Structural Mechanics (2)

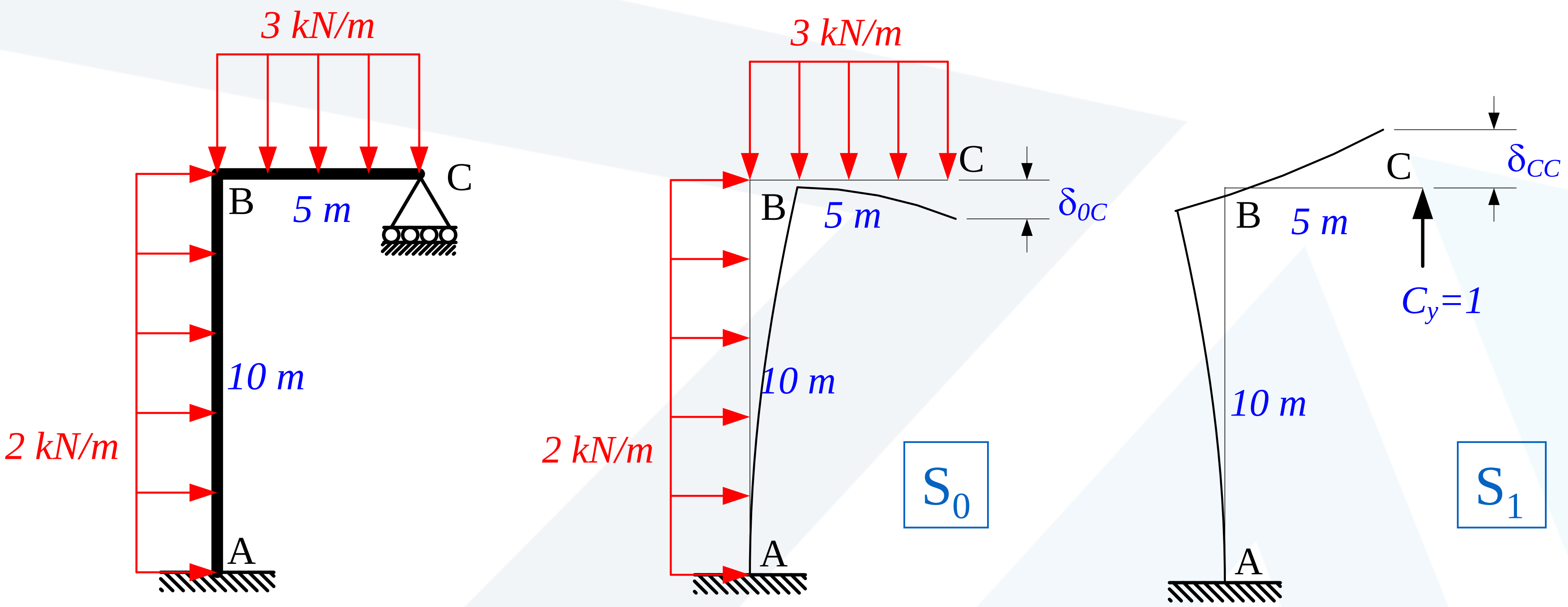
Lecture No-02

Analysis of Indeterminate Structures - Force Method

- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- **Structures with multiple Degrees of Indeterminacy**
- **Support Settlements**
- **Three-Moment Equation for Continuous Beams**

Force method: Frame with a single Degree of Indeterminacy

Compute the support reactions in the frame. $E = 200$ GPa, $I_c = 10^6$ mm^4 for the column & $I_b = 2I_c$, for the beam. Then draw the BM & SF diagrams

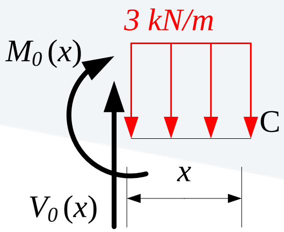
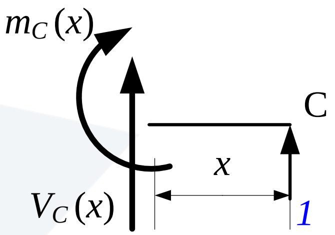
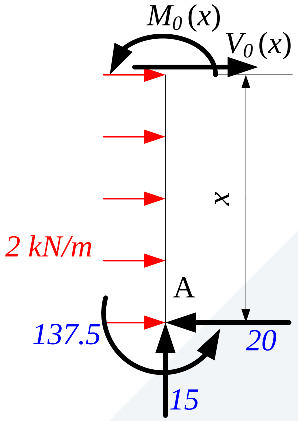
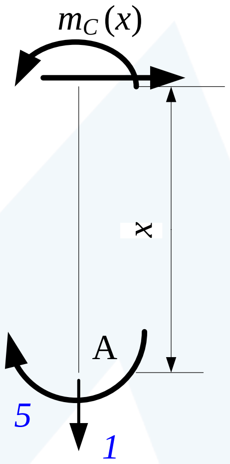


SOLUTION: The frame is statically indeterminate to degree one. Select C_y as the redundant. Draw the two determinate frames (S_0) & (S_1)

The compatibility equation is

$$\delta_{0C} + C_y \delta_{CC} = 0$$

Force method: Frame with a single Degree of Indeterminacy

Segment	FBD in S_0	$M_0(x)$	FBD in S_1	$m_C(x)$
CB $0 \le x \le 5$ $2EI$ is constant		$-3x^2/2$		x
AB $0 \le x \le 10$ EI is constant		$-x^2+20x-137.5$		5

$$\delta_{0C} = \frac{1}{2EI}\int_0^5 \left(-\frac{3x^2}{2}\right)(x)dx + \frac{1}{EI}\int_0^{10}(-x^2+20x-137.5)(5)dx = -\frac{234.375}{2EI} - \frac{3541.67}{EI} = -\frac{3658.85}{EI}$$

$$\delta_{CC} = \frac{1}{2EI}\int_0^5 x^2dx + \frac{1}{EI}\int_0^{10} 25dx = \frac{270.833}{EI} \qquad \Rightarrow C_y = 13.5\,\text{kN}$$

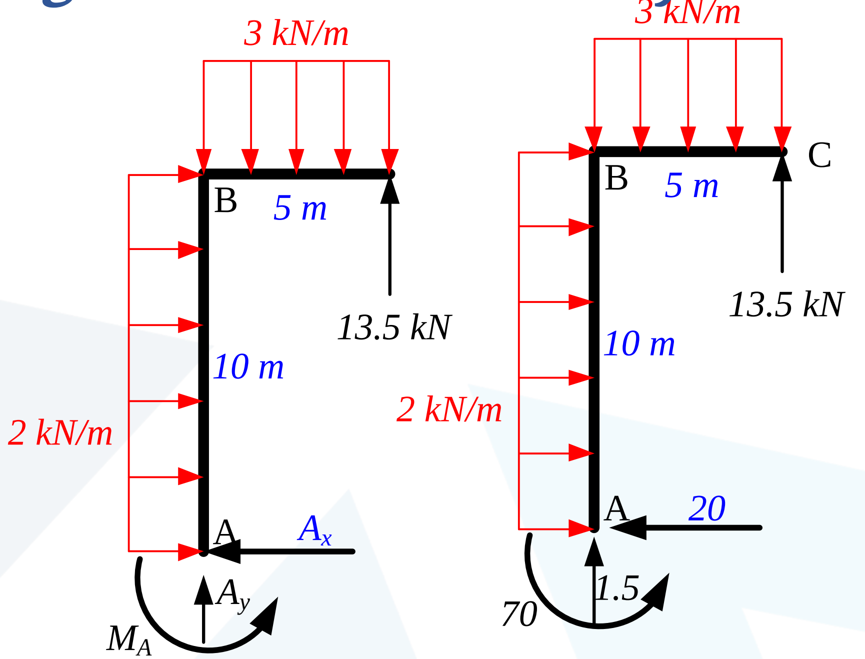
Force method: Frame with a single Degree of Indeterminacy

Computing the reactions

$$\uparrow + \sum F_y = 0 \Rightarrow A_y + 13.5 - 15 = 0 \Rightarrow A_y = 1.5kN$$

$$\overset{\rightarrow +}{\sum} F_x = 0 \Rightarrow A_x + 20 = 0 \Rightarrow A_x = -20kN = 20kN(\leftarrow)$$

$$(\downarrow\uparrow +)\sum M_A = 0 \Rightarrow M_A - (5)20 - (2.5)15 + (5)13.5 = 0 \Rightarrow M_A = 70\text{kN.m}$$



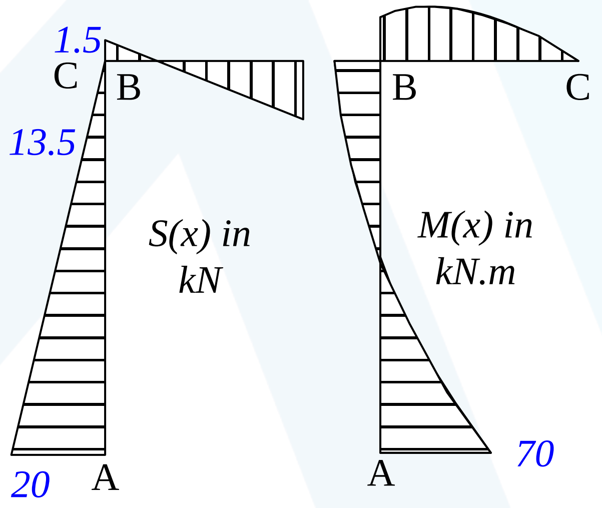
Drawing the BM & SF diagrams

In the segment AB: $0 < x < 10$,

$$M(x) = -x^2 + 20x - 70 \& S(x) = -2x + 20$$

In Segment CB: $0 < x < 5$,

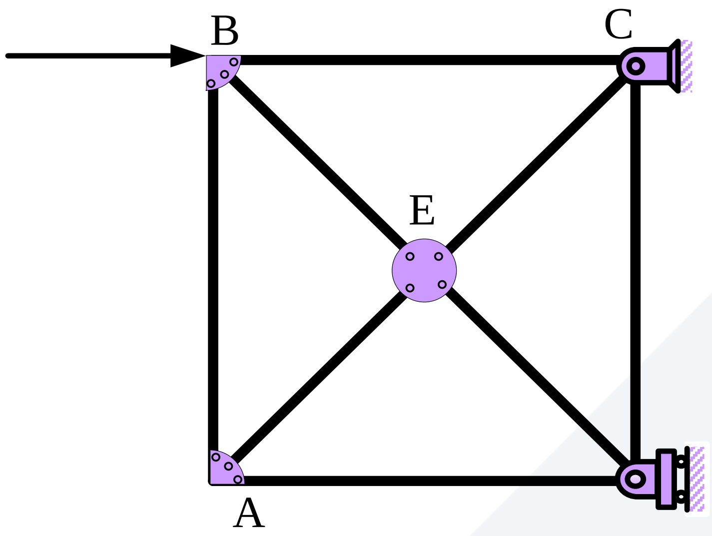
$$M(x) = -(3/2)x^2 + (27/2)x \& S(x) = 3x - 13.5$$

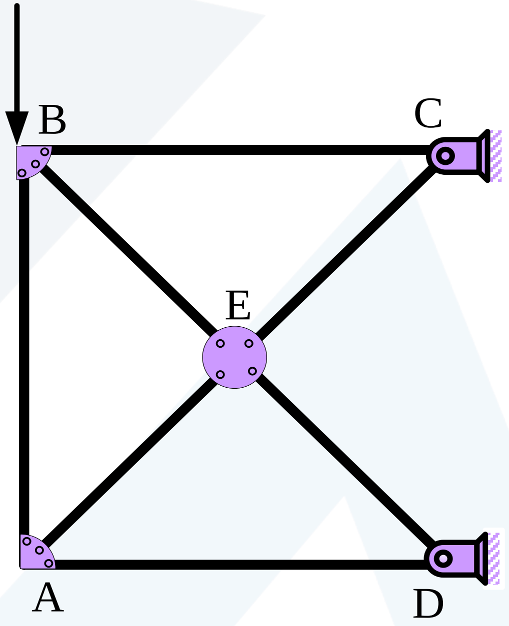


Force Method for Trusses that are statically indeterminate to degree one

Trusses can be statically indeterminate due to a variety of reasons:

- redundant support reactions (externally indeterminate),
- redundant members (internally indeterminate),
- or a combination of both.



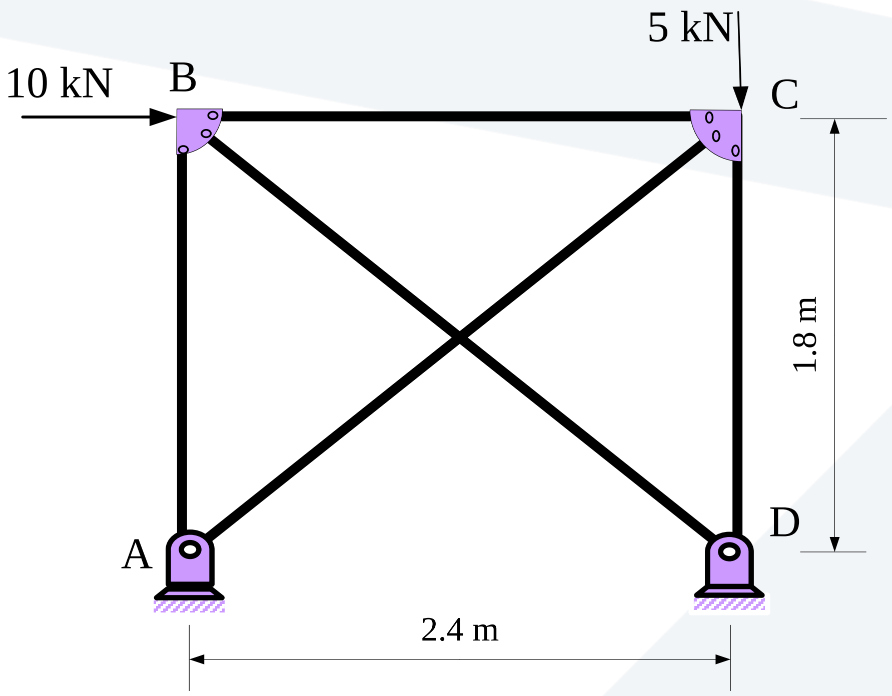


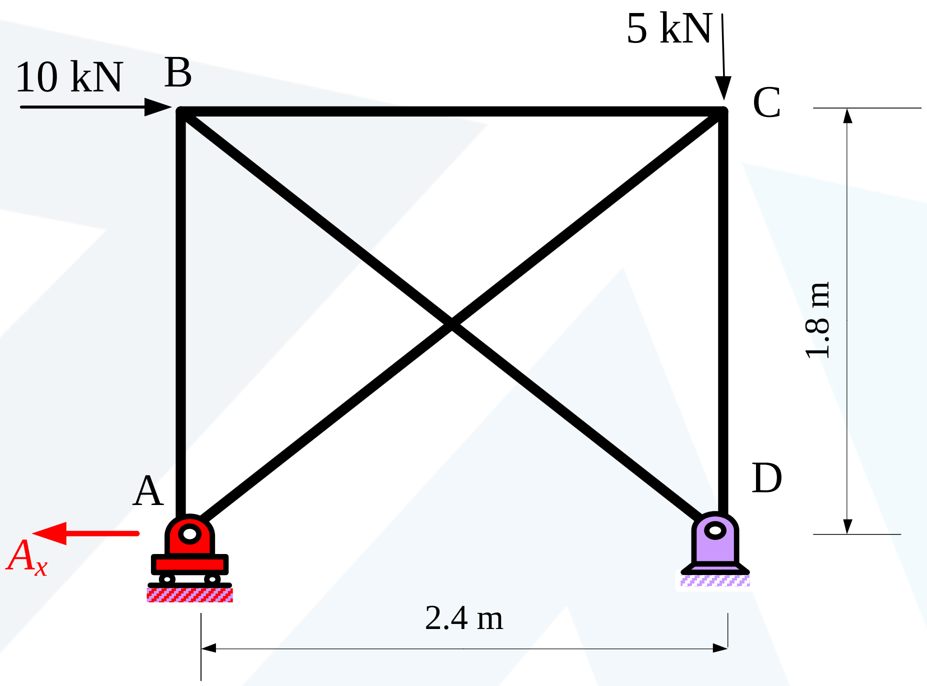
At the right, the truss is externally indeterminate $(m + r) - 2j = 7 + 4 - 2(5) = 1.$

At the left, the truss is internally indeterminate $(m + r) - 2j = 8 + 3 - 2(5) = 1.$

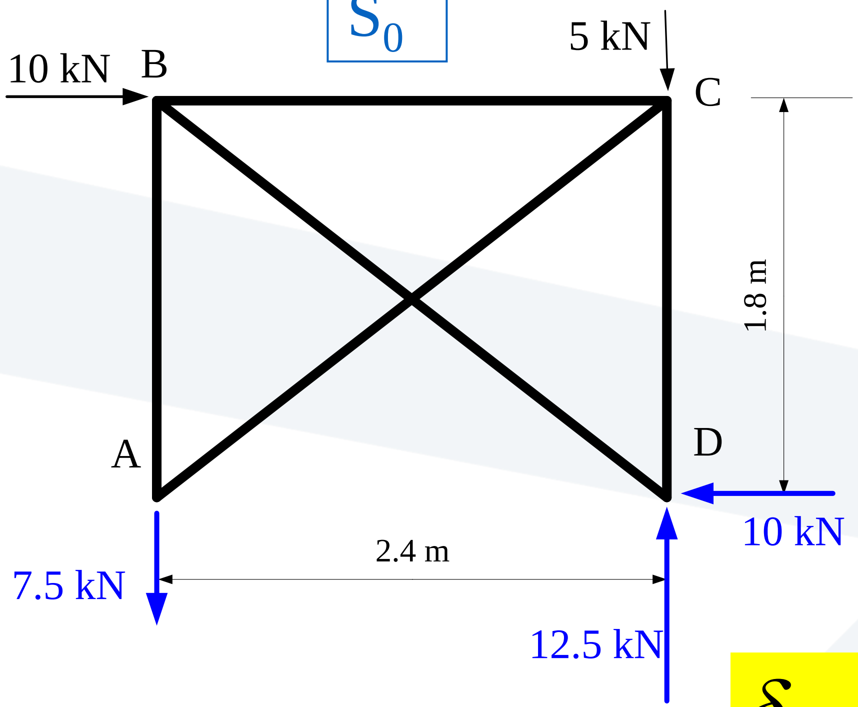
Example: Externally Indeterminate Planar Truss

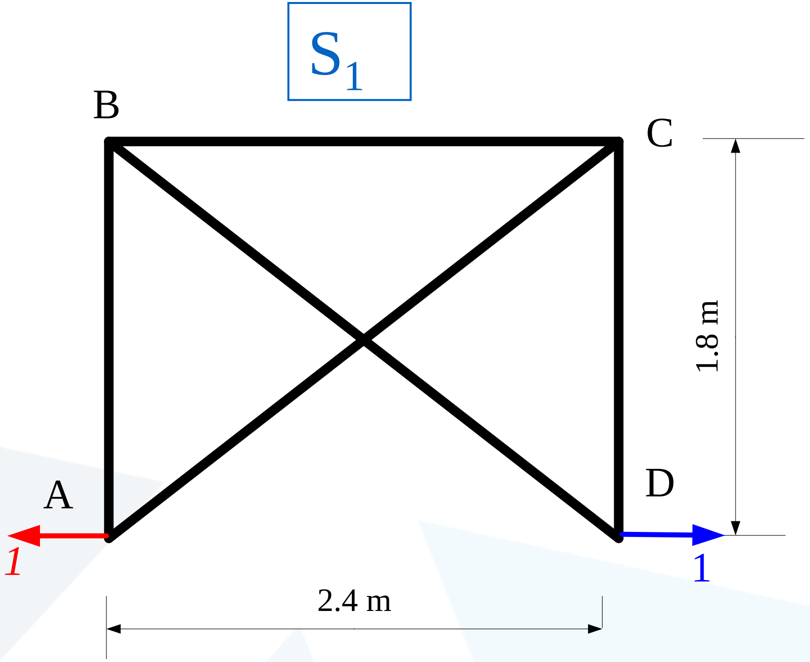
Compute the support reactions and the member forces for the truss in the next figure. Take E= 200 E+6 kN/m^2 and A= 400E-6 m^2. for all the members.





A word about the crossing members: If we do not label the joint, we mean that the members do not intersect and move independently





$$\delta_{0A} + A_x \delta_{AA} = 0$$

$$\delta_{0A} = \sum \frac{NnL}{EA} \quad \& \quad \delta_{AA} = \sum \frac{n^2 L}{EA}$$

Eqm. of joint A $\Rightarrow N_{AC}=0$ & $N_{AB}= +7.5$ kN

Eqm. of joint C $\Rightarrow N_{BC}=0$ & $N_{CD}= -5$ kN

Eqm. of joint B $\Rightarrow N_{BD}= -12.5$ kN

Eqm. of joint A $\Rightarrow n_{AC}=+1.25$ & $n_{AB}= -0.75$

Eqm. of joint C $\Rightarrow n_{BC}= -1$ & $n_{CD}= -0.75$

Eqm. of joint B $\Rightarrow n_{BD}= 1.25$

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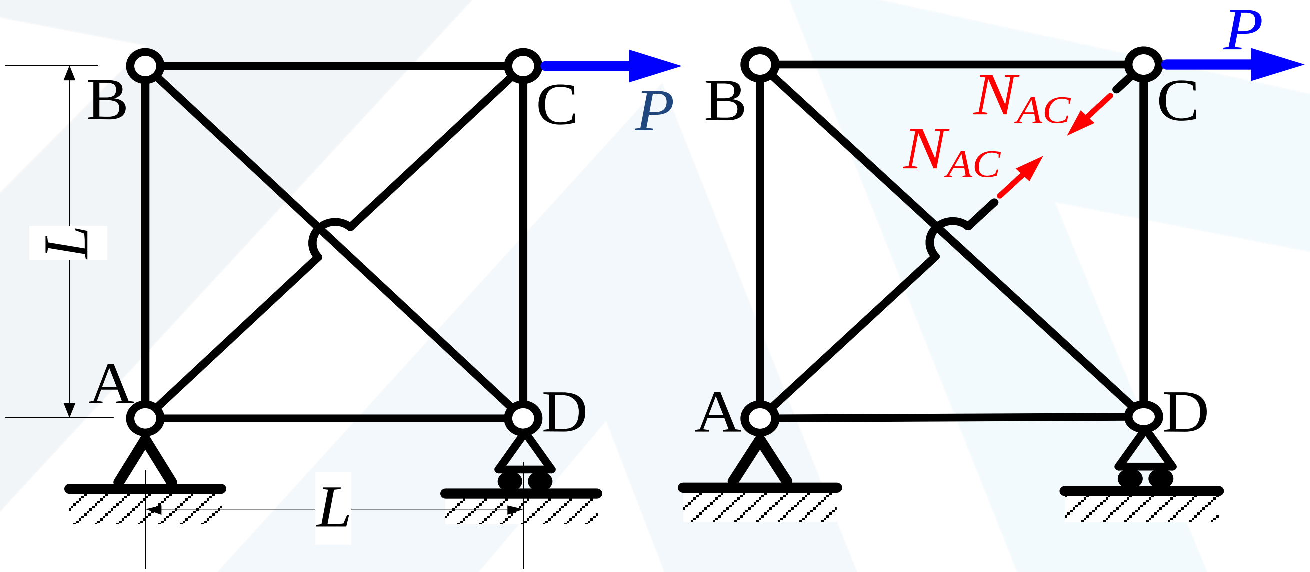
Computing Reactions and Axial Forces in the Indetrminate Truss								
Member	L [m]	A [m²]	N [kN]	n [--]	L/A [1/m]	NnL/A [kn/m]	n^2L/A [1/m]	$F = N + A_x n$ [kN]
AB	1.8	4.00E-04	7.5	-0.750	4.50E+03	-25313	2531	***4.7693***
AC	3.0	4.00E-04	0.0	1.250	7.50E+03	0	11719	***4.5513***
BC	2.4	4.00E-04	0.0	-1.000	6.00E+03	0	6000	***-3.6410***
BD	3.0	4.00E-04	-12.5	1.250	7.50E+03	-117188	11719	***-7.9488***
CD	1.8	4.00E-04	-5.0	-0.750	4.50E+03	16875	2531	***-7.7308***
					Sum	-125625	34500	
						$A_x = 125625/34500 = 3.64$ kN		
Reaction			R_N [kN]	R_n [--]				$R = R_N + A_x R_n$ [kN]
A_x			0	-1				***-3.6410***
A_y			-7.5	0.0				***-7.5000***
D_x			-10	1				***-6.3590***
D_y			12.5	0.0				***12.5000***

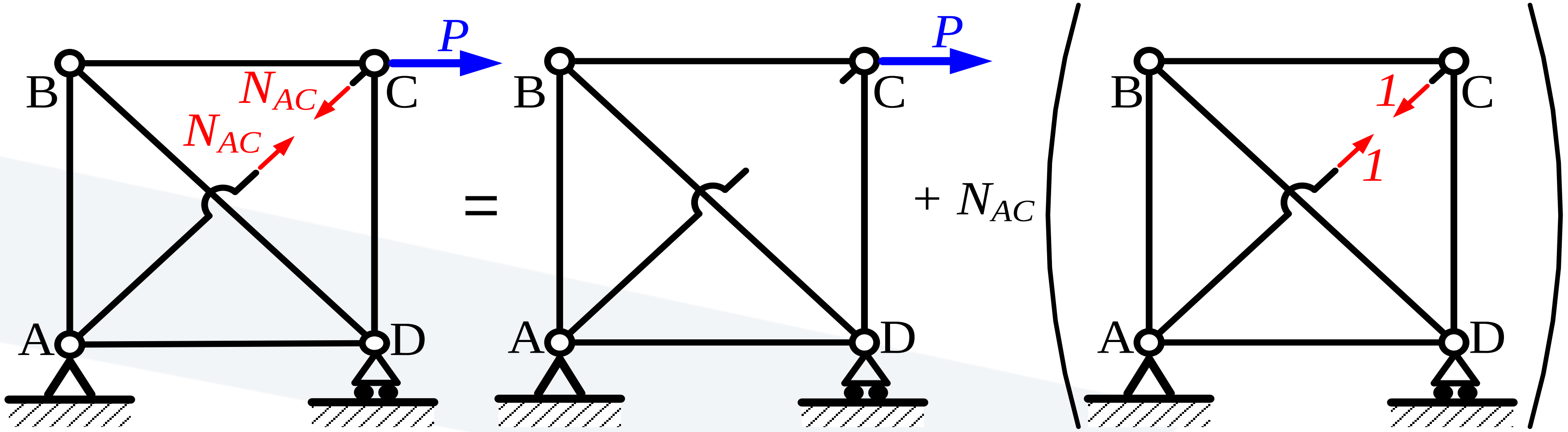
Example: Internally Indeterminate Planar Truss

Determine the forces in the members of the truss shown in Fig. The cross-sectional area, *A*, and, Young modulus, *E*, are the same for all members.

Introduction:

- Their is no intersection between AC and BD.
- There are 6 members, 3 reactions and 4 joints.
- The truss is internally indeterminate to the first order.
- There is one redundant member, like AC. The truss and the corresponding compatibility equation are





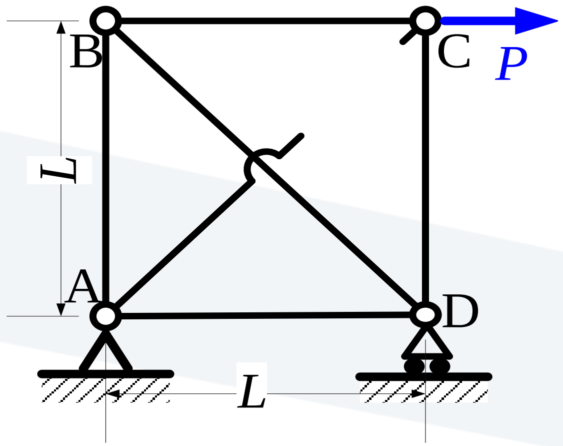
$$\Delta_{AC} = \delta^{0}_{AC} + N_{AC}\left(\delta^{1}_{AC}\right)$$

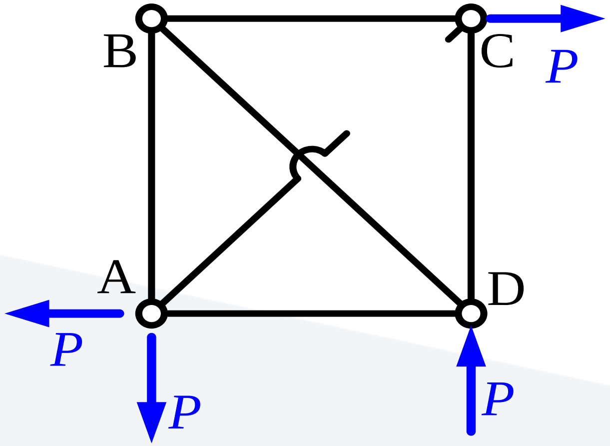
$$\Delta_{AC} = -\frac{N_{AC}L_{AC}}{E_{AC}A_{AC}}, \quad \delta^{0}_{AC} = \sum_{m=1}^{M-1}\frac{N^{0}_{m}n_{m}L_{m}}{E_{m}A_{m}}, \quad \delta^{1}_{AC} = \sum_{m=1}^{M-1}\frac{n_{m}n_{m}L_{m}}{E_{m}A_{m}}$$

As $N^{0}_{AC} = 0$, and as: $n_{AC} = 1$, $\Delta_{AC} = -\frac{N_{AC}L_{AC}}{E_{AC}A_{AC}} = -N_{AC}\left(\frac{n_{AC}n_{AC}L_{AC}}{E_{AC}A_{AC}}\right)$

The Compatibility equation becomes

$$\left(\sum_{m=1}^{M}\frac{N^{0}_{m}n_{m}L_{m}}{E_{m}A_{m}}\right) + N_{AC}\left(\sum_{m=1}^{M}\frac{n_{m}n_{m}L_{m}}{E_{m}A_{m}}\right) = 0$$





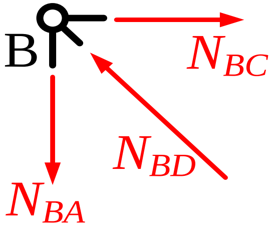
Analyzing S_0:

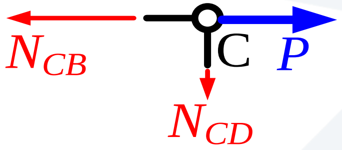
Eq. Eqs. of S^0:

$\Sigma F_x=0 \Rightarrow A_x=P(\leftarrow)$

$\Sigma M_A=0 \Rightarrow D_y=P(\uparrow)$

$\Sigma M_D=0 \Rightarrow A_y=P(\downarrow)$





Eq. Eqs. of C:

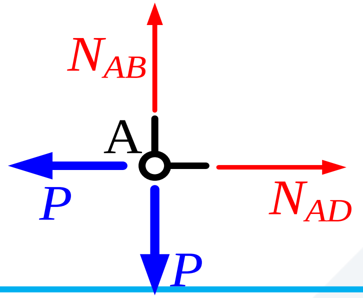
$\Sigma F_x=0 \Rightarrow N_{CB}=P(\mathrm{T})$

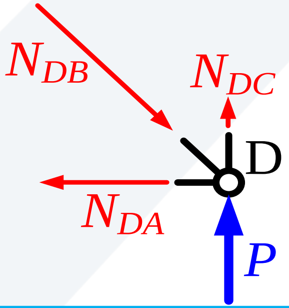
$\Sigma F_y=0 \Rightarrow N_{CD}=0$

Eq. Eqs. of B:

$\Sigma F_x=0 \Rightarrow N_{BD}=P\sqrt{2}\ (\mathrm{C})$

$\Sigma F_y=0 \Rightarrow N_{BA}=P(\mathrm{T})$

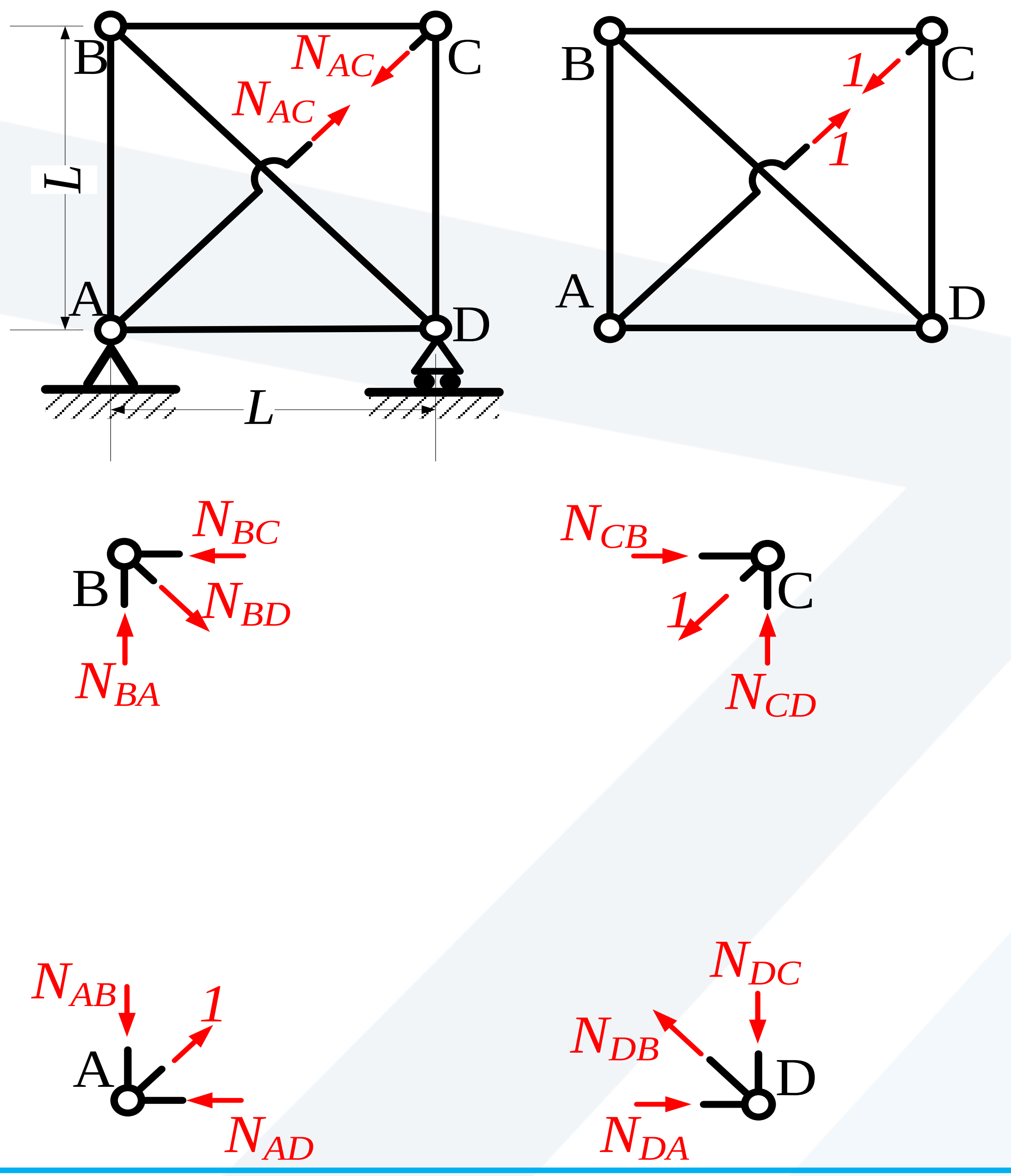




Eq. Eqs. of A:

$\Sigma F_x=0 \Rightarrow N_{AD}=P(\mathrm{T})$

$\Sigma F_y=0 \Rightarrow N_{AB}=P(\mathrm{T})$



Analyzing S_1:

Eq. Eqs. of S^1:

$\Sigma F_x=0 \Rightarrow A_x=0$

$\Sigma M_A=0 \Rightarrow D_y=0$

$\Sigma M_D=0 \Rightarrow A_y=0$

Eq. Eqs. of C:

$\Sigma F_x=0 \Rightarrow n_{CB}=0.707$ (C)

$\Sigma F_y=0 \Rightarrow n_{CD}=0.707$ (C)

Eq. Eqs. of A:

$\Sigma F_x=0 \Rightarrow n_{AB}=0.707$ (C)

$\Sigma F_y=0 \Rightarrow n_{AD}=0.707$ (C)

Eq. Eqs. of B:

$\Sigma F_x=0 \Rightarrow n_{BD}=1$(T)

$\Sigma F_y=0 \Rightarrow n_{BA}=0.707$ (C)

Example: Internally Indeterminate Planar Truss

Computing Axial Forces in the Indetrminate Truss							
Member	Length	area	N^0	n	N^0nL/A	n^2L/A	$N=N^0+N_{AC}n$
AB	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4\ P$
BC	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4\ P$
AD	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4\ P$
DC	L	A	0	$-1/\sqrt{2}$	0	$L/2A$	$-0.6\ P$
DB	$L\sqrt{2}$	A	$-P\sqrt{2}$	+1	$-2PL/A$	$L\sqrt{2}/A$	$-0.56\ P$
AC	$L\sqrt{2}$	A	0	+1	0	$L\sqrt{2}/A$	$0.85\ P$
Σ					$-4.121PL/A$	$4.828\ L/A$	
$N_{AC}=4.121P/4.828=0.85P$							