

Structural Mechanics (2)

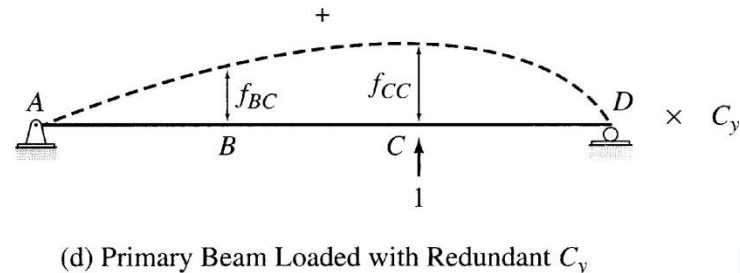
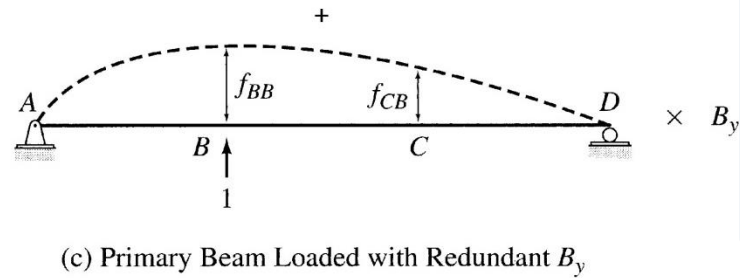
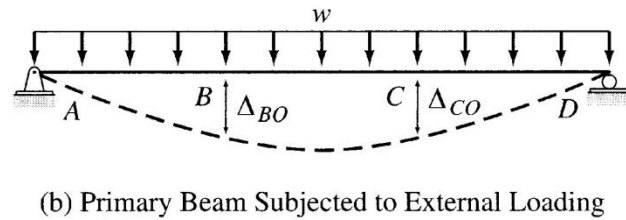
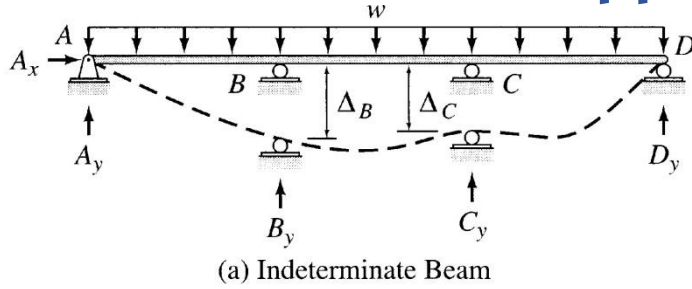
Lecture No-04

Part-01

Analysis of Indeterminate Structures - Force Method

- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- Structures with multiple Degrees of Indeterminacy
- **Support Settlements**
- **Three-Moment Equation for Continuous Beams**

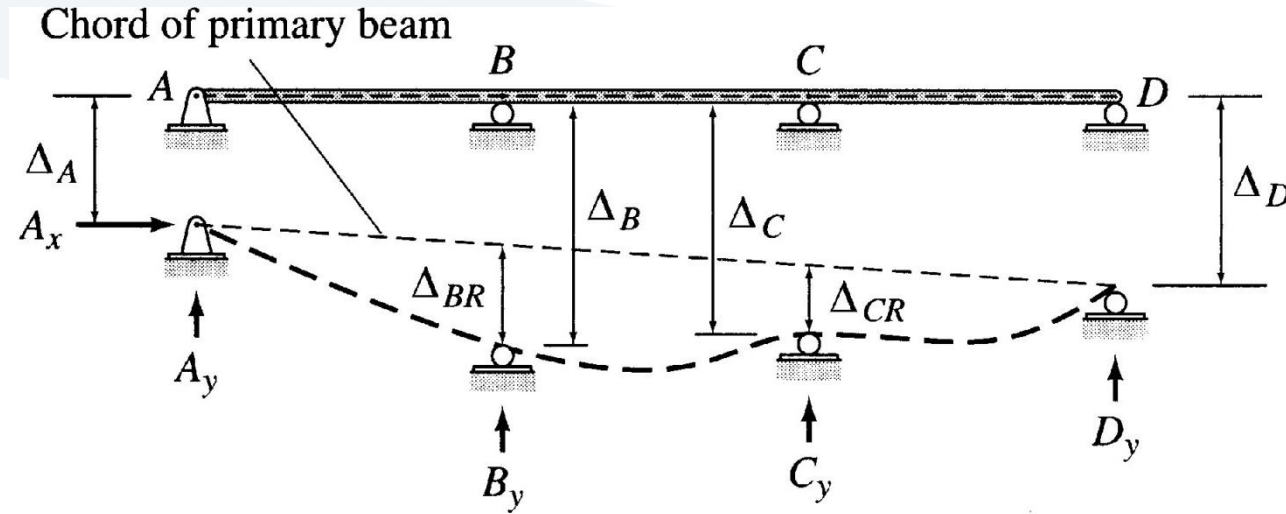
Support Settlements



$$\Delta_{BO} + f_{BB}B_y + f_{BC}C_y = \Delta_B$$

$$\Delta_{CO} + f_{CB}C_y + f_{CC}C_y = \Delta_C$$

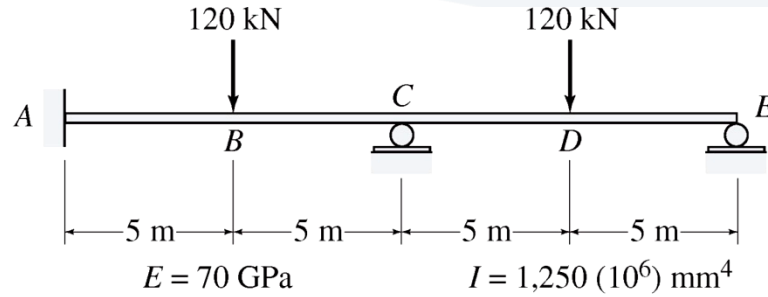
Relative Support Settlements



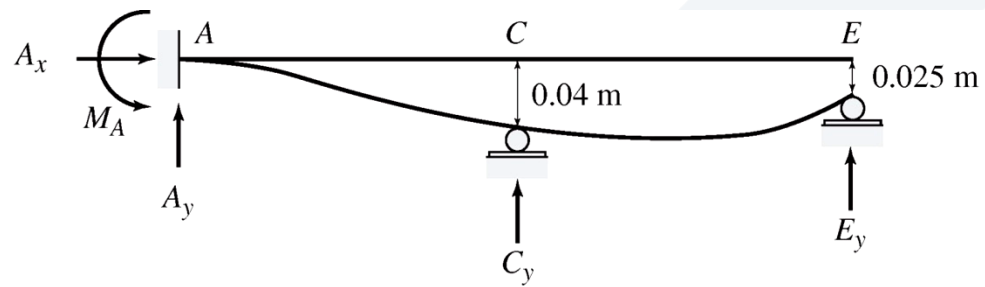
Compatibility equations:

$$f_{BB} B_y + f_{BC} C_y = \Delta_{BR}$$

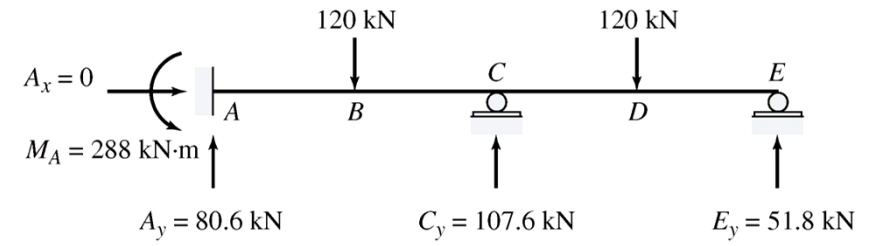
$$f_{CB} B_y + f_{CC} C_y = \Delta_{CR}$$



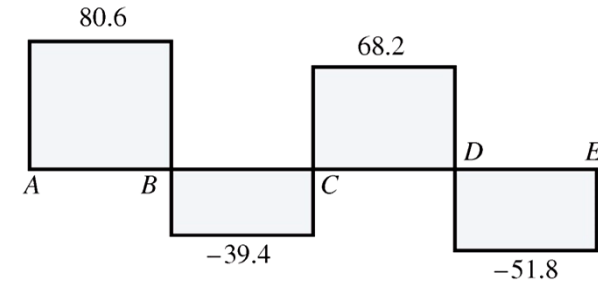
(a) Indeterminate Beam



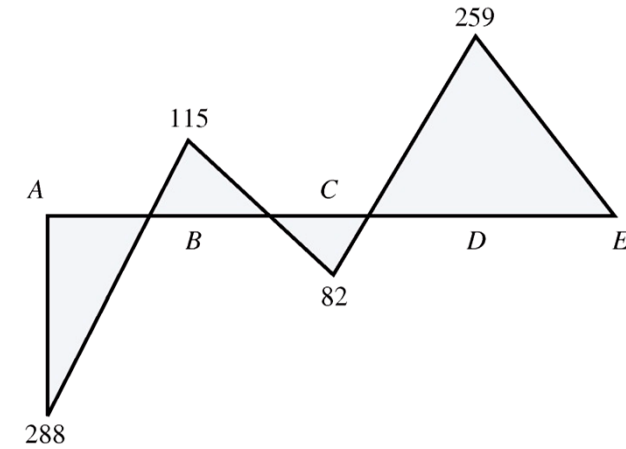
(b) Support Settlements



Reactions



Shear diagram (kN)



Bending moment diagram (kN-m)

Structural Mechanics (2)

Lecture No-04

Part-02

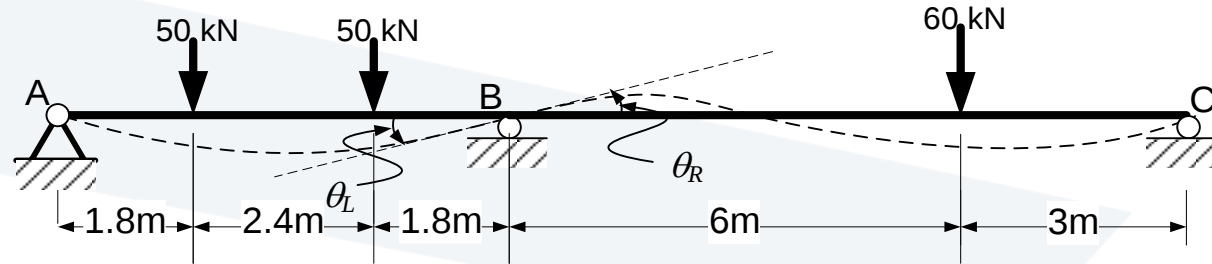
Analysis of Indeterminate Structures - Force Method

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- Three-Moment Equation for Continuous Beams

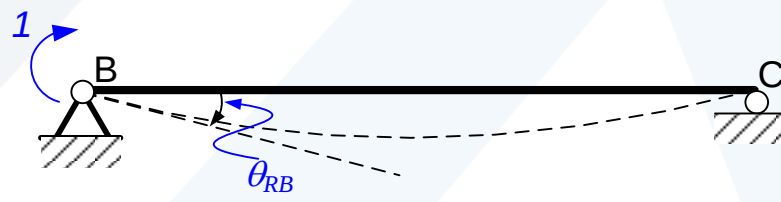
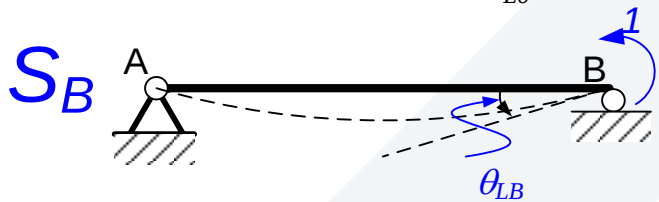
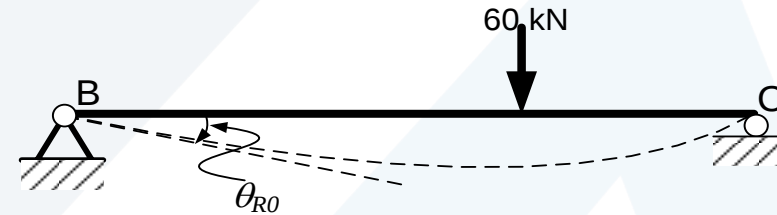
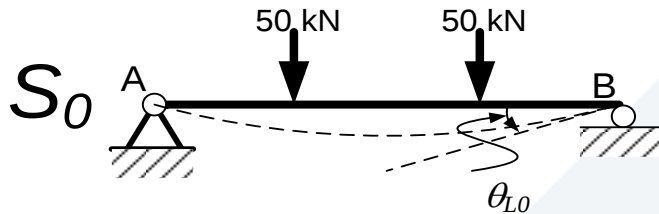
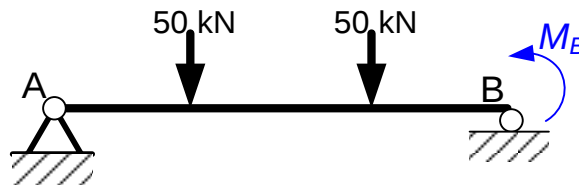
The Three-Moment Equation - Force Method for Continuous Beams

Introduction: Support Moments as Redundant

The analysis of structure with one degree of indeterminacy can also be done by choosing an internal force or moment as the redundant, provided that the released structure is determinate and stable



$$\theta_L = \theta_R$$



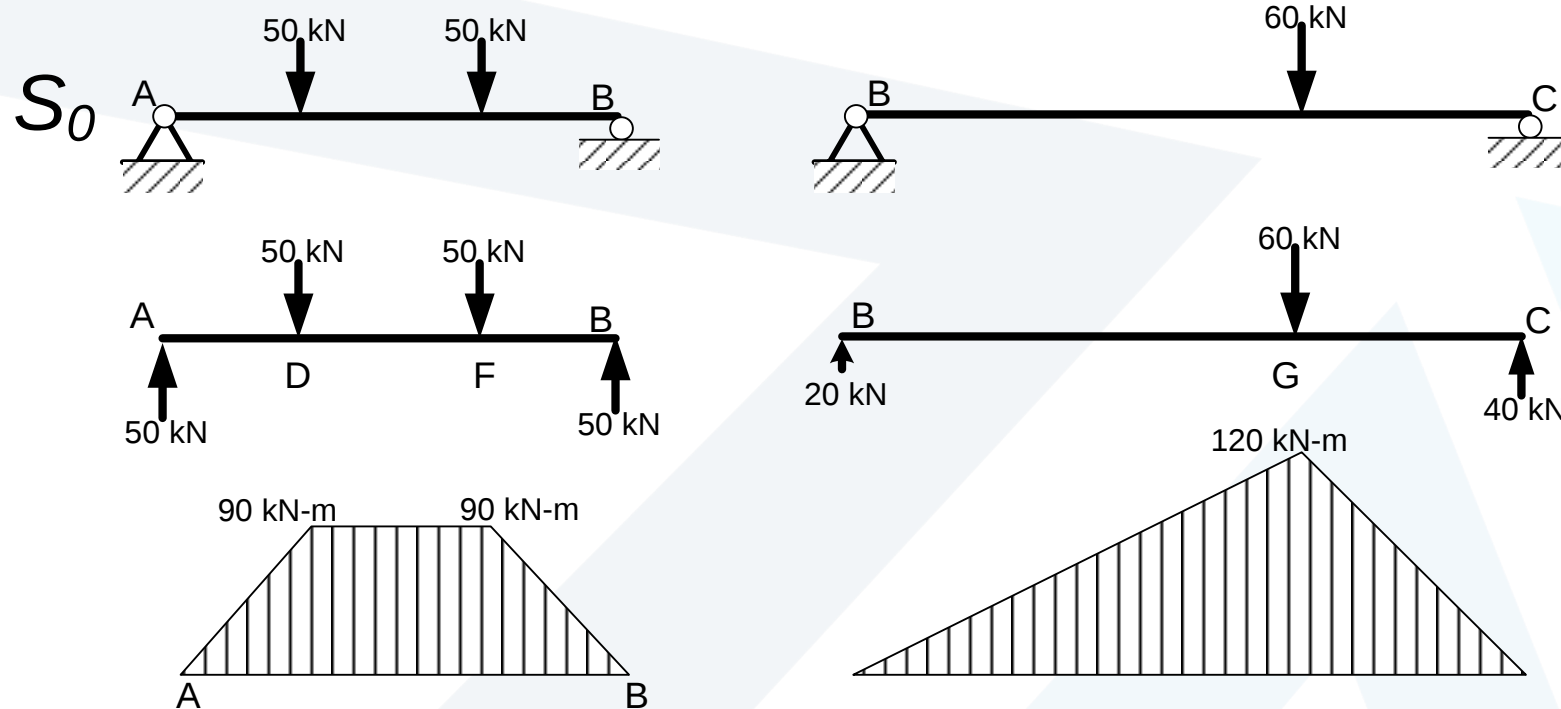
$$\theta_L = \theta_{L0} + M_B \theta_{LB}$$

$$\theta_R = \theta_{R0} + M_B \theta_{RB}$$

The Three-Moment Equation - Force Method for Continuous Beams

Introduction: Support Moments as Redundants

Analysis of S_0 Structure



$$M_{L0}$$

In AD: $M_{L0} = 50x$
 In DF: $M_{L0} = 90$
 In FB: $M_{L0} = 50(6 - x)$

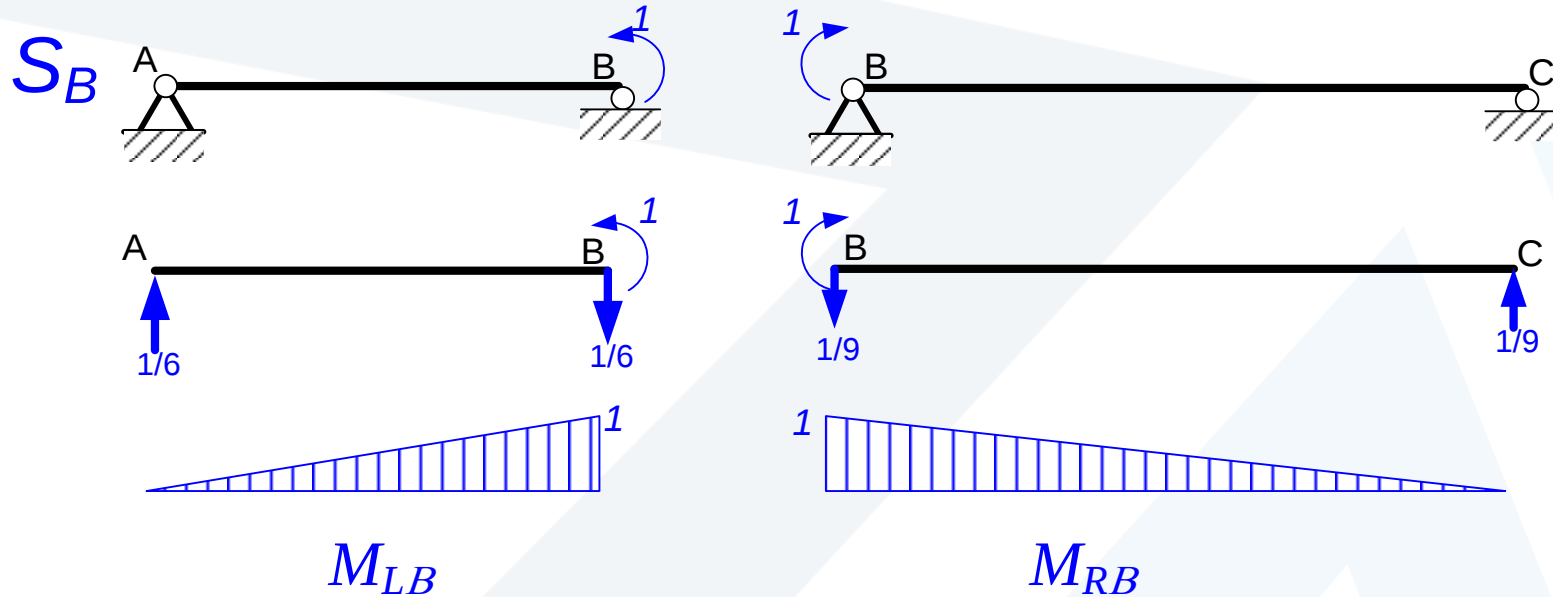
$$M_{R0}$$

In BG: $M_{R0} = 20x$
 In GC: $M_{R0} = 40(9 - x)$

The Three-Moment Equation - Force Method for Continuous Beams

Introduction: Support Moments as Redundants

Analysis of S_B Structure



In AB: $M_{LB} = x / 6$

In BC: $M_{RB} = (9 - x) / 9$

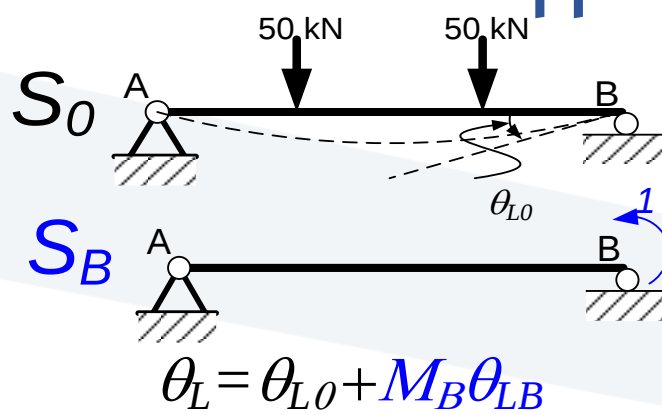
The Three-Moment Equation - Force Method for Continuous Beams

Introduction: Support Moments as Redundants

05/05/2025

B. Haidar

Structural Mechanics (2)



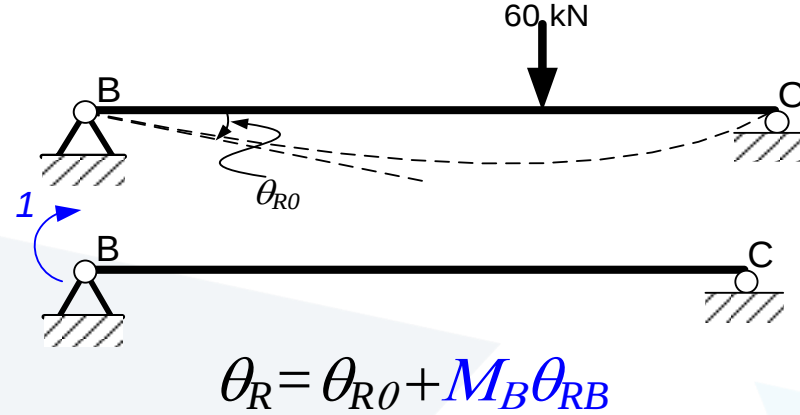
$$EI\theta_{L0} = \int_0^6 M_{L0} M_{LB} dx$$

$$= \int_0^{1.8} (50x)(x/6) dx$$

$$+ \int_{1.8}^{4.2} (90)(x/6) dx$$

$$+ \int_{4.2}^6 50(6-x)(x/6) dx$$

$$EI\theta_{L0} = 189 \quad +$$



$$EI\theta_{R0} = \int_0^9 M_{R0} M_{RB} dx$$

$$= \int_0^6 (20x)[(9-x)/9] dx$$

$$+ \int_6^9 [40(9-x)][(9-x)/9] dx$$

$$EI\theta_{R0} = 240 \quad +$$

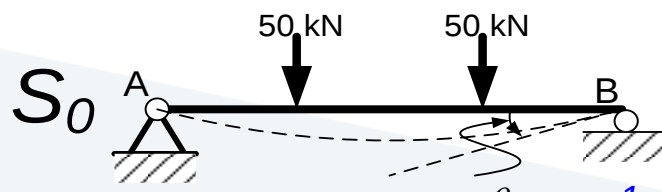
The Three-Moment Equation - Force Method for Continuous Beams

Introduction: Support Moments as Redundants

05/05/2025

B. Haidar

Structural Mechanics (2)



$$\theta_L = \theta_{L0} + M_B \theta_{LB}$$

$$EI\theta_{LB} = \int_0^6 M_{LB} M_{LB} dx$$

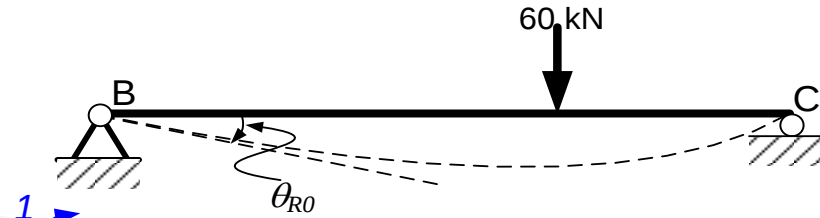
$$= \int_0^6 (x/6)(x/6) dx$$

$$EI\theta_{LB} = 2 \quad + \quad EI\theta_{L0} = 189 \quad +$$

$$\theta_L = \theta_{L0} + M_B \theta_{LB} \equiv \theta_R = -(\theta_{R0} + M_B \theta_{RB})$$

$$(\theta_{L0} + \theta_{R0}) + M_B(\theta_{LB} + \theta_{RB}) = 0$$

$$M_B(\theta_{LB} + \theta_{RB}) = -(\theta_{L0} + \theta_{R0}) \Rightarrow M_B = -429/5 = -85.5 \text{ kN.m}$$



$$\theta_R = \theta_{R0} + M_B \theta_{RB}$$

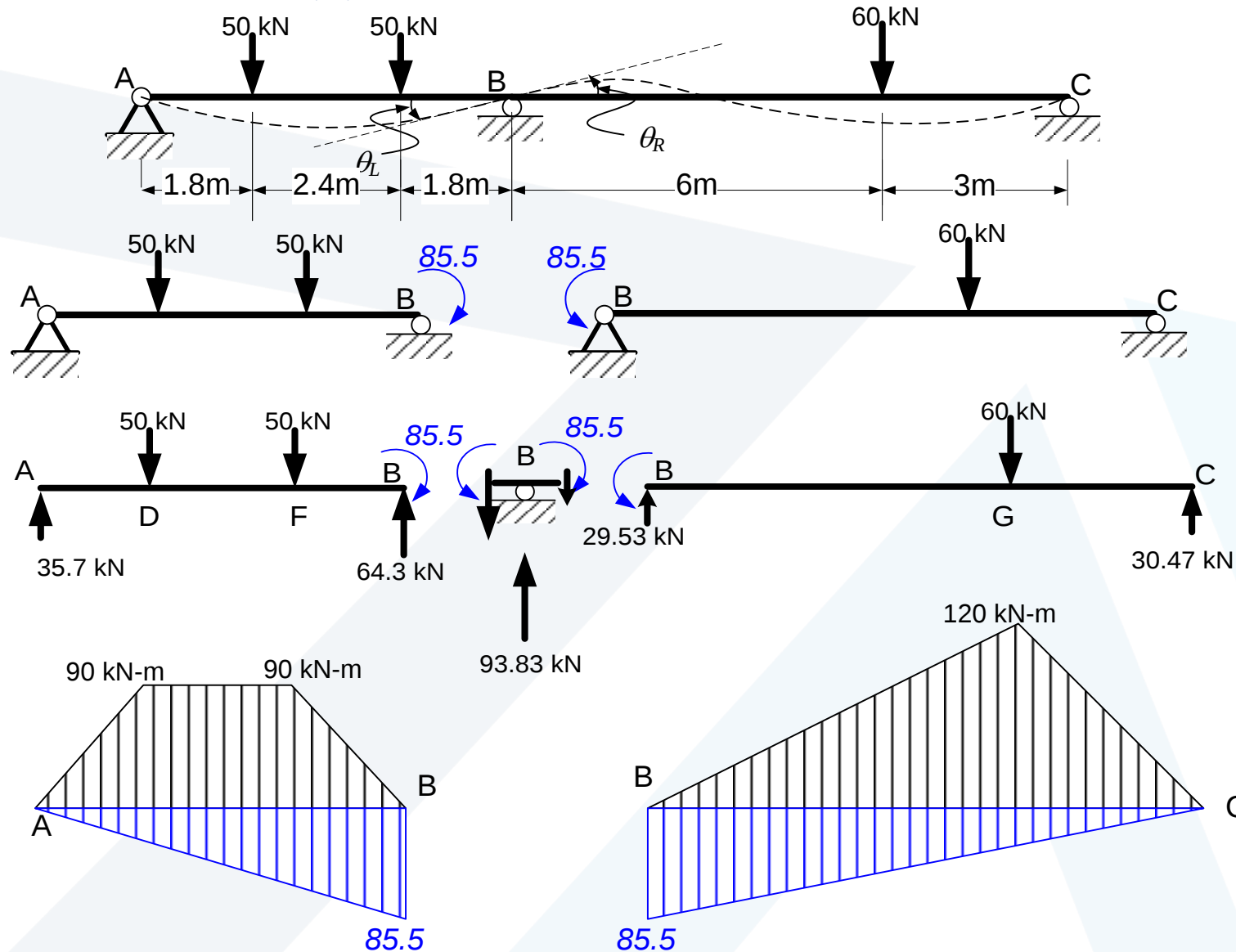
$$EI\theta_{RB} = \int_0^9 M_{RB} M_{RB} dx$$

$$= \int_0^9 [(9-x)/9][(9-x)/9] dx$$

$$EI\theta_{RB} = 3 \quad + \quad EI\theta_{R0} = 240 \quad +$$

The Three-Moment Equation - Force Method for Continuous Beams

Introduction: Support Moments as Redundants



The Three-Moment Equation - Force Method for Continuous Beams

Introduction: Support Moments as Redundants

