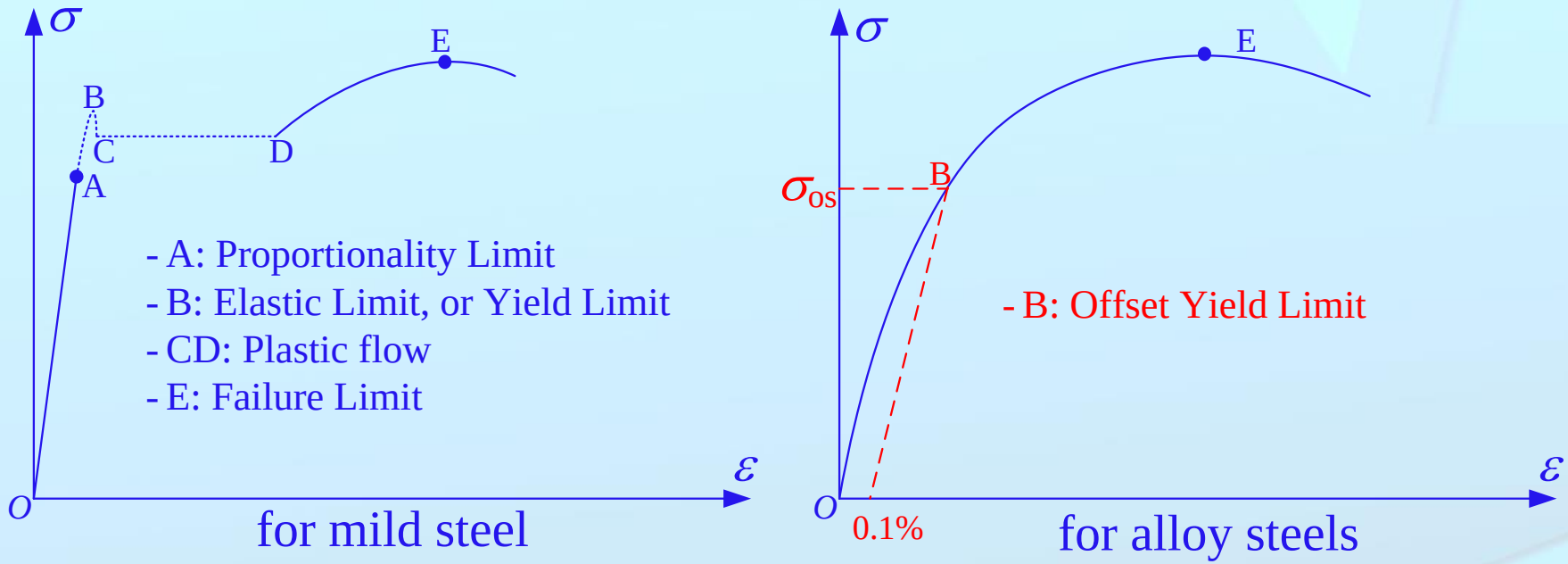
Plastic Behavior in Simple Tension and Compression

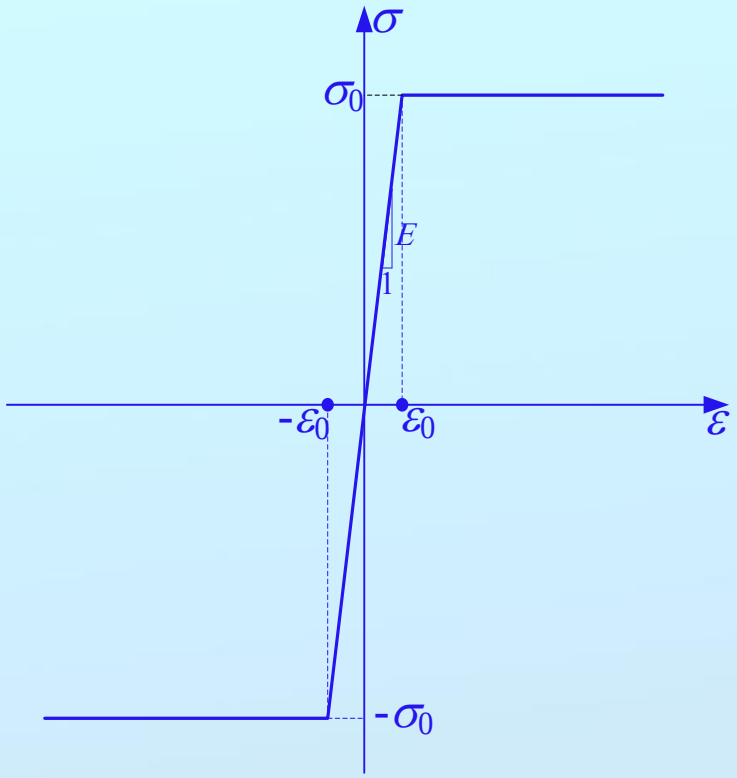
Monotonic Loading



stress-strain diagram in simple tension. Monotonic loading

Modeling of Uniaxial Behavior in Plasticity

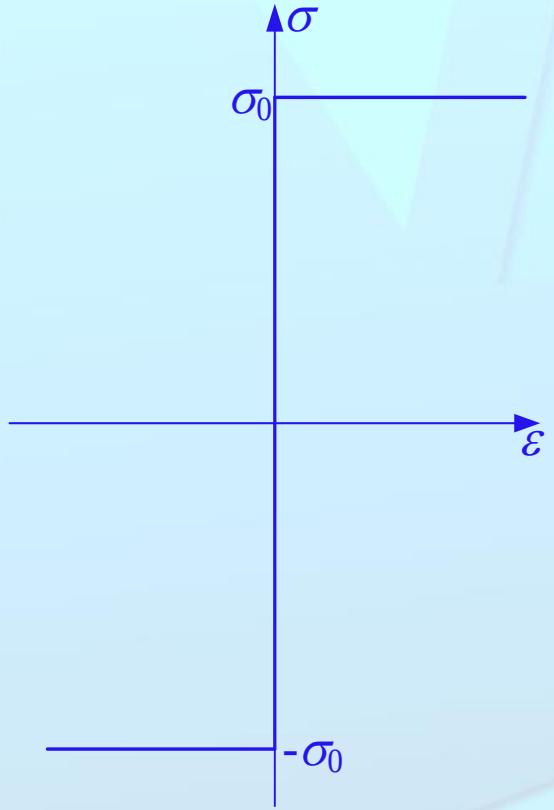
Simplified Uniaxial Stress-Strain Relations



elastic-perfectly plastic

$$\varepsilon = \frac{\sigma}{E} \quad \text{for} \quad \sigma < \sigma_0$$

$$\varepsilon = \frac{\sigma_0}{E} + \lambda \quad \text{for} \quad \sigma = \sigma_0$$

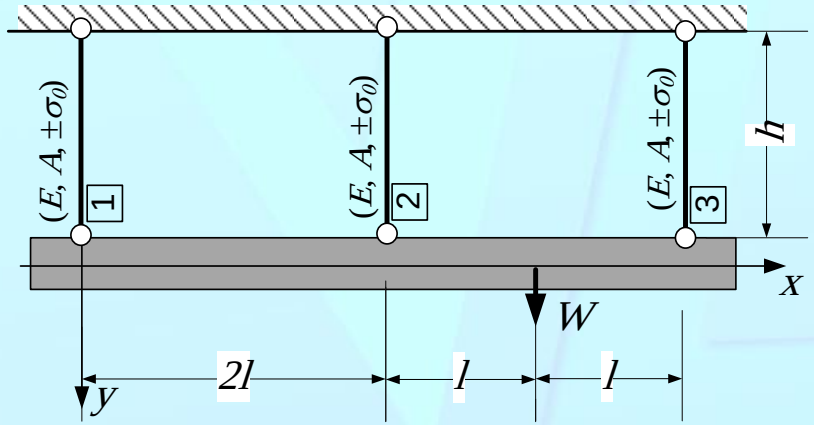


rigid-perfectly plastic

$$\varepsilon = 0 \quad \text{for} \quad \sigma < \sigma_0$$

$$\varepsilon = \lambda \quad \text{for} \quad \sigma = \sigma_0$$

Ex.1 Consider a rigid horizontal beam suspended to the roof by three identical bars as indicated on the figure. If the load W increases in magnitude. Determine the evolution of W to the total collapse.

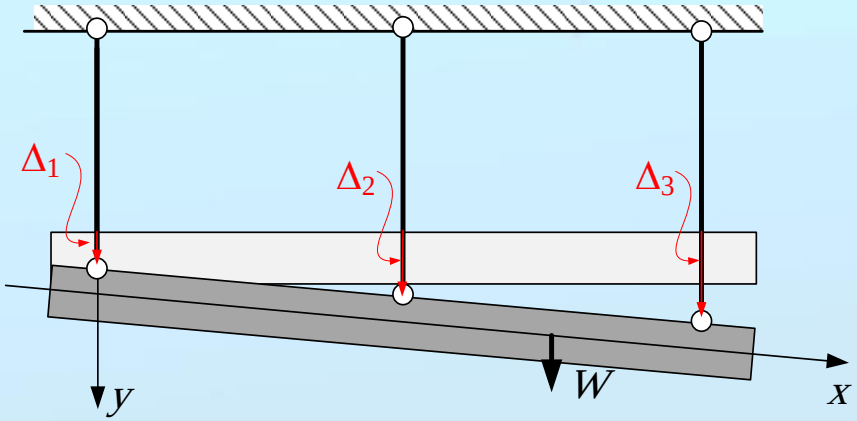


Solution:

Eq. Eqs.

$F_1 + F_2 + F_3 = W$ (1) & $3F_1 + F_2 - F_3 = 0$ (2)

Comp. Eq. $\Delta_2 = \dfrac{\Delta_1 + \Delta_2}{2} \Rightarrow \Delta_1 - 2\Delta_2 + \Delta_3 = 0$ (3)

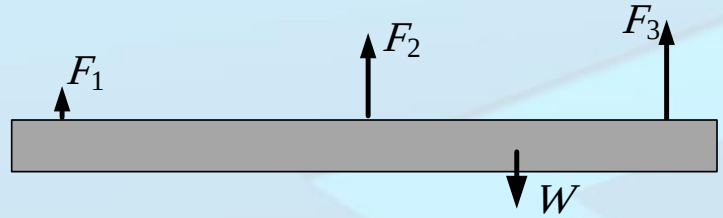


Beh. Eqs.

$$F_i = \frac{EA_i}{l_i}\Delta_i \ , if \ \Delta_i \le \frac{\sigma_0 l_i}{E} \quad or \quad F_i = \sigma_0 A_i \ , if \ \Delta_i > \frac{\sigma_0 l_i}{E}$$

Elastic Phase: مرحلة المرونة $F_i = \dfrac{EA_i}{l_i}\Delta_i$

$\Delta_1 = \dfrac{h}{EA}F_1, \quad \Delta_2 = \dfrac{h}{EA}F_2, \quad \Delta_3 = \dfrac{h}{EA}F_3$ ⟹ Eq. (3)



$\Rightarrow F_1 - 2F_2 + F_3 = 0$ (3′) Solving with (1) & (2) to get $F_1 = 0.0833W, F_2 = 0.333W, F_3 = 0.583W$

End of Elastic Phase When - نهاية مرحلة المرونة عندما : $Max(F_i) = A\sigma_0 \Rightarrow F_3 = A\sigma_0 \Rightarrow 0.583W = A\sigma_0 \Rightarrow W_1 = 1.714A\sigma_0$

Plastic Phase: مرحلة اللدونة

$$\Rightarrow W > W_1 = 1.714A\sigma_0 \;\&\; F_3 = A\sigma_0$$

Eq. Eqs.

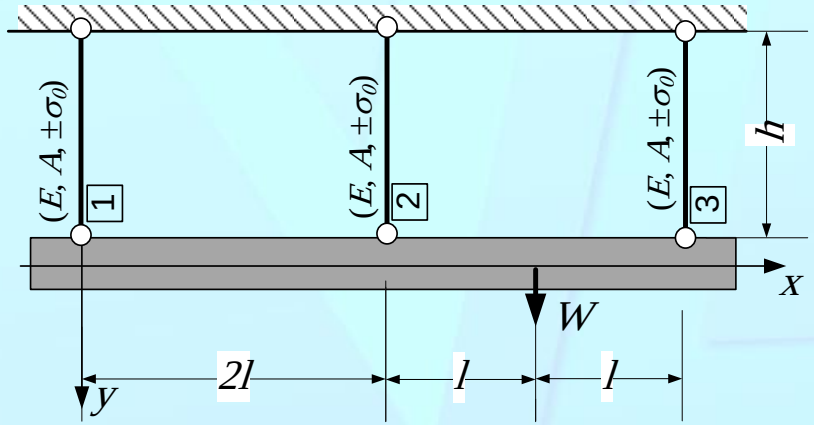
$$F_1 + F_2 + A\sigma_0 = W \;(4) \quad \& \quad 3F_1 + F_2 - A\sigma_0 = 0 \;(5)$$

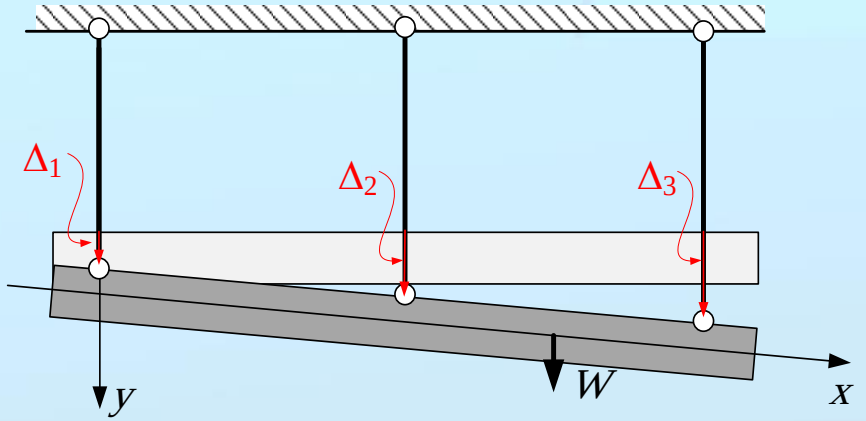
Solving with (4) & (5) to get

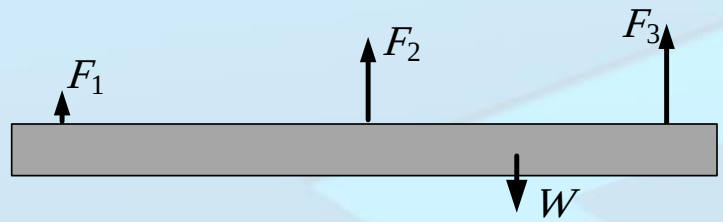
$$F_1 = -\frac{W}{2} + A\sigma_0 \qquad \& \qquad F_2 = \frac{3W}{2} - 2A\sigma_0$$

Total Yielding when: $F_2 = \frac{3W}{2} - 2A\sigma_0 = A\sigma_0$

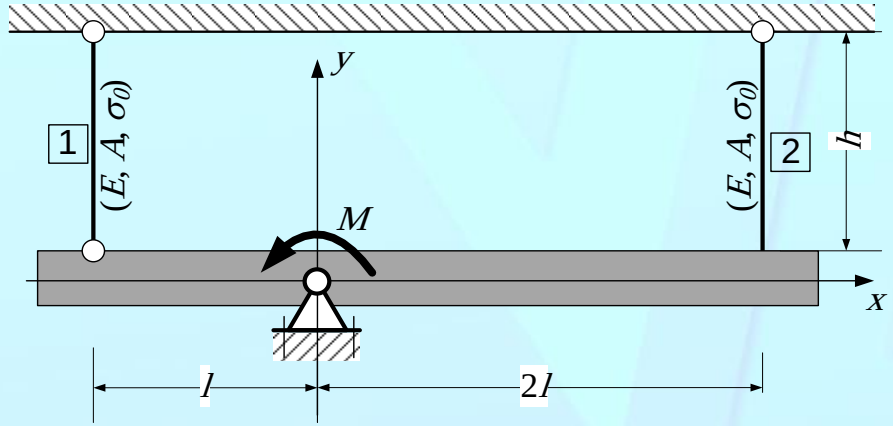
$$\Rightarrow W_C = 2A\,\sigma_0$$







Ex2. Consider a rigid horizontal beam suspended to the roof by two different bars as indicated on the figure, and rotates about the shown hinge by a small angle θ, under the action of a moment M, applied at the hinge. If the moment M increases in magnitude. Determine the evolution of M to the total collapse.



Solution: