

AVL Tree

In this lecture, you will learn what an avl tree is. Also, you will find working examples of various operations performed on an avl tree in C, C++, Java and Python.

AVL tree is a self-balancing binary search tree in which each node maintains extra information called a balance factor whose value is either -1, 0 or +1.

AVL tree got its name after its inventor Georgy Adelson-Velsky and Landis.

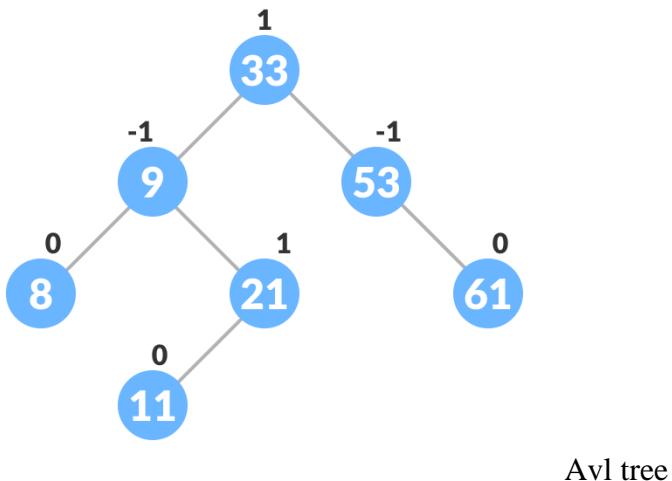
Balance Factor

Balance factor of a node in an AVL tree is the difference between the height of the left subtree and that of the right subtree of that node.

Balance Factor = (Height of Left Subtree - Height of Right Subtree) or (Height of Right Subtree - Height of Left Subtree)

The self balancing property of an avl tree is maintained by the balance factor. The value of balance factor should always be -1, 0 or +1.

An example of a balanced avl tree is:



Operations on an AVL tree

Various operations that can be performed on an AVL tree are:

Rotating the subtrees in an AVL Tree

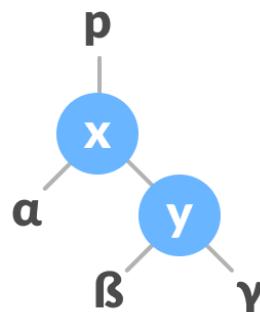
In rotation operation, the positions of the nodes of a subtree are interchanged.

There are two types of rotations:

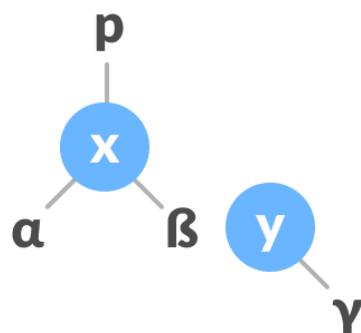
Left Rotate

In left-rotation, the arrangement of the nodes on the right is transformed into the arrangements on the left node.

Algorithm

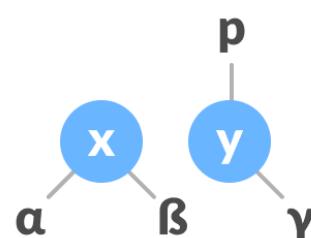


1. Let the initial tree be:
2. If y has a left subtree, assign x as the parent of the left subtree of y .



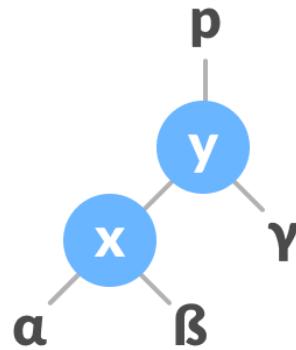
Assign x as the parent of the left subtree of y

3. If the parent of x is `NULL`, make y as the root of the tree.
4. Else if x is the left child of p , make y as the left child of p .



5. Else assign y as the right child of p .

Change the parent of x to that of y

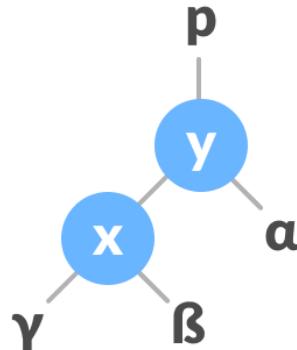


6. Make y as the parent of x .
parent of x .

Assign y as the

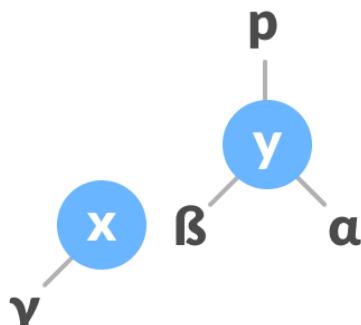
Right Rotate

In left-rotation, the arrangement of the nodes on the left is transformed into the arrangements on the right node.



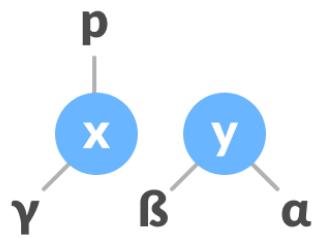
1. Let the initial tree be:
2. If x has a right subtree, assign y as the parent of the right subtree of x .

Initial tree



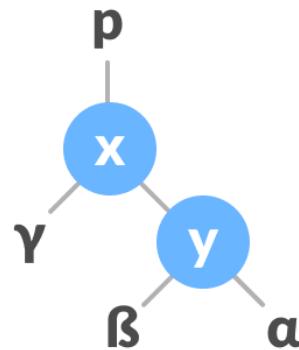
Assign y as the parent of the right subtree
of x

3. If the parent of y is `NULL`, make x as the root of the tree.
4. Else if y is the right child of its parent p , make x as the right child of p .



5. Else assign x as the left child of p .
parent of y as the parent of x .

Assign the



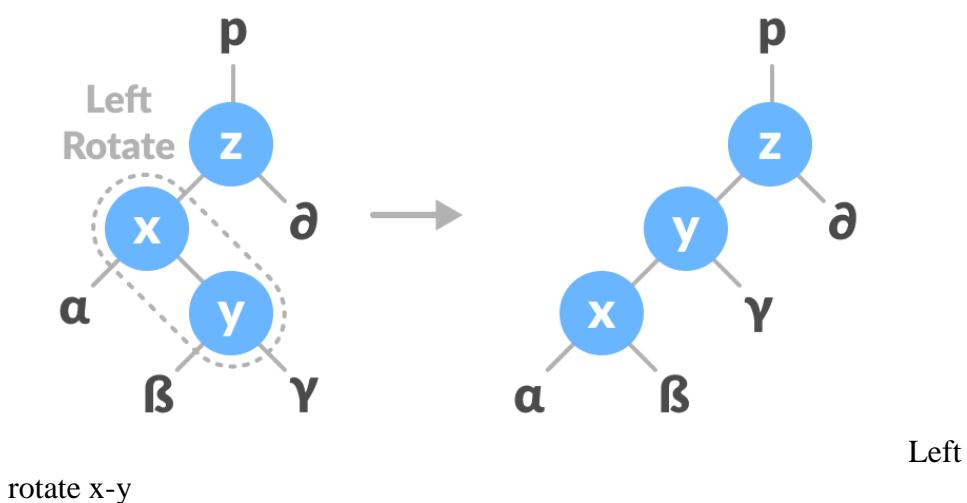
6. Make x as the parent of y .
parent of y

Assign x as the

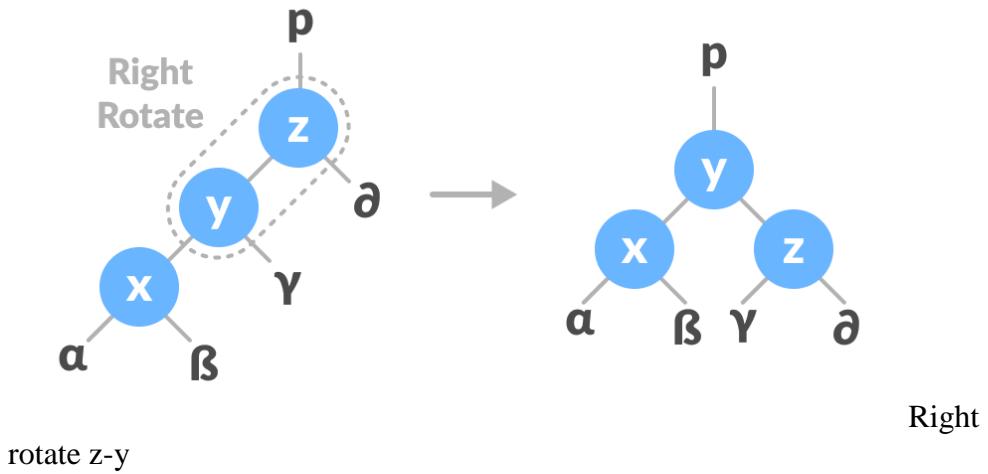
Left-Right and Right-Left Rotate

In left-right rotation, the arrangements are first shifted to the left and then to the right.

1. Do left rotation on x - y .

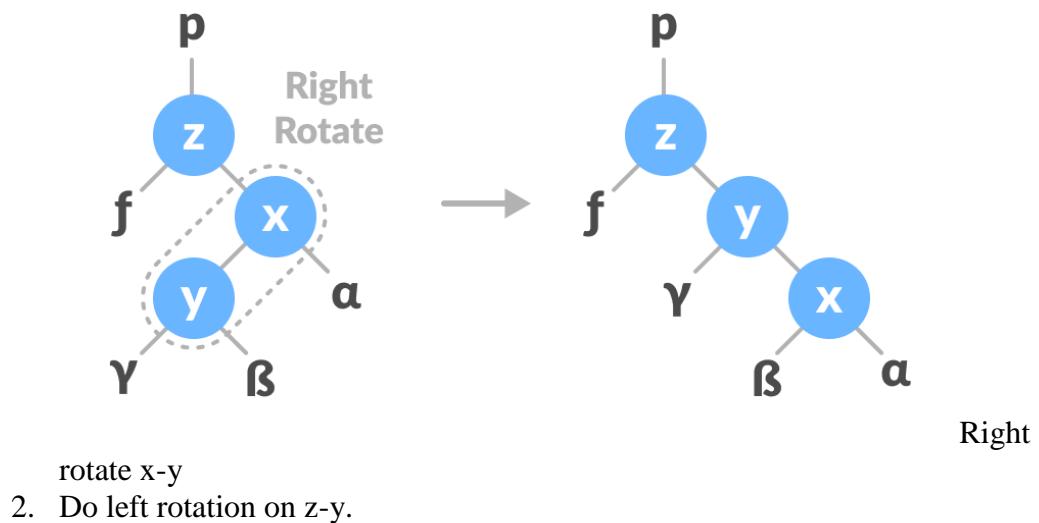


2. Do right rotation on y-z.

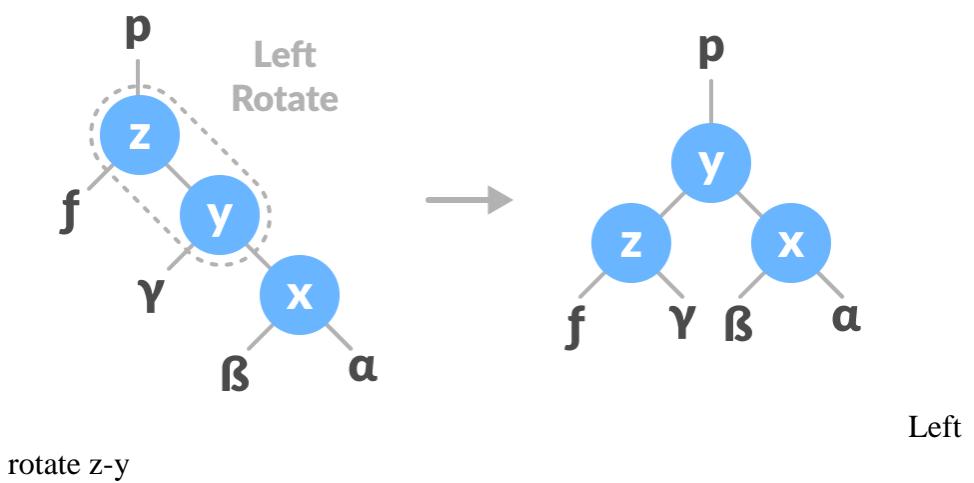


In right-left rotation, the arrangements are first shifted to the right and then to the left.

1. Do right rotation on x-y.



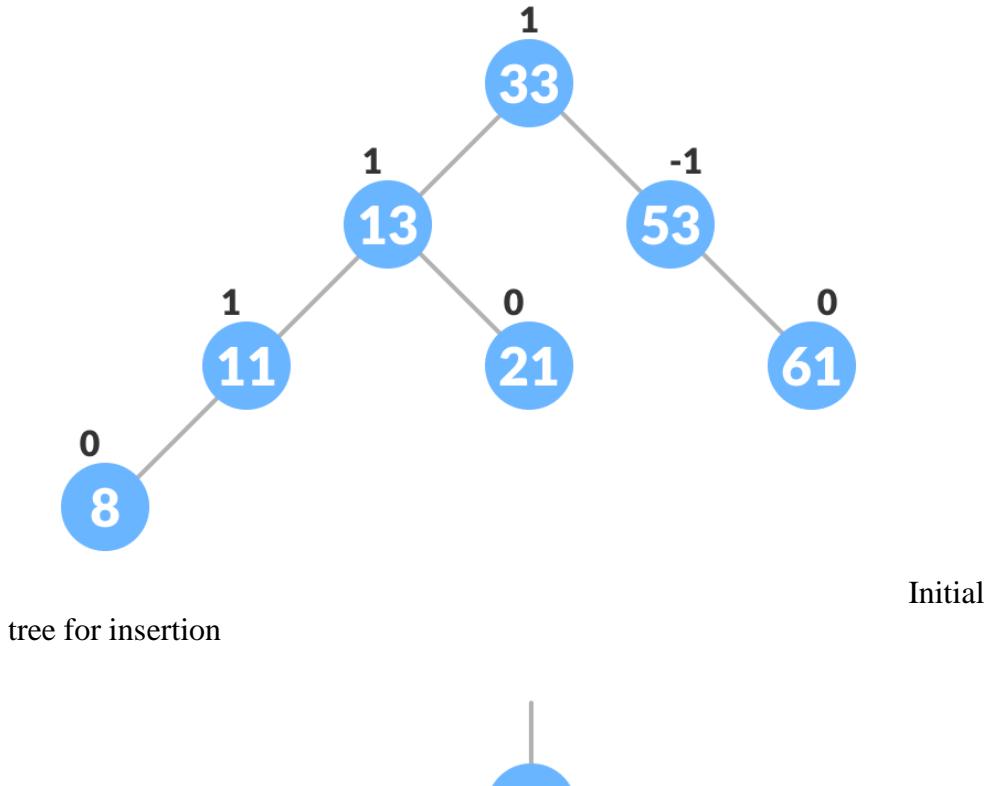
2. Do left rotation on z-y.



Algorithm to insert a newNode

A *newNode* is always inserted as a leaf node with balance factor equal to 0.

1. Let the initial tree be:

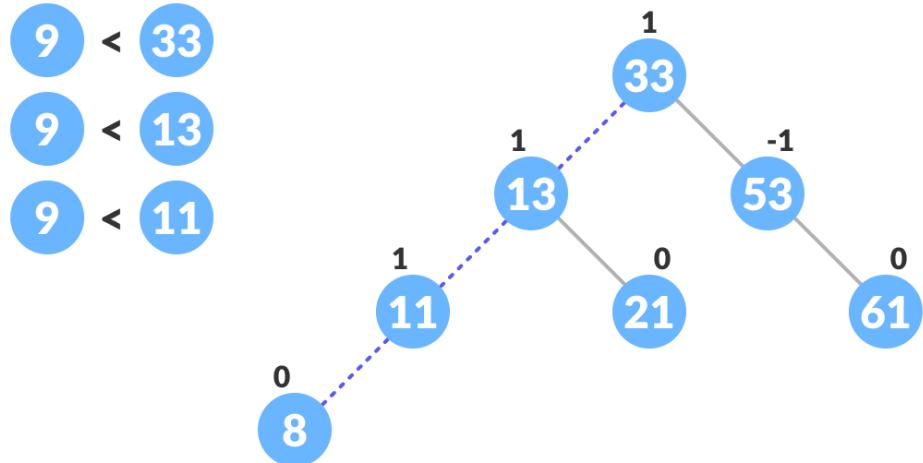


Let the node to be inserted be:

New node

2. Go to the appropriate leaf node to insert a *newNode* using the following recursive steps. Compare *newKey* with *rootKey* of the current tree.
 - a. If *newKey* < *rootKey*, call insertion algorithm on the left subtree of the current node until the leaf node is reached.
 - b. Else if *newKey* > *rootKey*, call insertion algorithm on the right subtree of current node until the leaf node is reached.

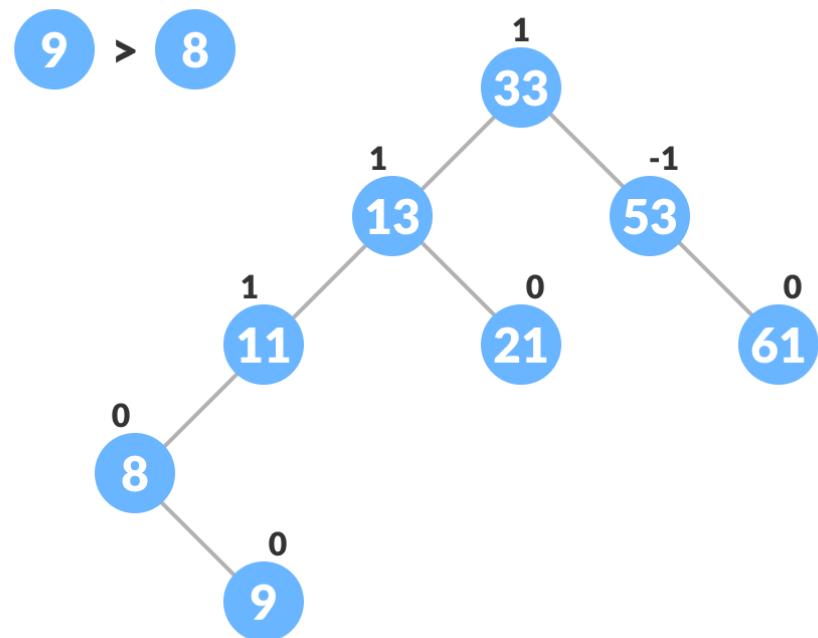
c. Else, return *leafNode*.



Finding the location to insert newNode

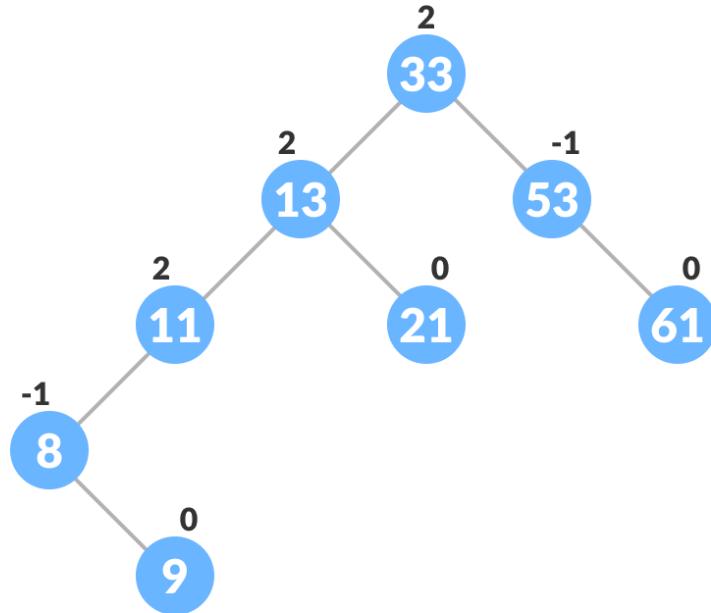
3. Compare *leafKey* obtained from the above steps with *newKey*:

- If *newKey* < *leafKey*, make *newNode* as the *leftChild* of *leafNode*.
- Else, make *newNode* as *rightChild* of *leafNode*.



Inserting the new node

4. Update *balanceFactor* of the nodes.

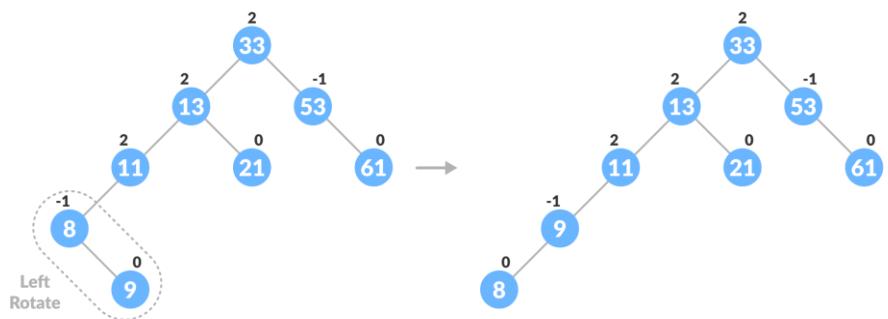


Updating the

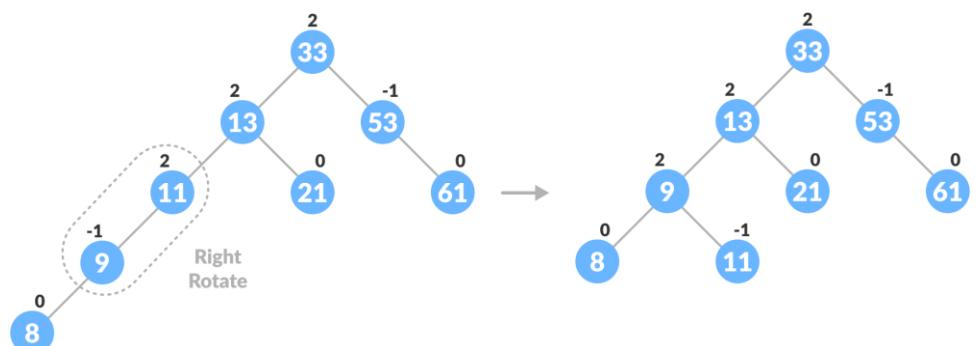
balance factor after insertion

5. If the nodes are unbalanced, then rebalance the node.

- a. If $balanceFactor > 1$, it means the height of the left subtree is greater than that of the right subtree. So, do a right rotation or left-right rotation
 - i. If $newNodeKey < leftChildKey$ do right rotation.
 - ii. Else, do left-right rotation.

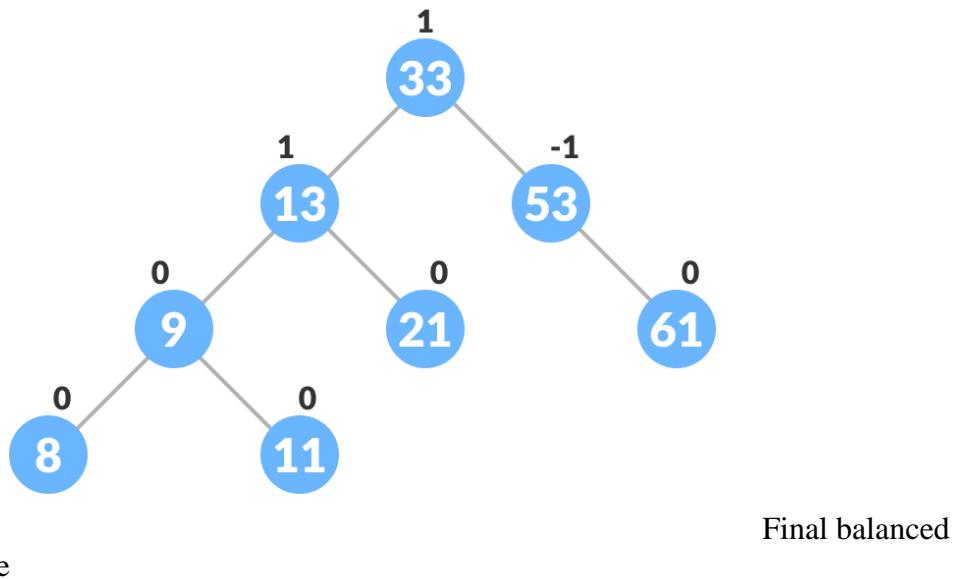


Balancing the tree with rotation



Balancing the tree with rotation

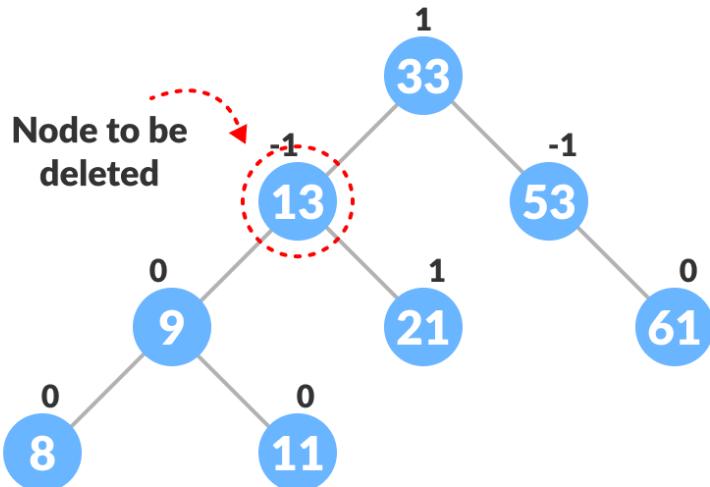
- b. If $balanceFactor < -1$, it means the height of the right subtree is greater than that of the left subtree. So, do right rotation or right-left rotation
 - i. If $newNodeKey > rightChildKey$ do left rotation.
 - ii. Else, do right-left rotation
- 6. The final tree is:



Algorithm to Delete a node

A node is always deleted as a leaf node. After deleting a node, the balance factors of the nodes get changed. In order to rebalance the balance factor, suitable rotations are performed.

1. Locate $nodeToBeDeleted$ (recursion is used to find $nodeToBeDeleted$ in the code used below).

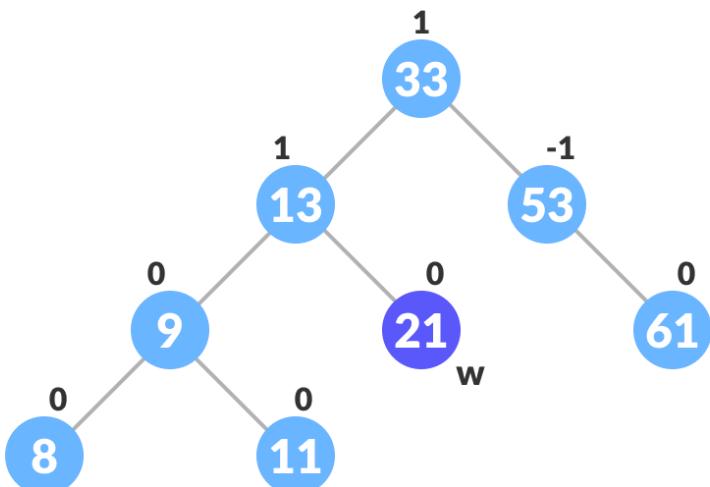


Locating the

node to be deleted

2. There are three cases for deleting a node:

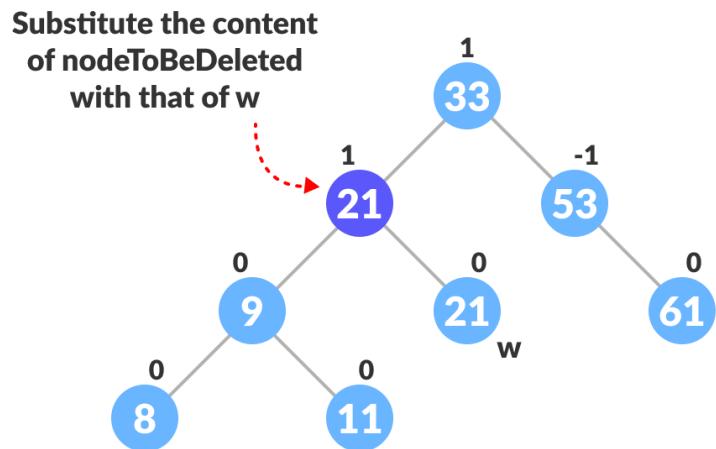
- If *nodeToDelete* is the leaf node (ie. does not have any child), then remove *nodeToDelete*.
- If *nodeToDelete* has one child, then substitute the contents of *nodeToDelete* with that of the child. Remove the child.
- If *nodeToDelete* has two children, find the inorder successor *w* of *nodeToDelete* (ie. node with a minimum value of key in the right subtree).



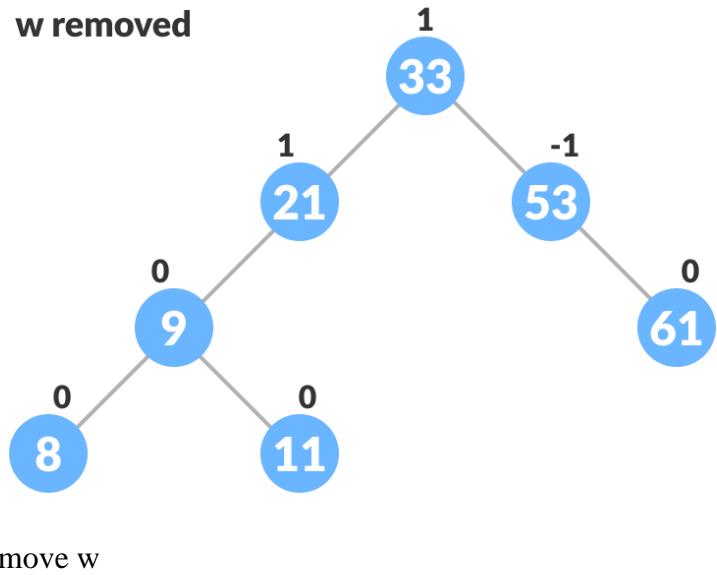
Finding

the successor

i. Substitute the contents of *nodeToDelete* with that of *w*.

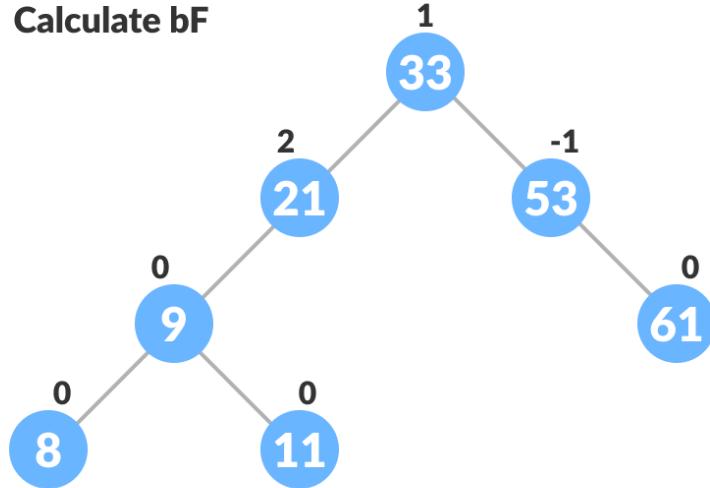


ii. Substitute the node to be deleted
ii. Remove the leaf node *w*.



3. Update *balanceFactor* of the nodes.

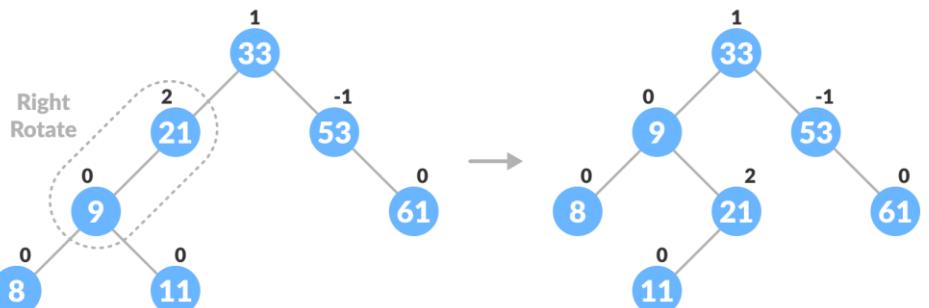
Calculate bF



Update bf

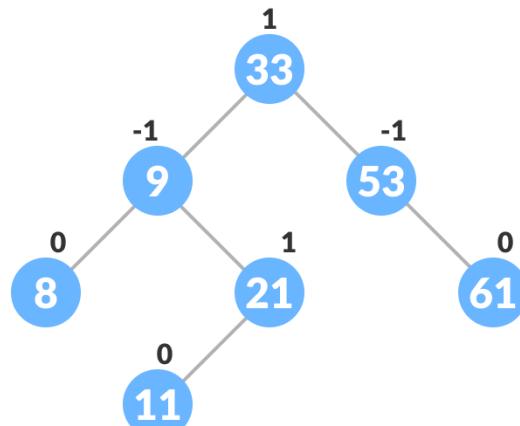
4. Rebalance the tree if the balance factor of any of the nodes is not equal to -1, 0 or 1.

- If *balanceFactor* of *currentNode* > 1,
 - If *balanceFactor* of *leftChild* >= 0, do right rotation.



Right-rotate for balancing the tree

- Else do left-right rotation.
- If *balanceFactor* of *currentNode* < -1,
 - If *balanceFactor* of *rightChild* <= 0, do left rotation.
 - Else do right-left rotation.



5. The final tree is:

Avl tree final

```

// AVL tree implementation in C++

#include <iostream>
using namespace std;

class Node {
public:
    int key;
    Node *left;
    Node *right;
    int height;
};

int max(int a, int b);

// Calculate height
int height(Node *N) {
    if (N == NULL)
        return 0;
    return N->height;
}

int max(int a, int b) {
    return (a > b) ? a : b;
}

// New node creation
Node *newNode(int key) {
    Node *node = new Node();
    node->key = key;
    node->left = NULL;
    node->right = NULL;
    node->height = 1;
    return (node);
}

// Rotate right
Node *rightRotate(Node *y) {
    Node *x = y->left;
    Node *T2 = x->right;
    x->right = y;
    y->left = T2;
    y->height = max(height(y->left),
                      height(y->right)) +
    1;
    x->height = max(height(x->left),
                      height(x->right)) +
    1;
    return x;
}

// Rotate left
Node *leftRotate(Node *x) {
    Node *y = x->right;
    Node *T2 = y->left;
    y->left = x;
    x->right = T2;
}

```

```

x->height = max(height(x->left),
                  height(x->right)) +
                  1;
y->height = max(height(y->left),
                  height(y->right)) +
                  1;
return y;
}

// Get the balance factor of each node
int getBalanceFactor(Node *N) {
    if (N == NULL)
        return 0;
    return height(N->left) -
           height(N->right);
}

// Insert a node
Node *insertNode(Node *node, int key) {
    // Find the correct position and insert the node
    if (node == NULL)
        return (newNode(key));
    if (key < node->key)
        node->left = insertNode(node->left, key);
    else if (key > node->key)
        node->right = insertNode(node->right, key);
    else
        return node;

    // Update the balance factor of each node and
    // balance the tree
    node->height = 1 + max(height(node->left),
                           height(node->right));
    int balanceFactor = getBalanceFactor(node);
    if (balanceFactor > 1) {
        if (key < node->left->key) {
            return rightRotate(node);
        } else if (key > node->left->key) {
            node->left = leftRotate(node->left);
            return rightRotate(node);
        }
    }
    if (balanceFactor < -1) {
        if (key > node->right->key) {
            return leftRotate(node);
        } else if (key < node->right->key) {
            node->right = rightRotate(node->right);
            return leftRotate(node);
        }
    }
    return node;
}

// Node with minimum value
Node *nodeWithMimumValue(Node *node) {
    Node *current = node;
    while (current->left != NULL)
        current = current->left;
    return current;
}

```

```

// Delete a node
Node *deleteNode(Node *root, int key) {
    // Find the node and delete it
    if (root == NULL)
        return root;
    if (key < root->key)
        root->left = deleteNode(root->left, key);
    else if (key > root->key)
        root->right = deleteNode(root->right, key);
    else {
        if ((root->left == NULL) ||
            (root->right == NULL)) {
            Node *temp = root->left ? root->left : root->right;
            if (temp == NULL) {
                temp = root;
                root = NULL;
            } else
                *root = *temp;
            free(temp);
        } else {
            Node *temp = nodeWithMimumValue(root->right);
            root->key = temp->key;
            root->right = deleteNode(root->right,
                                      temp->key);
        }
    }
    if (root == NULL)
        return root;

    // Update the balance factor of each node and
    // balance the tree
    root->height = 1 + max(height(root->left),
                           height(root->right));
    int balanceFactor = getBalanceFactor(root);
    if (balanceFactor > 1) {
        if (getBalanceFactor(root->left) >= 0) {
            return rightRotate(root);
        } else {
            root->left = leftRotate(root->left);
            return rightRotate(root);
        }
    }
    if (balanceFactor < -1) {
        if (getBalanceFactor(root->right) <= 0) {
            return leftRotate(root);
        } else {
            root->right = rightRotate(root->right);
            return leftRotate(root);
        }
    }
    return root;
}

// Print the tree
void printTree(Node *root, string indent, bool last) {
    if (root != nullptr) {
        cout << indent;
        if (last) {
            cout << "R----";
            indent += "    ";
        }
        cout << root->key << endl;
        printTree(root->left, indent + "    ", false);
        printTree(root->right, indent + "    ", true);
    }
}

```

```
    } else {
        cout << "L----";
        indent += "|  ";
    }
    cout << root->key << endl;
    printTree(root->left, indent, false);
    printTree(root->right, indent, true);
}
}

int main() {
    Node *root = NULL;
    root = insertNode(root, 33);
    root = insertNode(root, 13);
    root = insertNode(root, 53);
    root = insertNode(root, 9);
    root = insertNode(root, 21);
    root = insertNode(root, 61);
    root = insertNode(root, 8);
    root = insertNode(root, 11);
    printTree(root, "", true);
    root = deleteNode(root, 13);
    cout << "After deleting " << endl;
    printTree(root, "", true);
}
```