Structural Mechanics (1)

Lecture No-01

Part-01

What is this class?

- Introduction to Structural Analysis
- Loads on Structures
- Deflection in Determinate Structures
- Indeterminate Structures (Force Method)

Introduction to Structural Analysis

- Structural analysis is the prediction of the performance of a given structure under prescribed loads and/or other external effects, such as support movements, temperature changes and fabrication errors.
- The structure performance is measured by:
  - Displacements (deflections & rotations) & strains, and
  - Stresses & stress resultants (internal forces)
- The objective of the course is to present the methods for the analysis of structures in static equilibrium.

Historical Background

- In 1650s "engineers" began applying the knowledge of Mechanics (Math & Phy) in designing structures.
- Earlier structures were designed by trial and error and using past experience.

During his lifetime Leonardo was valued as an engineer. When he fled to Venice in 1499 he found employment as an engineer and devised a system of moveable barricades to protect the city from attack. He also had a scheme for diverting the flow of the Arno River.

In 1502, Leonardo produced a drawing of a single span 720-foot (220 m) bridge as part of a civil engineering project for Ottoman Sultan Beyazid II of Constantinople. The bridge was intended to span an inlet at the mouth of the Bosporus known as the Golden Horn. Beyazid did not pursue the project because he believed that such a construction was impossible. Leonardo's vision was resurrected in 2001 when a smaller bridge based on his design was constructed in Norway.

Historical Background

Galileo Galilei (1564–1642) is generally considered to be the originator of the theory of structures. In his book entitled Two New Sciences, which was published in 1638, Galileo analyzed the failure of some simple structures, including cantilever beams. Although Galileo's predictions of strengths of beams were only approximate, his work laid the foundation for future developments in the theory of structures and ushered in a new era of structural engineering, in which the analytical principles of mechanics and strength of materials would have a major influence on the design of structures.

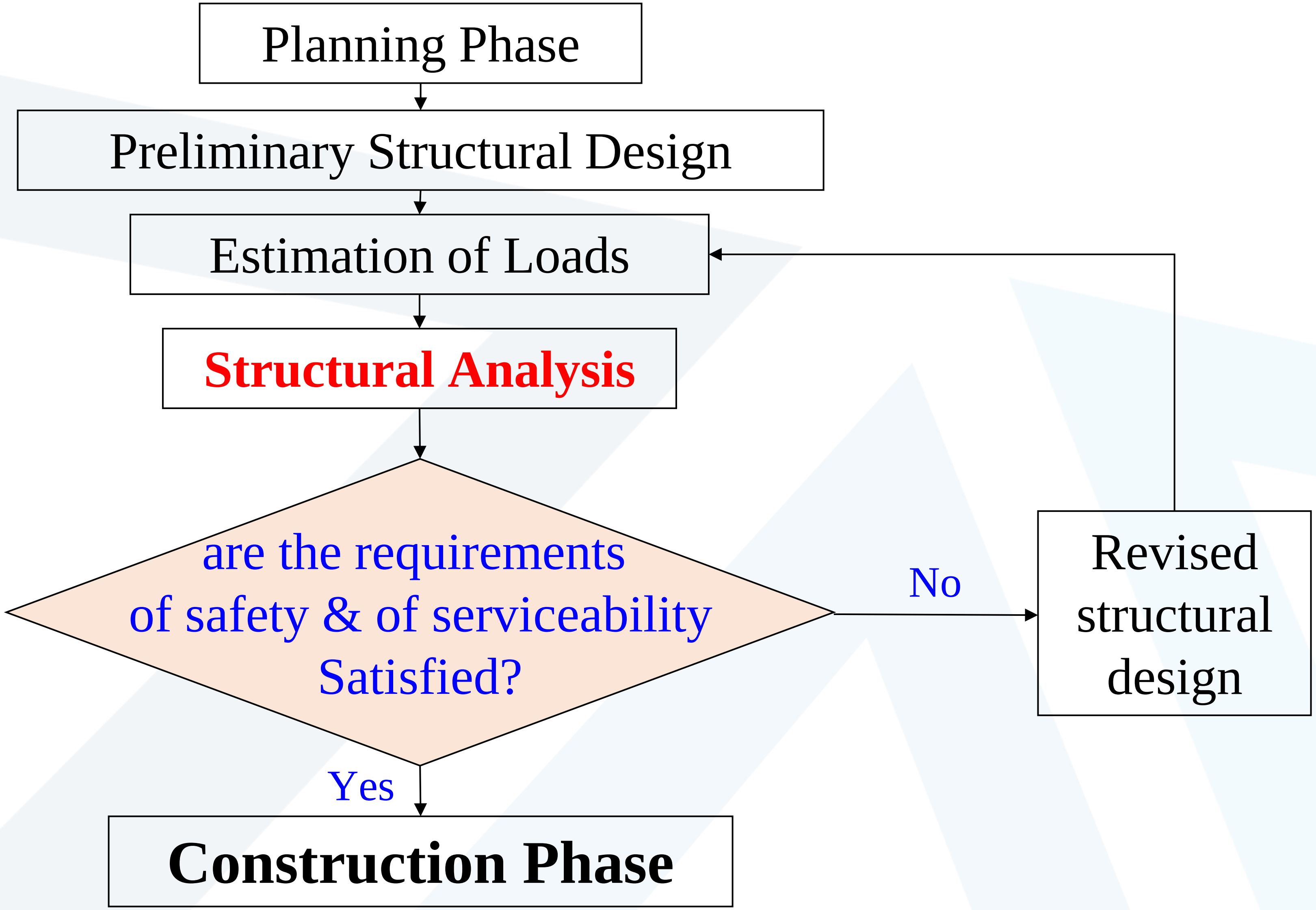
Among the notable investigators of that period were Robert Hooke (1635–1703), who developed the law of linear relationships between the force and deformation of materials (Hooke's law); Sir Isaac Newton (1642–1727), who formulated the laws of motion and developed calculus; John Bernoulli (1667–1748), who formulated the principle of virtual work; Leonhard Euler (1707–1783), who developed the theory of buckling of columns.

Historical Background

C.A. de Coulomb (1736–1806), who presented the analysis of bending of elastic beams.
In 1826 L. M. Navier (1785–1836) published a treatise on elastic behavior of structures, which is considered to be the first textbook on the modern theory of strength of materials. B. P. Clapeyron (1799–1864), who formulated the three-moment equation for the analysis of continuous beams; J. C. Maxwell (1831–1879), who presented the method of consistent deformations and the law of reciprocal deflections; Otto Mohr (1835–1918), who developed the conjugate-beam method for calculation of deflections and Mohr's circles of stress and strain; Alberto Castigliano (1847–1884), who formulated the theorem of least work; C. E. Greene (1842–1903), who developed the moment-area method; H. Muller-Breslau (1851–1925), who presented a principle for constructing influence lines; G. A. Maney (1888–1947), who developed the slope-deflection method, which is considered to be the precursor of the matrix stiffness method; and Hardy Cross (1885–1959), who developed the moment-distribution method in 1924.

John Smeaton (1724–1792) is the first to described himself as a 'civil engineer' in 1768. In doing so, he identified a new profession that was distinct from that of the military engineers.

Role of Structural Analysis in Structural Engineering Project



Classification Of Structures

Classification is not unique and not perfect

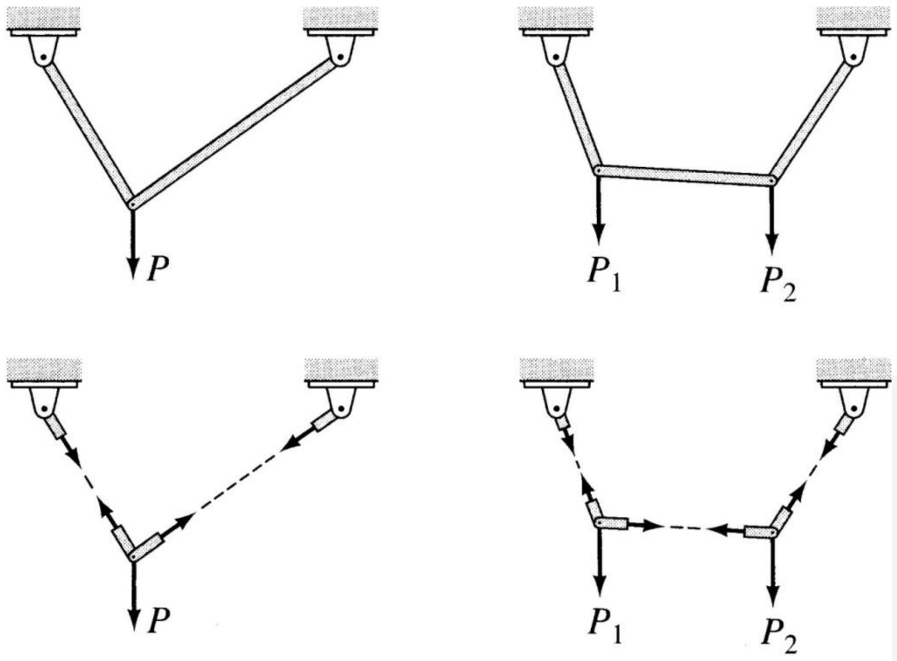
Classification of Structures According to State of Stress

1. Tension Structures

Classification Of Structures

Classification is not unique and not perfect

1-Tension Structures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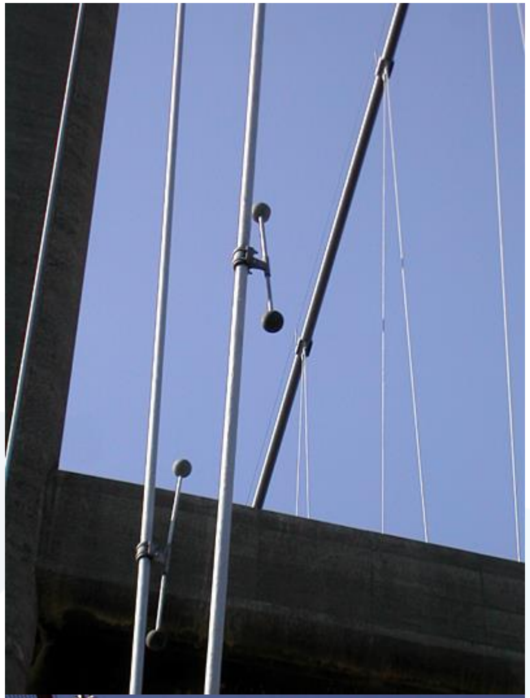
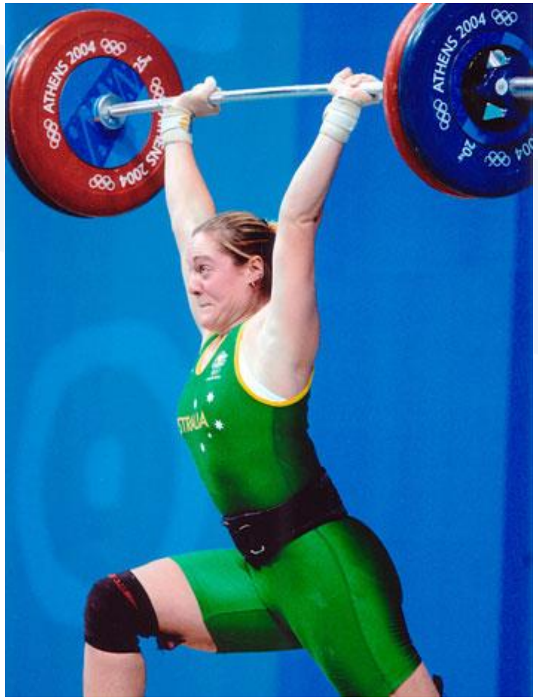
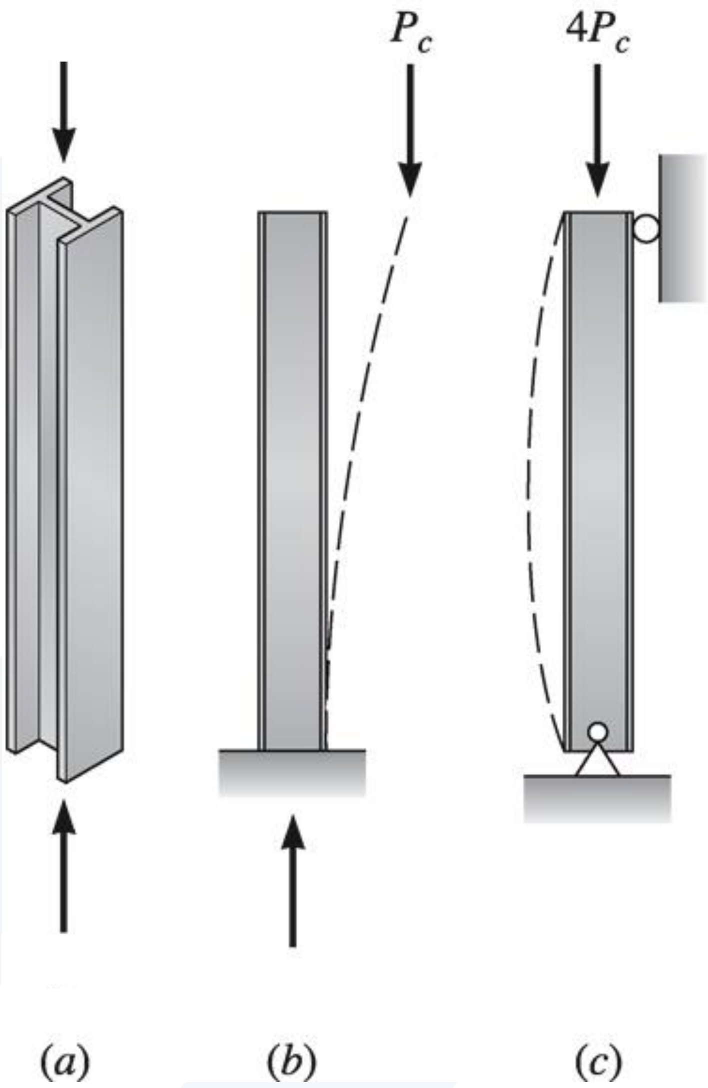
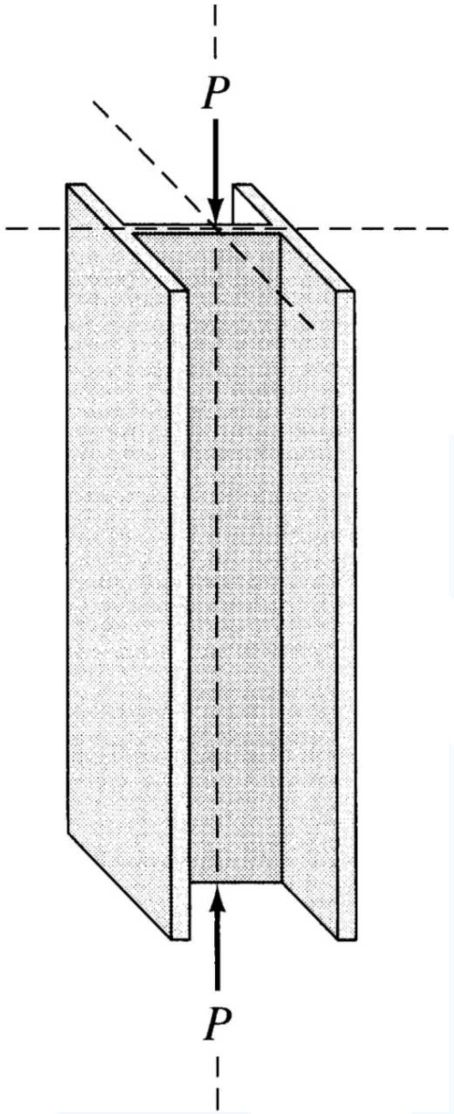


Classification Of Structures

2-Compression Structures



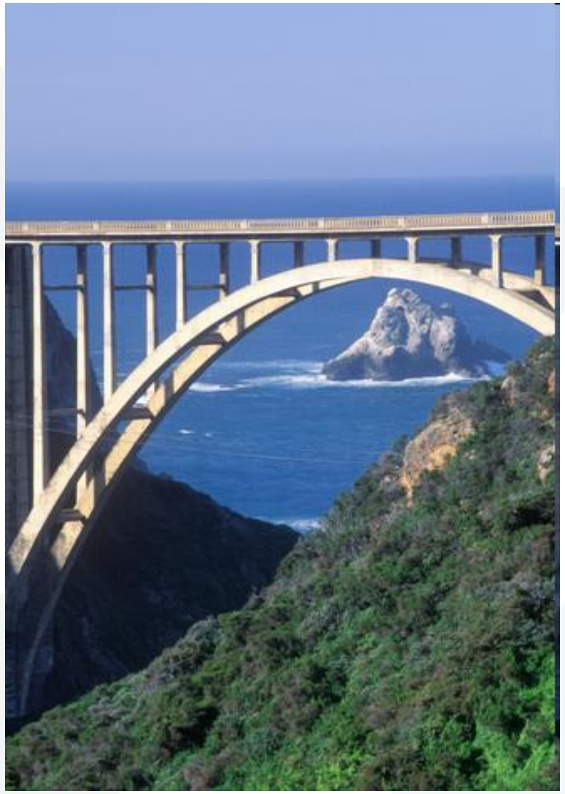




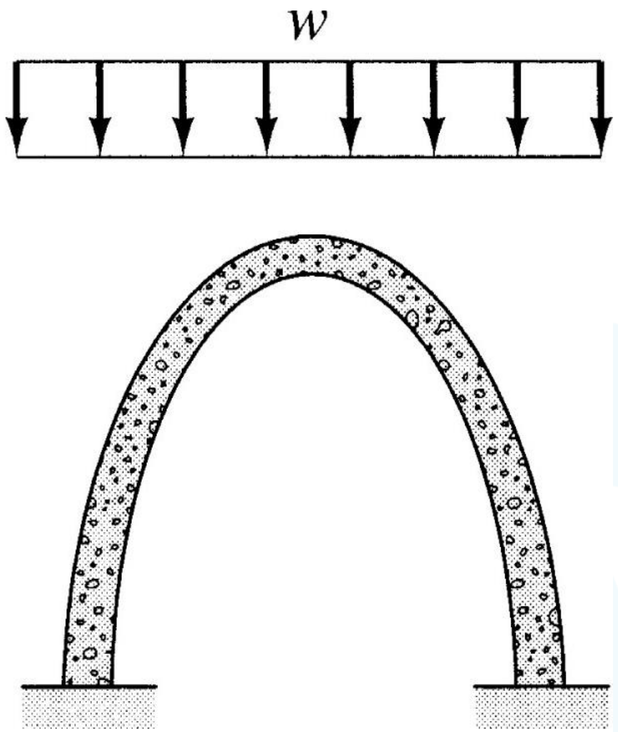
Classification Of Structures

2-Compression Structures

Wanxian Bridge, China.
Span ≈ 400m



Bixby Bridge California
Span ≈ 200m

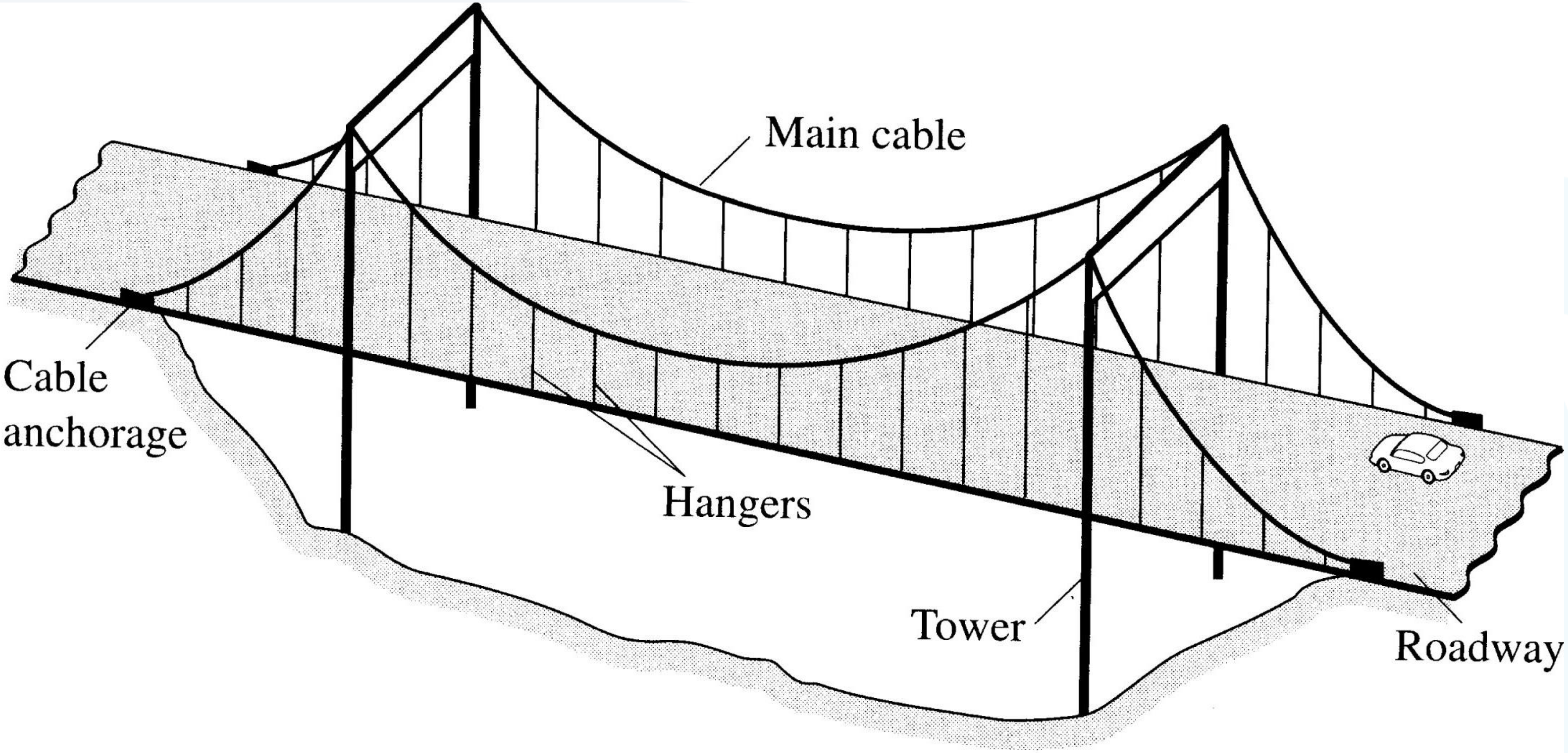


A proper shape
eliminates tension

Classification Of Structures

Combination of tension members with Compression members in the same Structure

Suspended Bridges



Classification Of Structures

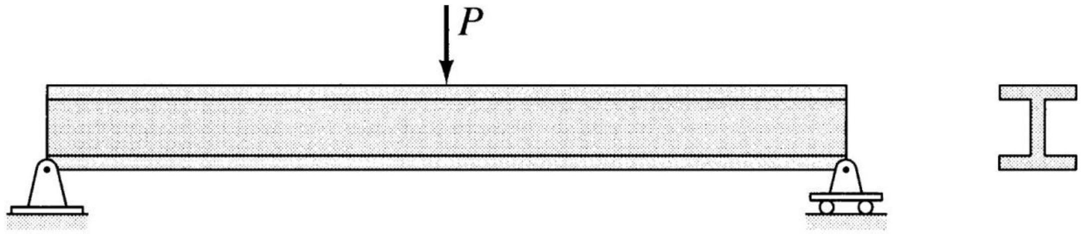
Combination of tension members with Compression members in the same Structure

Trusses

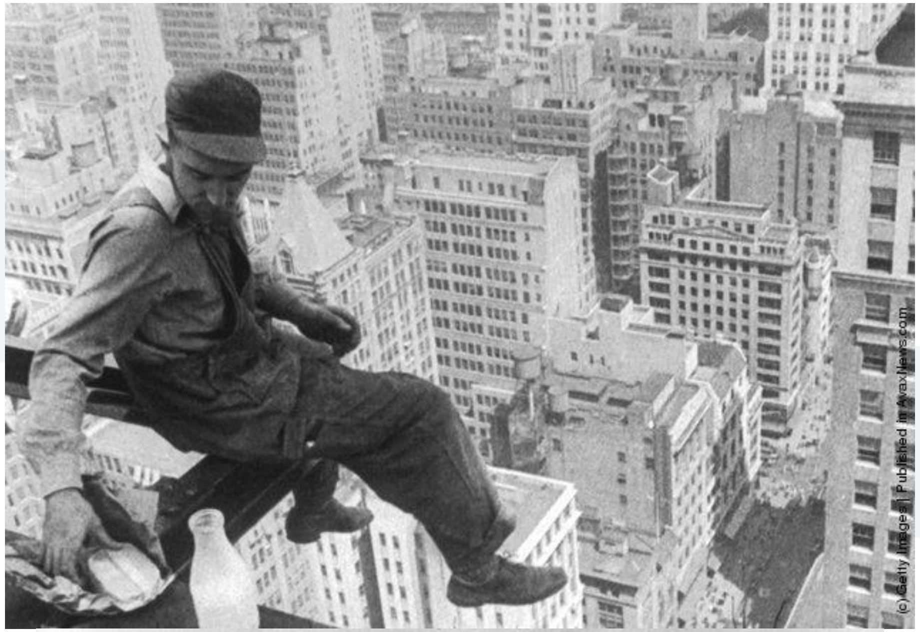
Trusses are very efficient structures

The ratio of the self weight to the load bearing capacity, is very small

Classification Of Structures



1. Bending Structures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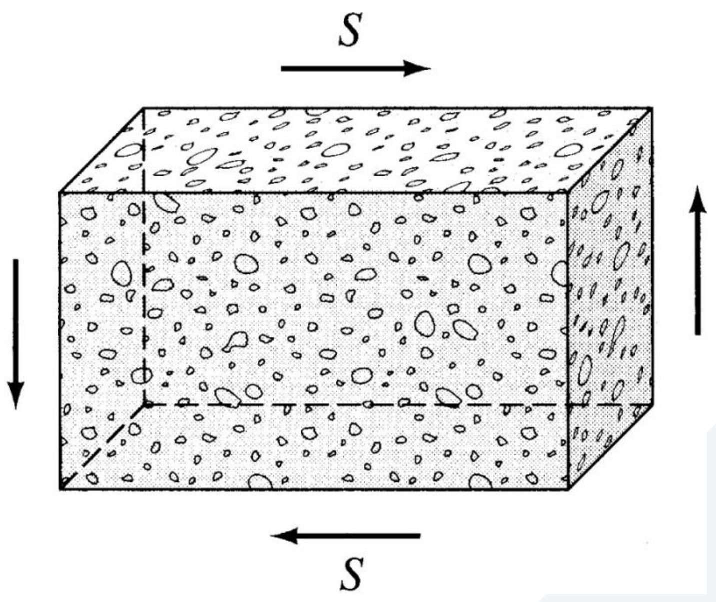
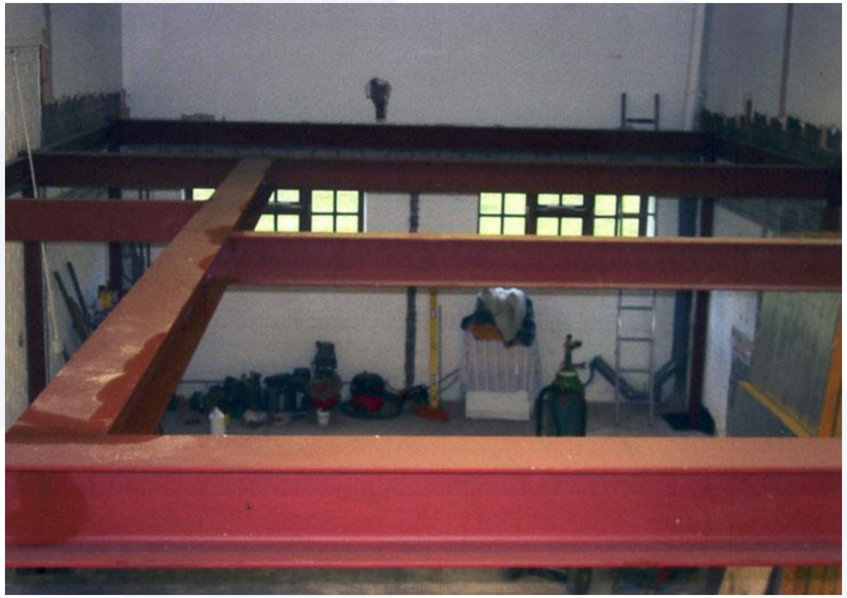
THE NEW YORK COLLECTION

4- Shear Structures



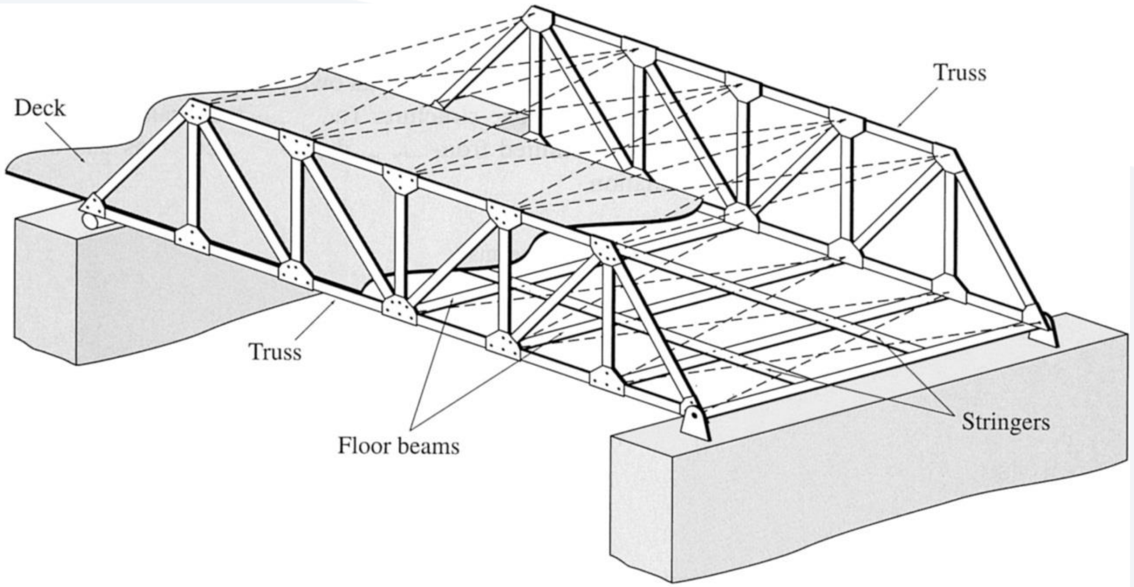
Loads on Structures

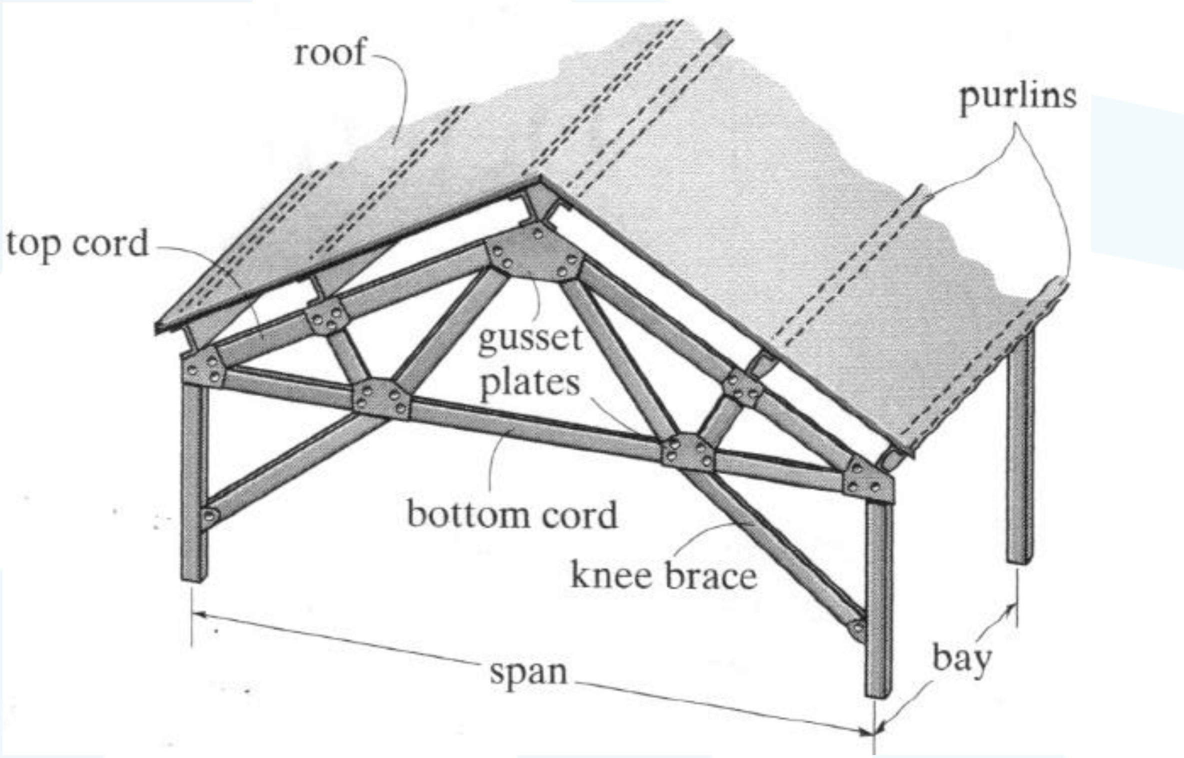
The main beam (or the girder) resting on a column or on basement walls & supporting all the other secondary beams (or the joists), which at their turn are support the slab.



The top surfaces of the main beams and of the secondary ones are at the same level. So, they are all supporting the slab directly

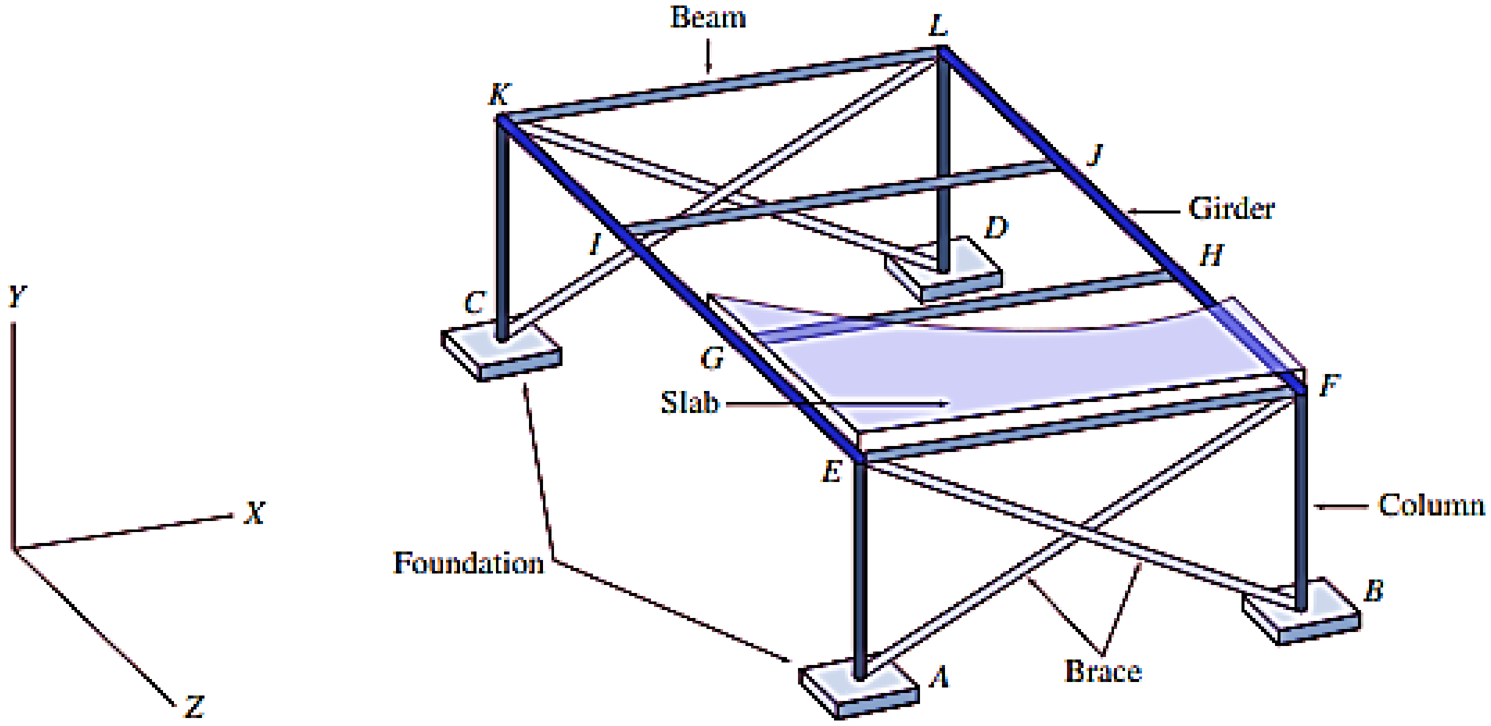
Loads on Structures





Loads on Structures

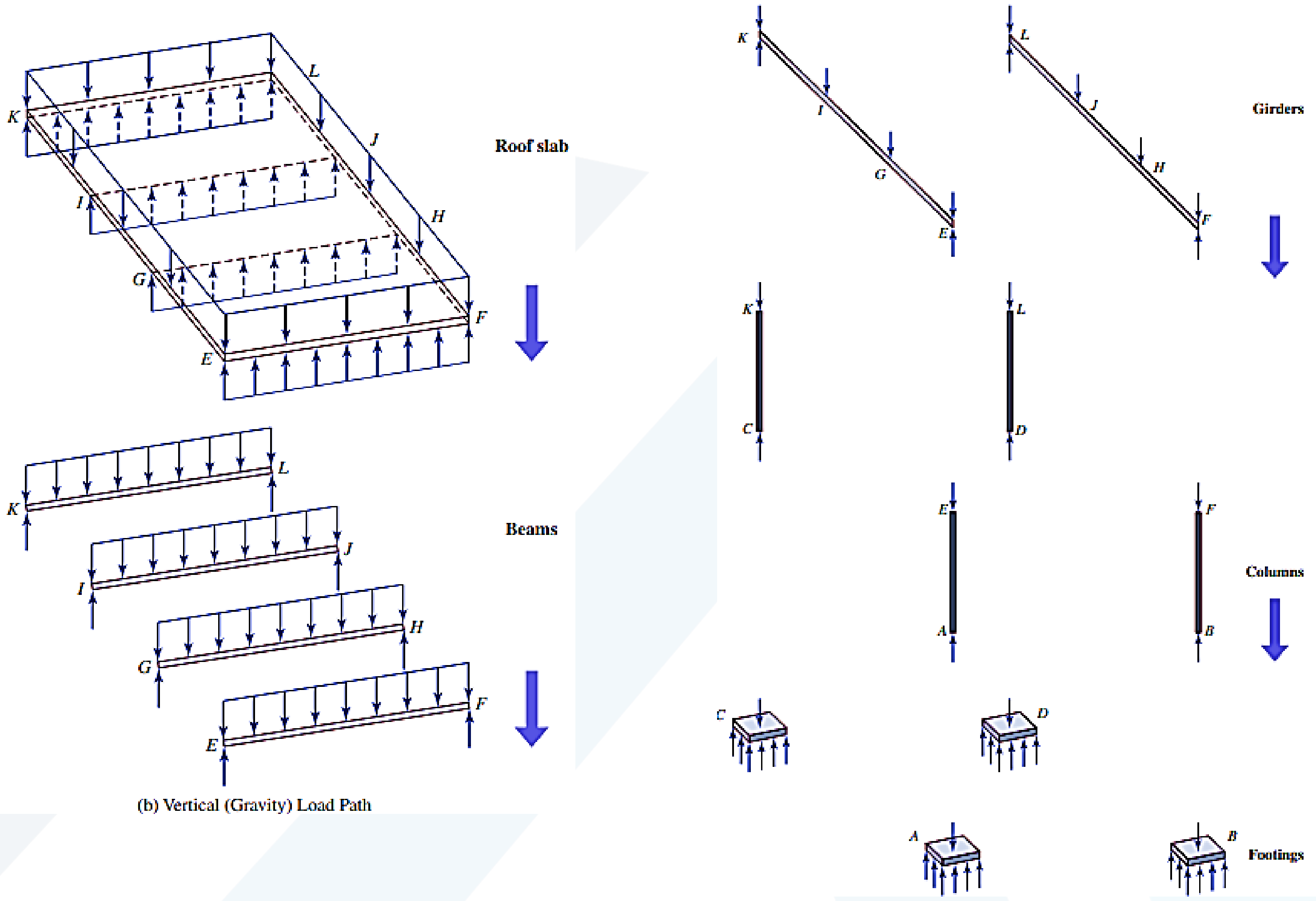
Structural Systems for Transmitting Loads



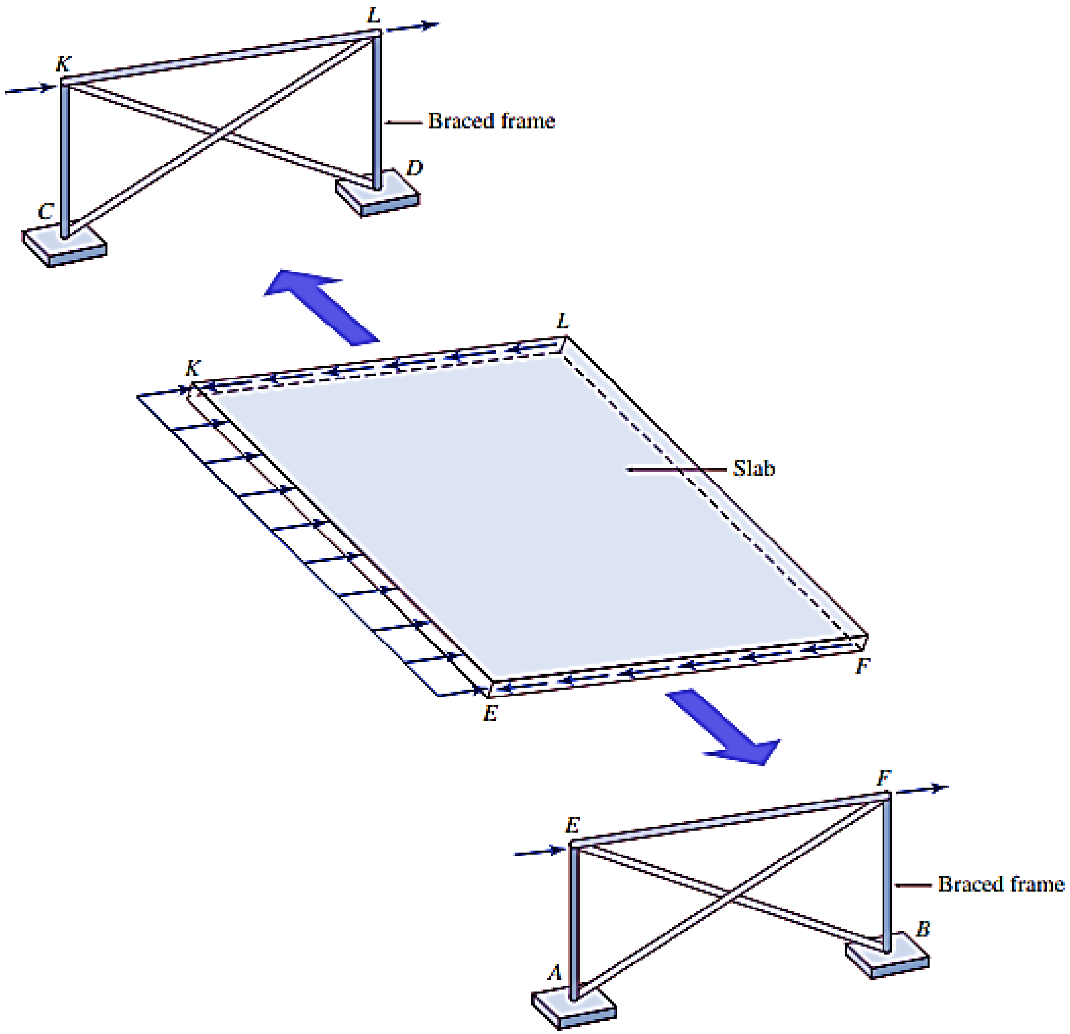
(a) Structural System for a Single-Story Building

Loads on Structures

Load Path: A term used to describe how a load acting on the building (or bridge) is transmitted through the various members of the structural system to the ground.



Loads on Structures



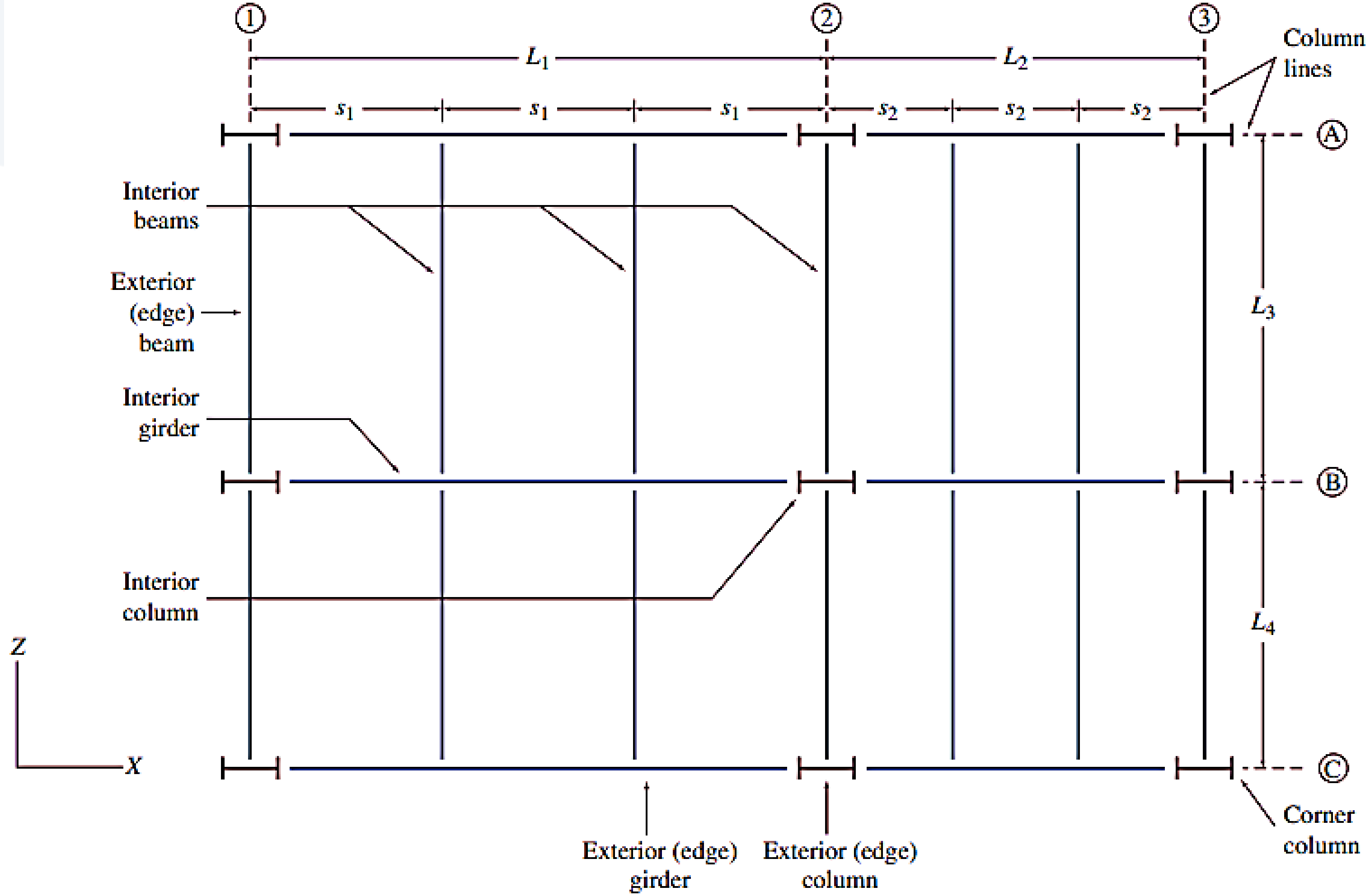
(c) Horizontal (Lateral) Load Path

Loads on Structures

Multistory Building with Braced Frames to Transmit Lateral Loads Due to Wind and Earthquakes

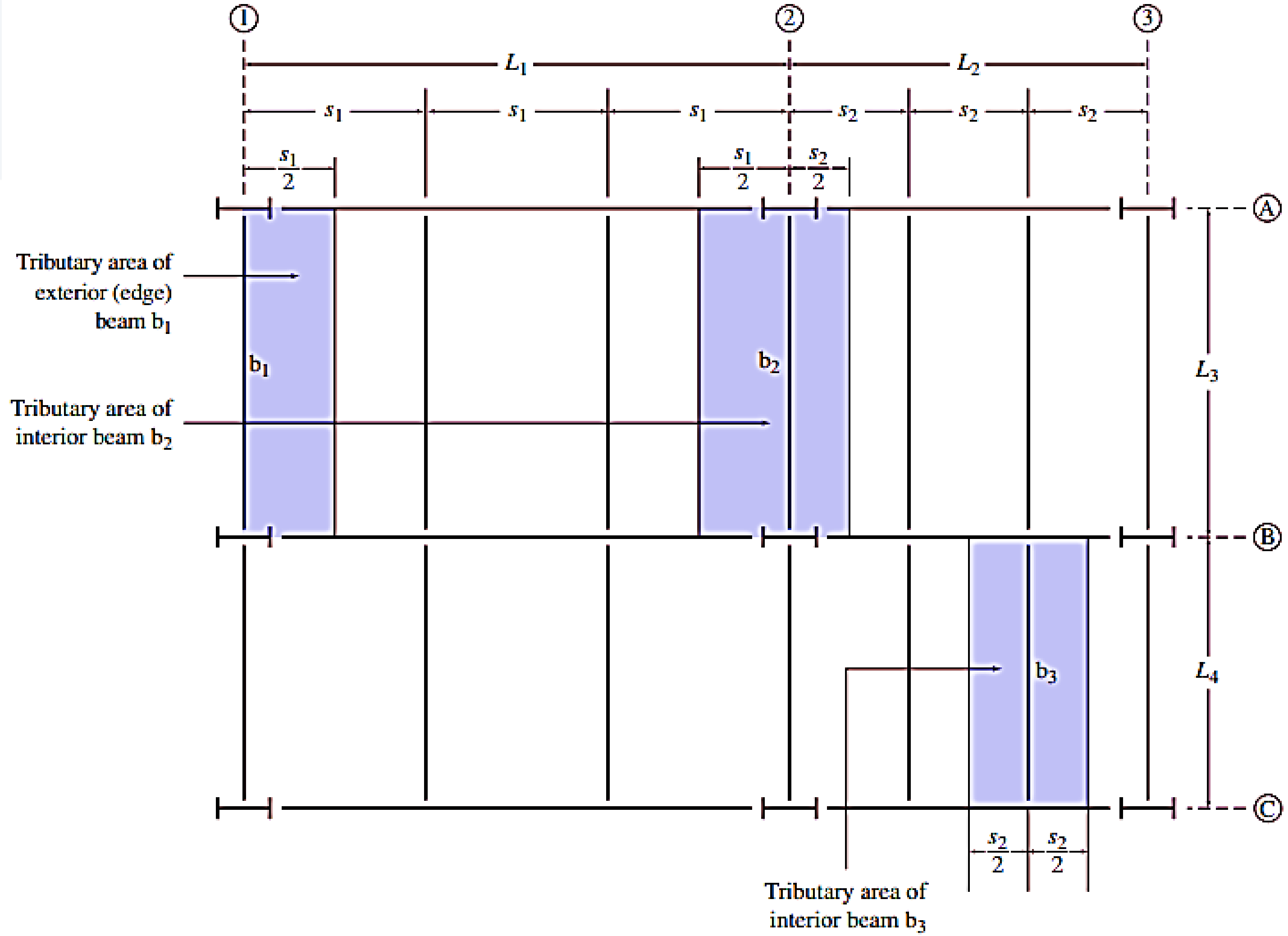
This Steel Frame Building Uses Masonry Shafts for Elevators and Stairs to Resist Lateral Loads Due to Wind and Earthquakes

Loads on Structures



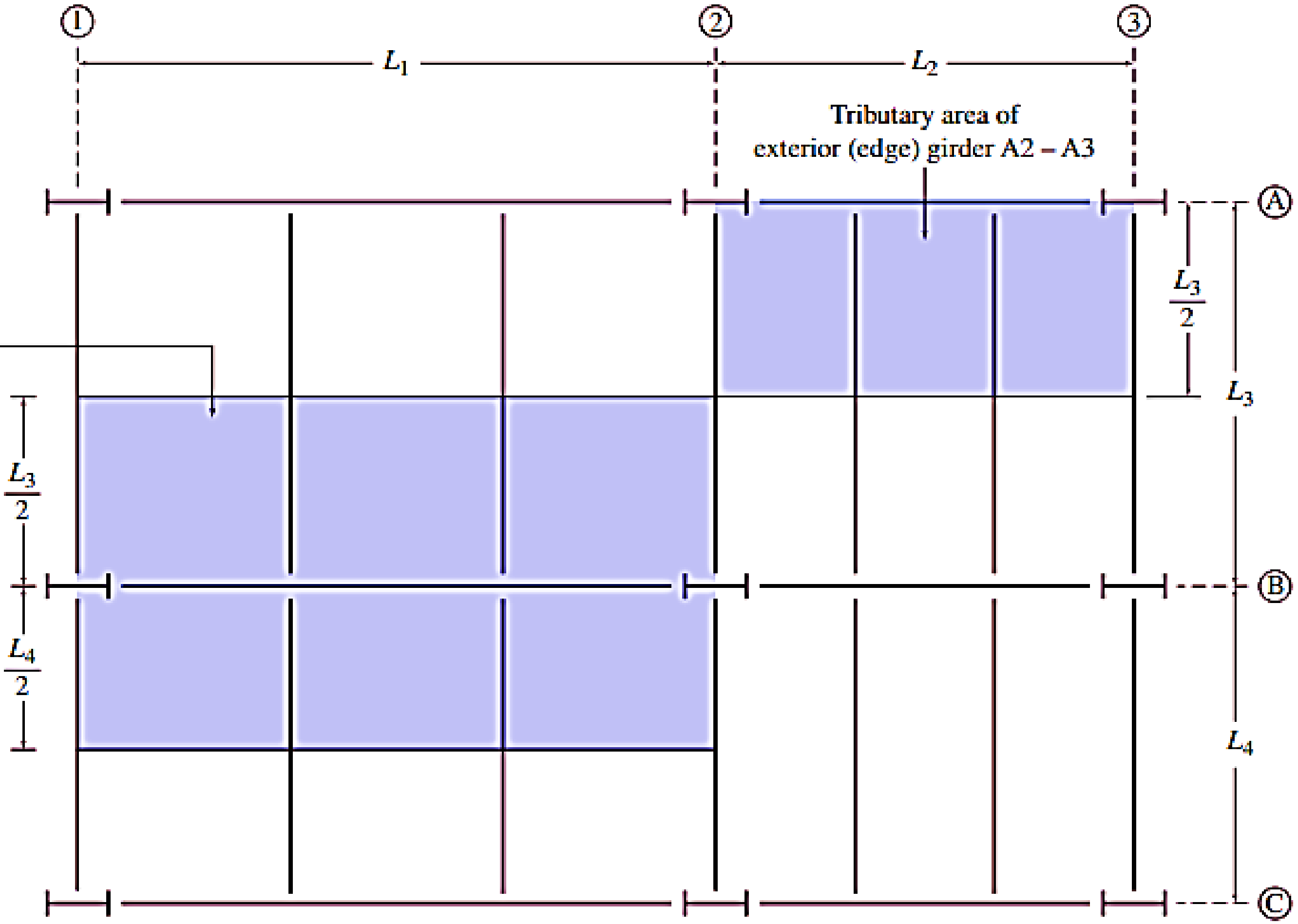
(a) A Typical Floor Framing Plan

Loads on Structures



(b) Tributary Areas of Beams

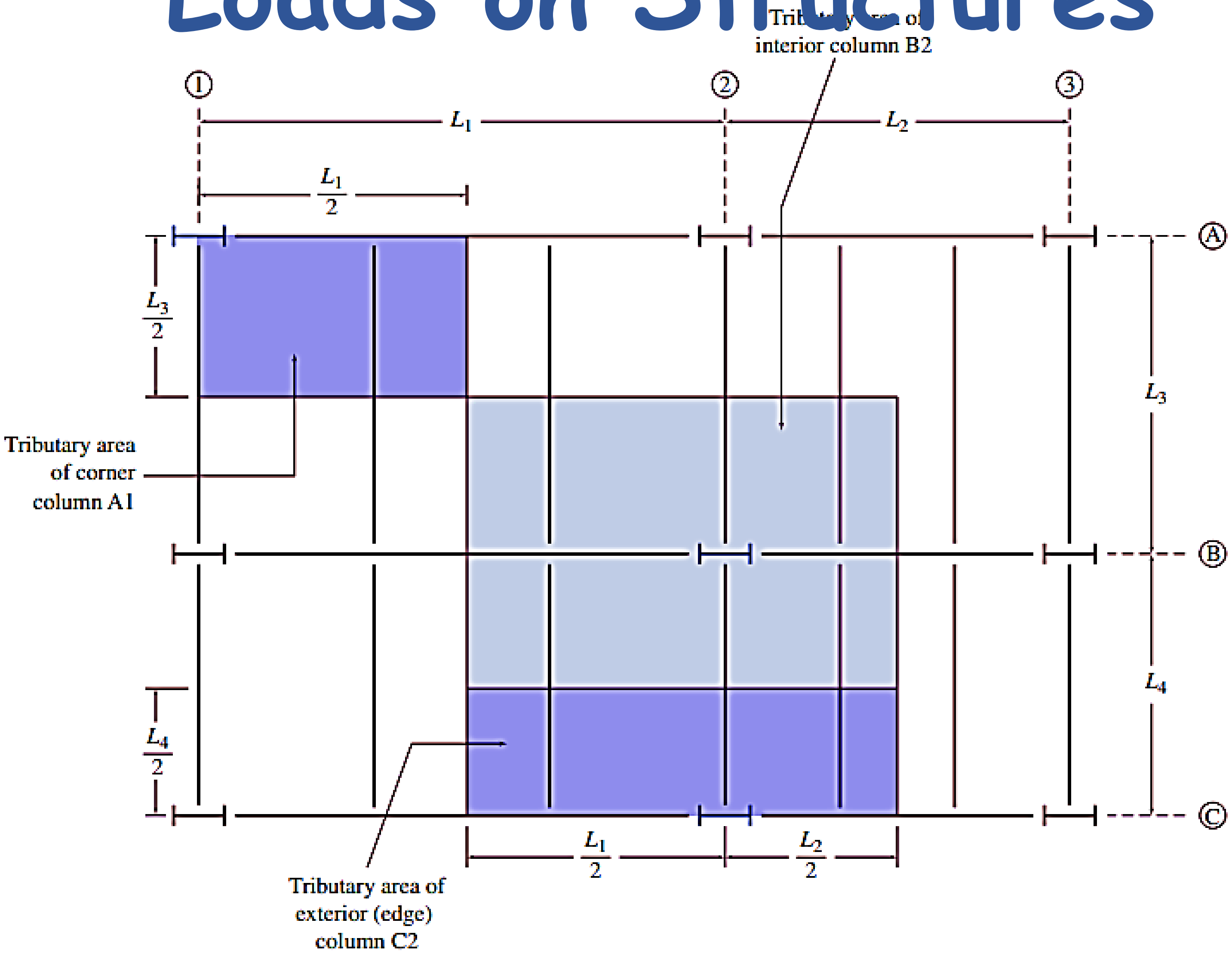
Loads on Structures



i)

(c) Tributary Areas of Girders

Loads on Structures



(d) Tributary Areas of Columns

Loads on Structures

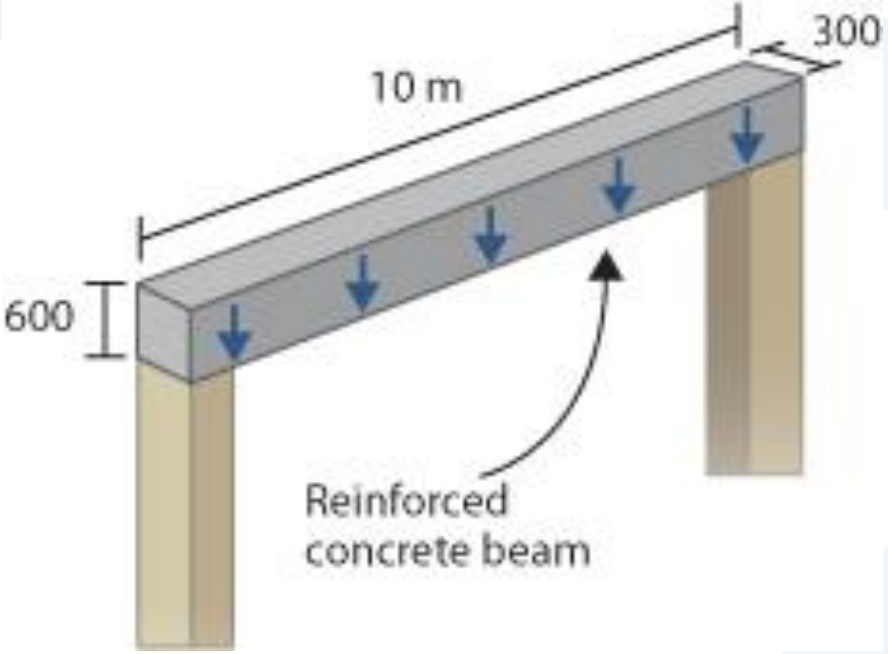
Dead Loads

Dead Loads are gravity loads of constant magnitudes and fixed positions that act permanently on the structure. Such loads consist of the weights of the structural system itself and of all other material and equipment permanently attached to the structural system.

Unit Weights of Construction Materials	
Materials	**Unit Weight [kN/m^3]**
Aluminum	**26**
Brick	**19**
Concrete, reinforced	**24**
Structural steel	**77**
Wood	**≈ 6**

Loads on Structures

Dead Loads



Volume of the beam: 10.0 x 0.6 x 0.3 = 1.8 m^3

Unit weight of reinforced concrete = 24 kN/m^3

Therefore, dead load on the beam = volume x unit weight

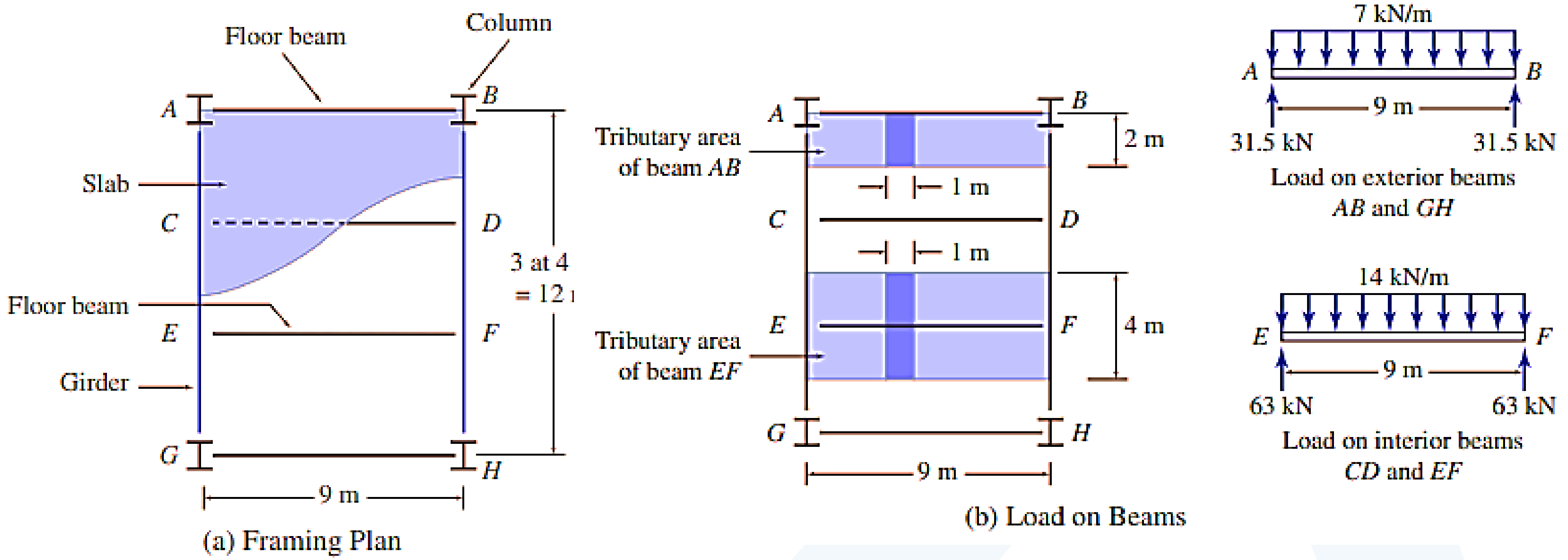
= 1.8 m^3 x 24 kN/m^3

= 43.2 kN

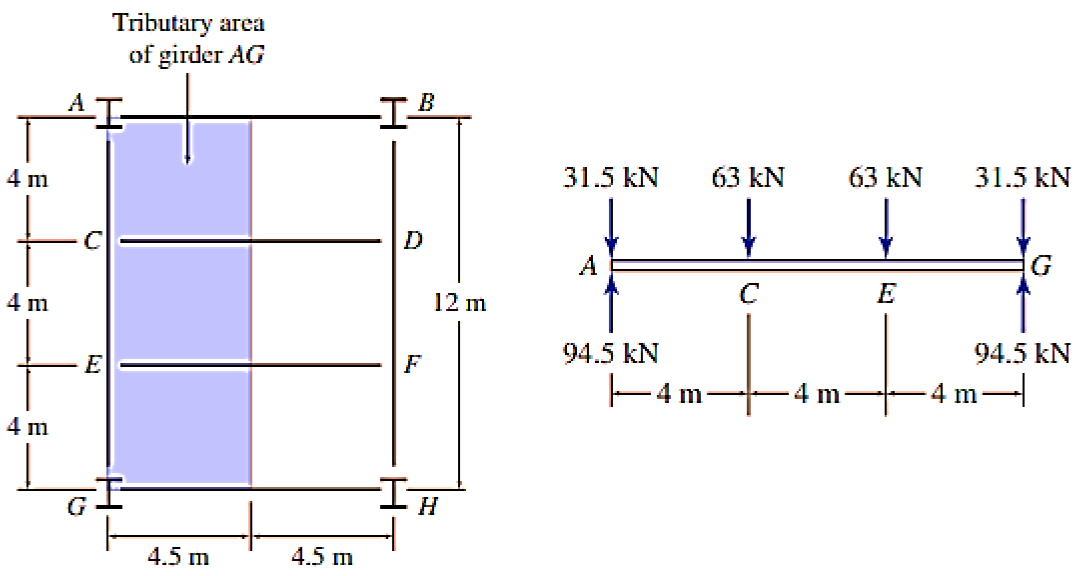
It is distributed uniformly with intensity: 43.2/10=4.32 kN/m

Loads on Structures

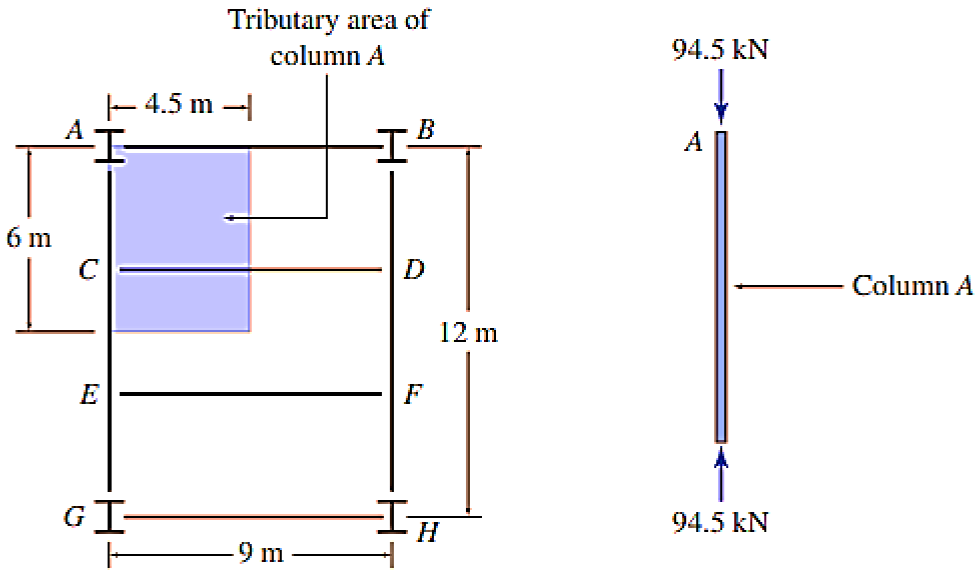
Tributary Area: The floor of a building, shown in following figure, is subjected to a uniformly distributed load of 3.5 kPa over its surface area. Determine the loads acting on all the members of the floor system.



Loads on Structures



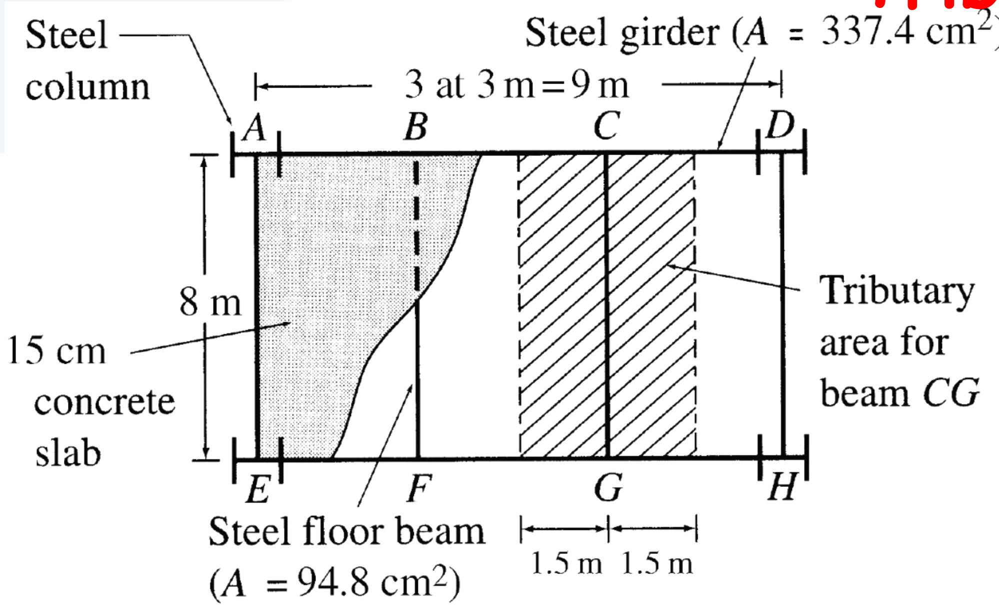
(c) Load on Girders AG and BH



(d) Compressive Axial Load on Columns A, B, G, and H

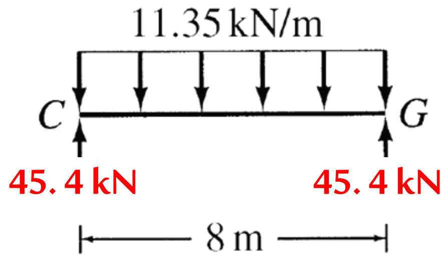
Loads on Structures

Tributary Area

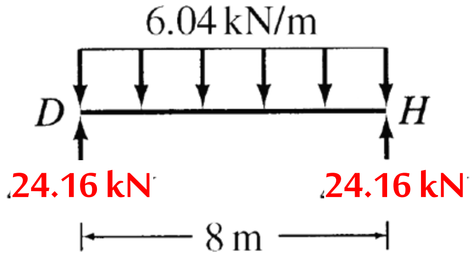


(a) Framing Plan

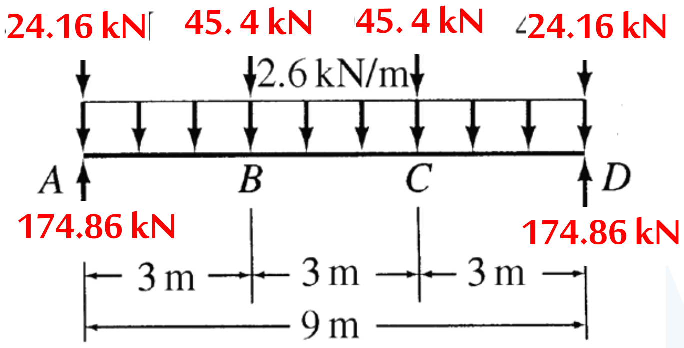
Tributary Area: The floor system of a building consists of a 15 cm thick RC slab resting on 4 steel floor beams, which in turn are supported by 2 steel girders. The cross-sectional areas of the floor beams and the girders are 94.8 cm^2 & 337.4 cm^2, respectively. Determine the dead loads acting on the beams CG and DH and the girder AD.



(b) Load on Beam CG



(c) Load on Beam DH



(d) Load on Girder AD

Loads on Structures

Live Loads

Live Loads are loads of varying magnitudes and/or positions caused by the use of the structure.

TABLE 2.2 MINIMUM FLOOR LIVE LOADS FOR BUILDINGS

Occupancy or Use	Live Load kPa
Hospital patient rooms, residential dwellings, apartments, hotel guest rooms, school classrooms	1.92
Library reading rooms, hospital operating rooms and laboratories	2.87
Dance halls and ballrooms, restaurants, gymnasiums	4.79
Light manufacturing, light storage warehouses, wholesale stores	6.00
Heavy manufacturing, heavy storage warehouses	11.97

Loads on Structures

Live Loads

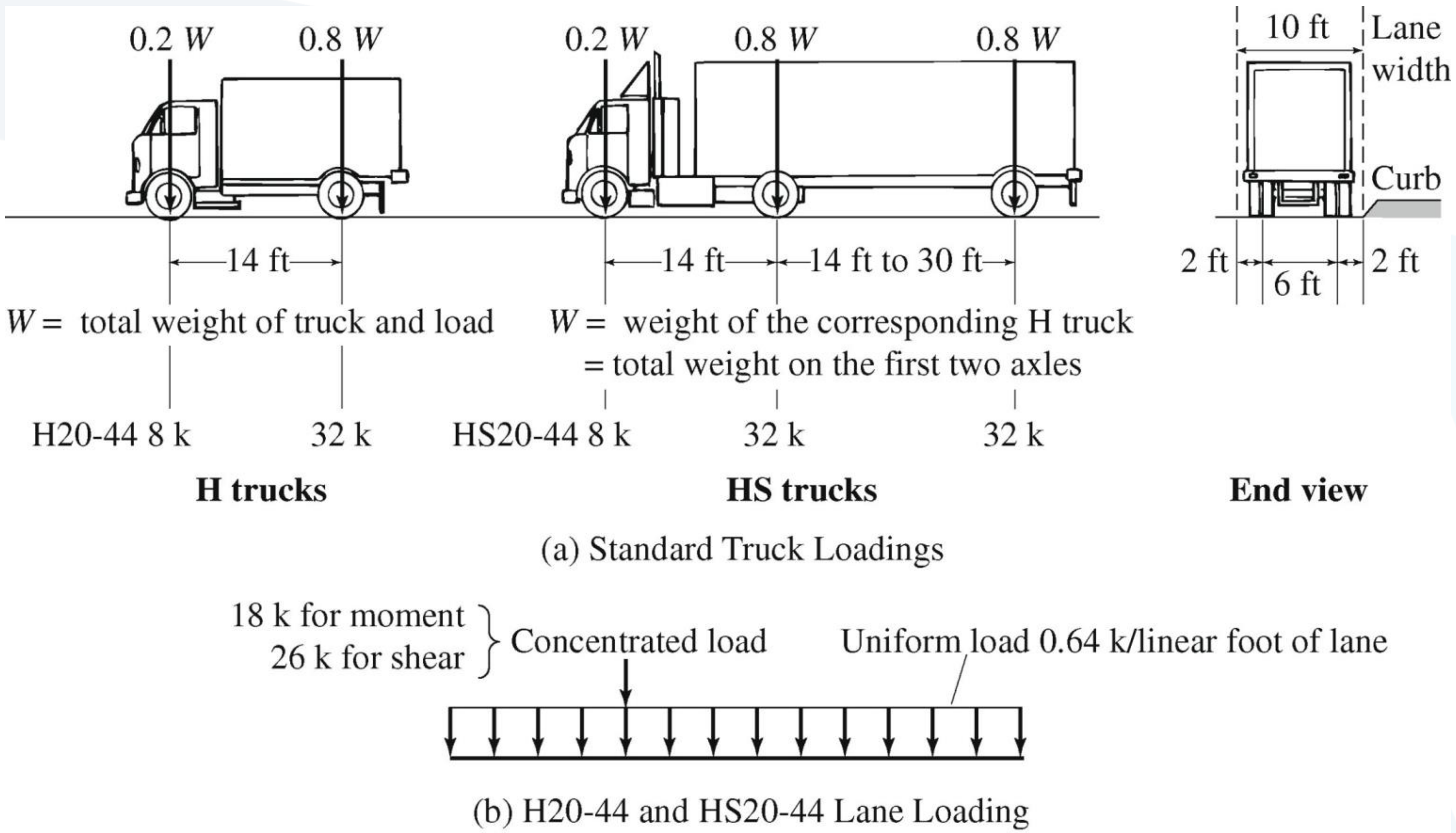


FIG. 2.2 Live Loads for Highway Bridges [1 foot = 0.305 m, 1 k = 4.45 kN]

Source: Taken from the *Standard Specifications for Highway Bridges*. Copyright 2002. American Association of State Highway and Transportation Officials, Washington, D.C. Used by permission.

Loads on Structures

Live Loads - Impact

When live loads are applied rapidly to a structure, they cause larger stresses than those that would be produced if the same loads would have been applied gradually. The dynamic effect of the load that causes this increase in stress in the structure is referred to as impact. To account for the increase in stress due to impact, the live loads are increased by certain impact percentages, or impact factors. For example:

- The *ASCE 7 Standard* specifies that all elevator loads for buildings be increased by 100% to account for impact.
- The *AASHTO Specification* gives the expression for the impact percentage of live load on highway bridges as

$$I = \frac{15}{L + 38.1} \leq 0.3$$

in which L is the length in m of the portion of the span loaded to cause the maximum stress in the member under consideration.

Loads on Structures

Live Loads - Wind

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in which L is the length in m of the portion of the span loaded to cause the maximum stress in the member under consideration.

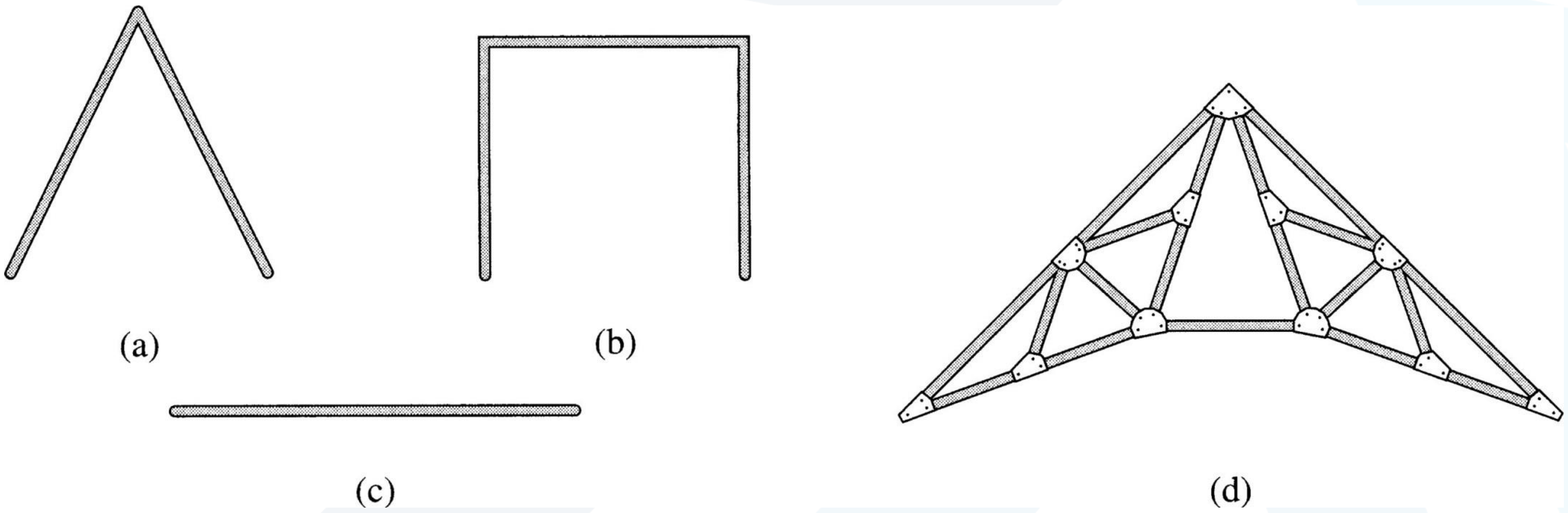
Structural Mechanics (1)

Lecture No-01

Part-02

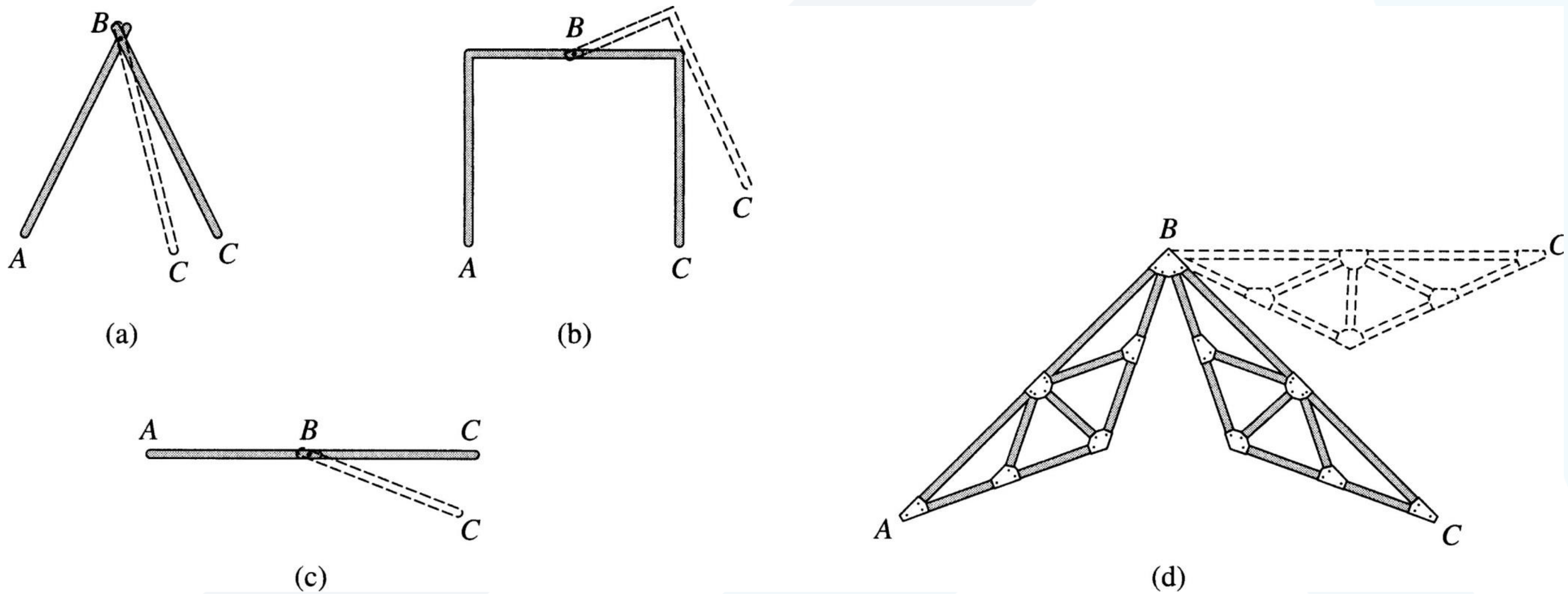
Static Instability

- A structure is **internally stable**, or **rigid**, if it maintains its shape and remains a rigid body when detached from the supports.



Static Instability

- Conversely, a structure is internally unstable (or nonrigid) if it cannot maintain its shape and may undergo large displacements under small disturbances when not supported externally.



Static Determinacy of Internally Stable Structures

- A structure is Statically Determinate (SD) if you can calculate all reactions and internal forces just using the equations of statics, i.e. equations of equilibrium.

- This means that you must have the same number of unknowns as you have equations of equilibrium.

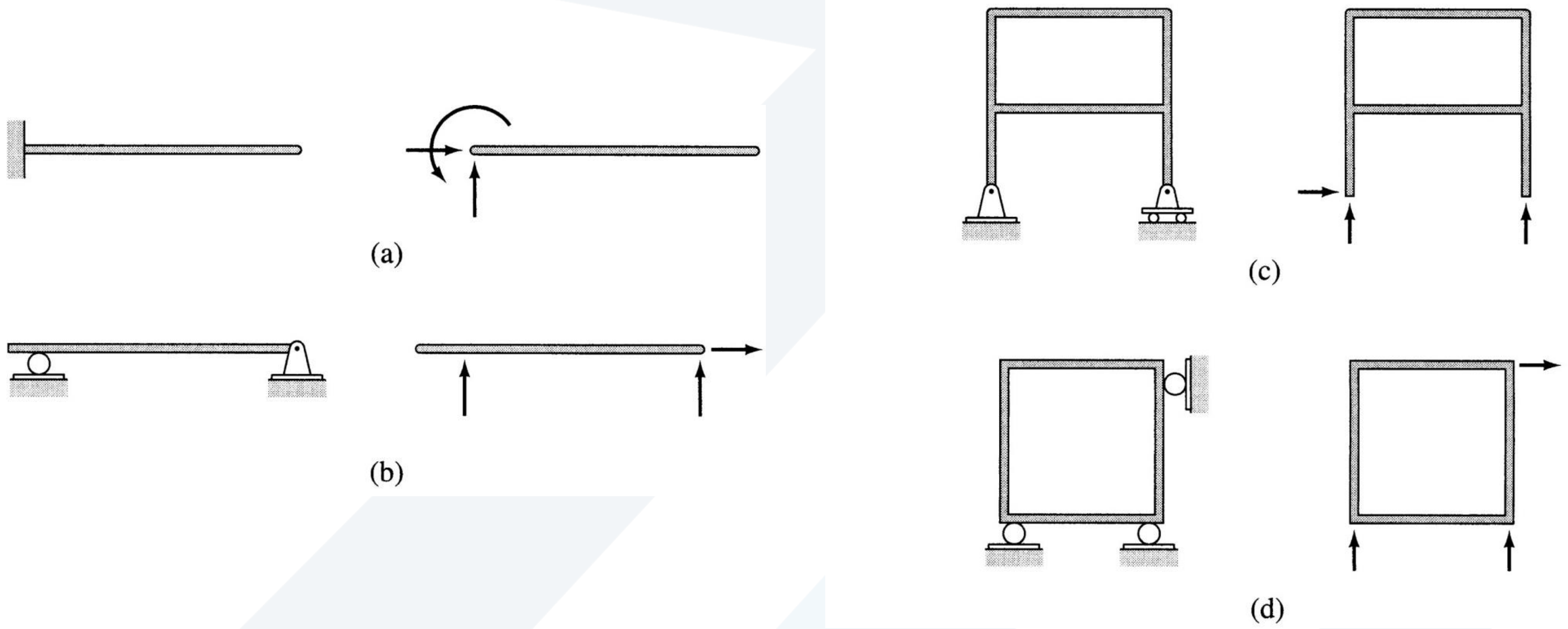
Number of unknowns=Number of equilibrium equations

- All problems we examined in Statics and most in Mechanics were statically determinate.

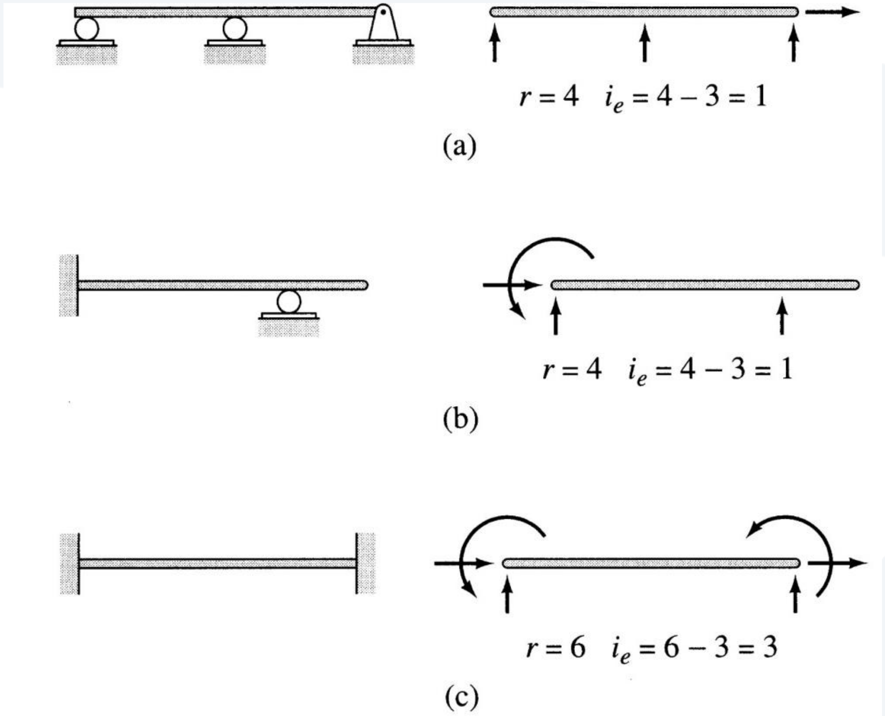
- Since a plane internally stable structure can be treated as a plane rigid body, it must be supported by at least three reactions that satisfy the three equations of equilibrium.

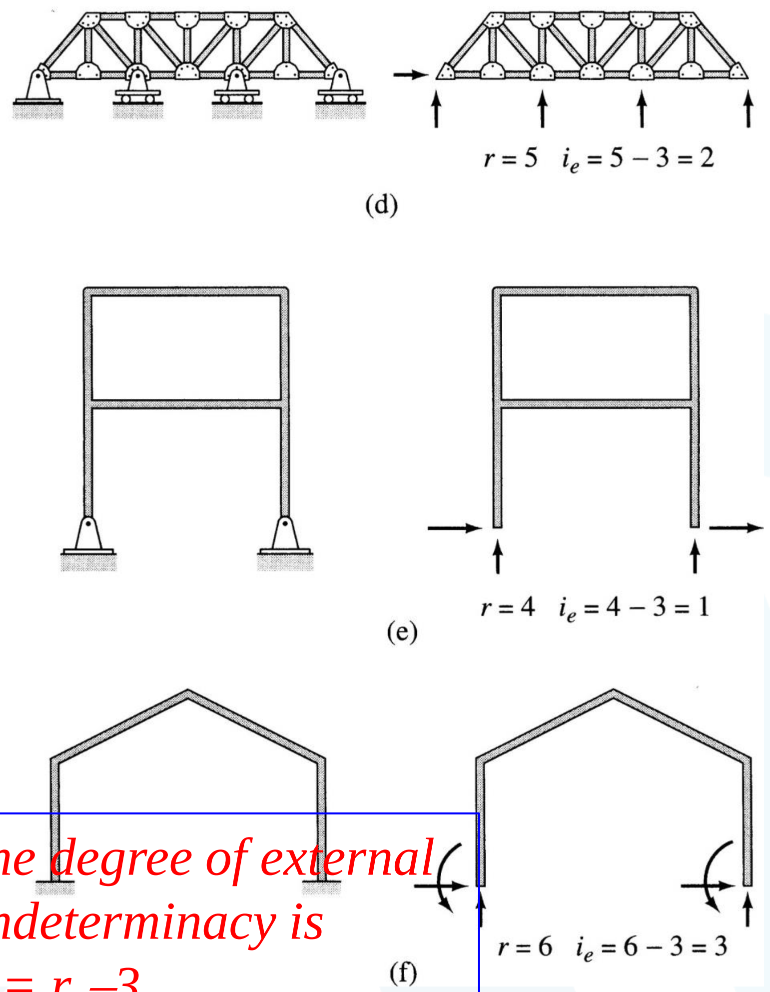
Static Determinacy of Internally Stable Structures

- Thus, a plane structure that is statically determinate externally must be supported by exactly three reactions.



Static Determinacy of Internally Stable Structures





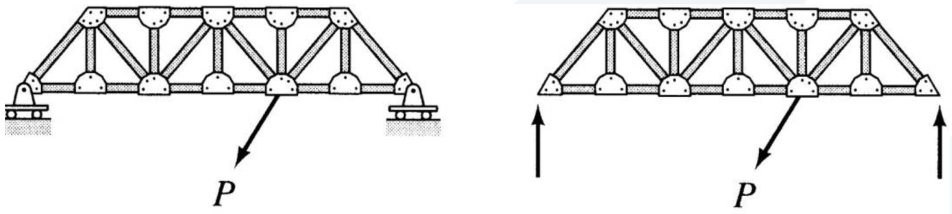
Thus, if a structure has r reactions, then if

$r < 3$ the structure is statically unstable externally
$r = 3$ the structure is statically determinate externally
$r > 3$ the structure is statically indeterminate externally

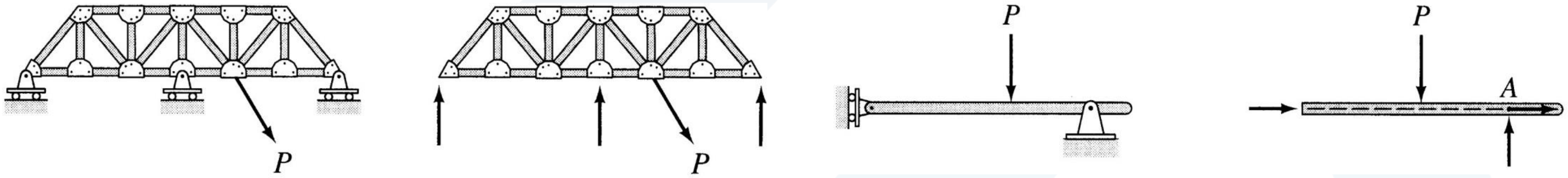
→ *the degree of external indeterminacy is* $i_e = r - 3$

Static Determinacy of Internally Stable Structures

It should be realized that the first of three conditions stated above is both necessary and sufficient in the sense that if $r < 3$, the structure is definitely unstable. However, the remaining two conditions, $r = 3$ & $r > 3$, although necessary, are not sufficient for static determinacy and indeterminacy, respectively. In other words, a structure may be supported by a sufficient number of reactions ($r \geq 3$) but may still be unstable due to improper arrangement of supports. Such structures are referred to as *geometrically unstable externally*



An Example of Externally Statically Unstable Plane Structure



Reaction Arrangements Causing External Geometric Instability in Plane Structures

Static Determinacy of Internally Stable Structures

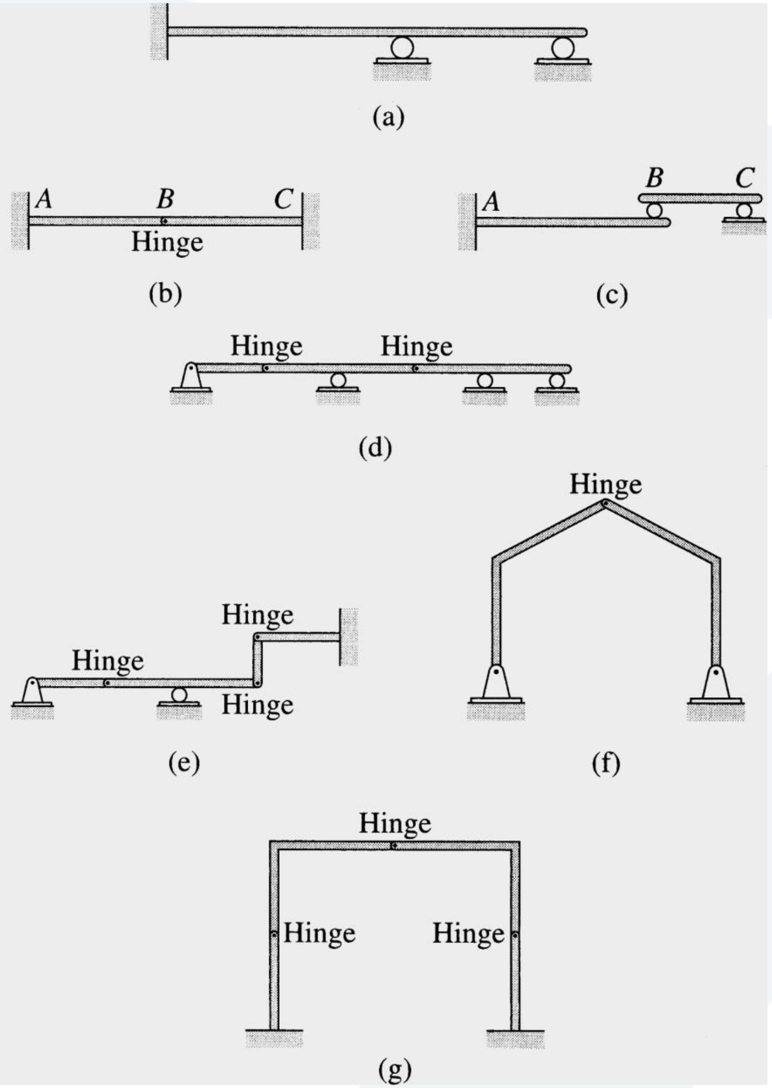
From the foregoing discussion, we can conclude that if there are e_c equations of condition (one equation for each internal hinge and two equations for each internal roller) for an internally unstable structure, which is supported by r external reactions, then if

$r < 3+e_c$	the structure is statically unstable externally
$r = 3+e_c$	the structure is statically determinate externally
$r > 3+e_c$	the structure is statically indeterminate externally

For an internally unstable structure, the degree of external indeterminacy is expressed as:

$$i_e = r - (3+e_c)$$

Classify each of the structures shown in the figure, as externally unstable, statically determinate, or statically indeterminate. If the structure is statically indeterminate externally, then determine the degree of external indeterminacy



Solution

(a) This beam is internally stable, $r = 5 > 3$. It is statically indeterminate externally with the degree of external indeterminacy of $i_e = r - 3 = 2$

(b) This beam is internally unstable. It is composed of two rigid members connected by an internal hinge. For this beam, $r = 6$ & $e_c = 1$. It is statically indeterminate externally with the degree of external indeterminacy of $i_e = r - (3 + e_c) = 2$.

(c) This structure is internally unstable with $r = 4$ & $e_c = 2$. $r < 3 + e_c$, it is statically unstable externally. This can be verified noting that the member BC is not restrained against movement in the horizontal direction.

(d) This beam is internally unstable, $r = 5$ and $e_c = 2$. So $i_e = r - (3 + e_c) = 0$, the beam is statically determinate externally

(e) This is an internally unstable structure, $r = 6$ & $e_c = 3$. $i_e = r - (3 + e_c) = 0$, the structure is statically determinate externally.

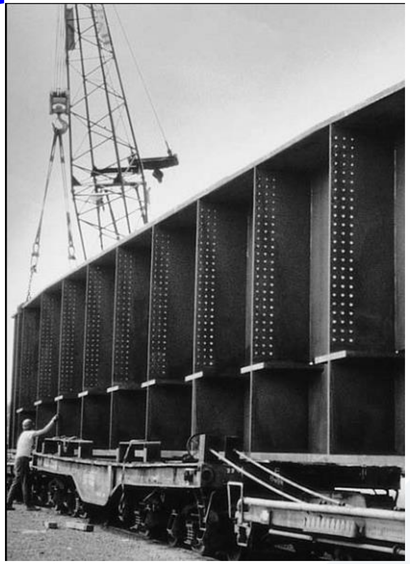
(f) This frame is internally unstable, $r = 4$ and $e_c = 1$. $i_e = r - (3 + e_c) = 0$
the frame is statically determinate externally.

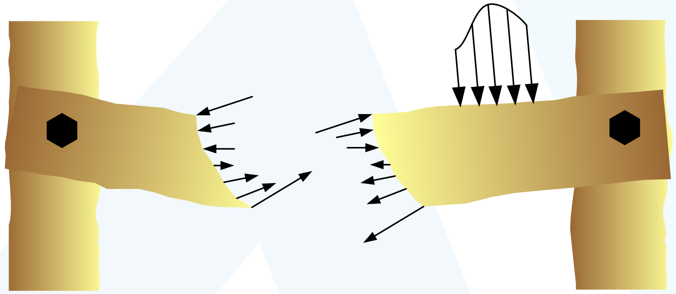
(g) This frame is internally unstable with $r = 6$ and $e_c = 3$. Since $i_e = r - (3 + e_c) = 0$, the frame is statically determinate externally

Beams & Frames. Shear Force & Bending Moment

In trusses, members are subjected to only axial forces. The members of rigid frames and beams may be subjected to *shear forces* and *bending moments* & *axial forces* under the action of external loads. Determining these internal forces is necessary for the design of such structures.

Our objective is to present the analysis of internal forces and moments that may develop in beams, and the members of plane frames, under the action of coplanar systems of external forces and couples.



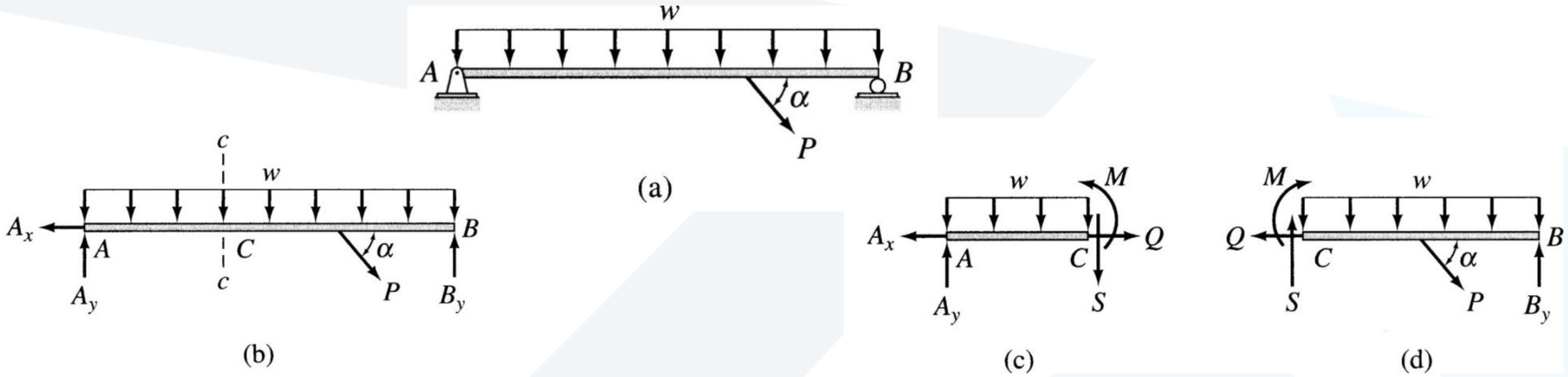


internal forces ≡ stress resultants

Stresses = functions of internal forces and section & material properties

Internal Forces

Definition of Internal Forces: **Internal forces can be defined as the forces and couples exerted on a portion of the structure by the rest of the structure.**



Consider, for example, the simply supported beam shown in Fig(a). The free body diagram of the entire beam is depicted in Fig. (b), which shows the external loads, as well as the reactions Ax and Ay, and By at supports A and B, respectively.

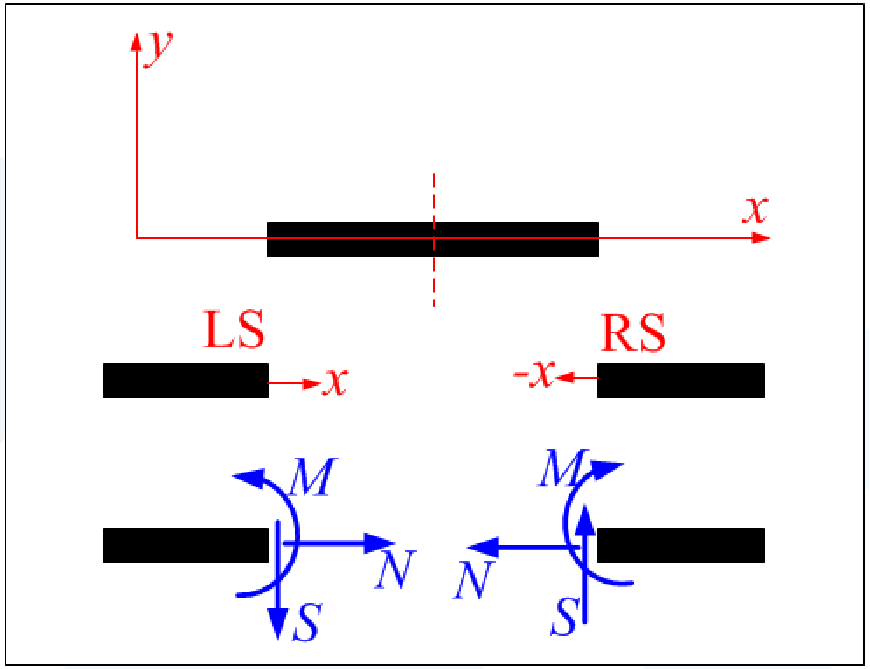
In order to determine the internal forces acting on the cross section of the beam at a point C, we pass an imaginary section cc through C, thereby cutting the beam into two parts, AC and CB, as shown in Figs.(c) & (d).

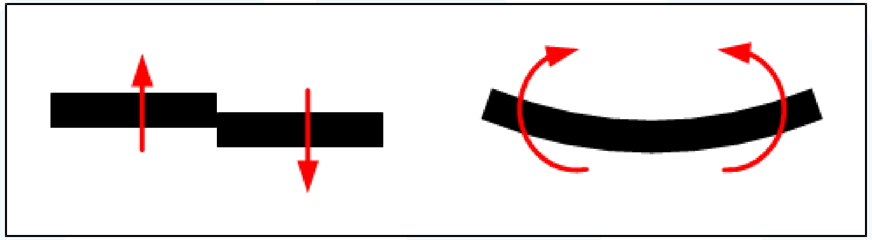
Internal Forces

Sign Convention: The internal axial force N is considered to be positive when the external forces acting on the member produce tension or have the tendency to pull the member apart at the section.

the shear force S is considered to be positive when the external forces tend to push the portion of the member on the left of the section upward with respect to the portion on the right of the section.

The bending moment M is considered to be positive when the external forces and couples tend to bend the beam concave upward, causing compression in the upper fibers and tension in the lower fibers of the beam at the section.





Internal Forces

Procedure of Analysis: The procedure for determining internal forces at a specified location on a beam can be summarized as follows:

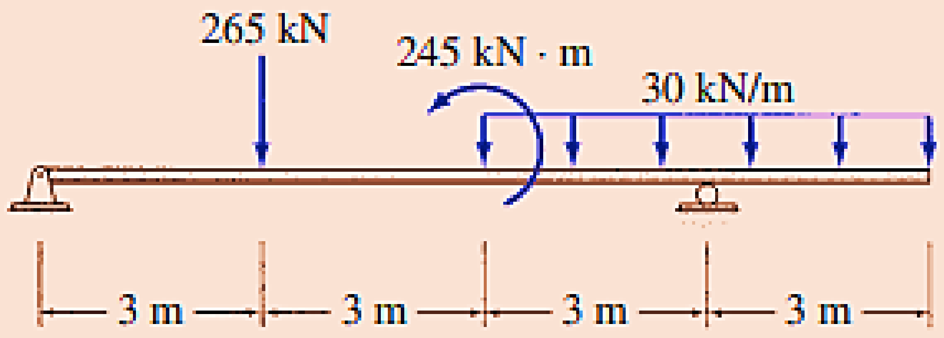
- Compute the support reactions, if needed.
- Make an imaginary cut of the beam into two portions at the point where the internal forces are desired.
- For computing internal forces, select the portion that will require the least amount of computational effort.
- Determine the axial force at the section by algebraically summing the components in the direction parallel to the axis of the beam of all the external loads and support reactions acting on the selected portion.
- Determine the shear at the section by algebraically summing the components in the direction perpendicular to the axis of the beam of all the external loads and reactions acting on the selected portion.
- . Determine the bending moment at the section by algebraically summing the moments about the section of all the external forces plus the moments of any external couples acting on the selected portion.
- check some values of the calculated internal forces.

Shear & Bending Moment Diagrams

Shear and bending moment diagrams depict the variations of these quantities along the length of the member. Such diagrams can be constructed by using the method of sections described in the preceding section. Proceeding from one end of the member to the other (usually from left to right), sections are passed, after each successive change in loading, along the length of the member to determine the equations expressing the shear and bending moment in terms of the distance of the section from a fixed origin. The values of shear and bending moments determined from these equations are then plotted as ordinates against the position with respect to a member end as abscissa to obtain the shear and bending moment diagrams. This procedure is illustrated by the following examples:

Shear & Bending Moment Diagrams

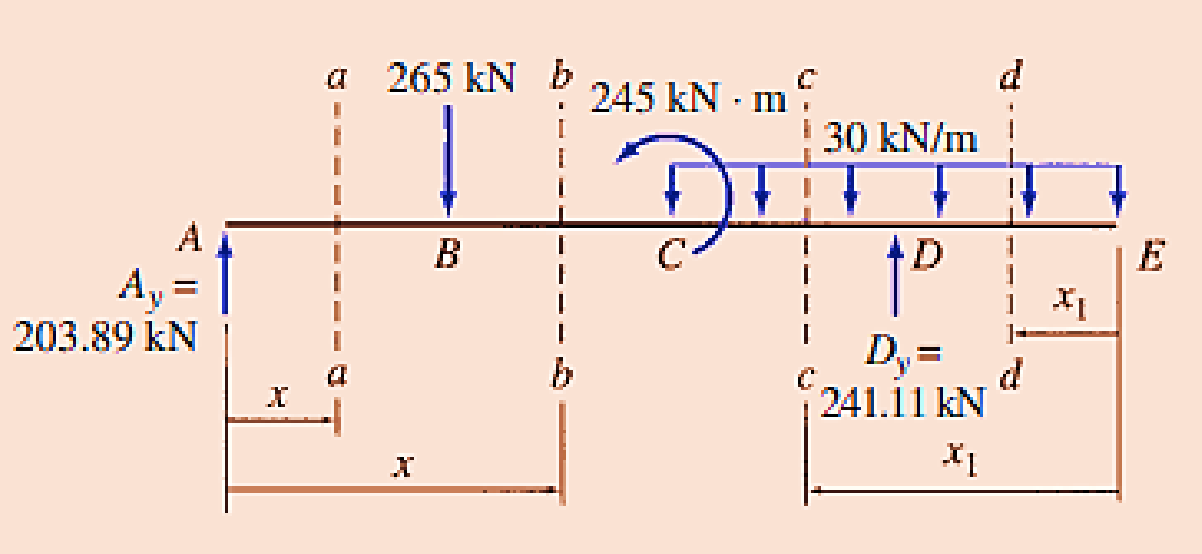
Example: Draw the shear and bending moment diagrams for the beam shown in (a).



(a)

Shear & Bending Moment Diagrams

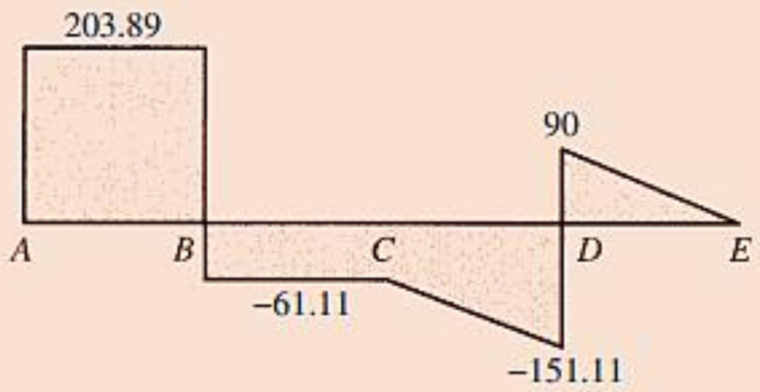
Example: Draw the shear and bending moment diagrams for the beam shown in (a).



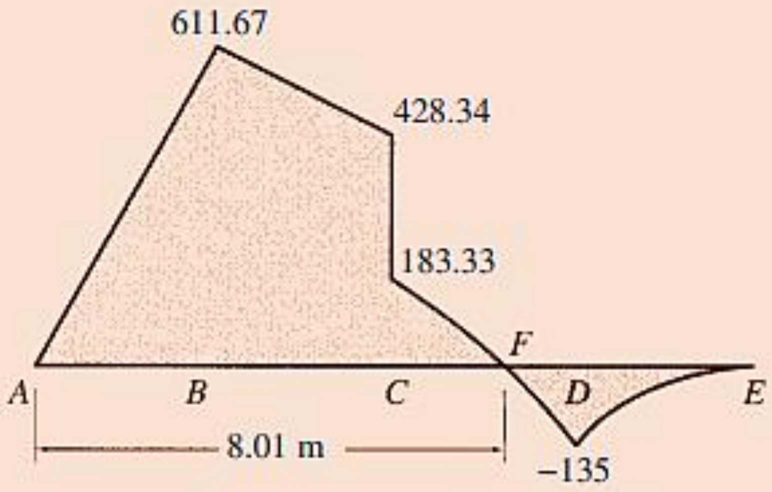
(b)

Shear & Bending Moment Diagrams

Example: Draw the shear and bending moment diagrams for the beam shown in (a).



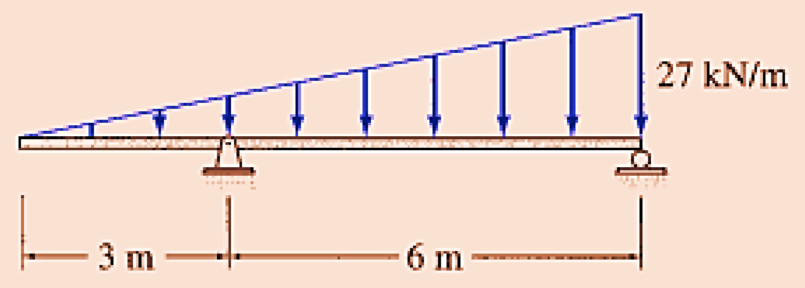
(c) Shear Diagram (kN)



(d) Bending Moment Diagram (kN · m)

Shear & Bending Moment Diagrams

Homework: Draw the shear and bending moment diagrams for the beam shown in (a).



(a)

Shear & Bending Moment Diagrams

Example: Draw the shear and bending moment diagrams for the beam shown in (a).

In Fig. (b): $w(x)= -3x$??, B_y=60.75 kN; C_y=60.75 kN.

In AB, $0\leq x \leq 3$,

$S(x)= -3x^2/2$; $S_A=0$, $S_{Bl} = -13.5$ kN

$M(x)= -x^3/2$; $M_A=0$, $M_{Bl} = -13.5$ kN.m

In BC, $3\leq x \leq 9$,

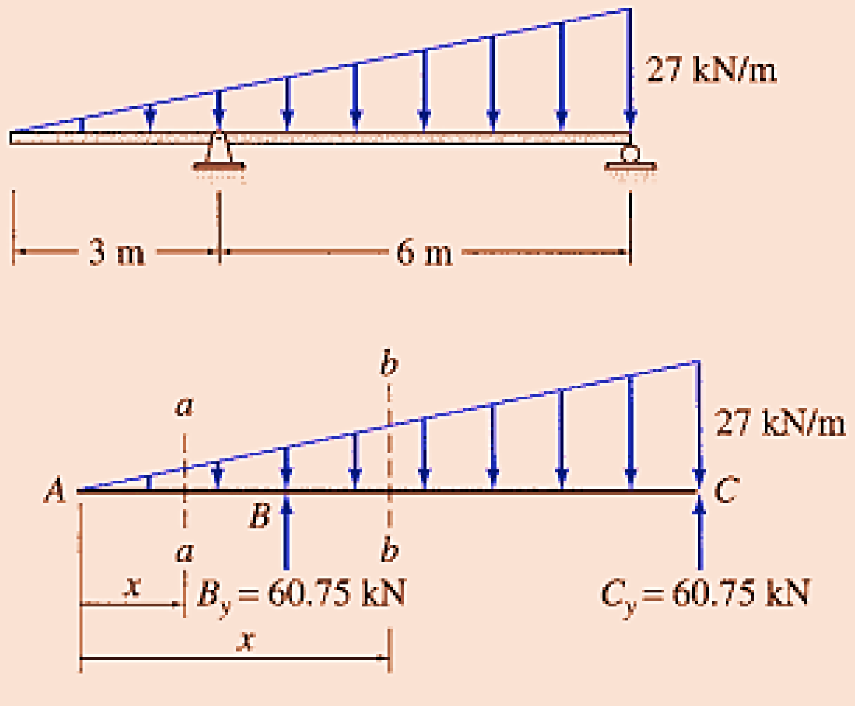
$S(x)= -3x^2/2+60.75$; S_{Br} =47.25 kN, $S_C = -60.75$ kN

The point D where $S(x)=0$, is at $x_D=(2\times 60.75/3)^{1/2}$=6.36m

$M(x)= -x^3/2+60.75(x-3)$; $M_{Br} = -13.5$ kN.m , $M_C = 0$

M_{max} occurs at x where $M'(x)= 0$, it coincides with D, S0

$M_{max}= -(6.36)^3/2+60.75(6.36-3)= 75.5$ kN.m



(b)

Shear & Bending Moment Diagrams

Example: **Draw the shear and bending moment diagrams for the beam shown in (a).**

In Fig. (b): $w(x)= -3x$??, B_y=60.75 kN; C_y=60.75 kN.

In AB, $0\leq x \leq 3$,

$S(x)= -3x^2/2$; $S_A=0$, $S_{Bl} = -13.5$ kN

$M(x)= -x^3/2$; $M_A=0$, $M_{Bl} = -13.5$ kN.m

In BC, $3\leq x \leq 9$,

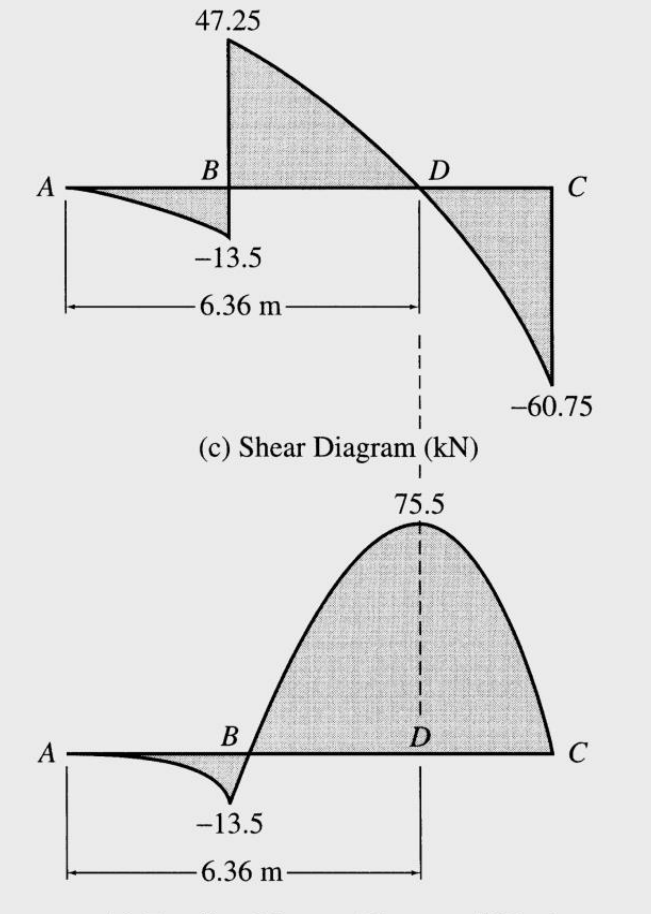
$S(x)= -3x^2/2+60.75$; S_{Br} =47.25 kN, $S_C = -60.75$ kN

The point D where $S(x)=0$, is at $x_D=(2\times 60.75/3)^{1/2}$=6.36m

$M(x)= -x^3/2+60.75(x-3)$; $M_{Br} = -13.5$ kN.m , $M_C = 0$

M_{max} occurs at x where $M'(x)= 0$, it coincides with D, S0

$M_{max}= -(6.36)^3/2+60.75(6.36-3)= 75.5$ kN.m

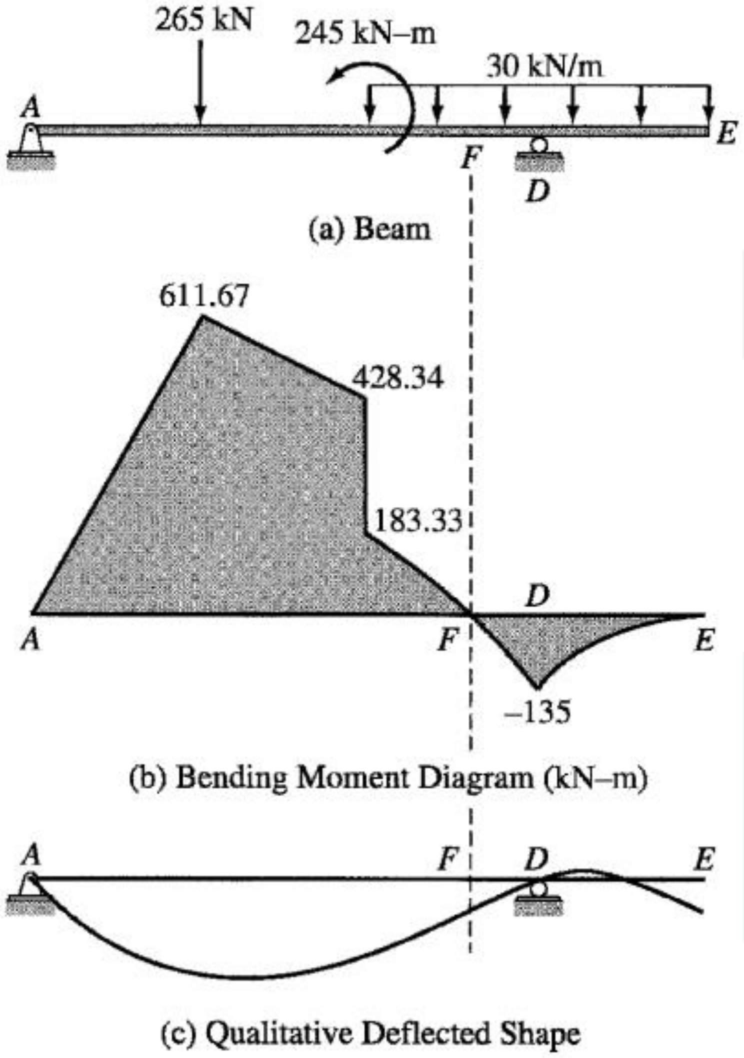


(c) Shear Diagram (kN)

(d) Bending Moment Diagram (kN–m)

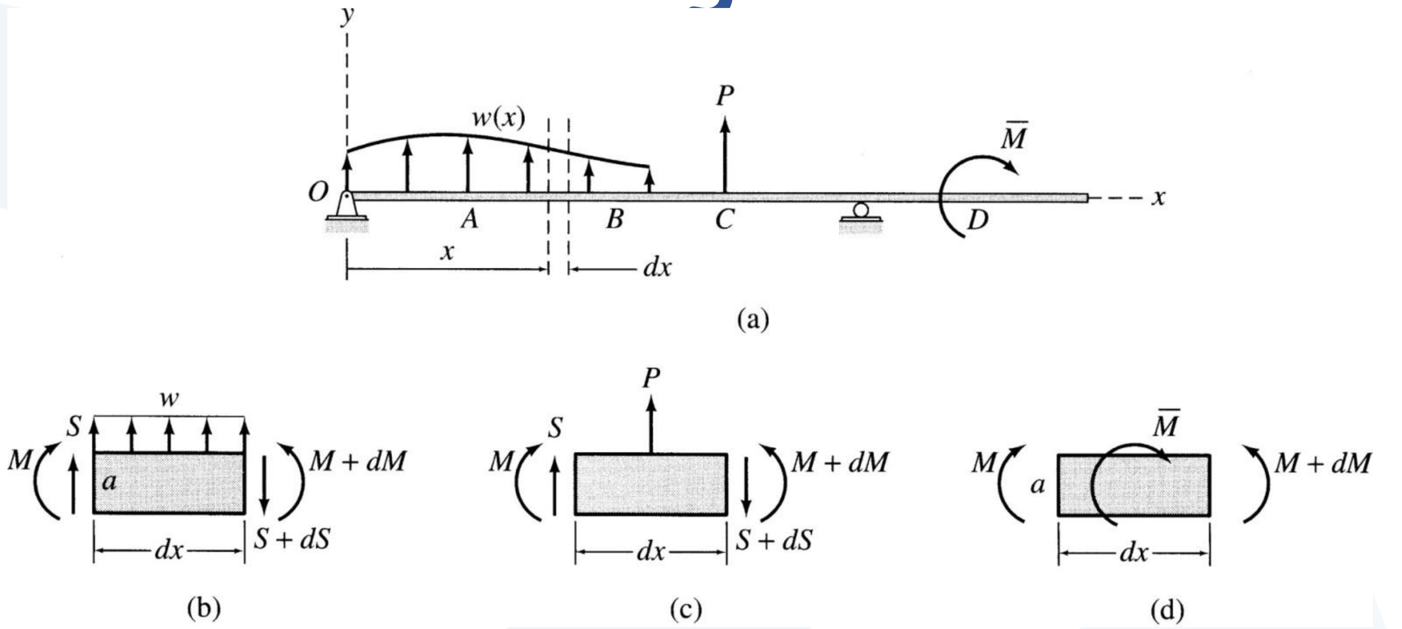
Qualitative Deflected Shape

A qualitative deflected shape of a structure is a simply exaggerated sketch of the neutral axis of the beam, in the deformed position, under the action of a given loading condition. Such sketches, which can be constructed without any knowledge of the numerical values of deflections, provide valuable insights into the behavior of beams and are often useful in computing the deflections.



According to the sign convention adopted in previous section, a (+) bending moment bends a beam concave upward, whereas a (-) bending moment bends a beam concave downward. Thus, the sign of the curvature at any point along the axis of a beam can be obtained from the bending moment diagram. Using the signs of curvatures, a qualitative deflected shape (elastic curve) of the beam, which is consistent with its support conditions, can be easily sketched.

Relationships between loads, shears &bending moments



$$\uparrow \sum F_y = 0 \Rightarrow S + w\,dx - (S + dS) = 0 \Rightarrow dS - w\,dx = 0 \Rightarrow \boxed{\frac{dS}{dx} = w(x)}$$

$$\downarrow\uparrow \sum M_{Z,a} = 0 \Rightarrow -M + w\,(dx)\left(\frac{dx}{2}\right) - (S + dS)\,dx + (M + dM) = 0 \Rightarrow \boxed{\frac{dM}{dx} = S}$$

Concentrated Loads: $dS = P$

Couples or Concentrated Moments: $dM = \bar{M}$

Statically Indeterminate Structures

- A structure is Statically Indeterminate (SI) if you cannot calculate all reactions and internal forces just using the equations of statics, i.e. equations of equilibrium.
- This means that you have more unknowns than you have equations of equilibrium.

Number of unknowns > Number of equilibrium equations

- You solved a few SI problems in your Mechanics class.
- But not many!
- In this class we solve SI problems!