Structural Mechanics (2)

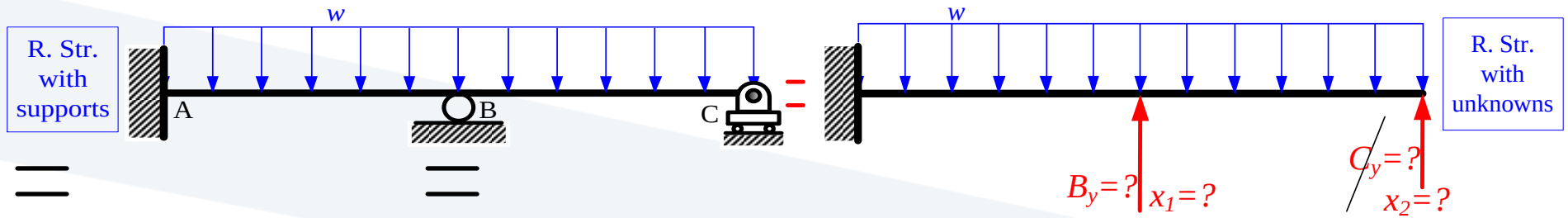
Lecture No-02

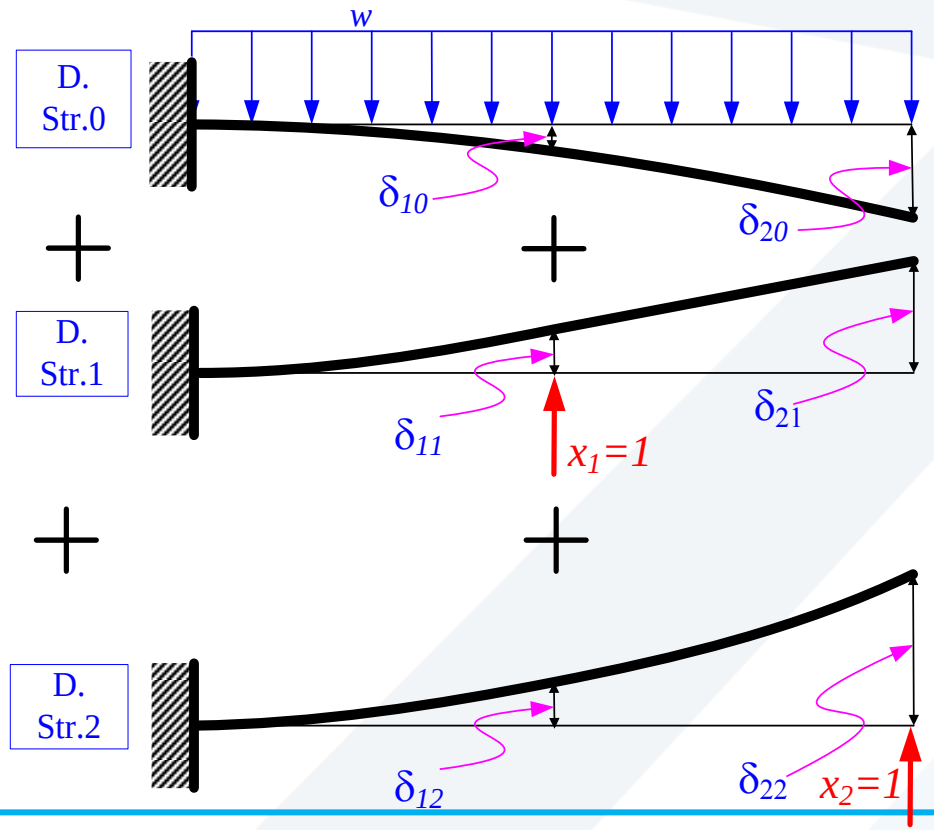
Part-01

Analysis of Indeterminate Structures - Force Method

- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- Structures with multiple Degrees of Indeterminacy
- Support Settlements
- Three-Moment Equation for Continuous Beams

Force Method for Higher degrees of indeterminacy





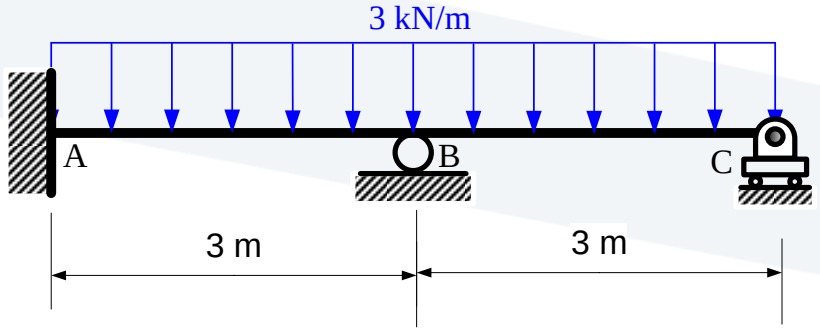
Two Compatibility Equations

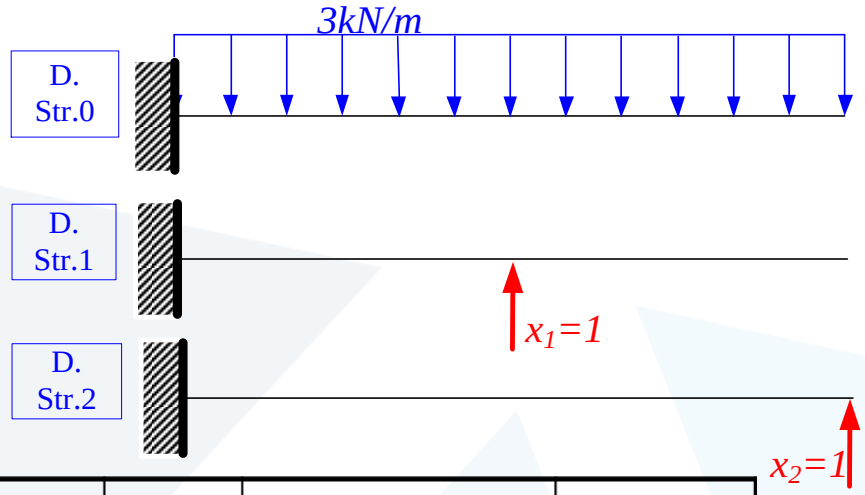
$$\delta_{10} + \delta_{11}x_1 + \delta_{12}x_2 = 0$$

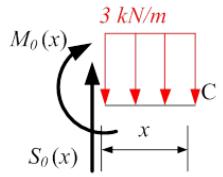
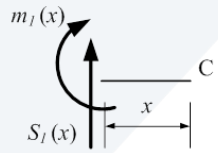
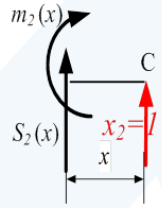
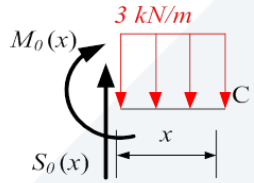
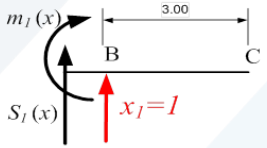
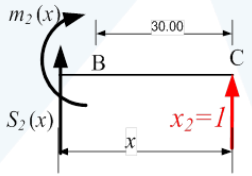
$$\delta_{20} + \delta_{21}x_1 + \delta_{22}x_2 = 0$$

B. Haidar

Ex1. Compute the supports reactions, then draw the shear force & bending moment diagrams, EI is constant





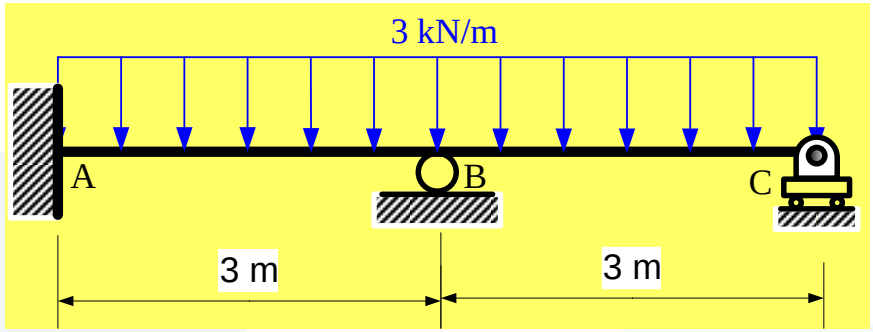
Segment	FBD in D.S.0	$M_0(x)$	FBD in D.S.1	$m_1(x)$	FBD in D.S.2	$m_2(x)$
CB $0 \le x \le 3$ EI		$-3x^2/2$		0		x
BA $3 \le x \le 6$ EI		$-3x^2/2$		$x-3$		x

$$\delta_{10} = -172.125, \delta_{11} = 9, \delta_{12} = 22.5, \delta_{20} = -486, \delta_{21} = 22.5, \delta_{22} = 72.$$

Ex1. Compute the supports reactions, then draw the shear force & bending moment diagrams, EI is constant

$$\left.\begin{aligned} 9x_1 + 22.5x_2 &= 172.125 \\ 22.5x_1 + 72x_2 &= 486 \end{aligned}\right\}$$

$$\Rightarrow x_1 = 10.29\,\text{kN} \;\&\; x_2 = 3.54\,\text{kN}$$



Segment	$M_0(x)$	$m_1(x)$	$m_2(x)$	$M(x)$ [kN-m]	$S(x)$ [kN]
CB $0 \le x \le 3$ EI	$-3x^2/2$	0	x	$-1.5x^2 + 3.54x$; $M_C=0$, $M(1.5)= 1.935$, $M_B= -2.88$; $M_{max}=2.09$	$-3x + 3.54$, $S_C= +3.54$, $S=0$ at $x = 1.18$m
BA $3 \le x \le 6$ EI	$-3x^2/2$	$x-3$	x	$-1.5x^2 + 13.83x - 30.87$ $M_B= -2.88$, $M(4.5)=0.99$, $M_A= -1.89$; $M_{max}=1.$	$-3x + 13.83$, $S_B= +4.93$, $S=0$ at $x = 4.61$m

Ex2. Compute the supports reactions, and the member internal forces in the next truss.

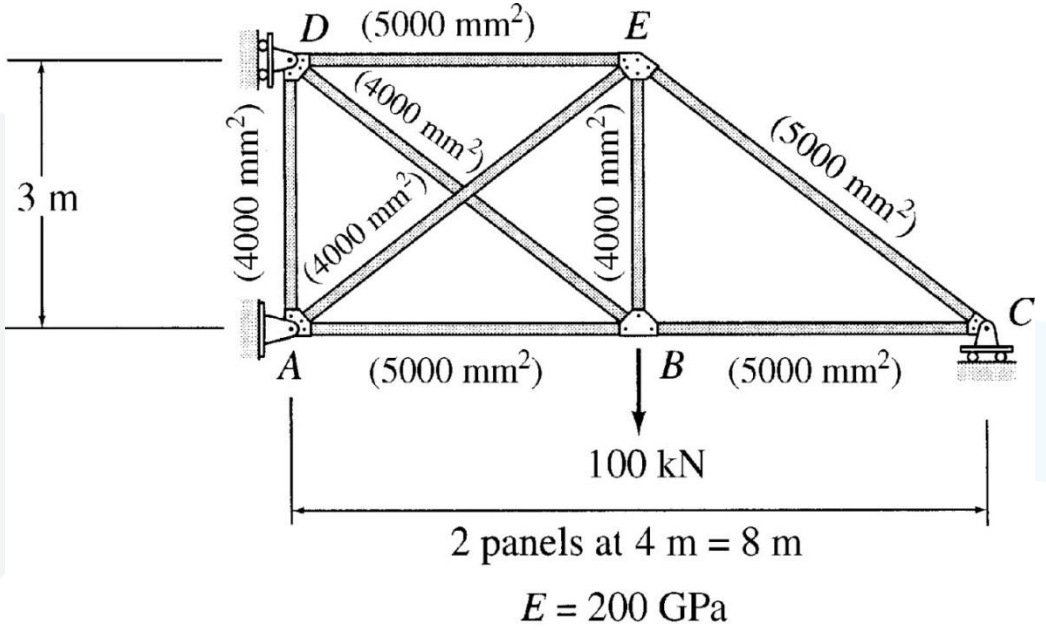
Solution:

The truss is indeterminate to the second degree, once externally and once internally.

C_y is taken as redundant reaction

N_{AE} is taken as redundant member

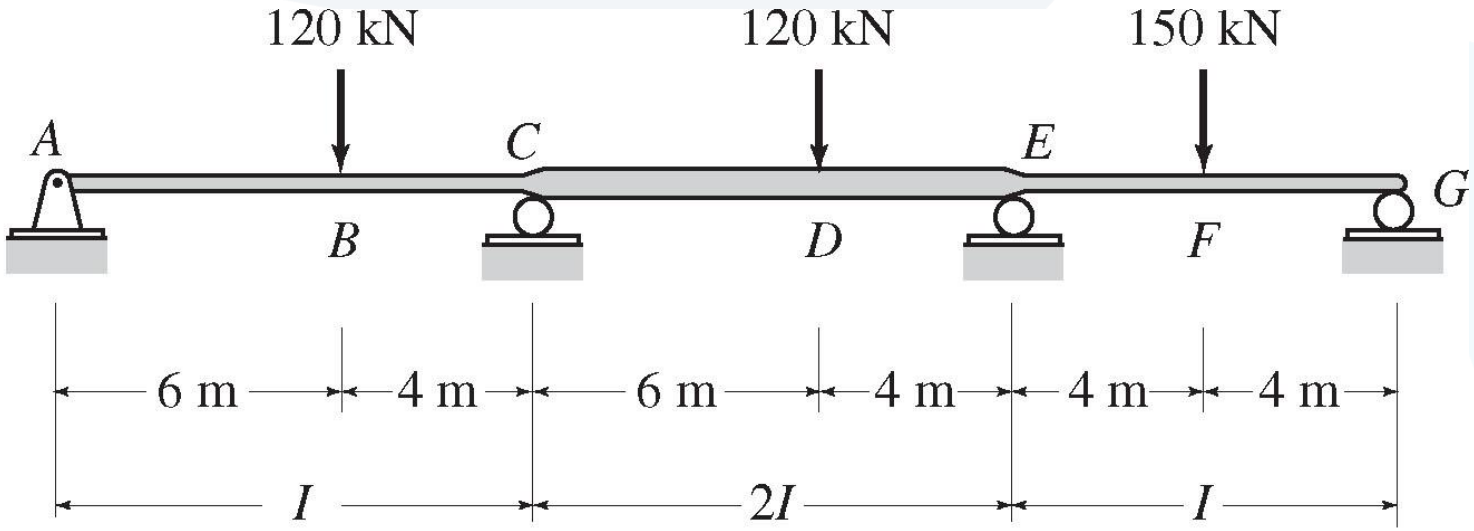
Analysis of S_0, S_1 & S_2 to get N_0, N_1 & N_2. Then use the next table to determine the coefficients of the two compatibility equations



Member	L	EA	N_0	N_1	N_2	N_0N_1L/EA	N_0N_2L/EA	N_1N_1L/EA	N_1N_2L/EA	N_2N_2L/EA

Homework

Pr-01: Using force method, determine the reactions and draw the shear and bending moment diagrams for the beam shown in the following figure.



$E = 200$ Gpa

$I = 500\ (10^6)\ \text{mm}^4$

Homework

Pr-02: Using force method, determine the reactions and draw the shear and bending moment diagrams for the frame shown in the following figure.