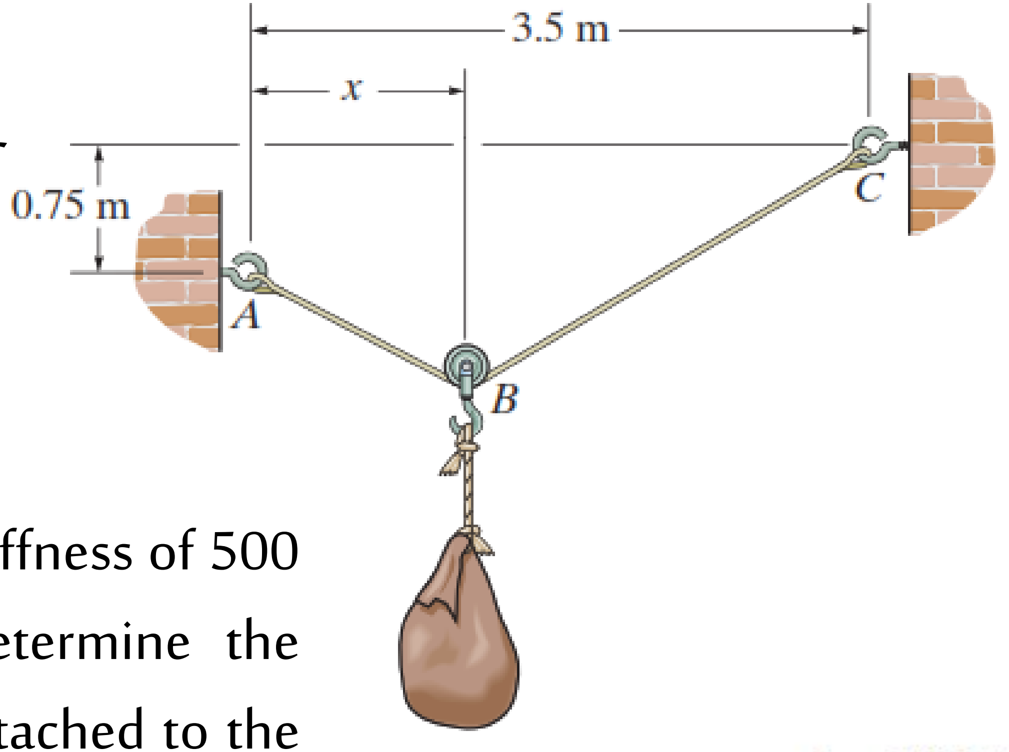




Ex.8. (1) Determine the tension developed in each cord required for equilibrium of the 20-kg lamp.

(2) Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 600 N.

Ex. 9. Cable *ABC* has a length of 5 m. Determine the position *x* and the tension developed in *ABC* required for equilibrium of the 100-kg sack. Neglect the size of the pulley at *B*.



Ex.10. (1) The springs *BA* and *BC* each have a stiffness of 500 N/ m and an unstretched length of 3 m. Determine the horizontal force *F* applied to the cord which is attached to the small ring *B* so that the displacement of the ring from the wall is $d = 1.5$ m.

(2) The springs *BA* and *BC* each have a stiffness of 500 N/ m and an unstretched length of 3 m. Determine the displacement *d* of the cord from the wall when a force $F = 175$ N is applied to the cord.