

# Structural Mechanics (1)

Lecture No-04

Part-01

# Deflection in Determinate Structures

## Deflections of Trusses, Beams, & Frames: Work-Energy Methods

- Deflection of trusses by Work & Strain energy principle
- Principle of Virtual Work
- Deflections of Trusses by the V. W. M.
- Deflections of Beams by the V. W. M.
- Deflections of Frames by the V. W. M.

# Deflections of Trusses Under External Loads and Other Effects

The virtual work method is used to determine the deflections of trusses under the action of external load, *and temperature change or fabrication errors*.

Let us assume that we want to determine the vertical deflection  $\Delta$ , at joint B of the truss due to the given external Loads  $P_1$  &  $P_2$ .

If  $N$  represents the internal axial force in an arbitrary member  $j$  of the truss then from the axial deformation,  $\delta$ , of this member is given by:  $\delta = NL/EA$ , where  $L$ ,  $A$  &  $E$ , denote respectively, the length, cross-section and elastic modulus of member  $j$ .

$$W_{ve} = 1_v(\Delta) \quad U_{vi} = \sum N_v(NL/EA) = \sum N_v(\delta_r)$$

1) Under the action of external loads:

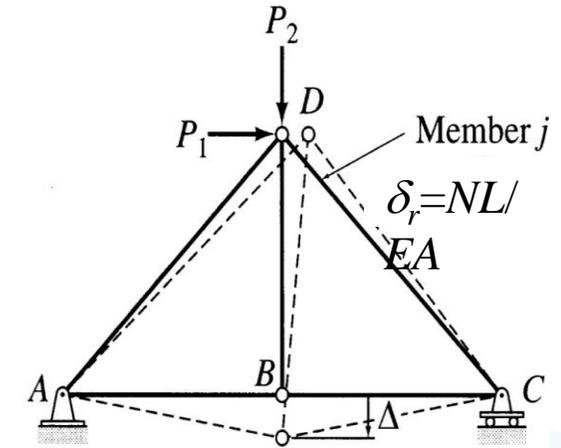
$$1_v(\Delta_r) = \sum N_v(N_r L/EA)$$

2) Under the action of a temperature change  $\Delta T$ :

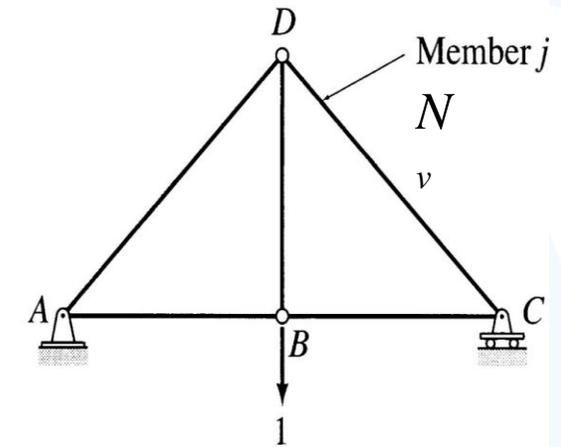
$$1_v(\Delta_r) = \sum N_v[\alpha(\Delta T)_r L]$$

3) Under the action of a fabrication error  $\delta_r$ :

$$1_v(\Delta) = \sum N_v(\delta_r)$$

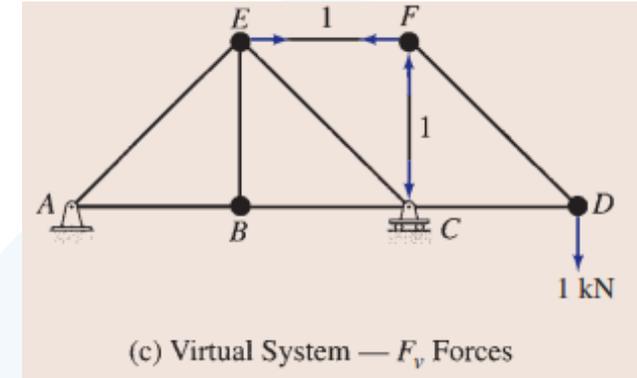
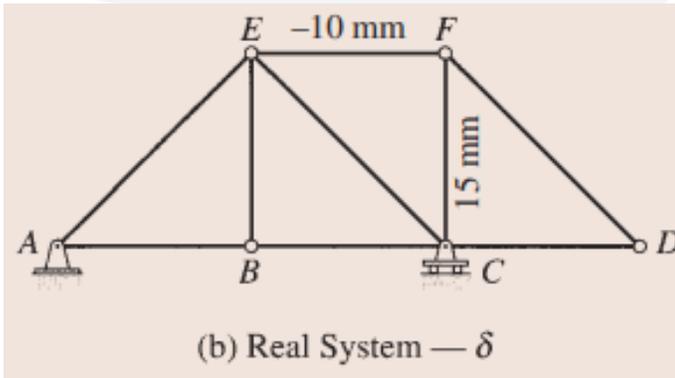
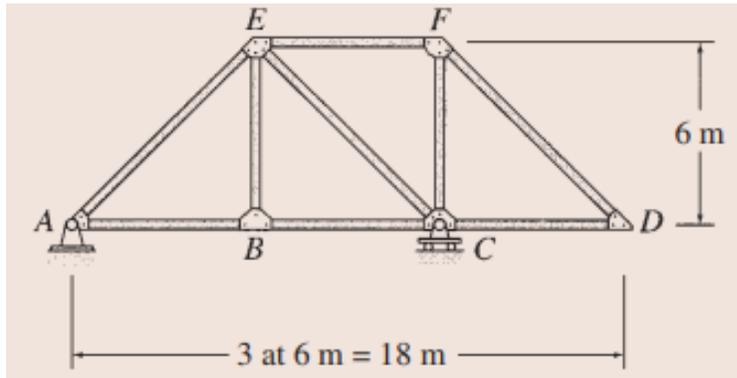


(a) Real System



(b) Virtual System

**Example 01:** Use the virtual work method to determine the vertical deflection at joint D of the truss shown in the following figure if member CF is 15 mm too long and member EF is 10 mm too short.



**Example 01:** The member axial forces due to the real system ( $N$ ) and this virtual system ( $N_{v1}$ ) are then tabulated as shown in the following table:.

Member	$\delta$ (mm)	$N_v$ (kN)	$N_v \delta$ (kn.mm)
EF	15	-1	-15
CF	-10	1	-10
			-25

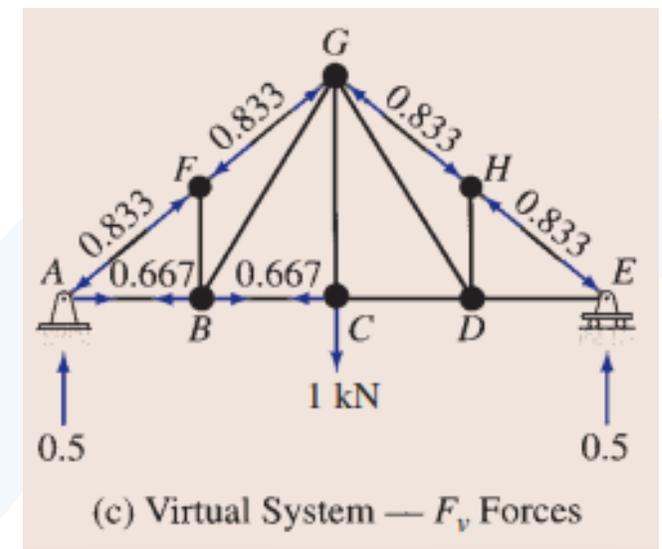
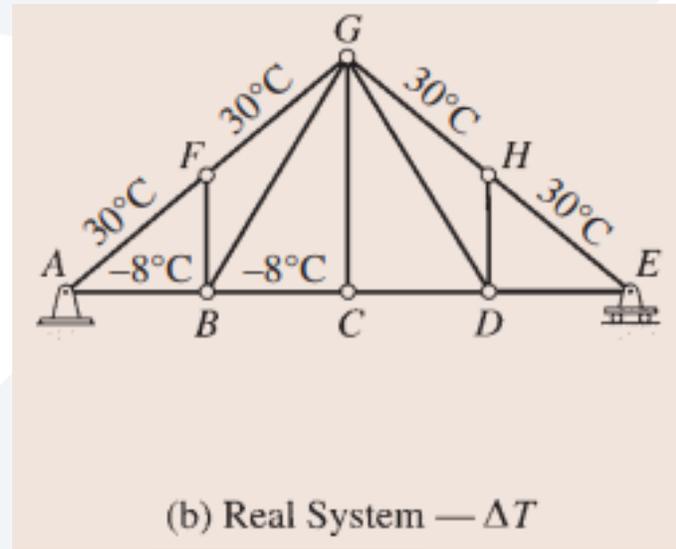
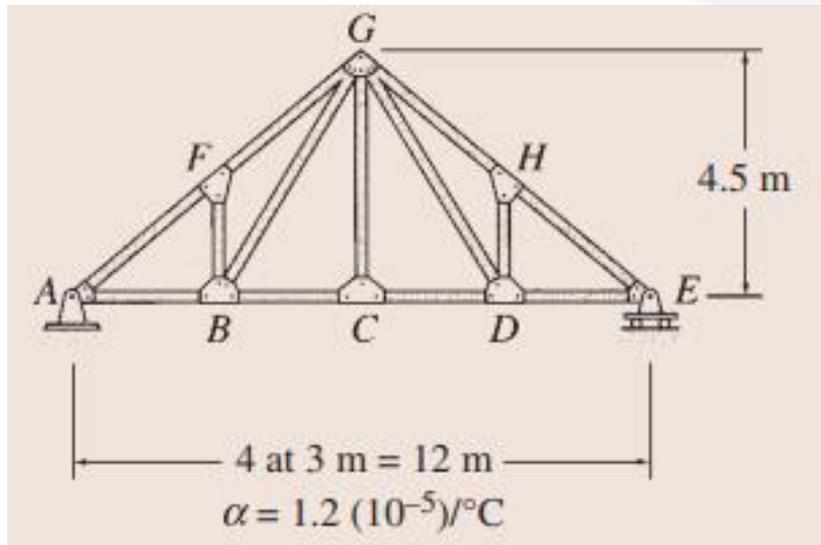
$$1(\Delta_D) = \sum N_v \delta$$

$$(1 \text{ kN})\Delta_D = -25 \text{ kN}\cdot\text{mm}$$

$$\Delta_D = -25 \text{ mm}$$

$$\Delta_D = 25 \text{ mm } \uparrow$$

**Example 02:** Use the virtual work method to determine the vertical deflection at joint C of the truss shown in the following due to a temperature drop of  $8^{\circ}\text{C}$  in members AB and BC and a temperature increase of  $30^{\circ}\text{C}$  in members AF, FG, GH, and EH.



**Example 02:** The member axial forces due to the real system ( $N$ ) and this virtual system ( $N_{v1}$ ) are then tabulated as shown in the following table:

Member	L (m)	$\Delta T$ ( $^{\circ}\text{C}$ )	$N_v$ (kN)	$N_{v1}(\Delta T)L$ (kN- $^{\circ}\text{C}$ -m)
AB	3	-8	0.667	-16.0
BC	3	-8	0.667	-16.0
AF	3.75	30	-0.833	-93.7
FG	3.75	30	-0.833	-93.7
GH	3.75	30	-0.833	-93.7
EH	3.75	30	-0.833	-93.7
				<b>-406.8</b>

$$1(\Delta_C) = \alpha \sum N_v(\Delta T)L$$

$$(1 \text{ kN})\Delta_C = 1.2 (10^{-5})(-406.8)$$

$$\Delta_C = -0.00488 \text{ m}$$

$$\Delta_C = 4.88 \text{ mm } \uparrow$$