# Lecture Outline

- Concurrent 3-Dimensional Force Systems.
- Expressing 3D Force as Cartesian Vector.
- Position Vectors and Relative Position Vectors.
- Examples [(F.B.D), (Eq. Eqs.), (Geometry)]

- القوى المتلاقية في الفضاء ثلاثي الأبعاد.
- العبارة التحليلية الديكارتية للقوة في الفضاء ثلاثي الأبعاد.
- أشعة الموضع وأشعة الموضع النسبية.
- أمثلة: مخطط الجسم الحر، معادلات التوازن، القوانين المادية، الجيومتري.

Last week Outline

- Condition for the Equilibrium of a Particle. $\sum F = 0$
- Concurrent Coplanar Systems. $\sum F_x = 0$ & $\sum F_y = 0$
- The Free-Body Diagram.
- Examples & Exercises.

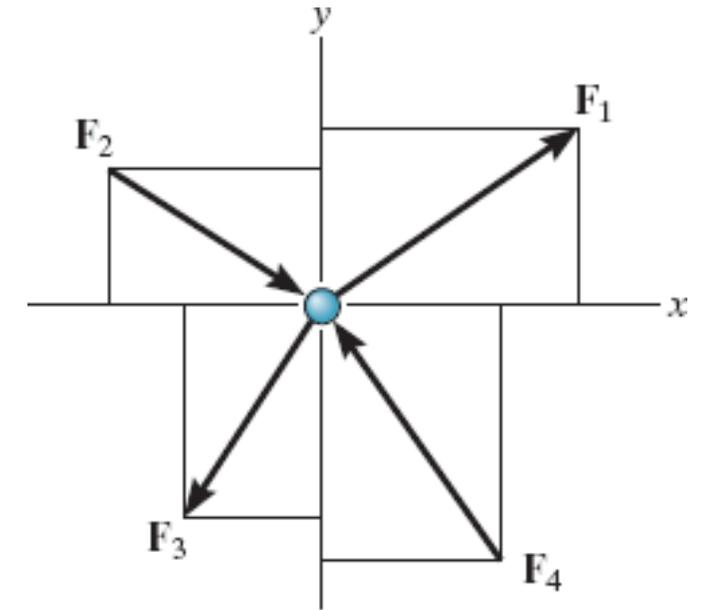Engineering Mechanics (Statics)

Geometry

Instrumenet Laws

Eq. Equations
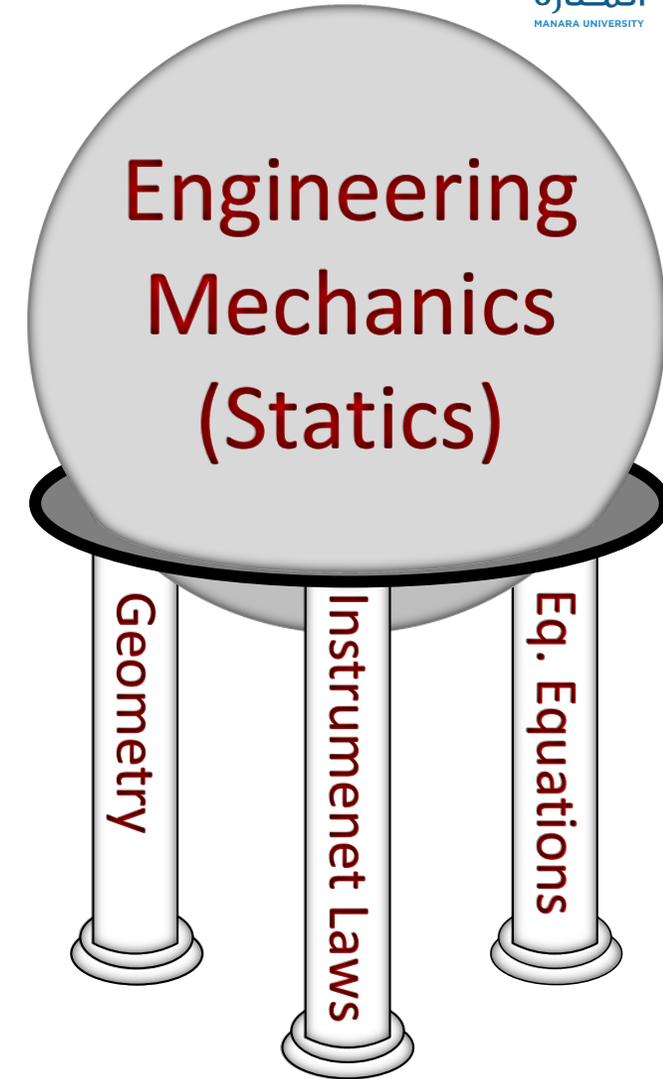
1. Concurrent *Three-Dimensional* Force Systems.
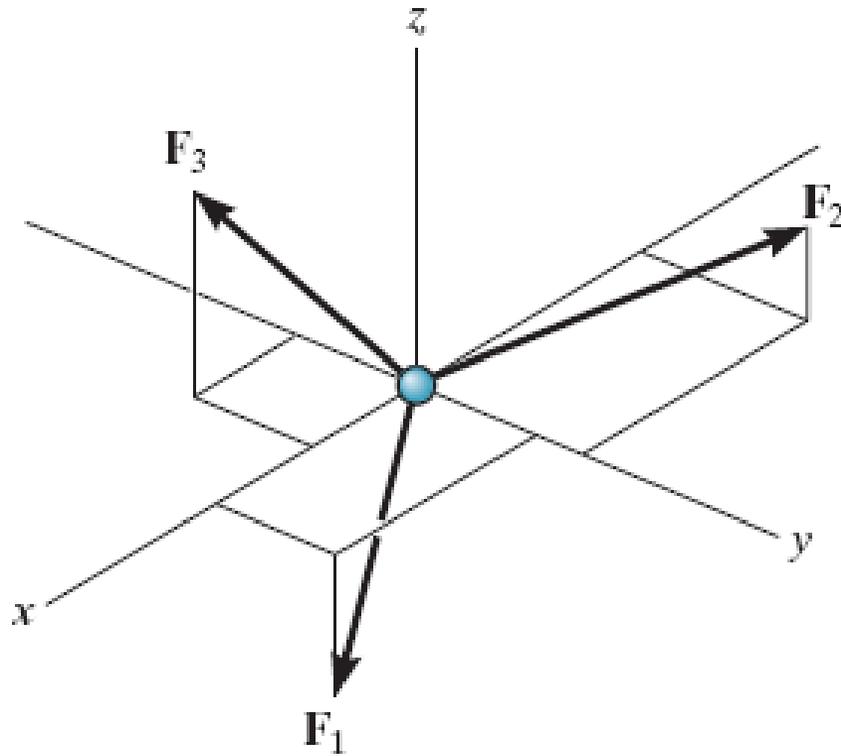
The vector equilibrium condition is: $\sum \boldsymbol{F} = 0$

Resolve into *x* , *y* and *z* components for equilibrium

$$\sum \boldsymbol{F} = 0 \Leftrightarrow \left[\sum F_x = 0 \ , \ \sum F_y = 0 \ \& \ \boxed{\sum F_z = 0} \right]$$

Concurrent Coplanar Systems

$$\sum \boldsymbol{F} = 0 \Leftrightarrow \sum F_x = 0 \ \& \ \sum F_y = 0$$

Expressing 3D Force as Cartesian Vector

Direction of the Force Vector **F** is defined by the coordinate direction angles α, β & γ
Measured between the tail of **F** and the positive *x, y* and *z* axes. $0° \leq \alpha, \beta$ & $\gamma \leq 180°$.

From the figure, $\boldsymbol{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_Z\mathbf{k},$

The *magnitude* of **F** is: $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$

And its *direction cosines* are: $\cos\alpha = \dfrac{F_x}{F}, \quad \cos\beta = \dfrac{F_y}{F}, \quad \cos\gamma = \dfrac{F_z}{F}.$

The components in terms of *direction cosines & magnitude,* are:
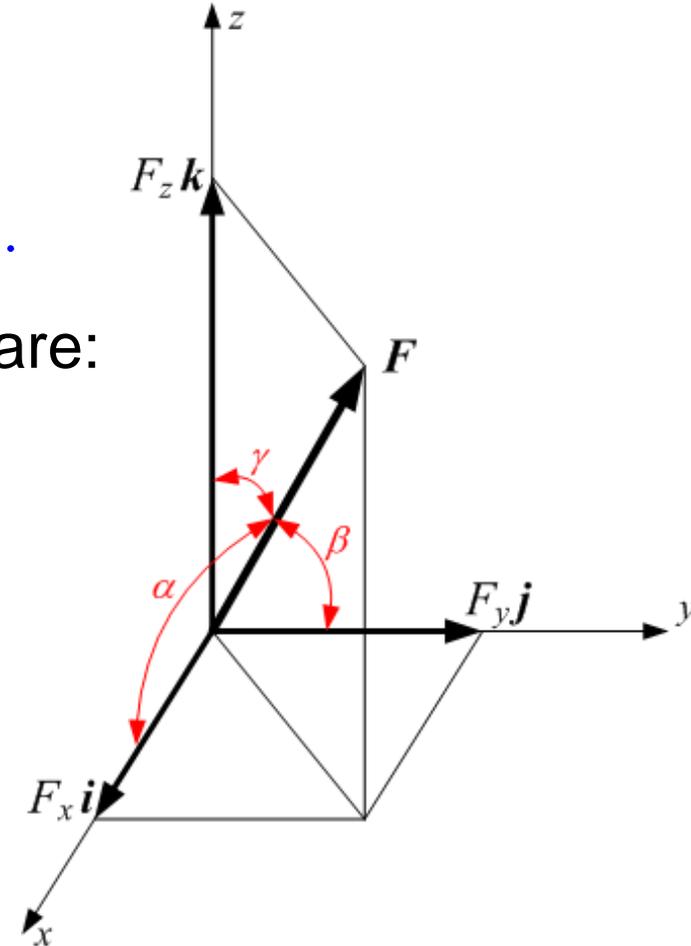
$$F_X = F\cos\alpha, \qquad F_y = F\cos\beta, \qquad F_z = F\cos\gamma.$$

then, $\boldsymbol{F} = F(\cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}) = F\,\boldsymbol{u_F}$

Where $\boldsymbol{u_F}$ is a unity vector given by:

$\boldsymbol{u_F} = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}.$

then, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Example

Express the shown force **F** as Cartesian vector.

Solution:

Since two angles are specified, the third angle is found by: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, so
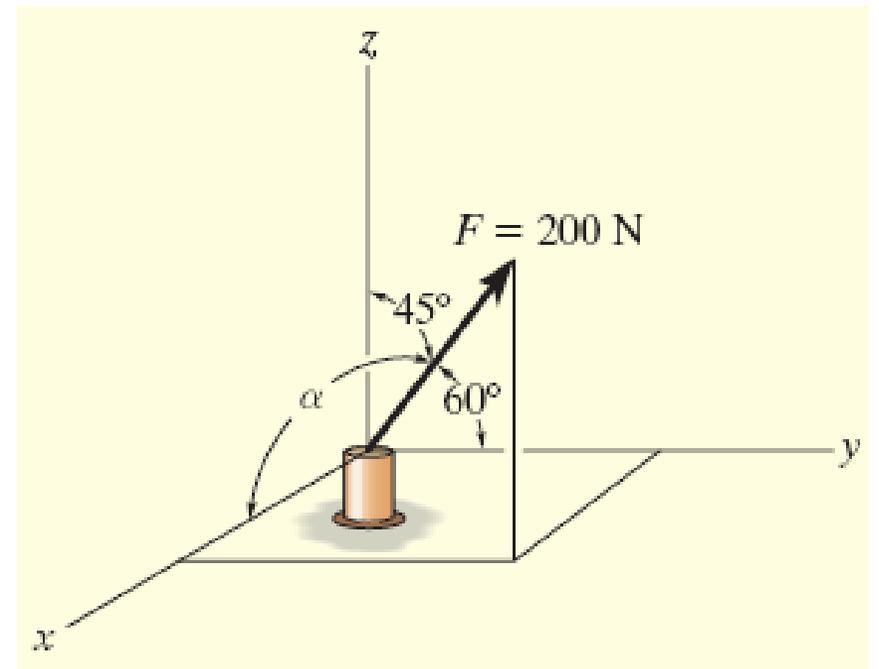
$\cos^2\alpha + \cos^2 60° + \cos^2 45° = 1$

then, $\cos\alpha = (1 - \cos^2 60° - \cos^2 45°)^{1/2} = \pm 0.5$

Two possibilities exit , namely: (1) $\alpha = \cos^{-1}(0.5) = 60°$, (2) $\alpha = \cos^{-1}(-0.5) = 120°$,

But by visual inspection, $\alpha = 60^0$ since F_x is in the +x direction

Given $F = 200$N,

then $\boldsymbol{F} = F\cos\alpha\ \boldsymbol{i} + F\cos\beta\ \boldsymbol{j} + F\cos\gamma\ \boldsymbol{k} = 200[N](\cos60^0\boldsymbol{i} + \cos60^0\boldsymbol{j} + \cos45^0\boldsymbol{k})$

$= \{100\ [N]\ \boldsymbol{i} + 100\ [N]\ \boldsymbol{j} + 141\ [N]\ \boldsymbol{k}\}$

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Position Vector and Relative Position Vector

We introduce the position vector and the relative position vector because they are used in cartesian formulation of any force vector directed between two points in space.

Position Vector

Position vector **r** is defined as a fixed vector which locates a point in space relative to the coordinates origin

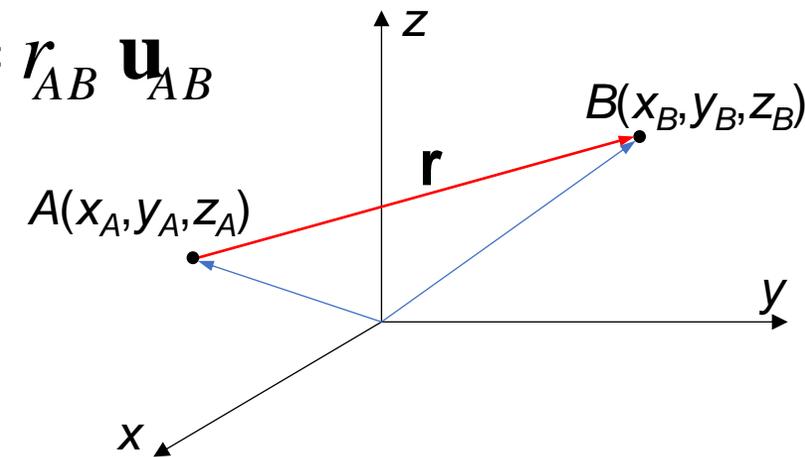$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$
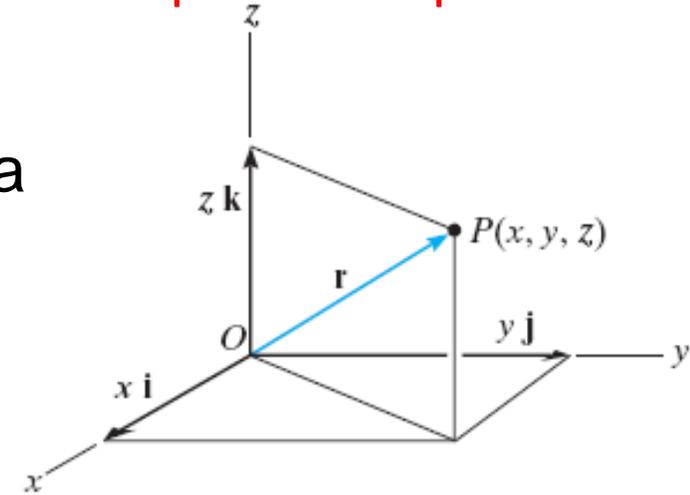
Relative Position Vector from *A* to *B*

$$\mathbf{r} = \mathbf{r}_{AB} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k} = r_{AB}\,\mathbf{u}_{AB}$$

$$r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

$$\mathbf{u}_{AB} = \left(\frac{x_B - x_A}{r_{AB}}\right)\mathbf{i} + \left(\frac{y_B - y_A}{r_{AB}}\right)\mathbf{j} + \left(\frac{z_B - z_A}{r_{AB}}\right)\mathbf{k}$$

Example 1.

Determine the force developed in each cable used to support the crate.

Solution:

$$r_{AB} = \sqrt{(3-0)^2 + (-4+0)^2 + (8-0)^2} = 9.434\,\text{m}$$

$$\mathbf{u}_{AB} = \frac{-3}{9.434}\mathbf{i} + \frac{-4}{9.434}\mathbf{j} + \frac{+8}{9.434}\mathbf{k} = -0.318\mathbf{i} - 0.424\mathbf{j} + 0.848\mathbf{k}$$

$$r_{AC} = \sqrt{(3-0)^2 + (4+0)^2 + (8-0)^2} = 9.434\,\text{m}$$

$$\mathbf{u}_{AC} = \frac{-3}{9.434}\mathbf{i} + \frac{+4}{9.434}\mathbf{j} + \frac{+8}{9.434}\mathbf{k} = -0.318\mathbf{i} + 0.424\mathbf{j} + 0.848\mathbf{k}$$
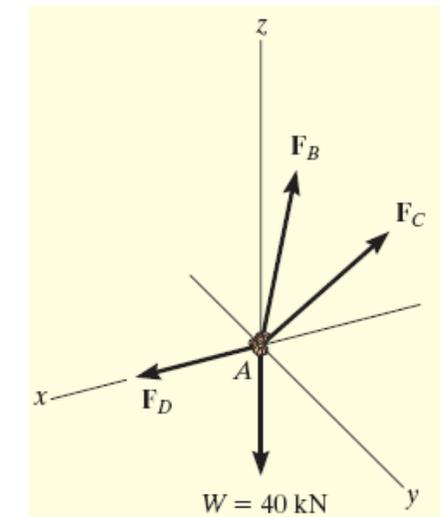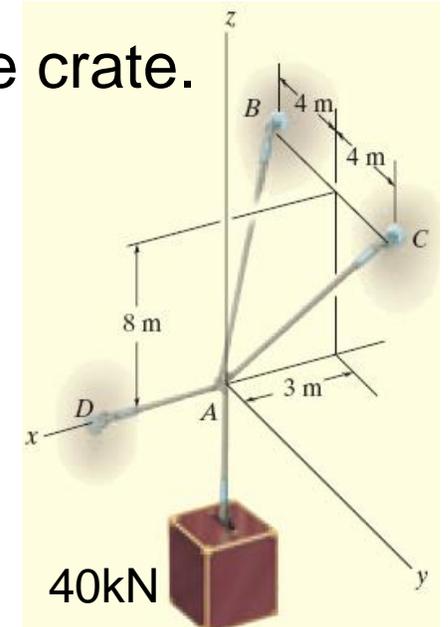
$$\mathbf{u}_{AD} = \mathbf{i}$$

Expressing the four forces in Cartesian vectors,

$$\boldsymbol{F}_B = F_B\,\boldsymbol{u}_{AB} = -0.318F_B\,\mathbf{i} - 0.424F_B\,\mathbf{j} + 0.848F_B\,\mathbf{k}$$

$$\boldsymbol{F}_C = F_C\,\boldsymbol{u}_{AC} = -0.318F_C\,\mathbf{i} + 0.424F_C\,\mathbf{j} + 0.848F_C\,\mathbf{k}$$

$$\boldsymbol{F}_D = F_D\,\mathbf{i}$$

$$\boldsymbol{W} = -40\mathbf{k}$$

40kN

FBD at Point A

$F_B = F_B\, u_{AB} = -0.318F_B\, \mathbf{i} - 0.424F_B\, \mathbf{j} + 0.848F_B\, \mathbf{k}$

$F_C = F_C\, u_{AC} = -0.318F_C\, \mathbf{i} + 0.424F_C\, \mathbf{j} + 0.848F_C\, \mathbf{k}$

$F_D = F_D\, \mathbf{i}$

$W = -40\mathbf{k}$

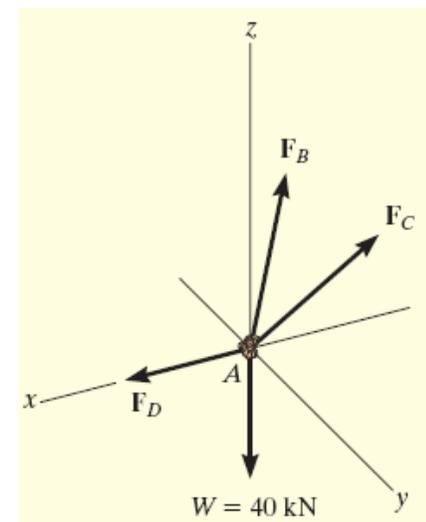For equilibrium,

$\sum F = 0$; $F_B + F_C + F_D + W = 0$

Eq. Eqs.:

$\sum F_x = 0$: $-0.318F_B - 0.318F_C + F_D = 0$

$\sum F_y = 0$: $-0.424F_B + 0.424F_C \qquad = 0$

$\sum F_z = 0$; $0.848F_B + 0.848F_C \qquad = 40$

Solving, $F_B = F_C = 23.6\text{kN}$ $F_D = 15.0\text{kN}$

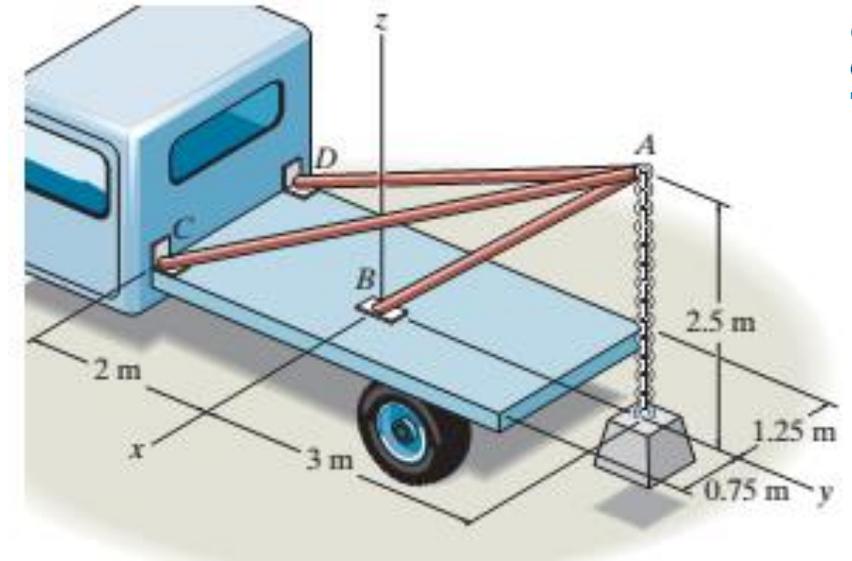$\mathbf{u}_{AB} = -0.318\mathbf{i} - 0.424\mathbf{j} + 0.848\mathbf{k}$

$\mathbf{u}_{AC} = -0.318\mathbf{i} + 0.424\mathbf{j} + 0.848\mathbf{k}$

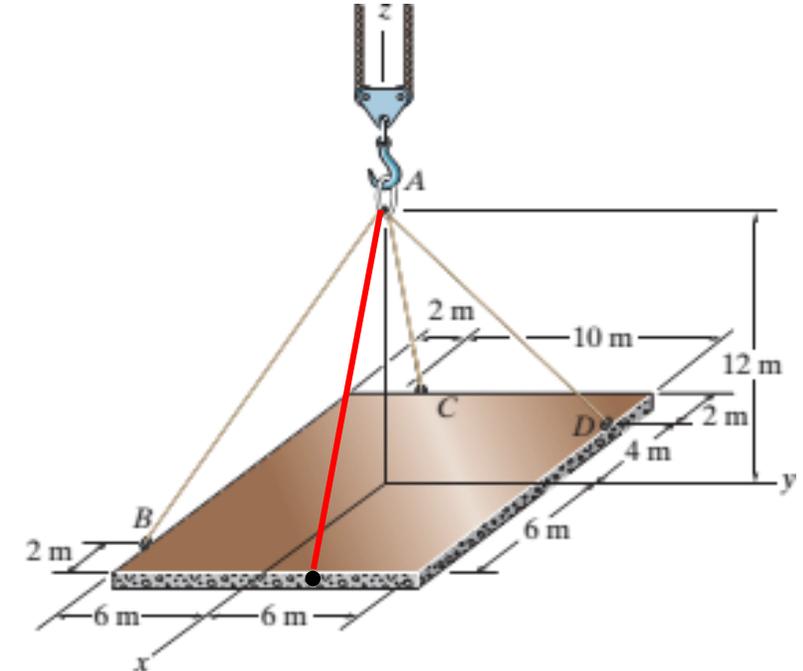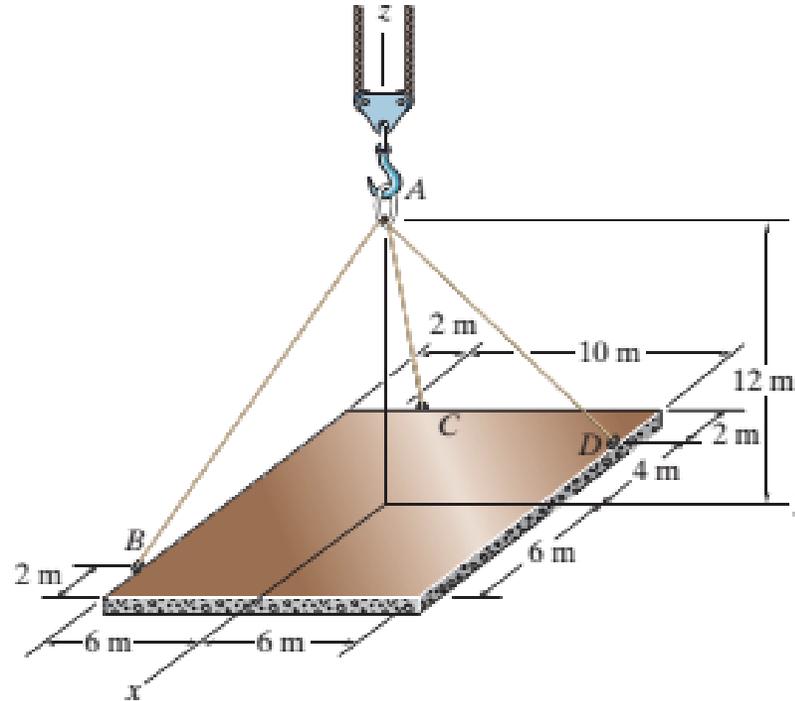$\mathbf{u}_{AD} = \mathbf{i}$

$\mathbf{u}_W = -\mathbf{k}$

Example 2.

Determine the force acting along the axis of each of the three struts needed to support the 500-kg block

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Example 3.

The ends of the three cables are attached to a ring at A and to the edge of the uniform plate. Determine the largest mass the plate can have if each cable can support a maximum tension of 15 kN.

Example 4. The joint of a space frame is subjected to four member forces. Member OA lies in the x-y plane and member OB lies in the y-z plane. Determine the forces acting in each of the members required for equilibrium of the joint.

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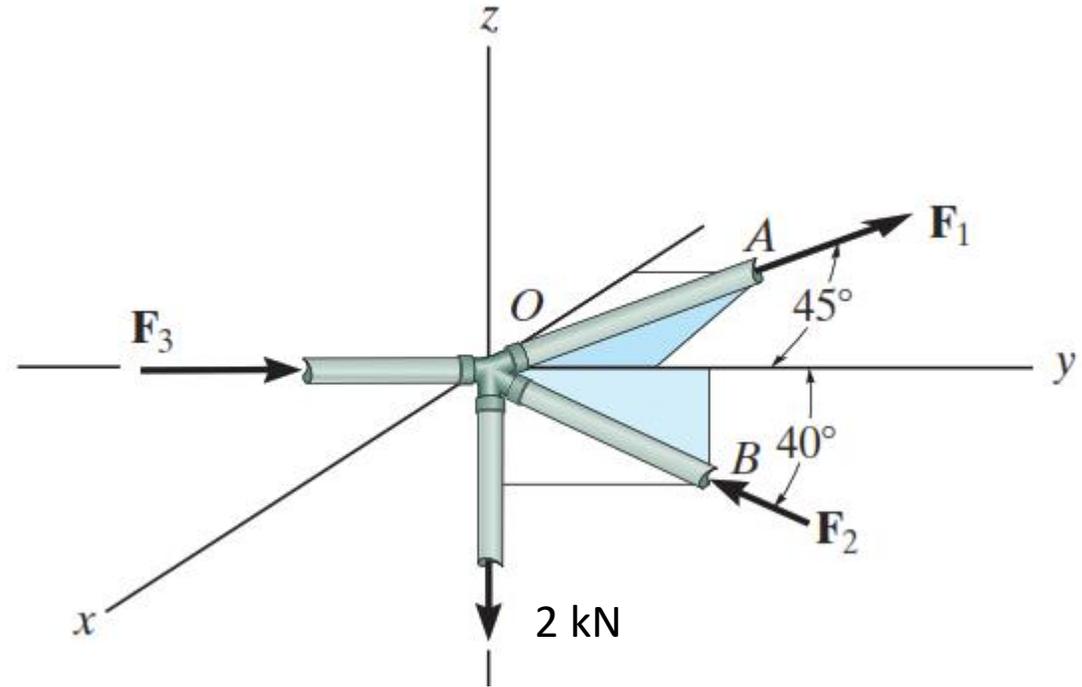
Example 5.

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of k = 300 N/ m.
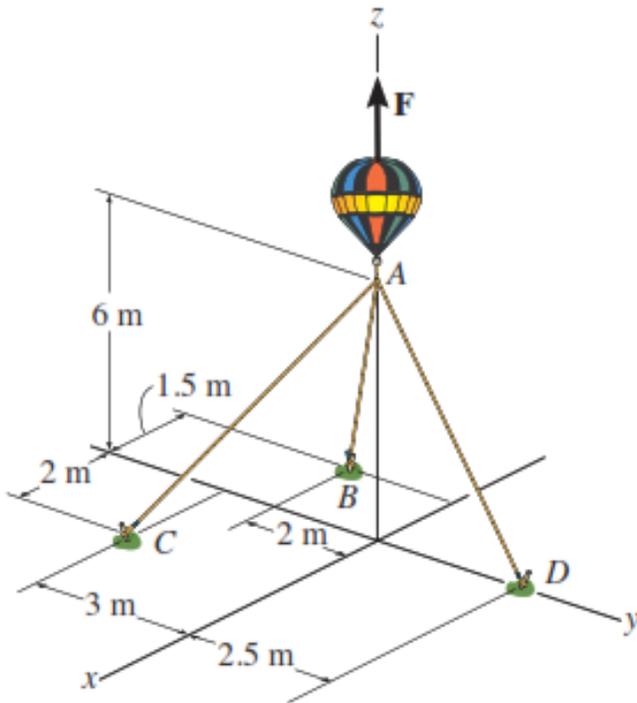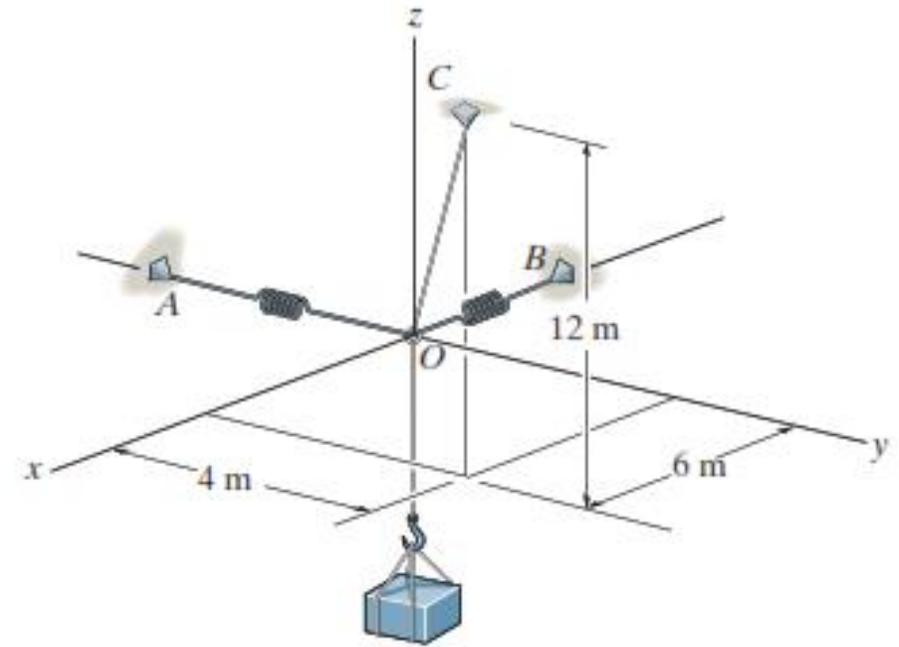
Example 6.

(1) If the balloon is subjected to a net uplift force of F = 800 N, determine the tension developed in ropes AB , AC , AD .
(2) If each one of the ropes will break when it is subjected to a tensile force of 450 N, determine the maximum uplift force F the balloon can have before one of the ropes breaks.

Example 7.(1) The lamp has a mass of 15 kg and is supported by a pole AO and cables AB and AC. If the force in the pole acts along its axis, determine the forces in AO, AB, and AC for equilibrium.

(2) If Cables AB and AC can sustain a maximum tension of 500 N, and the pole can support a maximum compression of 300 N. Determine the maximum weight of the lamp that can be supported in the position shown.