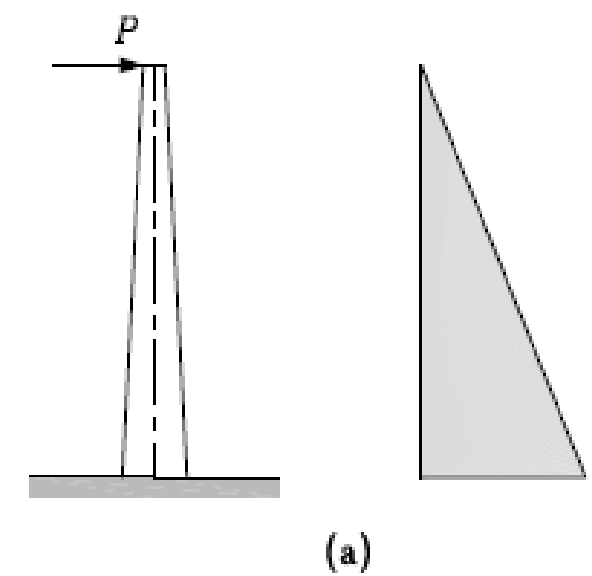
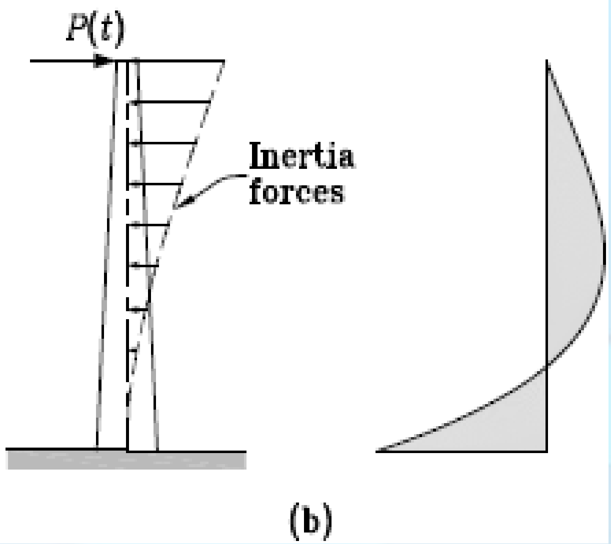
Aim: To study and control vibrations of structures caused by dynamic loadings that vary over time as opposed to static loadings.

The distinctive nature of a dynamic problem comes from the presence of *inertia forces,* which oppose the accelerations generated by the applied dynamic loading.

Difference between static and dynamic loading



static loading & corresponding bending moment diagram



dynamic loading &corresponding bending moment diagram

Dynamic loading

A ***dynamic load*** has intensity, direction, and point of application that can vary in time. It can be divided into *periodic loadings and non-periodic loadings*.

Sources of dynamic loadings			
Periodic		Non-periodic	
Simple harmonic	Arbitrary periodic	Arbitrary	Impulsive
Rotating machine	Reciprocating machine	Construction	Construction
	Walking, jogging	Wind	Impact
	Wind	Waves	Explosion
		Earthquakes	Loss of support
		Traffic	Rupture of an element

Permanent and live loads that are applied slowly compared to the period of vibration of structures are generally considered static loadings, as are dead loads.

Periodic loadings

A periodic loading repeats itself after a regular time interval, *T, called the period.*
Periodic loadings can be divided into simple harmonic loadings and arbitrary periodic loadings.

Harmonic loadings

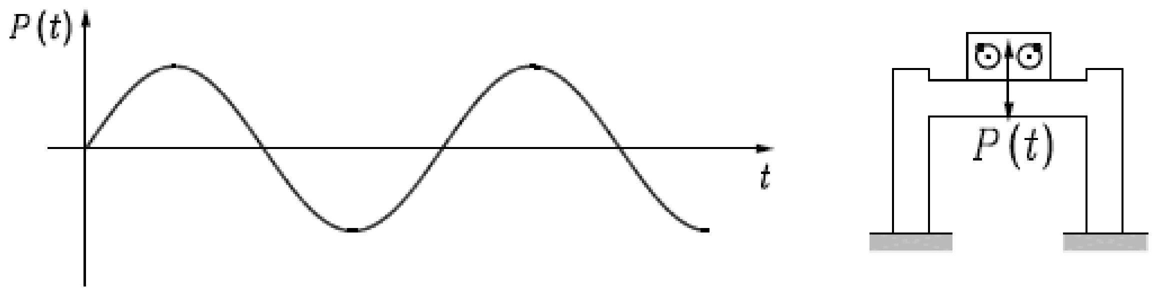


Figure 1.3. *Harmonic loading applied by a rotating machine*

Resonance phenomenon occurs when the excitation period matches the structure's natural period of vibration.

Arbitrary periodic loadings

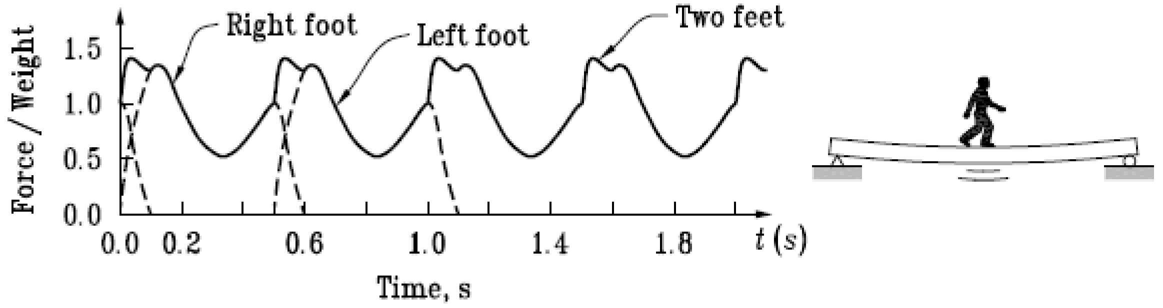


Figure 1.4. *Periodic loading caused by the steps of a person crossing a pedestrian bridge*

Non-periodic loadings

Non-periodic loadings vary arbitrarily in time without periodicity. Non-periodic loadings can be divided into impulsive short-duration loadings and arbitrary long-duration transient loadings.

Impulse loads have a very short duration with respect to the vibration period of the structures and are caused by explosions, shock, failure of structural elements, support failure, etc.

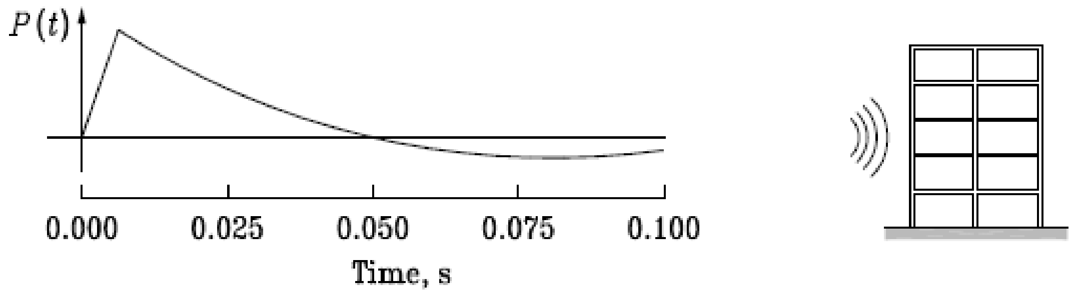


Figure 1.5. *Impulse loading caused by an explosion*

Arbitrary loads are of long duration and are caused by earthquakes, wind, waves, etc. Figure 1.6 shows the time variation of the acceleration that occurs at the base of a structure during an earthquake, giving rise to time-varying inertia forces over the structure's height.

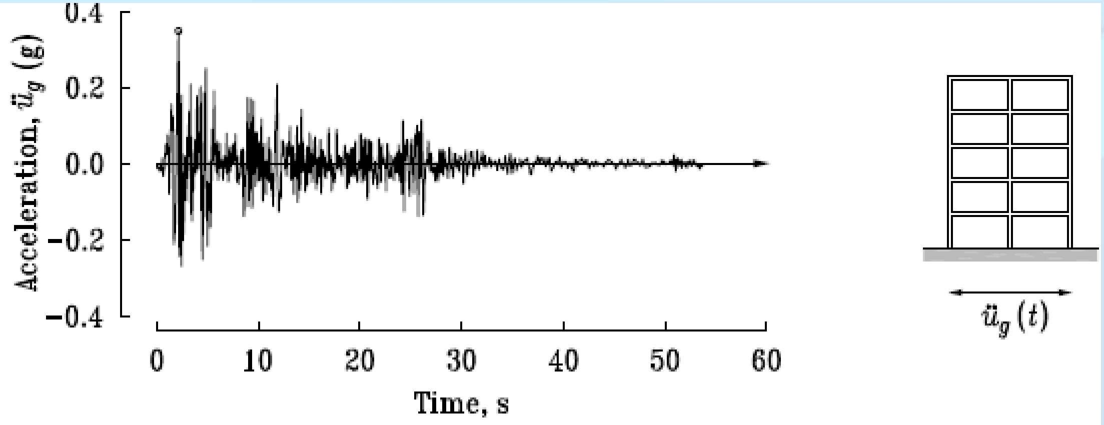


Figure 1.6. *Long-duration arbitrary load caused by an earthquake*

Dynamic response

dynamic response expresses the response (displacements, strains, stresses…) of the structure with time, it is also called the ***time history***.

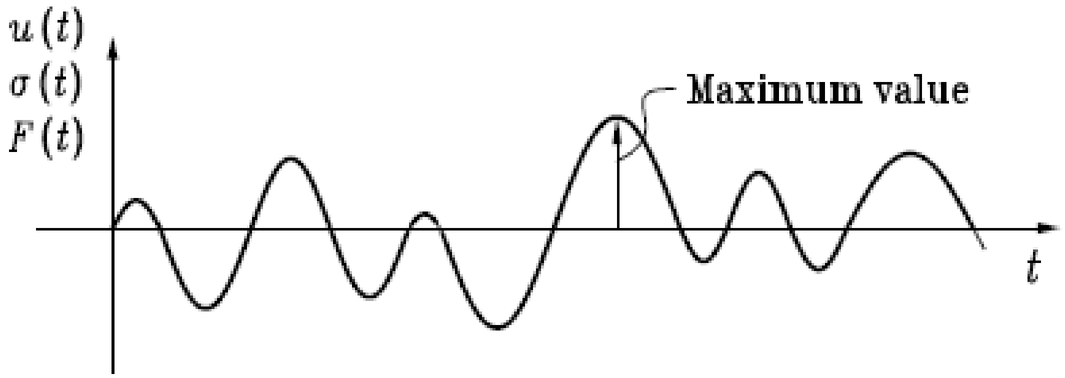


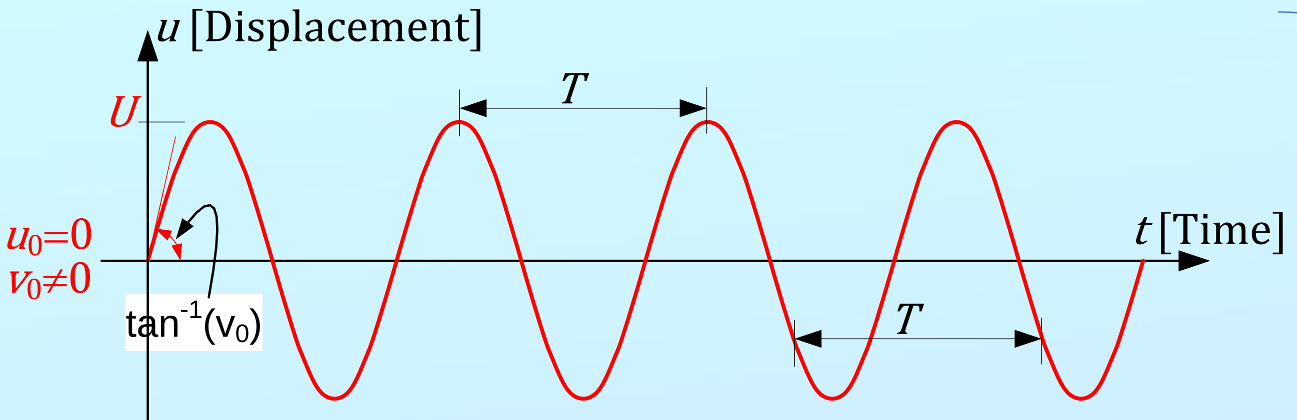
Figure 1.2. *Response time history: displacements, stresses or forces*

For design or verification of linear systems:
Maximum dynamic response is added to the *maximum static response*

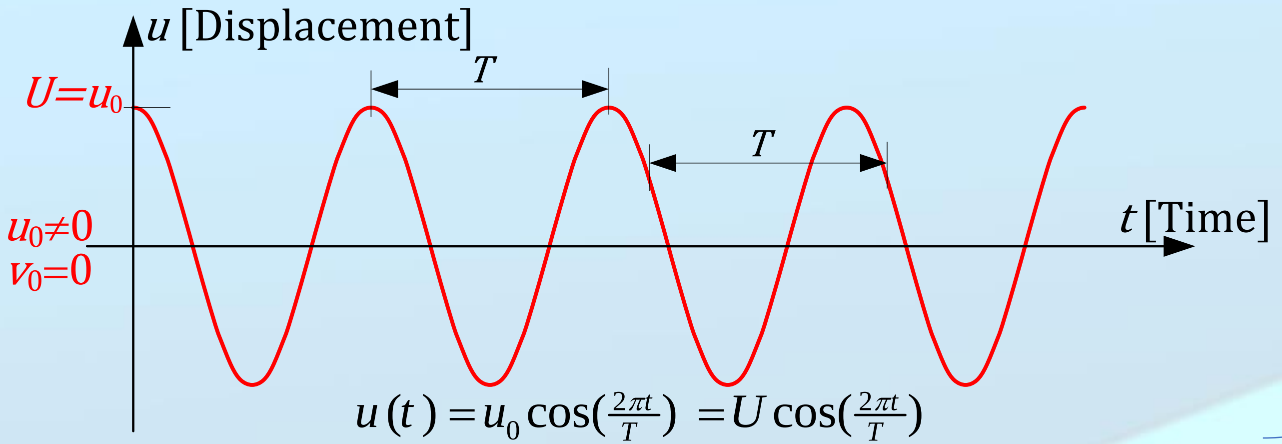
For a nonlinear system:
Static effects need to be calculated first and combined with the dynamic effects to determine the total nonlinear response

Dynamic response

The simplest of the dynamic response is the harmonic one. It is described by a combination of trigonometric functions of the same period



$$u(t) = (v_0 T/2\pi)\sin(\tfrac{2\pi t}{T}) = U\sin(\tfrac{2\pi t}{T})$$



$$u(t) = u_0\cos(\tfrac{2\pi t}{T}) = U\cos(\tfrac{2\pi t}{T})$$

In both cases using the $(\dots)^{\bullet}$ as symbol for derivation with respect to time. We find that

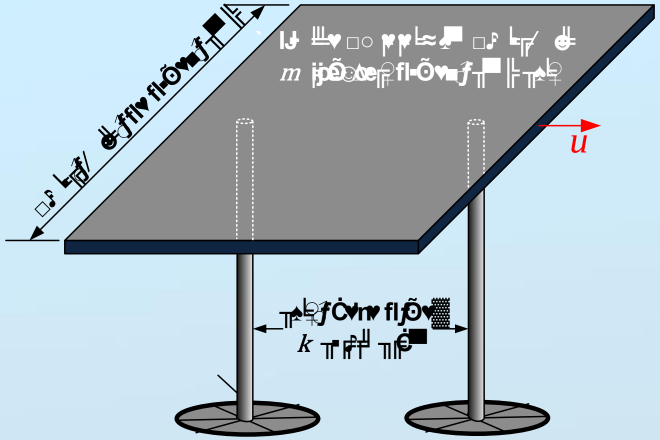
$$\ddot{u}(t) + (2\pi/T)^2 u(t) = 0$$

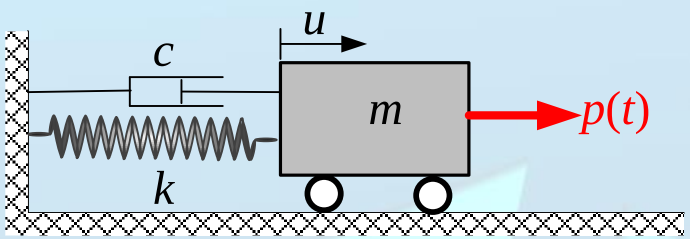
or more simply

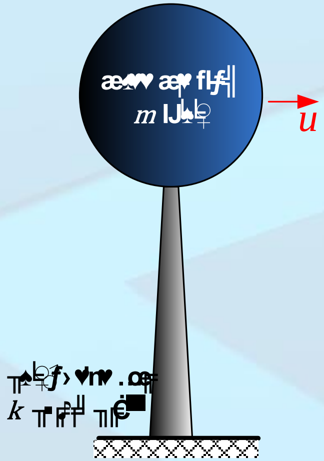
$$\ddot{u} + (2\pi/T)^2 u = 0$$

which is a differential equation

Dynamic degrees of freedom

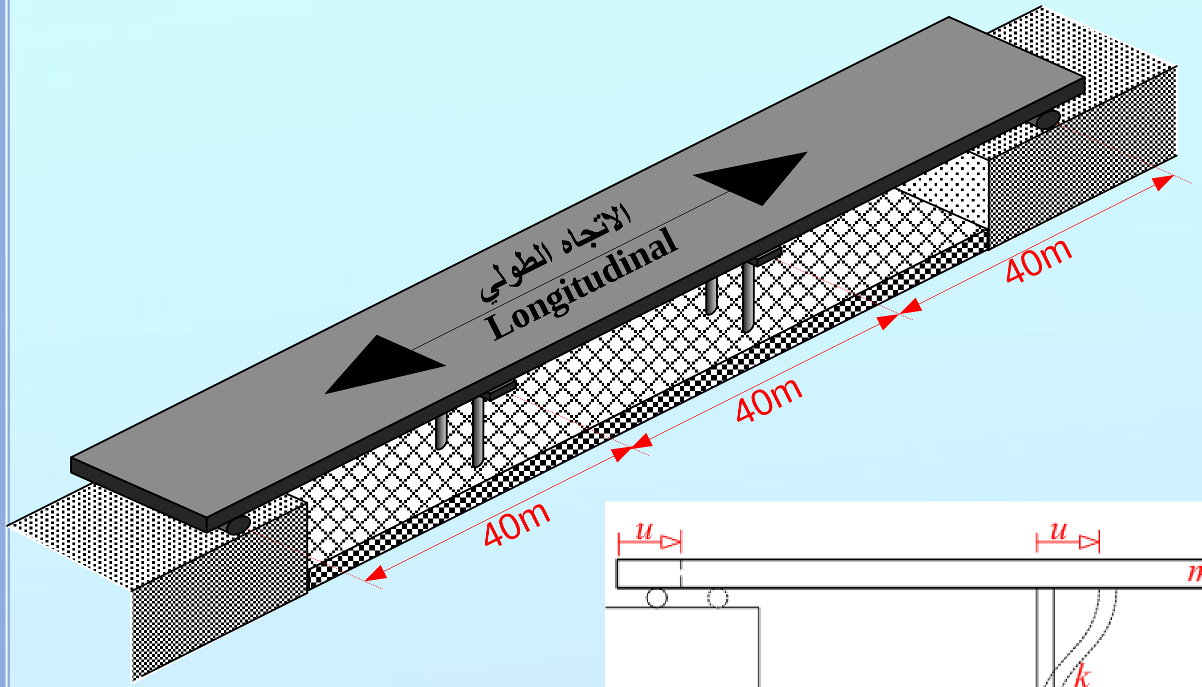


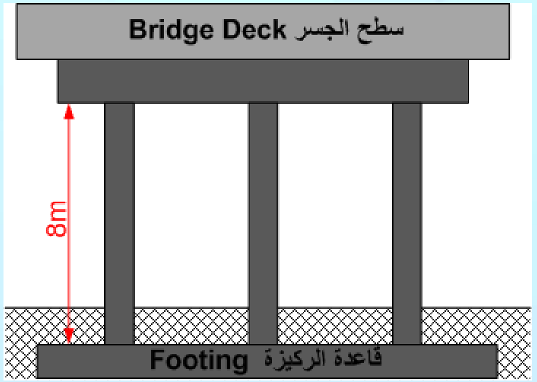


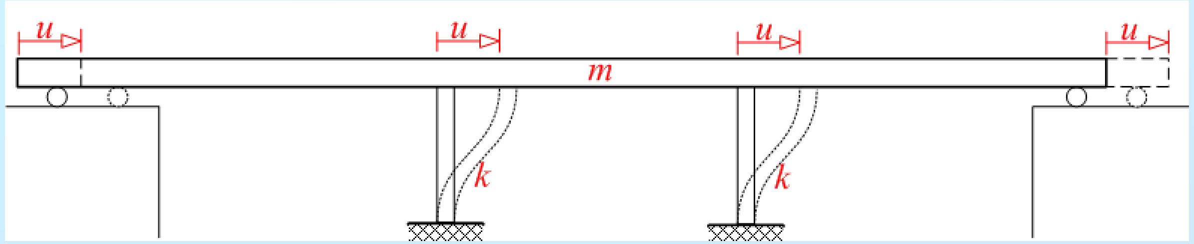


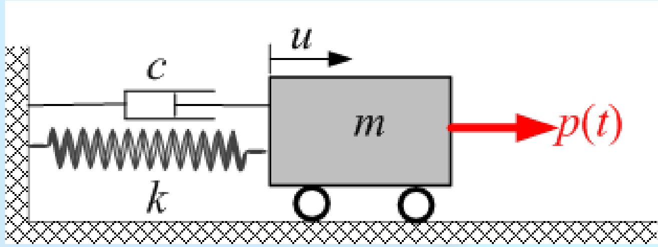
Dynamic degrees of freedom

Single Degree of Freedom Systems: SDOF

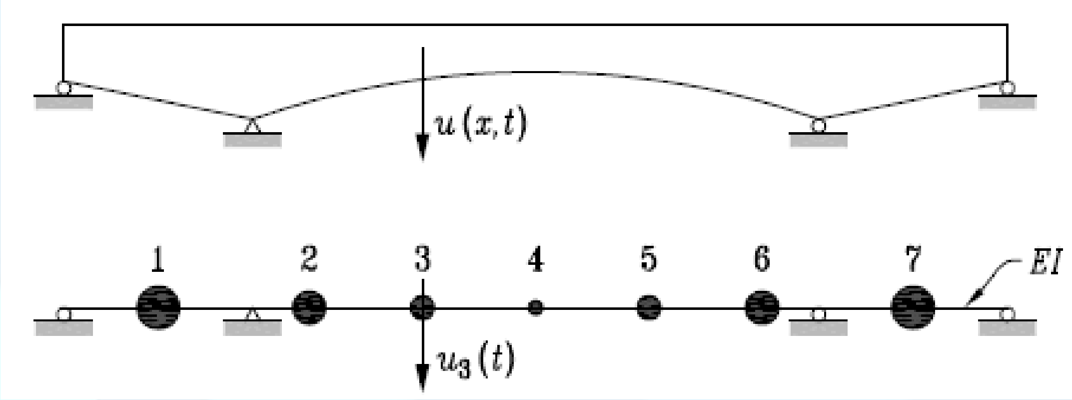






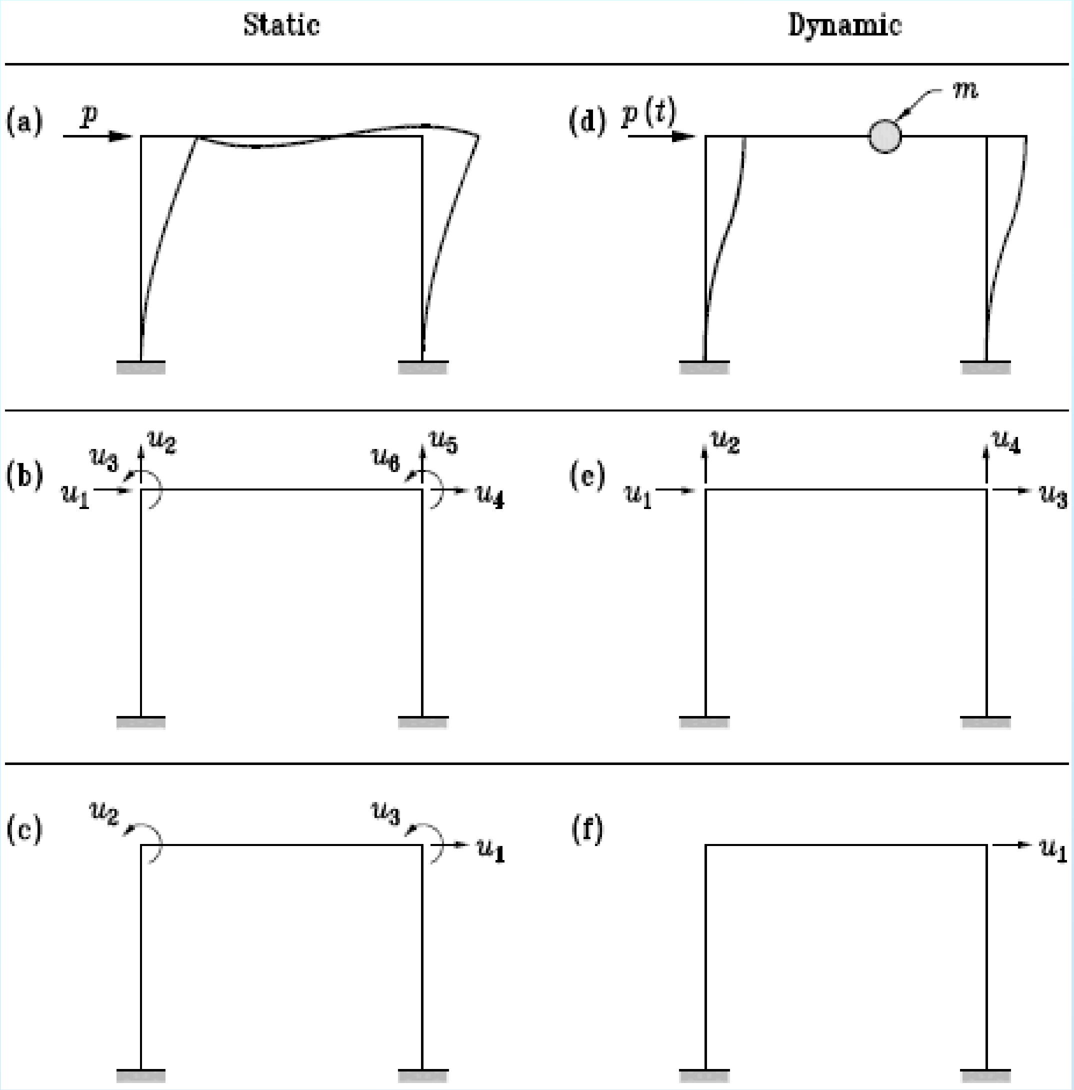


Multi-Degrees of Freedom Systems: MDOF

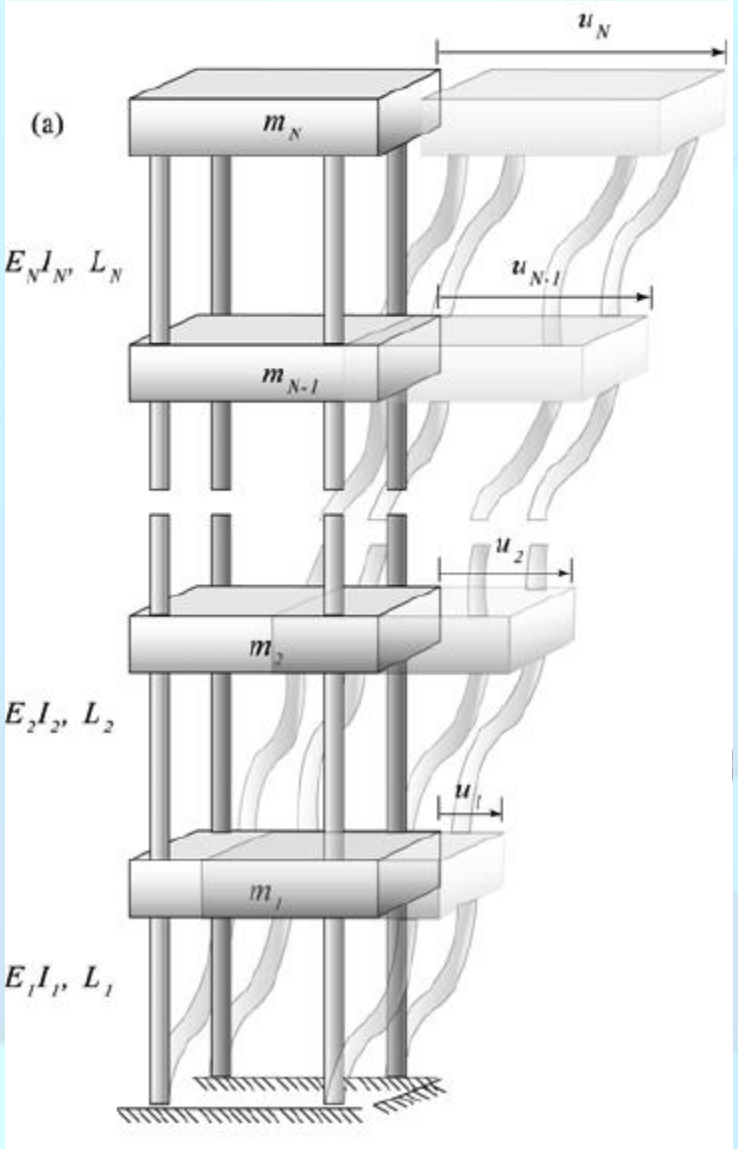


Dynamic degrees of freedom

Single Degree of Freedom Systems: SDOF



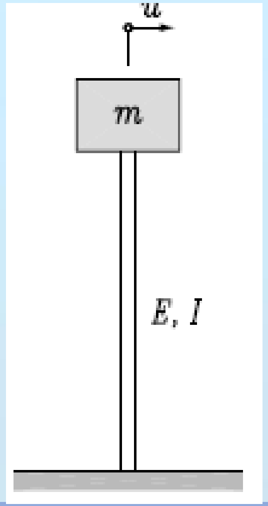
Multi-Degrees of Freedom Systems: MDOF

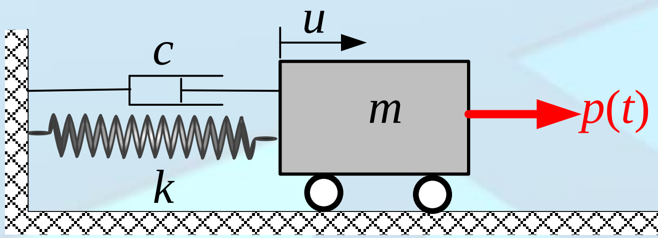


Single Degree of Freedom Systems

The dynamic response of many simple structures can be described by the analysis of SDOF system. For example the one story frame:

Elementary system: (a) SDOF structure and (b) equivalent mass-spring-damper model





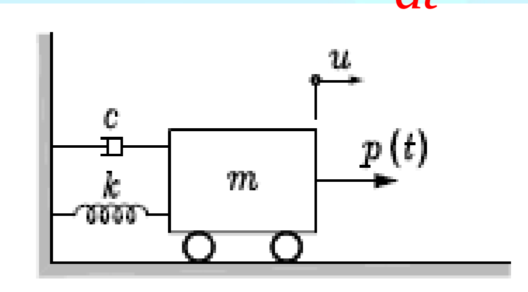
Single Degree of Freedom Systems Equation of Motion

The motion of this SDOF system is described with the following three parameters:

The mass displacement: $u(t)$

The mass velocity: $\dot{u}(t) = \dfrac{du(t)}{dt}$

The mass accelaration: $\ddot{u}(t) = \dfrac{d^2u(t)}{dt^2}$



where the dots represent differentiation with time

Formulation of the equation of motion

Newton's second law of motion :

Newton's second law of motion states that the rate of change of momentum of a mass particle *m is equal to the sum of forces acting onto it, that is*

$$p(t) = \frac{d}{dt}\left(m\frac{du}{dt}\right)$$

Assuming the mass does not vary with time, this law can be written as

$$p(t) = m\frac{d^2u}{dt^2} = m\ddot{u}$$

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The mass displacement: $u(t)$

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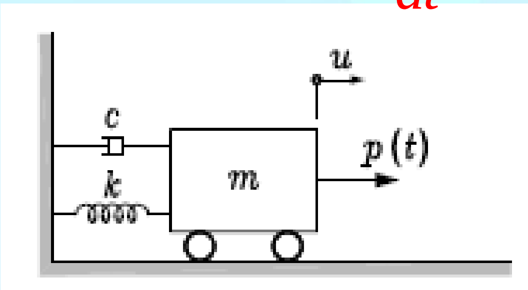
where the dots represent differentiation with time

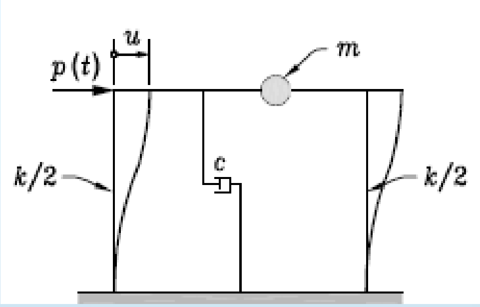


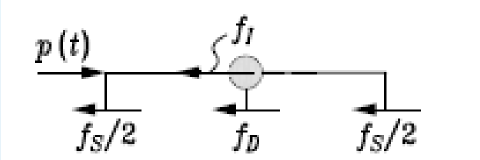
Formulation of the equation of motion

D'Alembert's principle

Transposing the right-hand side of Newton's second law to the left, we obtain

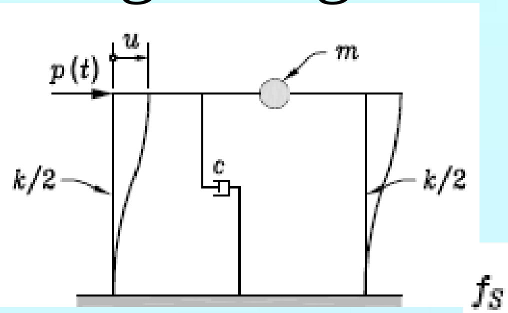
$$p(t) = m\ddot{u} \Rightarrow p(t) - m\ddot{u} = 0$$

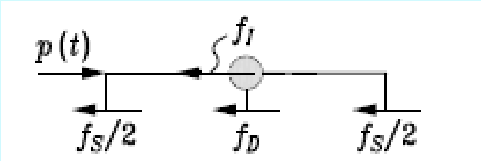




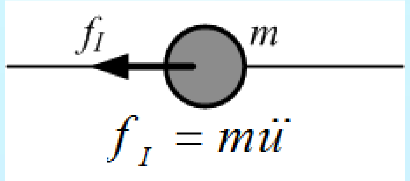
$$f_I(t) + f_D(t) + f_S(t) = p(t)$$

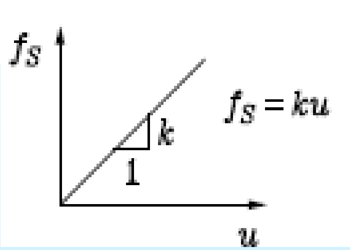
Single Degree of Freedom Systems Equation of Motion

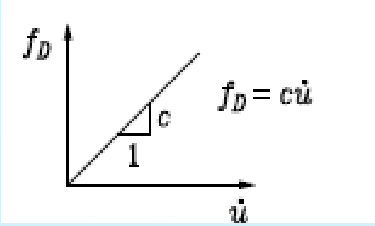




$$f_I(t)+f_D(t)+f_S(t)=p(t)$$







$$m\ddot{u}+c\dot{u}+ku=p(t)$$

Equation of Motion (Effect of gravity forces)

$$f_I(t)+f_D(t)+f_S(t)=p(t)+W$$

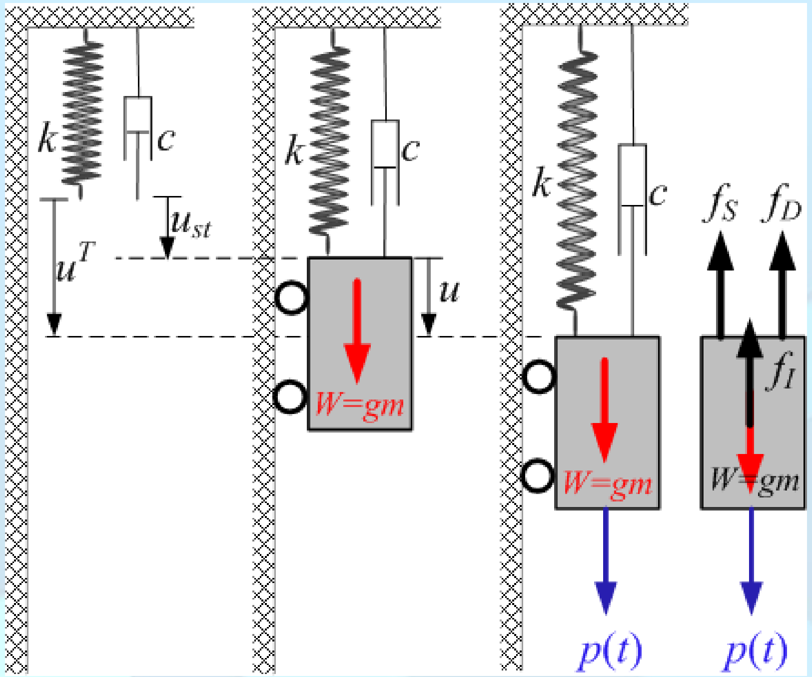
$$m\ddot{u}^T+c\dot{u}^T+ku^T=p(t)+W$$

But: $u^T(t)=u(t)+u_{st}$, $\dot{u}^T(t)=\dot{u}(t)$,

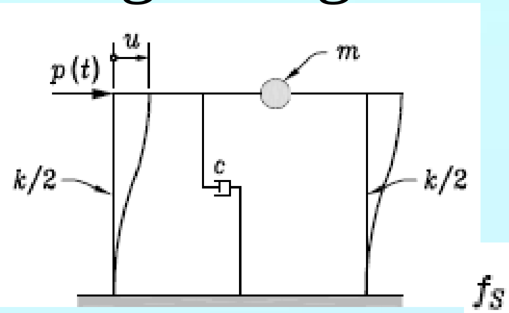
and $\ddot{u}^T(t)=\ddot{u}(t)$.

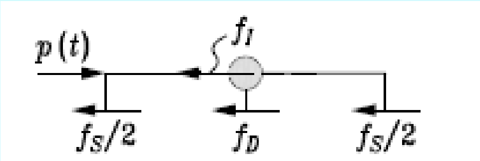
And: $ku_{st}=W$.

Then: $m\ddot{u}+c\dot{u}+ku=p(t)$

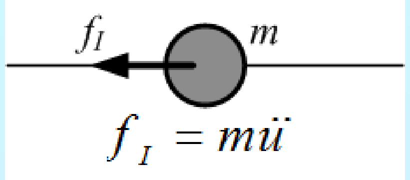


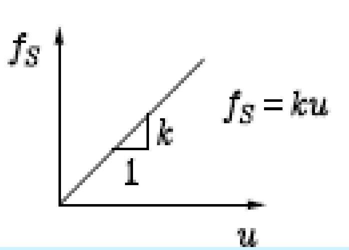
Single Degree of Freedom Systems Equation of Motion

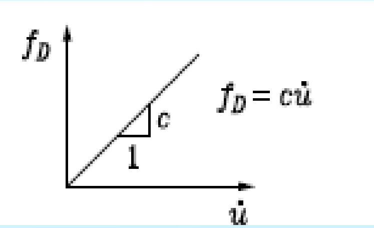




$$f_I(t)+f_D(t)+f_S(t)=p(t)$$







$$m\ddot{u}+c\dot{u}+ku=p(t)$$

Equation of Motion (**Motion of the support**)

$$u^T(t)=u(t)+u_g(t)$$

$$f_I(t)+f_D(t)+f_S(t)=0$$

$$f_I(t)=m\ddot{u}^T(t),$$

$$f_D(t)=c\dot{u}(t),\quad f_S(t)=ku(t).$$

$$m\ddot{u}(t)+m\ddot{u}_g(t)+c\dot{u}(t)+ku(t)=0$$

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=-m\ddot{u}_g(t)=p_{eff}(t)$$