Structural Mechanics (1)

Lecture No-02

Deflection in Determinate Structures

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Deflections of Trusses, Beams, & Frames: Work-Energy Methods

- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

Introduction

In this section we are going to introduce the use of conservation of energy for the calculation of displacements. The methods you are familiar with are called “geometric methods”. Energy methods are more powerful and more general.

- So far, the primary methods you used for calculating displacements have been “geometric methods” - The beam differential equation and the moment-area principles are examples
- We now need to turn our attention toward more powerful methods of calculating displacements
- Methods that are based upon **conservation of energy** are among the most powerful

Introduction

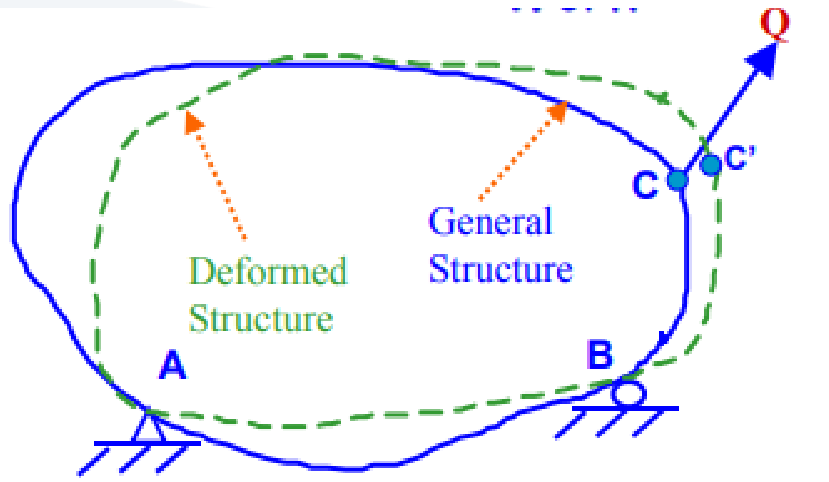
Recall: Definition: If a load corresponds to a displacement this means they are at the same point and have the same line of action.

- A translation corresponds to a concentrated force and a rotation corresponds to a concentrated moment.
- Recall also that a change in slope, a kink, corresponds to an internal moment and a step corresponds to an internal shear force.

Another way of looking at the concept of corresponding force and displacement is from the point of view of work.

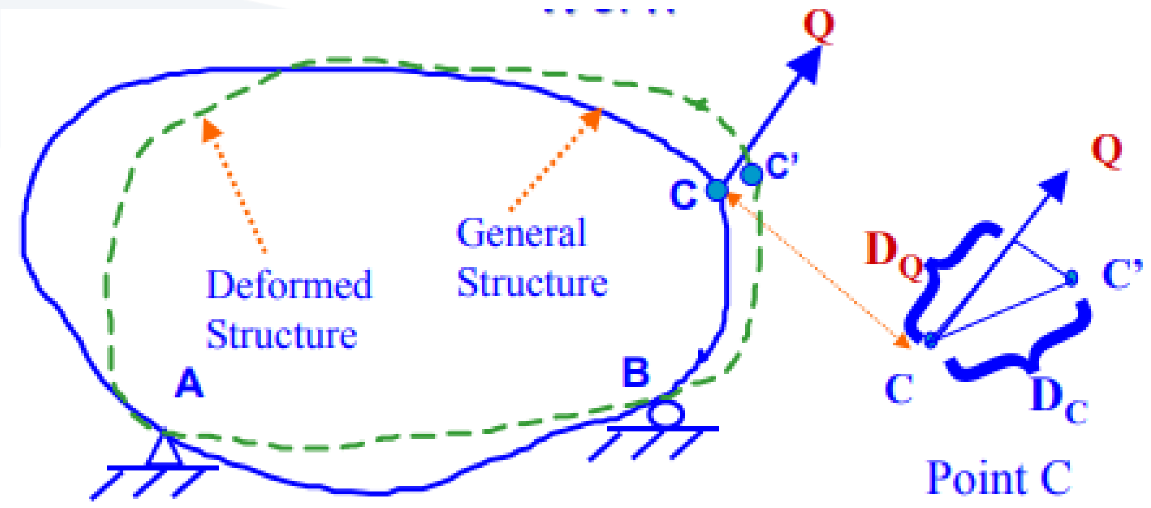
- We recall from physics that work is force times the distance through which that force moves
- The force moves through its corresponding displacement.
- Let's examine this idea by looking at the following problem.

Work



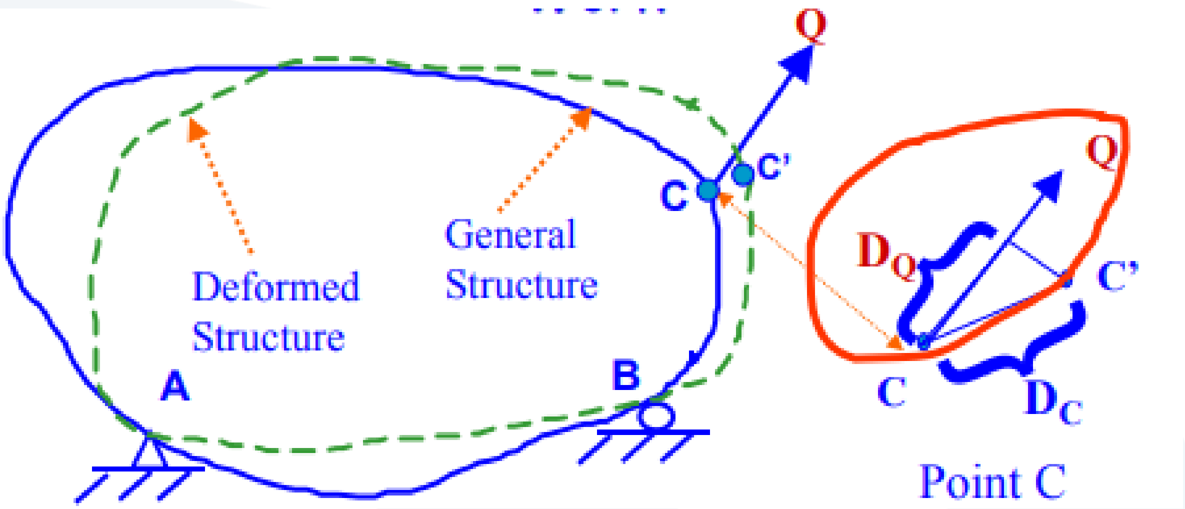
- We start off with a general structure with a load Q applied at point C
- The structure deforms and point C goes to C′
- Let′s look at point C:

Work



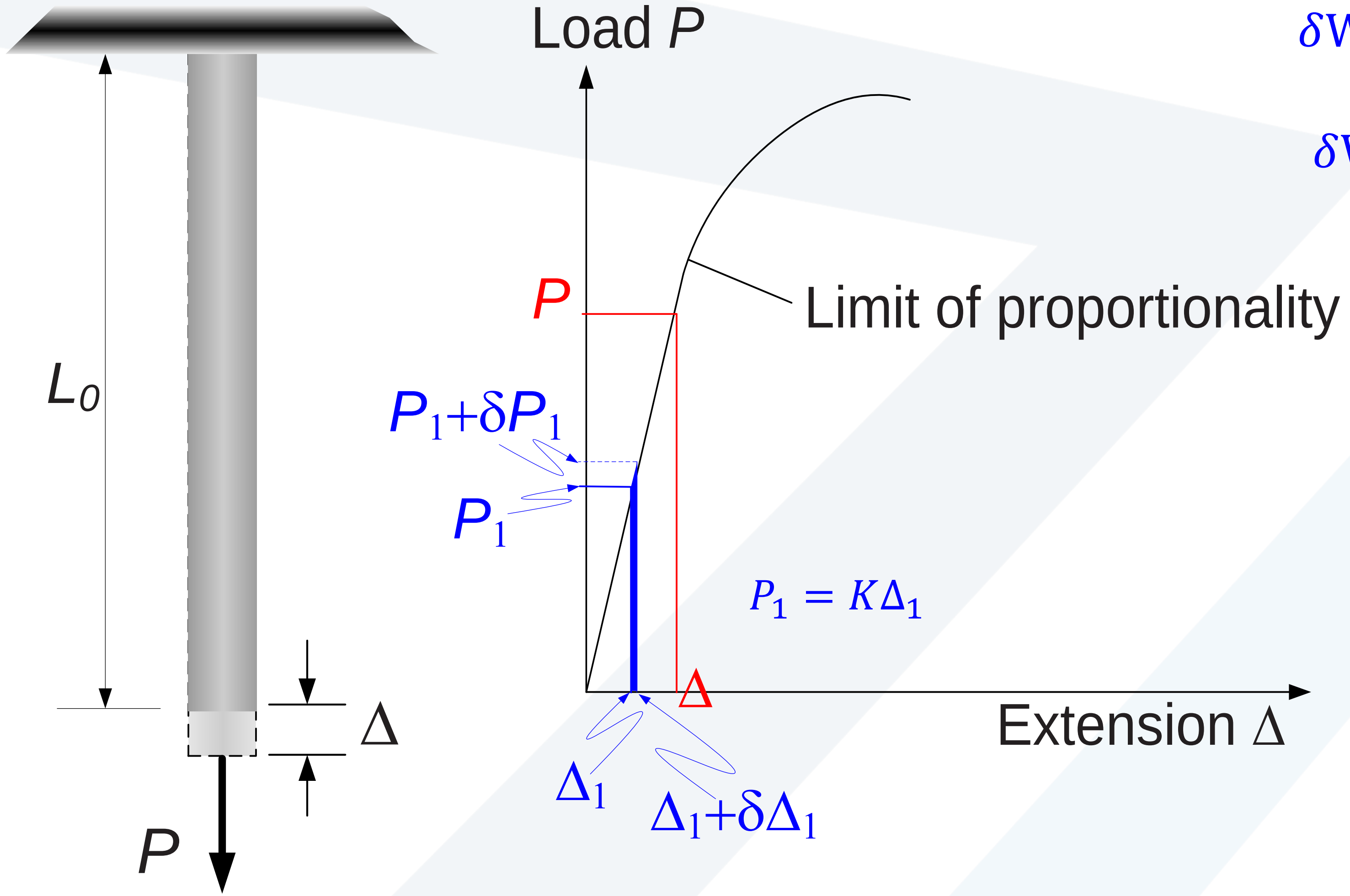
- D_C is the total displacement at C and D_Q is the component of that displacement along the line of the load Q

Work



- D_Q is the displacement that corresponds to the load Q
- Remember, if they correspond, the load and the displacement are at the same point and have the same line of action

External work & Strain Energy in axially Loaded members



$$\delta W_e = \{[P_1 + (P_1 + \delta P_1)]/2\}\delta\Delta_1$$

$$\delta W_e = P_1\delta\Delta_1 + \cdots \approx P_1\delta\Delta_1$$

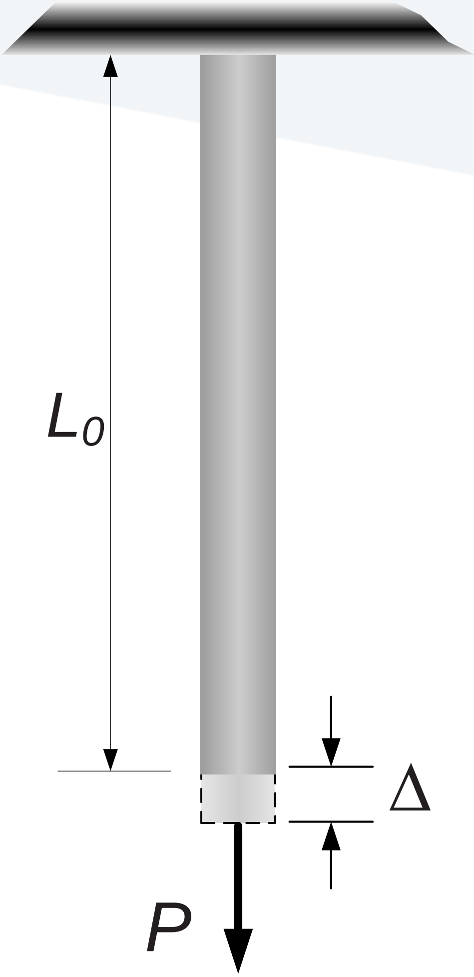
$$P_1 = K\Delta_1$$

$$W_e = \int_0^\Delta \delta W_e = \int_0^\Delta P_1\delta\Delta_1$$

$$W_e = \int_0^\Delta K\Delta_1\delta\Delta_1 = \frac{1}{2}K\Delta^2$$

$$W_e = \frac{1}{2}K\Delta^2 = \frac{1}{2}K\Delta\Delta = \frac{1}{2}P\Delta$$

External work & Strain Energy in axially Loaded members



In this case *N*, the axial internal force, is given from the Eq. Eq. as $N = P$

The strain energy is:

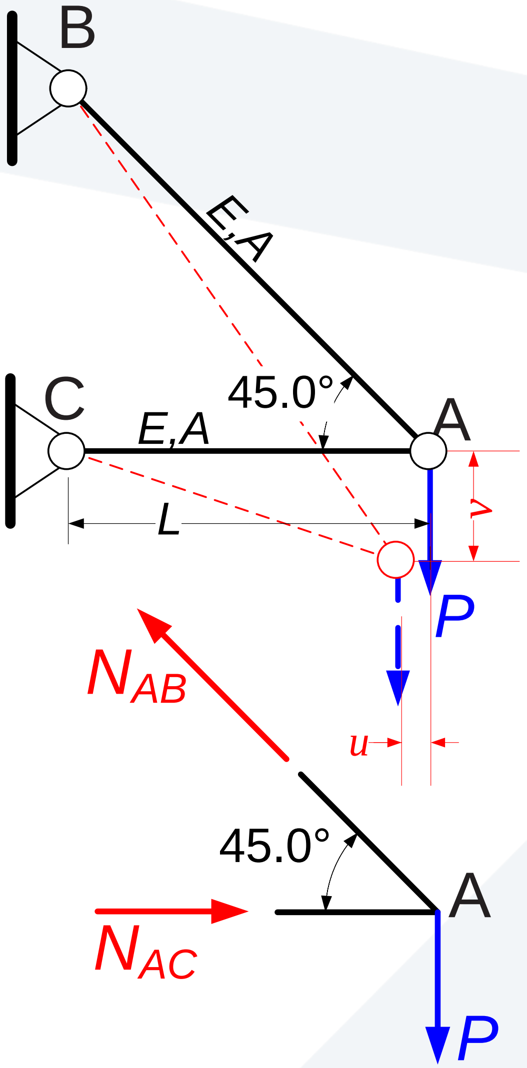
$$U = \iiint_V \frac{1}{2}\sigma_x \varepsilon_x dV = \frac{1}{2}\sigma\varepsilon A L_0 = \frac{1}{2}E\varepsilon^2 A L_0 = \frac{\sigma^2}{2E} A L_0 = \frac{N^2 L_0}{2EA}$$

Low of Conservation of Energy: $W_e = U = \frac{1}{2}P\Delta$

$$U = \frac{1}{2}P\Delta = \frac{1}{2}\sigma\varepsilon A L_0 = \frac{1}{2}E\varepsilon^2 A L_0 = \frac{\sigma^2}{2E} A L_0 = \frac{N^2 L_0}{2EA}$$

$$\frac{1}{2}P\Delta = \frac{N^2 L_0}{2EA}$$

DEFLECTION OF A SIMPLE TRUSS by We=U



Considering the vertical equil. at A:

$N_{AB} \cos 45^{o} - P = 0 \Rightarrow N_{AB} = 1.41P$ (T)

Considering the horizontal equil. at A:

$-N_{AB} \cos 45^{o} + N_{AC} = 0 \Rightarrow N_{AC} = P$ (C)

The strain energy in each member is

$U_{AB} = (1.41P)^2 \times 1.41L/2EA = 1.41P^2L/EA$

$U_{AC} = (-P)^2 \times L/2EA = P^2L/2EA$

The external work- strain energy principle

$W_e = U = U_{AB} + U_{AC}$

$(1/2)Pv = U_{AB} + U_{AC} = 3.82P^2L/2EA$

$v = 3.82PL/EA$ $\qquad$ $u = ?$

Ex01. Determine the horizontal deflection at A in the truss shown next. The cross-sectional area of the tension members is 80mm^2 while that of the compression members is150mm^2. E= 200 000N/mm^2.

From the FBD of joint A

N_{AC} = 15/sinα= 39kN (C) &
N_{AB} = 39 cosα = 36kN (T)

From the FBD of joint B

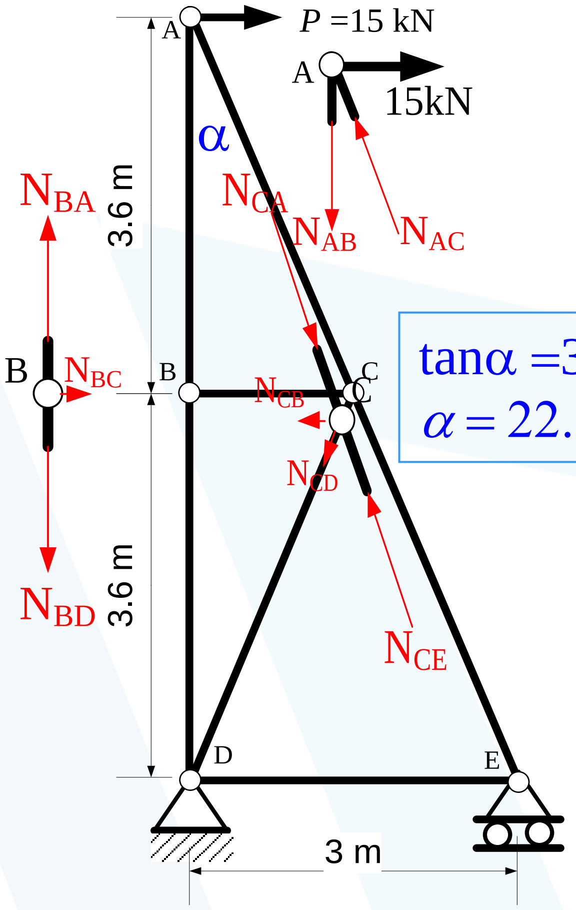
$N_{BA} = N_{BD}$ = 36kN (T) & $N_{BC} = 0$

From the FBD of joint C

$N_{CA} = N_{CE}$ = 39kN (C) &$N_{CB} = N_{CD} = 0$

$$(½)Pu_A = \sum (N^2_i L_i / 2E_iA_i) \Rightarrow u_A \cong 68 \text{ mm}$$

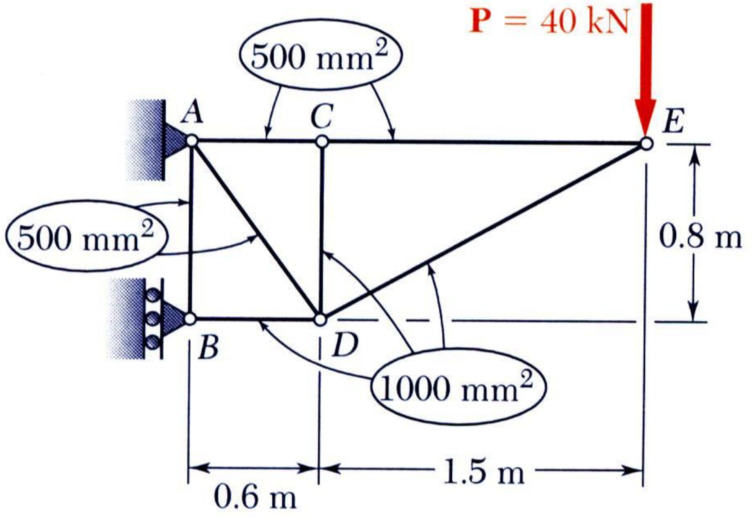
But, how to compute the u_B?



tanα =3/7.2
$\alpha = 22.62$

Example 02:

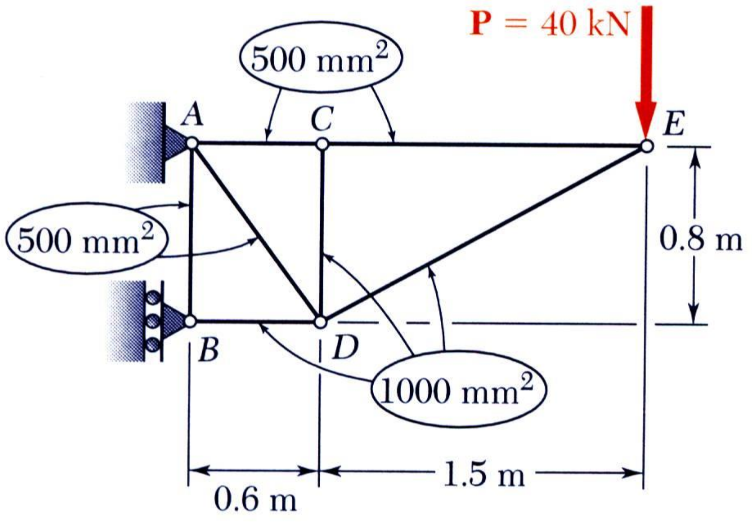
Members of the truss shown consist of sections of aluminum pipe with the cross-sectional areas indicated.
Using *E* = 73 GPa, determine the vertical deflection of the point *E* caused by the load P.

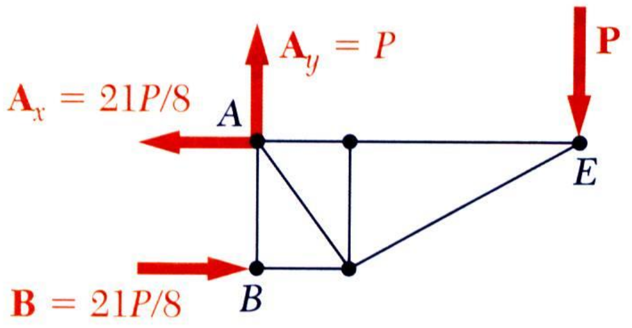


Solution

Members of the truss shown consist of sections of aluminum pipe with the cross-sectional areas indicated.

Using E = 73 GPa, determine the vertical deflection of the point E caused by the load P.

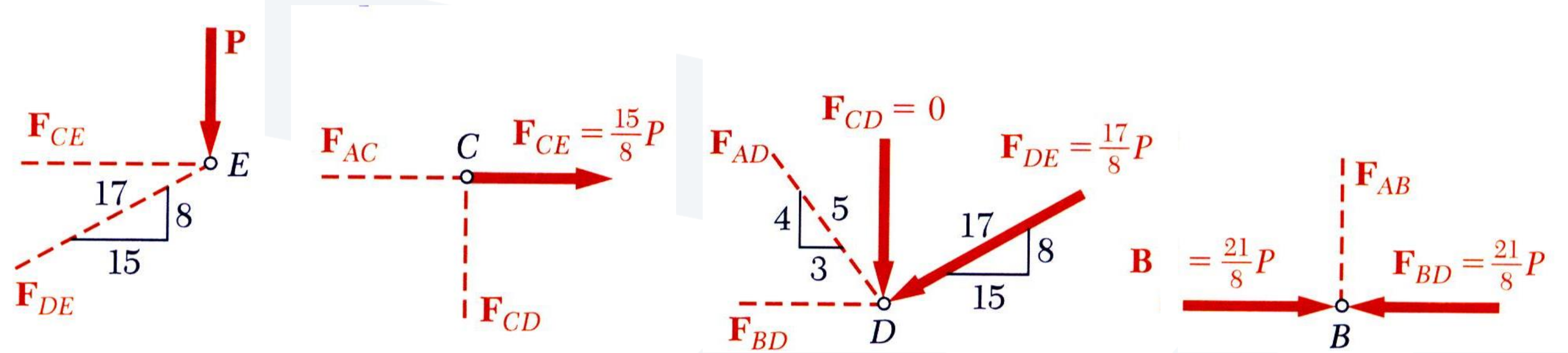




- Find the reactions at A and B from a free-body diagram of the entire truss.

$$A_x = -21P/8 \qquad A_y = P \qquad B_x = 21P/8$$

- Apply the method of joints to determine the axial force in each member.



$$F_{DE} = -\frac{17}{8}P \qquad F_{AC} = +\frac{15}{8}P \qquad F_{DA} = \frac{5}{4}P \qquad F_{AB} = 0$$

$$F_{CE} = +\frac{15}{8}P \qquad F_{CD} = 0 \qquad F_{DB} = -\frac{21}{8}P$$

$$(½)P\, v_E = \sum (N^2_i L_i / 2E_i A_i) \Rightarrow v_E \cong 16.27 \text{ mm} \downarrow$$