

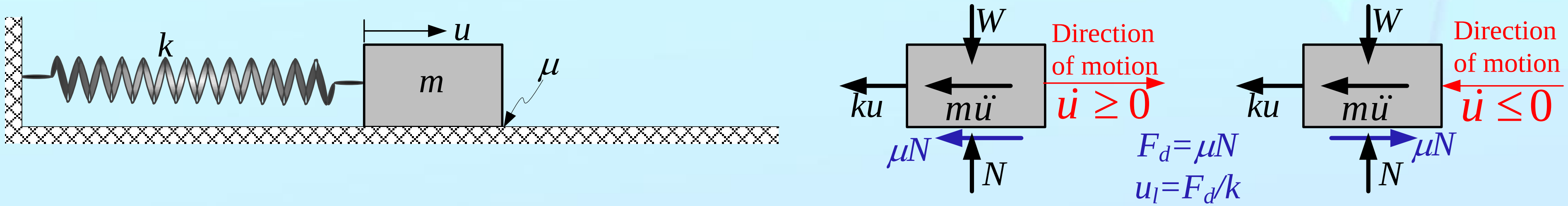



Bracing members
Pall Friction Damper


Coulomb Damping:

Viscous damping is convenient mathematically but not realistic and is not the only type of dissipative mechanism present in modern structural systems.

One such mechanism is sliding friction known as Coulomb Damping. Coulomb damping is a force used to model dry friction that takes place between two surfaces in contact and moving relative to each other.



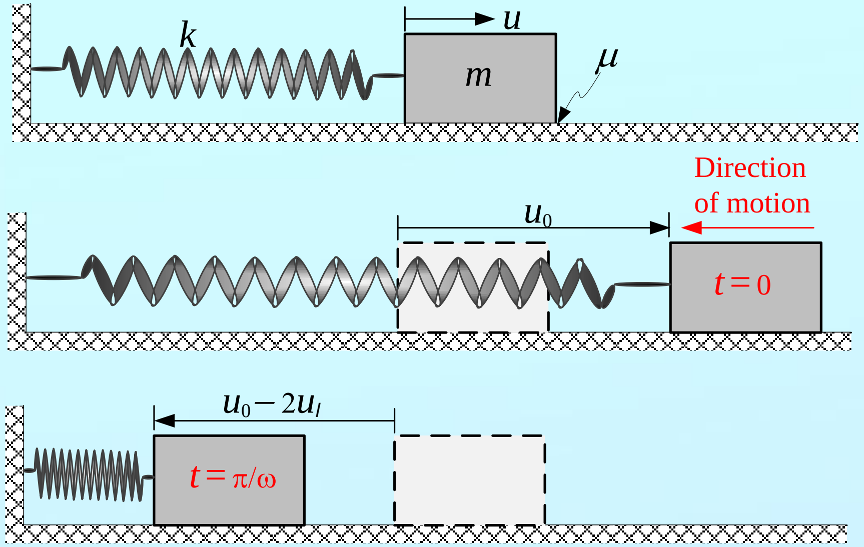
Any initial displacement greater than the locking one, $u_l = F_d / k$, can put the system in free vibration regime.

The Eq. of motion depends on its direction: $m\ddot{u} + ku = -F_d$ or, $m\ddot{u} + ku = +F_d$

Both forms can be written as: $\ddot{u} + \omega^2 u = -(\text{sign}(\dot{u}))(F_d / m)$, where $\omega^2 = k / m$

The complete solution of this ODE, is : $u(t) = A\cos(\omega t) + B\sin(\omega t) - (\text{sign}(\dot{u}))u_l$

The constants *A* and *B*, are determined according to the initial conditions (ICs). One possible scenario is to displace the mass to the right by $u_0 > u_l$, and let free it to vibrate.



$$u(t) = A\cos(\omega t) + B\sin(\omega t) - (\text{sign}(\dot{u}))u_l$$

First half cycle starts at $t = 0$: $u(0) = u_0 > u_l$, or : $ku_0 > F_d$.

So motion is to left and velocity at $t=0$, is null. Solution (displacement & velocity) has the form:

$$u(t) = A\cos(\omega t) + B\sin(\omega t) + u_l$$

$$\dot{u}(t) = -A\omega\sin(\omega t) + B\omega\cos(\omega t)$$

with $\dot{u}(0) = 0,\ B = 0$, & with $u(0) = u_0,\ A = u_0 - u_l$

So the displacement and velocity functions are:

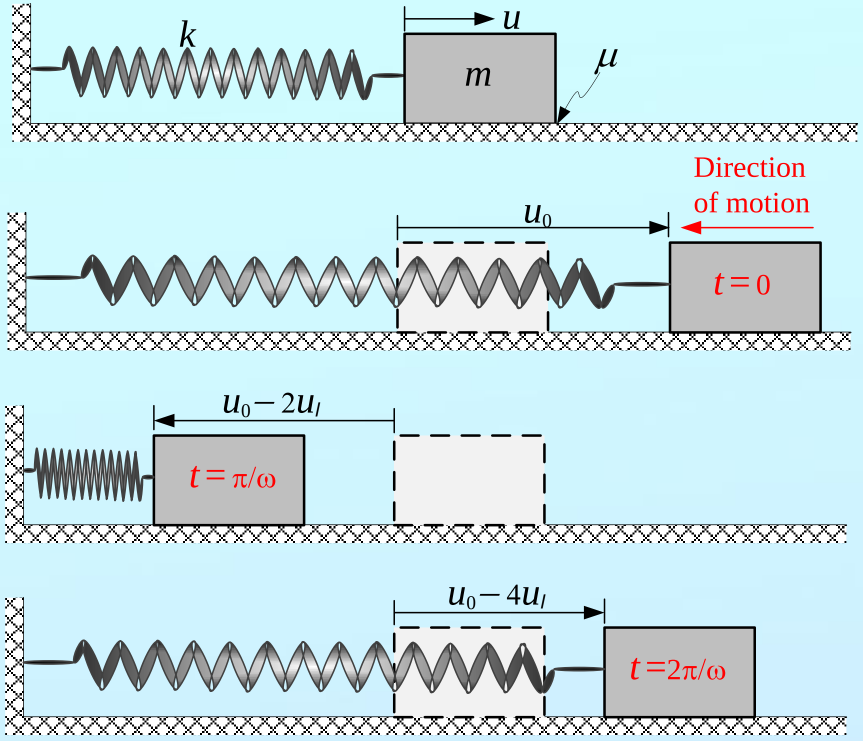
$$u(t) = (u_0 - u_l)\cos(\omega t) + u_l, \ \&\ \dot{u}(t) = -\omega(u_0 - u_l)\sin(\omega t)$$

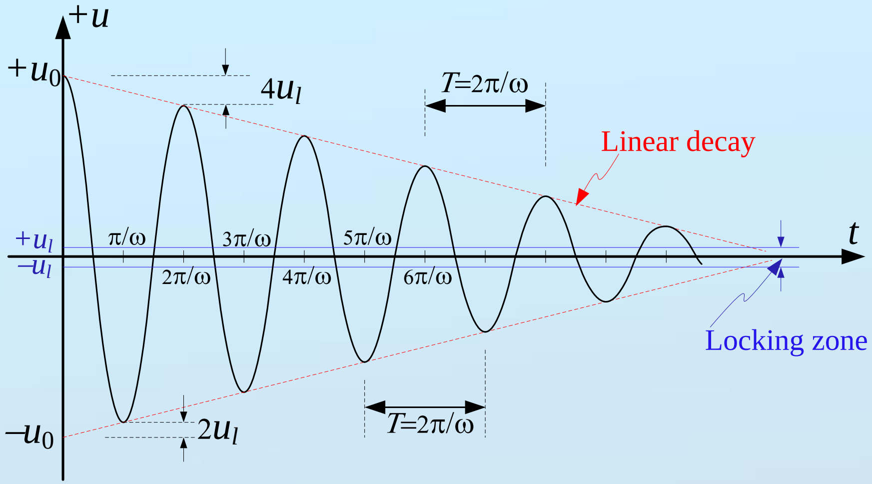
The motion continues to the left until:

$$\dot{u}(t) = 0 \Rightarrow \sin(\omega t) = 0, \quad \text{when} \quad t = \pi/\omega$$

Where the displacement reaches the value:

$$u(\pi/\omega) = (u_0 - u_l)\cos(\pi) + u_l = -u_0 + 2u_l$$





$$u(t)=A\cos(\omega t)+B\sin(\omega t)-(\text{sign}(\dot{u}))u_l$$

If $|u(\pi/\omega)|>u_l$, the motion restarts to right with:

$$u(t)=A\cos(\omega t)+B\sin(\omega t)-u_l$$

$$\dot{u}(t)=-A\omega\sin(\omega t)+B\omega\cos(\omega t)$$

with $\dot{u}(\pi/\omega)=0,\ B=0,$

& with $u(\pi/\omega)=-u_0+2u_l,\ A=u_0-3u_l$

So the displacement and velocity functions are:

$$u(t)=(u_0-3u_l)\cos(\omega t)-u_l,$$

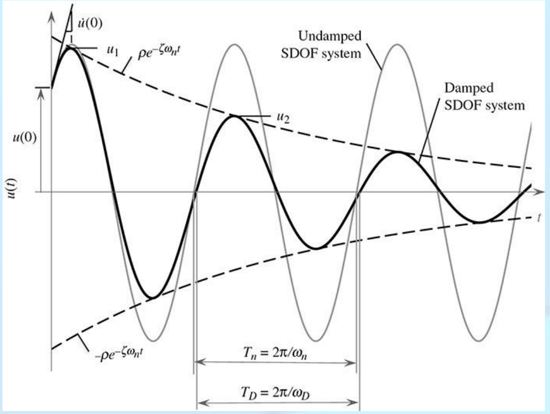
$$\&\ \dot{u}(t)=-\omega(u_0-3u_l)\sin(\omega t).$$

The motion continues to the right until:

$$\dot{u}(t)=0\Rightarrow\sin(\omega t)=0,\quad \text{when}\quad t=2\pi/\omega$$

Where the displacement reaches the value:

$$u(2\pi/\omega)=(u_0-3u_l)\cos(2\pi)-u_l=u_0-4u_l$$



Final Result:

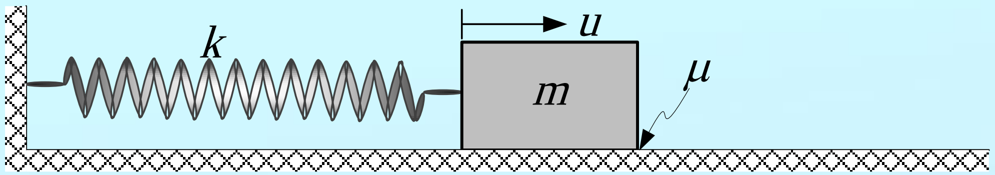
For every half-cycle of motion the amplitude loss is $2u_l=2F_d/k$, and for a complete cycle the loss is $4u_l$.

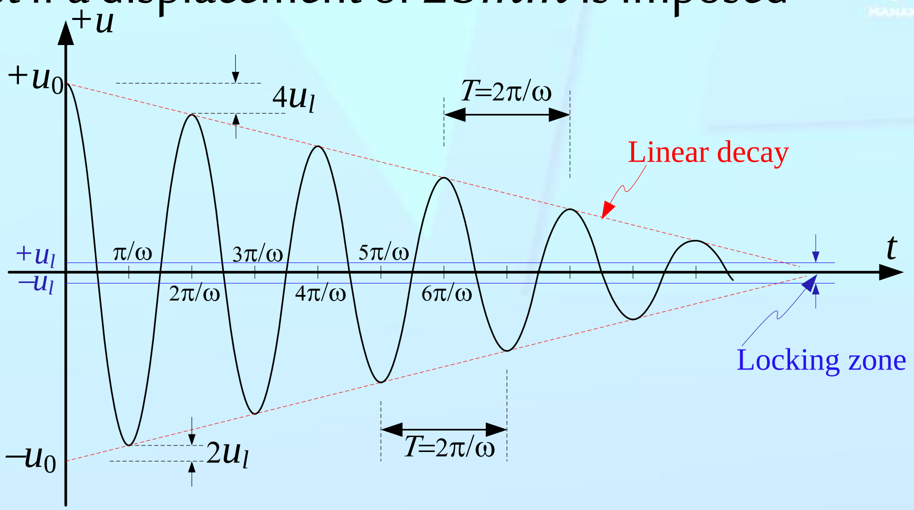
So for n cycle ($2n$ half-cycles) this loss is $4nu_l$.

When the amplitude becomes less than u_l, the motion stops and the mass is trapped in the locking zone.

EX. 1. A block of mass $m = 10kg$ is restrained by a spring of stiffness $k = 5000\ N/m$ and rests on a rough surface with coefficient of friction $\mu = 0.10$.

Calculate the number of half-cycles required for the mass to come to rest if a displacement of $25mm$ is imposed on the block and released with zero velocity.





SOLUTION:

1. The lock displacement u_l is:

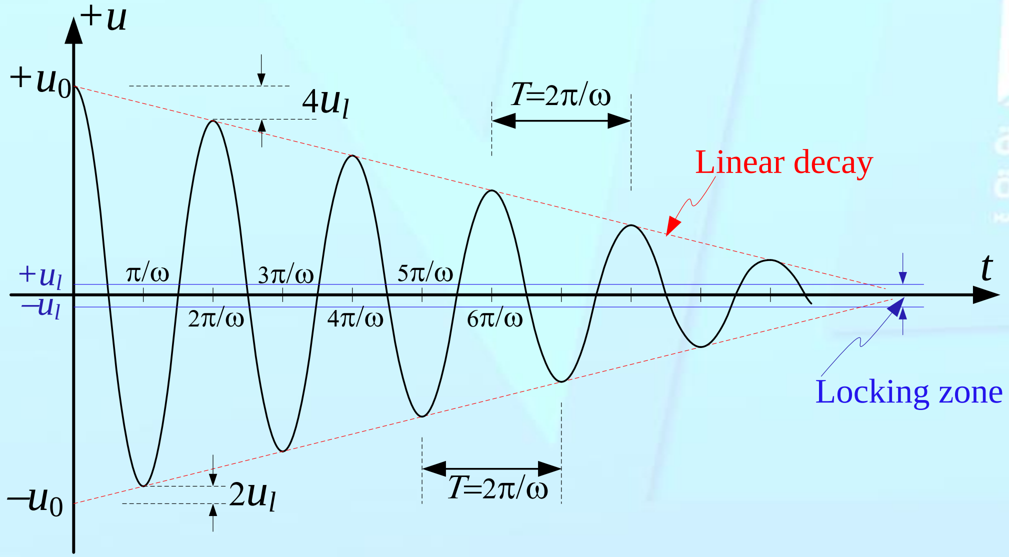
$$u_l = \frac{F_d}{k} = \frac{\mu g m}{k} = \frac{0.1 \times 9.81 \times 10}{5000} = 1.96 \times 10^{-3}\ \text{m}$$

2. The number of half-period of motion n, is given by

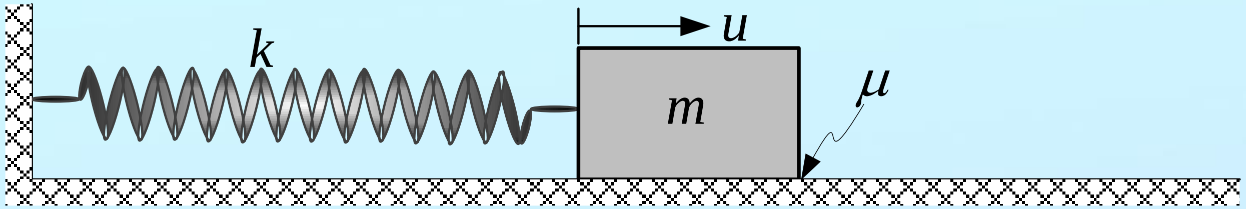
$$-u_l \leq u_0 - 2nu_l \leq u_l$$

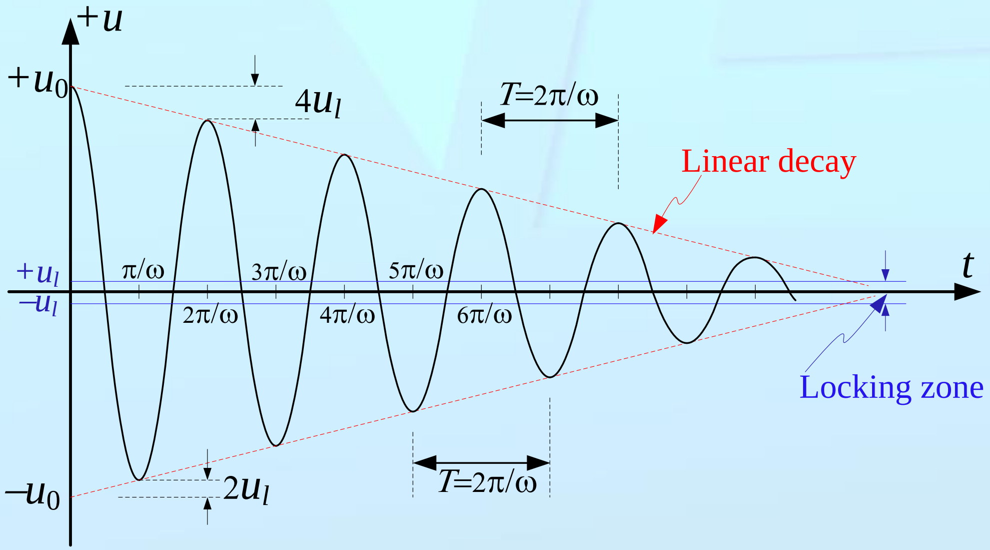
EX. 2. A block of mass $m = 200kg$ is restrained by a spring of stiffness $k = 6000\ N/m$ and rests on a rough surface with coefficient of friction $\mu = 0.15$.

Calculate the number of half-cycles required for the mass to come to rest if a displacement of $20mm$ is imposed on the block and released with zero velocity.

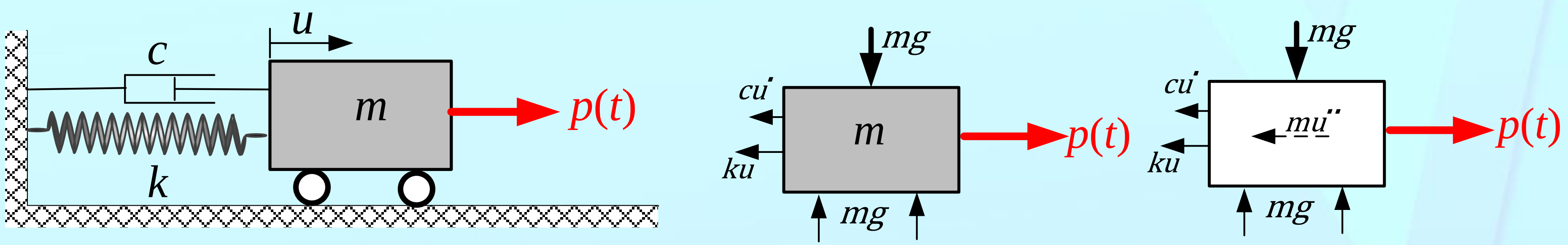


Ex. 3. For the system shown in figure, $m = 500$ kg, $k = 400$ kN/m , $\mu = 0.15$ and the initial conditions are: $u_0 = 16$ cm, $\dot{u}_0 = 0$. Determine the amplitude after 8 half-cycles and the number of half-cycles of motion completed before the mass comes to rest





Response to Harmonic Excitation



An harmonic excitation can be described either by means of a sine function: $p(t)=p_0\sin\Omega t$, or by means of a cosine function: $p(t)=p_0\cos\Omega t$.

Equation of Motion (E.o.M.):

$$m\ddot{u}+c\dot{u}+ku=p(t)=\begin{cases} p_0\cos\Omega t \\ p_0\sin\Omega t \end{cases}$$

The complete response (solution) will be the sum of the transient (homogenous) and steady-state (particular)components.

$$u(t)=\underbrace{e^{-\xi\omega t}(A\cos\omega_D t+B\sin\omega_D t)}_{\text{transient}}+\underbrace{C\cos\Omega t+D\sin\Omega t}_{\text{steady state}}$$

Find C & D, for the cosine and sin functions of harmonic excitation

Response to Harmonic Excitation

Undamped harmonic vibrations

$$m\ddot{u} + ku = p_0 \cos \Omega t \qquad \ddot{u} + \omega_n^2 u = (p_0 / m) \cos \Omega t \qquad \ddot{u} + \omega_n^2 u = \omega_n^2 u_{st} \cos \Omega t$$

A steady-state response (A particular solution) can be

$$u_p(t) = C \cos \Omega t \qquad \ddot{u}_p(t) = -C \Omega^2 \cos \Omega t \qquad (-C\Omega^2 + C\omega_n^2) \cos \Omega \mathrm{t} = \omega_n^2 u_{st} \cos \Omega t$$

$$(\omega_n^2 - \Omega^2) C = \omega_n^2 u_{st} \qquad C = \frac{\omega_n^2 u_{st}}{(\omega_n^2 - \Omega^2)} \qquad C = \frac{u_{st}}{(1 - r^2)} \quad \text{where} \quad r = \frac{\Omega}{\omega_n}$$

The transient response (The homogeneous solution) is

$$u_h(t) = A \cos \omega_n t + B \sin \omega_n t$$

The complete solution is(A homogeneous solution) is

$$u(t) = A \cos \omega_n t + B \sin \omega_n t + \frac{\omega_n^2 u_{st}}{(\omega_n^2 - \Omega^2)} \cos \Omega \mathrm{t}$$

Response to Harmonic Excitation

Undamped harmonic vibrations

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{\omega_n^2 u_{st}}{(\omega_n^2 - \Omega^2)}\cos\Omega t$$

By means of the initial conditions given by , $u(0) = u_0 \;\; \& \;\; \dot{u}(0) = \dot{u}_0$

the constants *A* and *B* can be calculated as follows:

$$A = u_0 - \frac{\omega_n^2 u_{st}}{(\omega_n^2 - \Omega^2)} \quad \& \quad B = \frac{\dot{u}_0}{\omega_n}$$

$$u(t) = \left(u_0 - \frac{\omega_n^2 u_{st}}{(\omega_n^2 - \Omega^2)}\right)\cos\omega_n t + \left(\frac{\dot{u}_0}{\omega_n}\right)\sin\omega_n t + \frac{\omega_n^2 u_{st}}{(\omega_n^2 - \Omega^2)}\cos\Omega t$$

For two null initial conditions the complete solution is,

$$u(t) = \frac{\omega_n^2 u_{st}}{(\omega_n^2 - \Omega^2)}\left(\cos\Omega t - \cos\omega_n t\right)$$

Response to Harmonic Excitation

Undamped harmonic vibrations

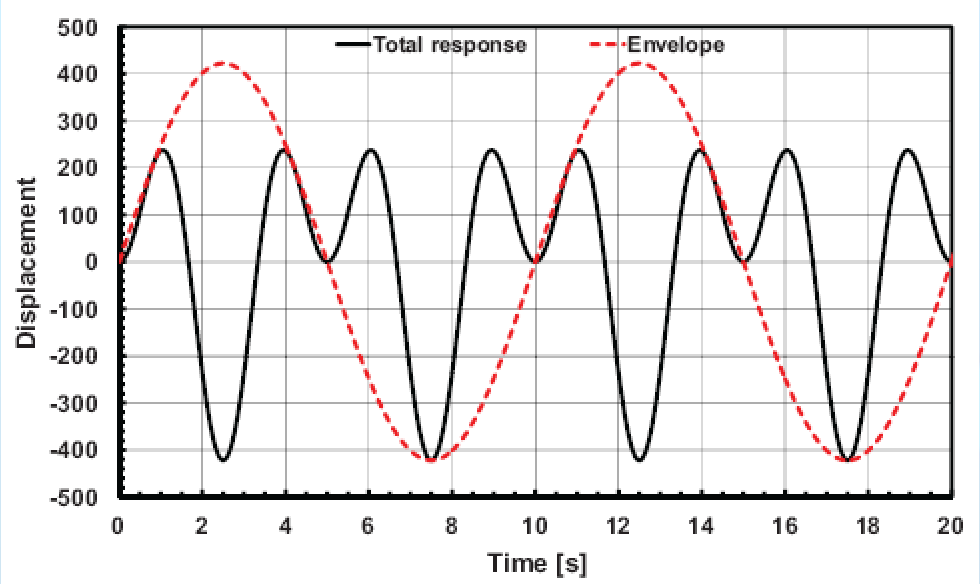
$$u(t)=\frac{\omega_n^2 u_{st}}{(\omega_n^2-\Omega^2)}(\cos\Omega t-\cos\omega_n t)$$

Using the trigonometric identity

$$\cos\alpha-\cos\beta=-2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$$

$$u(t)=\frac{2\omega_n^2 u_{st}}{(\Omega^2-\omega_n^2)}\sin\left(\frac{\Omega-\omega_n}{2}t\right)\sin\left(\frac{\Omega+\omega_n}{2}t\right)$$

Case 1: Natural frequency SDoF 0.2 Hz, excitation frequency 0.4 Hz. $\Omega/\omega_n=2$

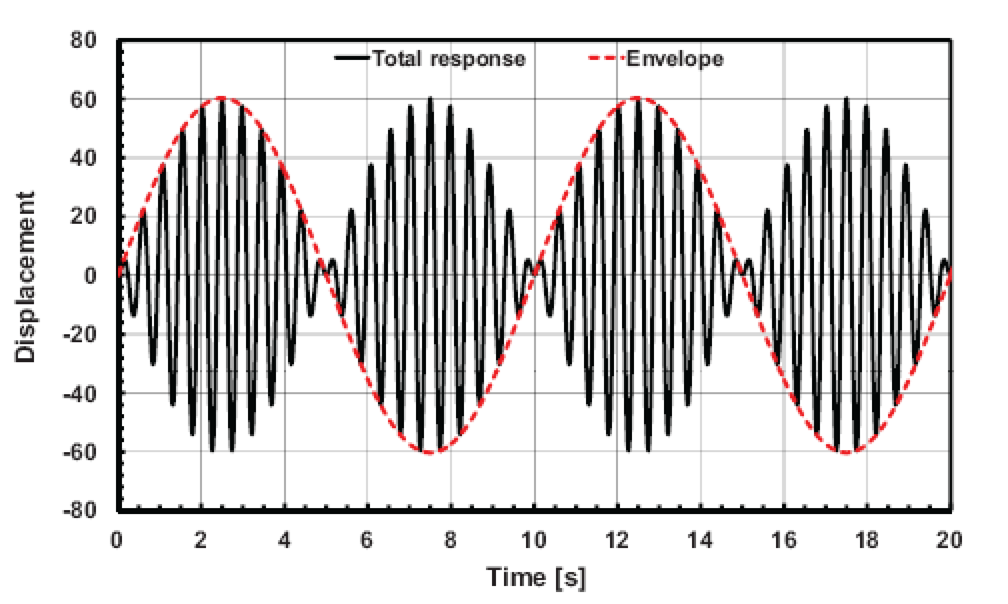


Response to Harmonic Excitation

Undamped harmonic vibrations

$$u(t) = \frac{2\omega_n^2 u_{st}}{(\Omega^2 - \omega_n^2)} \sin\left(\frac{\Omega - \omega_n}{2} t\right) \sin\left(\frac{\Omega + \omega_n}{2} t\right)$$

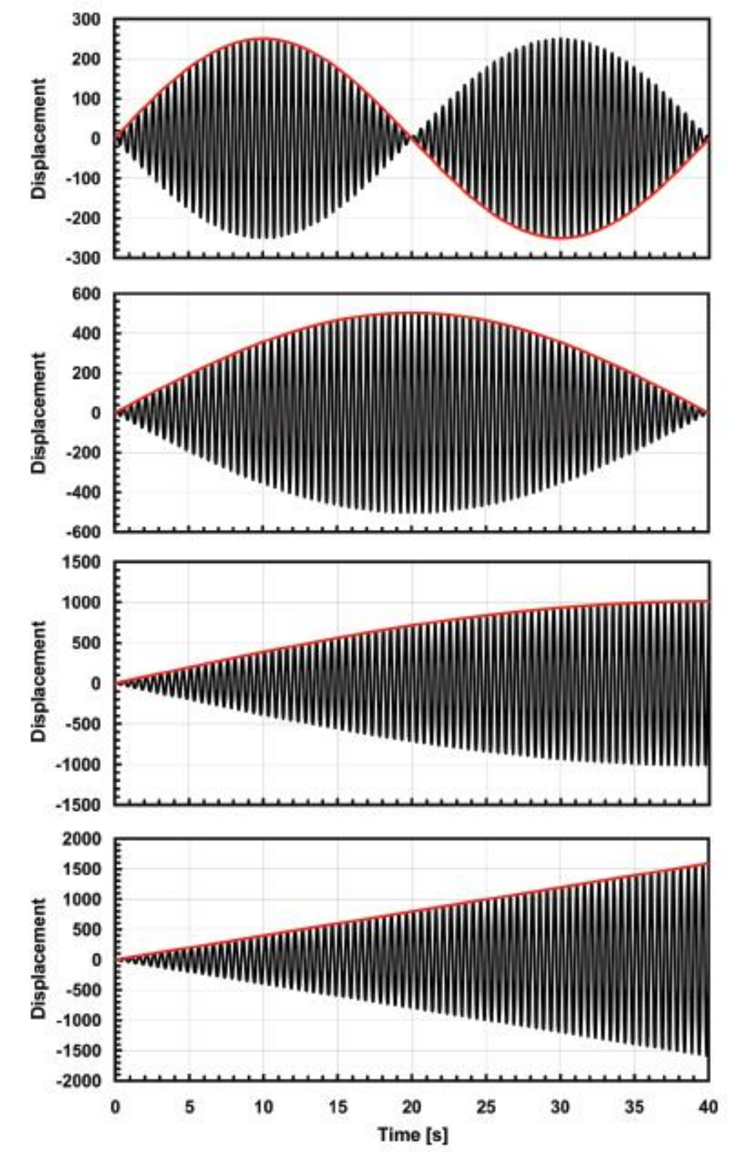
Case 2: Natural frequency SDoF 2.0 Hz, excitation frequency 2.2 Hz. $\Omega / \omega_n = 1.1$



A beat is always present, but is more evident when the natural frequency of the SDoF system and the excitation frequency are close

Response to Harmonic Excitation

Undamped harmonic vibrations



$\Omega / \omega_n = 1.025$

$\Omega / \omega_n = 1.0125$

$\Omega / \omega_n = 1.00625$

$\Omega / \omega_n = 1$
Resonance

Response to Harmonic Excitation

Undamped harmonic vibrations

Resonant excitation ($\Omega = \omega_n$)

$$\ddot{u} + \omega_n^2 u = \omega_n^2 u_{st} \cos \Omega t \qquad \ddot{u} + \omega_n^2 u = \omega_n^2 u_{st} \cos \omega_n t$$

A steady-state response (A particular solution) could not be $u_p(t) = C \cos \omega_n t$

Another possible choice is $u_p(t) = Ct \sin \omega_n t$

$$\dot{u}_p(t) = C \sin \omega_n t + Ct\,\omega_n \cos \omega_n t$$

$$\ddot{u}_p(t) = 2C\,\omega_n \cos \omega_n t - Ct\,\omega_n^2 \sin \omega_n t$$

Substituting into the E. o. M.

$$2C\,\omega_n \cos \omega_n t - Ct\,\omega_n^2 \sin \omega_n t + Ct\,\omega_n^2 \sin \omega_n t = \omega_n^2 u_{st} \cos \omega_n t \qquad 2C = \omega_n u_{st}$$

So the particular solution is $u_p(t) = \left(\dfrac{\omega_n u_{st}}{2} \right) t \sin \omega_n t$

Response to Harmonic Excitation

Undamped harmonic vibrations

The transient response (The homogeneous solution) is

$$u_h(t) = A\cos\omega_n t + B\sin\omega_n t$$

The complete solution is(A homogeneous solution) is

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \left(\frac{\omega_n u_{st}}{2}\right) t\sin\omega_n t$$

By means of the initial conditions given by , $u(0) = u_0 \;\&\; \dot{u}(0) = \dot{u}_0$

the constants *A* and *B* can be calculated as follows: $A = u_0 \;\&\; B = \dot{u}_0 / \omega_n$

$$u(t) = u_0\cos\omega_n t + \left(\frac{\dot{u}_0}{\omega_n}\right)\sin\omega_n t + \left(\frac{\omega_n u_{st}}{2}\right) t\sin\omega_n t$$

For two null initial conditions the homogeneous part of the solution falls away and the complete solution reduces to the particular solution,

$$u(t) = \left(\frac{\omega_n u_{st}}{2}\right) t\sin\omega_n t$$

Response to Harmonic Excitation

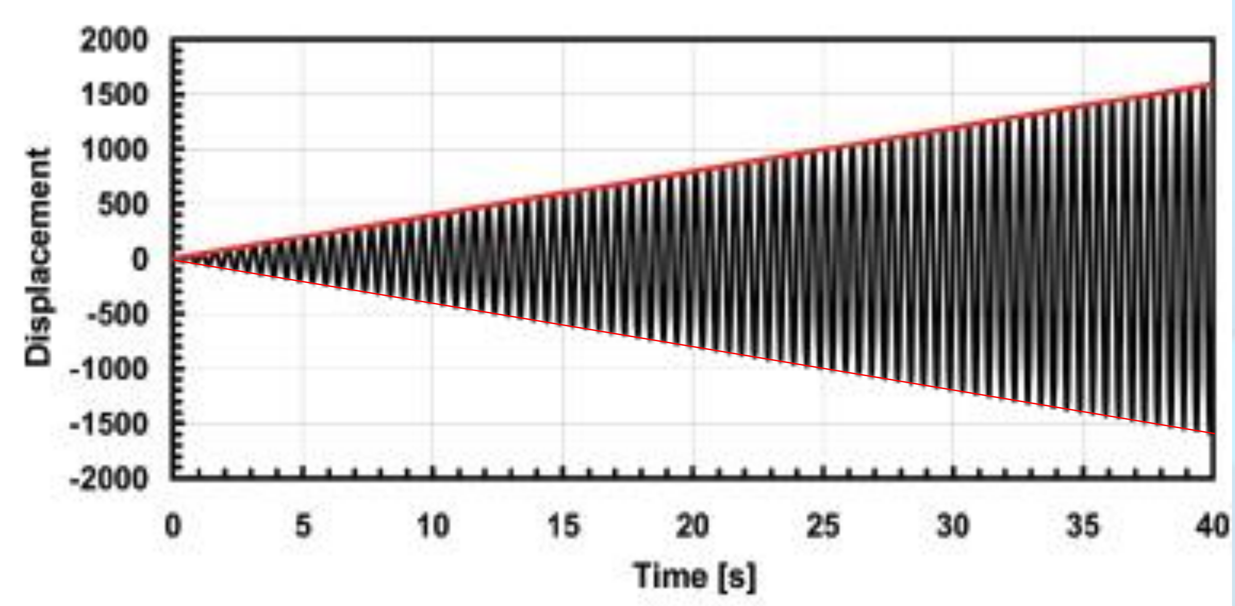
Undamped harmonic vibrations

$$u(t) = \left(\frac{\omega_n u_{st}}{2} \right) t \sin \omega_n t$$

This is a sinusoidal vibration with increasing amplitude: $C = (\omega_n u_{st}/2)t$.

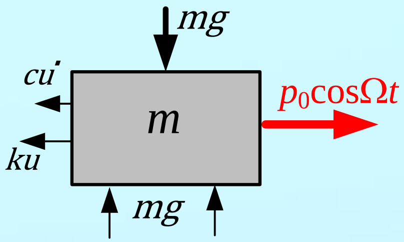
The amplitude grows linearly with time and when: $t \to \infty$, $C \to \infty$.

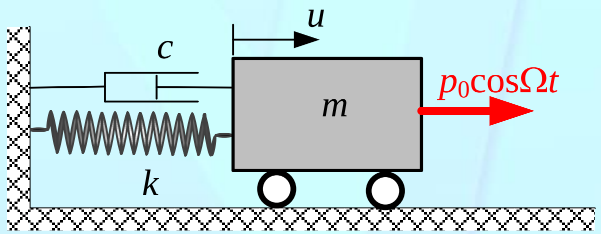
After infinite time the amplitude of the vibration is infinite as well.



Response to Harmonic Excitation

Damped harmonic vibrations





$$m\ddot{u} + c\dot{u} + ku = p_0\cos\Omega t \qquad \ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = (p_0/m)\cos\Omega t$$

Canonical E. o. M. $$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = \omega_n^2 u_{st}\cos\Omega t$$

Particular solution $$u_p(t) = C\cos\Omega t + D\sin\Omega t$$

$$\dot{u}_p(t) = -C\Omega\sin\Omega t + D\Omega\cos\Omega t$$

$$\ddot{u}_p(t) = -C\Omega^2\cos\Omega t - D\Omega^2\sin\Omega t$$ **Sub. Into E. o. M.**

$$-C\Omega^2\cos\Omega t - D\Omega^2\sin\Omega t + 2\xi\omega_n\left(-C\Omega\sin\Omega t + D\Omega\cos\Omega t\right) + \omega_n^2 u = \omega_n^2 u_{st}\cos\Omega t$$

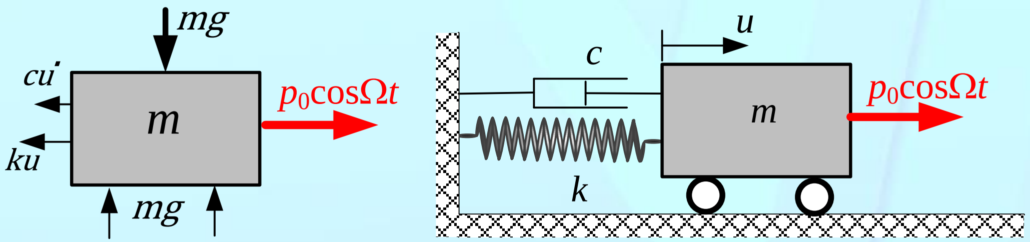
$$\left(\left(\omega_n^2 - \Omega^2\right)C + 2\xi\omega_n\Omega D\right)\cos\Omega t + \left(-2\xi\omega_n\Omega C + \left(\omega_n^2 - \Omega^2\right)D\right)\sin\Omega t = \omega_n^2 u_{st}\cos\Omega t$$

This is true at any time t, So

$$\left.\begin{aligned}\left(\omega_n^2 - \Omega^2\right)C + 2\xi\omega_n\Omega D &= \omega_n^2 u_{st}\\ -2\xi\omega_n\Omega C + \left(\omega_n^2 - \Omega^2\right)D &= 0\end{aligned}\right\} \Rightarrow \begin{cases} C = \omega_n^2 u_{st}\dfrac{\left(\omega_n^2 - \Omega^2\right)}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2} \\ D = \omega_n^2 u_{st}\dfrac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}\end{cases}$$

Response to Harmonic Excitation

Damped harmonic vibrations



Canonical E. o. M. $\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = \omega_n^2 u_{st}\cos\Omega t$

Particular solution $u_p(t) = C\cos\Omega t + D\sin\Omega t$

$$C = \omega_n^2 u_{st}\frac{\left(\omega_n^2 - \Omega^2\right)}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2} \quad \text{and} \quad D = \omega_n^2 u_{st}\frac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}$$

$$u_h(t) = e^{-\xi\omega_n t}\left(A\cos\omega_D t + B\sin\omega_D t\right) \quad \text{with} \quad \omega_D = \omega_n\sqrt{1-\xi^2}$$

By means of the initial conditions the constants *A* and *B*, can be determined

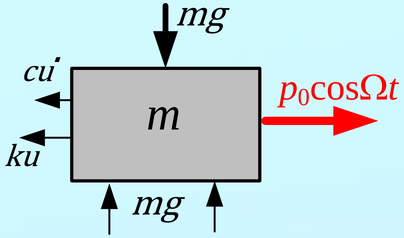
Denominations:

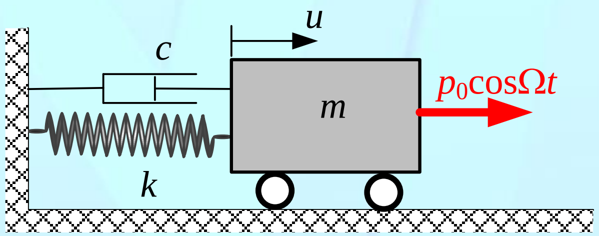
- Homogeneous part of the solution: **"transient"**
- Particular part of the solution: **"steady-state"**

Visualization of the solution is illustrated in the next example

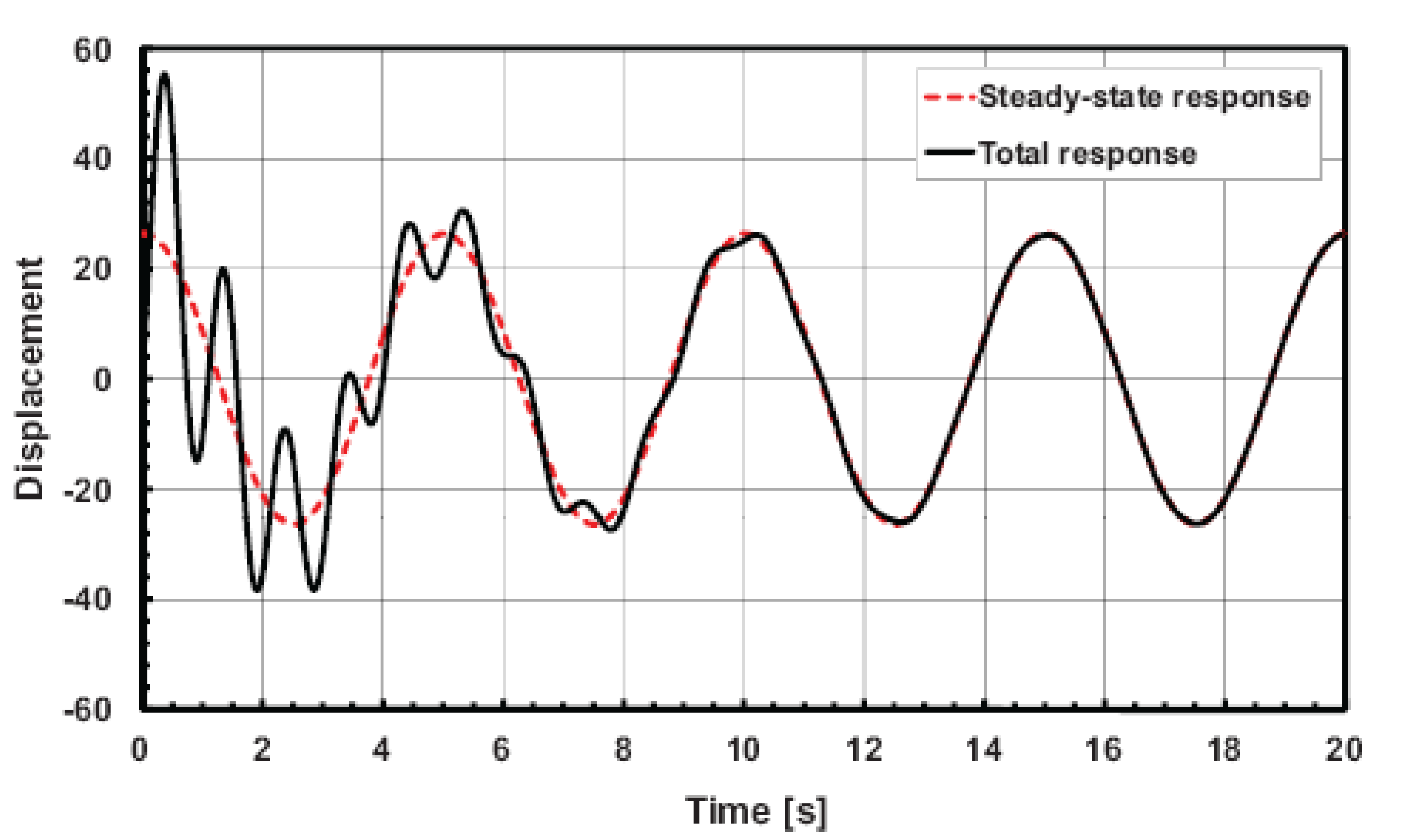
Response to Harmonic Excitation

Damped harmonic vibrations



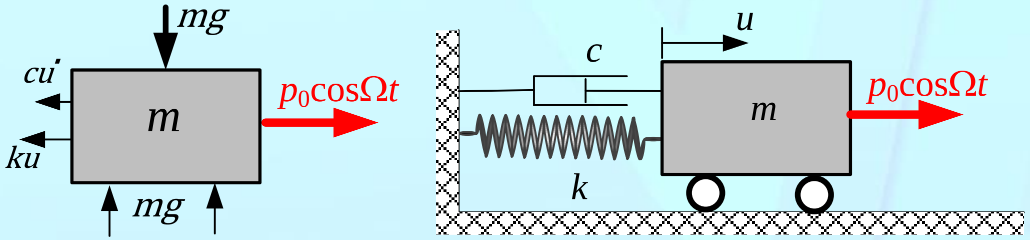


Example 1: $\omega_n = 2\pi$ [rad/sec], $\Omega = 0.4\pi$ [rad/sec], $\xi = 5\%$, u_{st}=25mm, u_0=0, $\dot{u}_0 = u_{st}\,\omega_n$



Response to Harmonic Excitation

Damped harmonic vibrations



Resonant excitation ($\Omega = \omega_n$)

Canonical E. o. M. $$\ddot{u} + 2\xi\omega_n\dot{u} + \omega_n^2 u = \omega_n^2 u_{st}\cos\Omega t$$

$$u_p(t) = C\cos\Omega t + D\sin\Omega t$$

$$C = \omega_n^2 u_{st}\frac{\left(\omega_n^2 - \Omega^2\right)}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2} \quad \text{and} \quad D = \omega_n^2 u_{st}\frac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}$$

By substituting **($\Omega = \omega_n$)** in thee two expressions constants C and D, becomes:

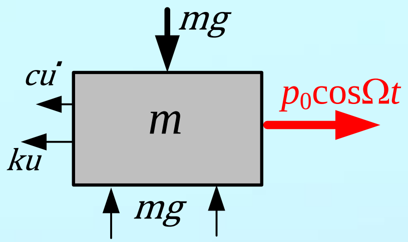
$$C = 0 \quad \text{and} \quad D = \frac{u_{st}}{2\xi}$$

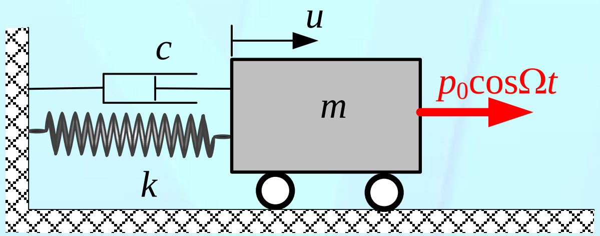
This means that if damping is present, the resonant excitation is not a special case any more, and the complete solution of the differential equation is:

$$u(t) = e^{-\xi\omega_n t}\left(A\cos\omega_D t + B\sin\omega_D t\right) + \frac{u_{st}}{2\xi}\sin\omega_n t$$

Response to Harmonic Excitation

Damped harmonic vibrations





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$$C = \omega_n^2 u_{st} \frac{\left(\omega_n^2 - \Omega^2\right)}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2} \quad \text{and} \quad D = \omega_n^2 u_{st} \frac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}$$

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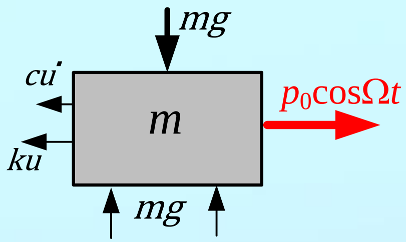
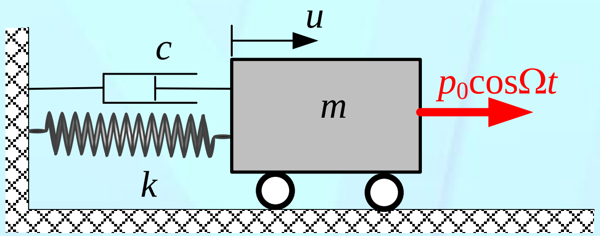
Response to Harmonic Excitation

Damped harmonic vibrations





Resonant excitation ($\Omega = \omega_n$)

$$u(t) = e^{-\xi\omega_n t}(A\cos\omega_D t + B\sin\omega_D t) + \frac{u_{st}}{2\xi}\sin\omega_n t$$

By means of the initial conditions the constants *A* and *B*, can be determined. For example in the special case, $u_0 = 0$ & $\dot{u}_0 = 0$, *A* & *B*, are

$$A = 0 \quad \text{and} \quad B = -\frac{u_{st}}{2\xi\sqrt{1-\xi^2}}$$

$$u(t) = \frac{u_{st}}{2\xi}\left(\sin\omega_n t - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin\omega_D t\right)$$

After a certain time, the homogeneous part of the solution vanishes and what remains is a sinusoidal oscillation of the maximum limited amplitude: ($u_{max}=u_{st}/2\xi$)

For small damping ratios ($\xi < 0.2$), $\omega_n \approx \omega_D$ and $(1-\xi)^{1/2} \approx 1$, hence $u(t)$ becomes:

$$u(t) = u_{max}\left(1 - e^{-\xi\omega_n t}\right)\sin\omega_n t$$

Response to Harmonic Excitation

Damped harmonic vibrations

Dynamic Amplification Factor

The steady-state displacement of a system due to harmonic excitation is the dominant part of its response. This steady-state response is given by

$$u_p(t) = C\cos\Omega t + D\sin\Omega t$$

Where $C = \omega_n^2 u_{st} \dfrac{\left(\omega_n^2 - \Omega^2\right)}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}$ and $D = \omega_n^2 u_{st} \dfrac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}$

By means of the trigonometric identity:

$$a\cos\alpha + b\sin\alpha = (a^2 + b^2)^{1/2}\cos(\alpha - \beta) \quad \text{with} \quad \tan\beta = b/a$$

The steady-state response can be transformed as follows

$$u_p(t) = u_{max}\cos(\Omega t - \varphi)$$

It is a cosine vibration with the maximum dynamic amplitude u_{max}, given by

$$u_{max} = (C^2 + D^2)^{1/2}$$

and the phase angle φ obtained from:

$$\tan\varphi = D/C$$

Response to Harmonic Excitation

Damped harmonic vibrations

Dynamic Amplification Factor

Substitution of C and D, in $u_{\max}$ expression gives

$$u_{\max} = \sqrt{\left[\omega_n^2 u_{st} \frac{\left(\omega_n^2 - \Omega^2\right)}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}\right]^2 + \left[\omega_n^2 u_{st} \frac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}\right]^2}$$

$$u_{\max} = \omega_n^2 u_{st} \sqrt{\left[\frac{\left(\omega_n^2 - \Omega^2\right)}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}\right]^2 + \left[\frac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}\right]^2}$$

$$u_{\max} = \omega_n^2 u_{st} \sqrt{\frac{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}{\left[\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2\right]^2}}$$

$$u_{\max} = \omega_n^2 u_{st} \frac{1}{\sqrt{\left(\omega_n^2 - \Omega^2\right)^2 + \left(2\xi\omega_n\Omega\right)^2}}$$

$$u_{\max} = u_{st} \frac{1}{\sqrt{\left(1 - \left(\Omega/\omega_n\right)^2\right)^2 + \left(2\xi\left(\Omega/\omega_n\right)\right)^2}}$$

$$\text{DAF} = \frac{u_{\max}}{u_{st}} = \frac{1}{\sqrt{\left(1 - \left(\Omega/\omega_n\right)^2\right)^2 + \left(2\xi\left(\Omega/\omega_n\right)\right)^2}}$$

Response to Harmonic Excitation

Damped harmonic vibrations

Dynamic Amplification Factor

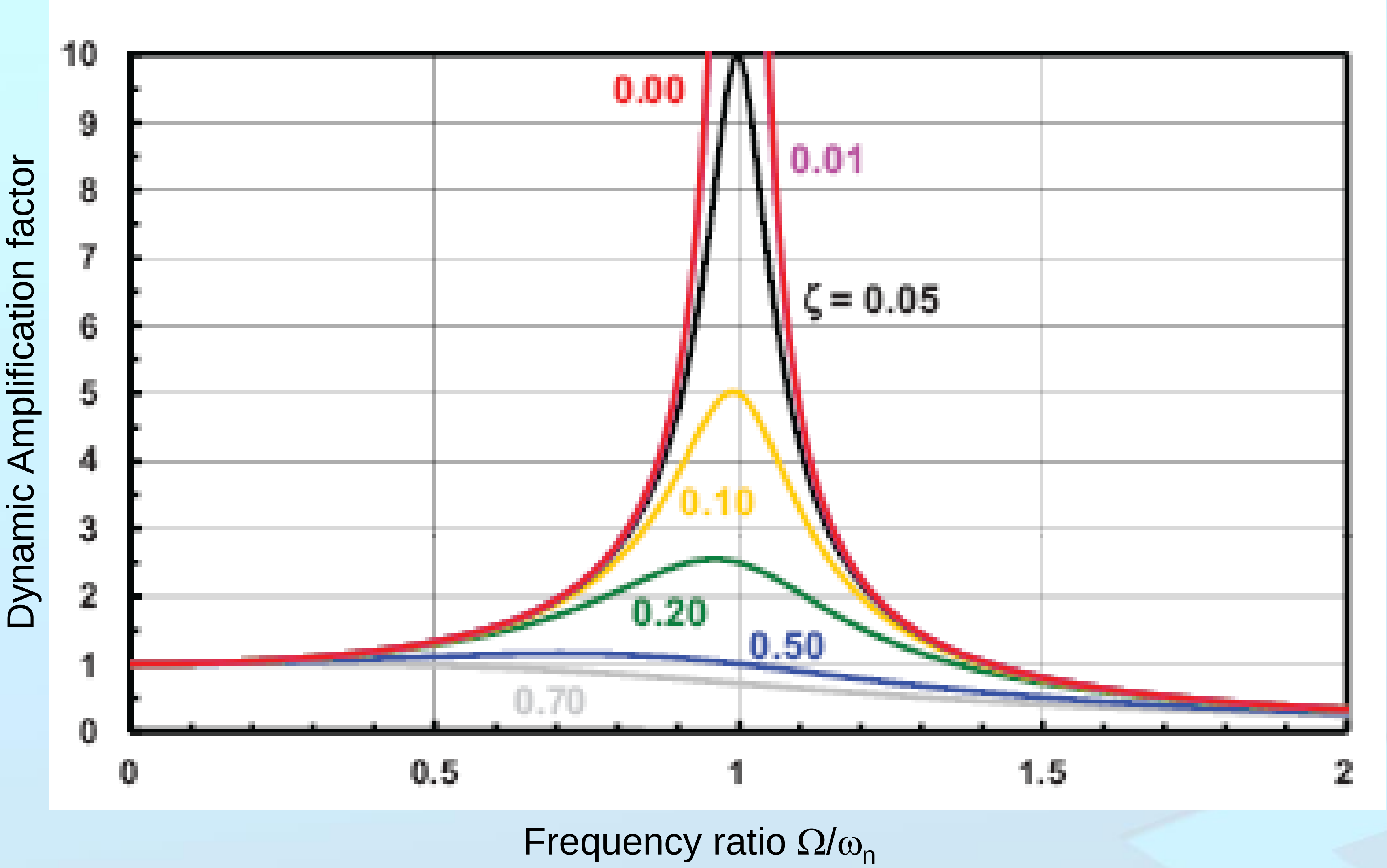
Substitution of C and D, in $\tan\varphi$ expression gives

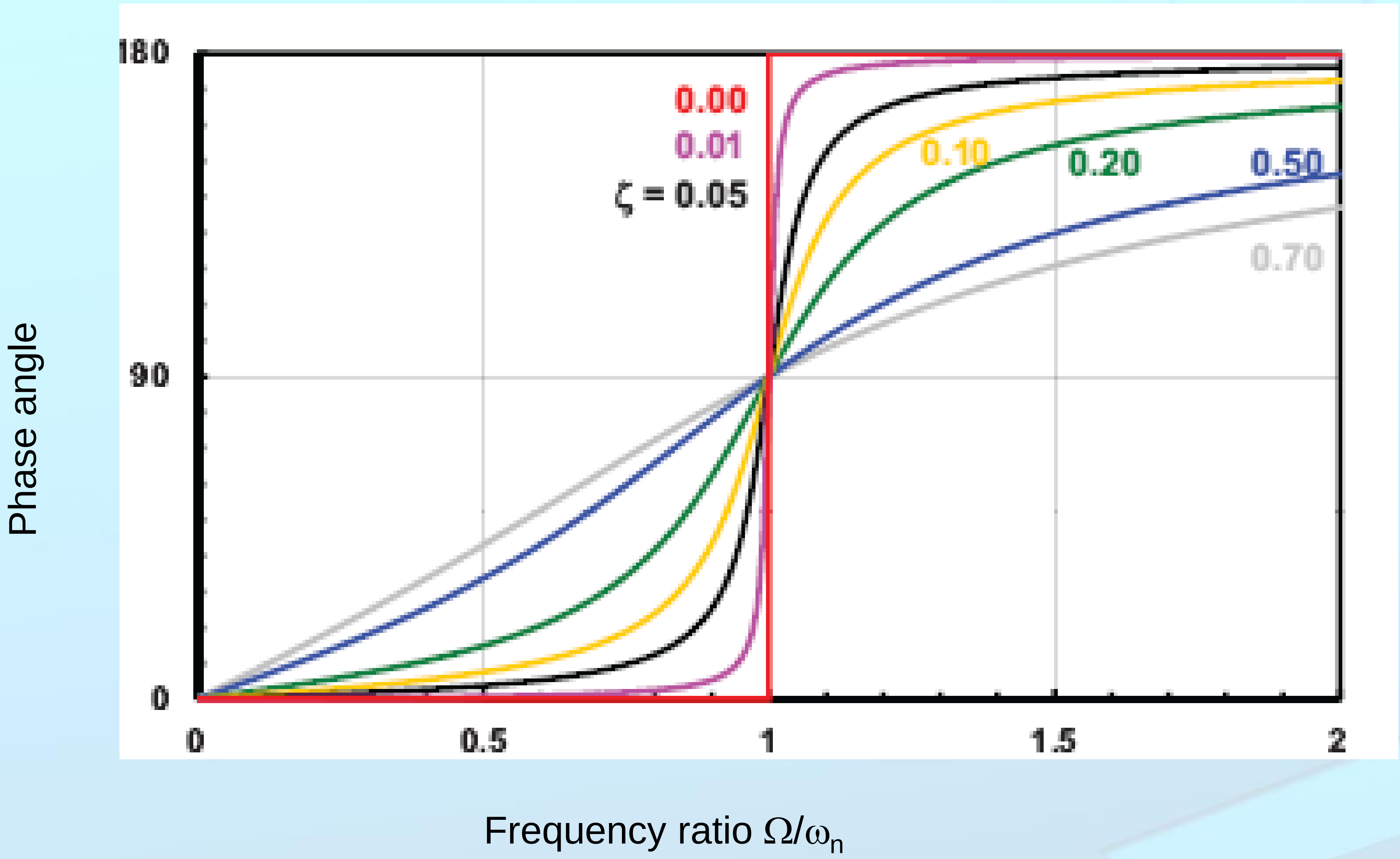
$$\tan\varphi = \frac{D}{C} = \frac{2\xi\omega_n\Omega}{\left(\omega_n^2 - \Omega^2\right)} = \frac{2\xi\left(\Omega/\omega_n\right)}{1-\left(\Omega/\omega_n\right)^2}$$

Defining the ratio $r = \Omega/\omega_n$, the two expressions simplify to

$$\text{DAF} = \frac{u_{\max}}{u_{st}} = \frac{1}{\sqrt{\left(1-r^2\right)^2 + \left(2\xi r\right)^2}}$$

$$\tan\varphi = \frac{2\xi r}{1-r^2}$$


Dynamic Amplification factor
Frequency ratio Ω/ωn
0.00
0.01
ζ = 0.05
0.10
0.20
0.50
0.70
0
1
2
3
4
5
6
7
8
9
10
0
0.5
1
1.5
2


Phase angle
180
90
0
0.00
0.01
ζ = 0.05
0.10
0.20
0.50
0.70
0
0.5
1
1.5
2
Frequency ratio Ω/ωn

Ex. 1. An undamped oscillator is driven by an harmonic loading. If the static displacement is u_{st}= 0.05m, determine the displacement response amplitude for the following frequency ratios: r = 0.2, 0.9, 1.1, 1.8 & 3.0.

Ex. 2 An undamped system consisting of a 10 kg mass and a spring of stiffness k =4 kN/m is acted upon by a harmonic force of magnitude P_0=0.5kN. The displacement amplitude of the steady-state response was observed to be 11 cm. Determine the frequency of the excitation force.

Ex. 3. An undamped system having a mass of 50 kg is excited by a harmonic force with magnitude P_0=100 N and an operating frequency of 10 Hz. The displacement amplitude of the steady-state response was observed to be 3.2 mm. Determine the spring constant k of the system.

Ex. 4. An undamped system having a mass of 10 kg and a spring of constant of k =8 N/mm is excited by a harmonic force with magnitude F_0=200 N and an operating frequency of 35 rad/sec. If the initial displacement is 21 mm and the initial velocity is 175mm/sec, determine the total displacement, velocity and acceleration of the mass at (a) t =2 sec, (b) t =4 sec and (c) t =6 sec.

Ex. 5. A portable eccentric mass shaker is sometimes used to evaluate the *in situ* dynamic properties of a structure, using two different frequencies and measuring the displacement amplitudes as well as the phase angles. Such a test was carried out on a single story building and the following responses were recorded:

(1) at Ω_1=18.30 rad/s, P_{o1}=837 kN, u_{max1}= 1.39mm & φ_1=8°;

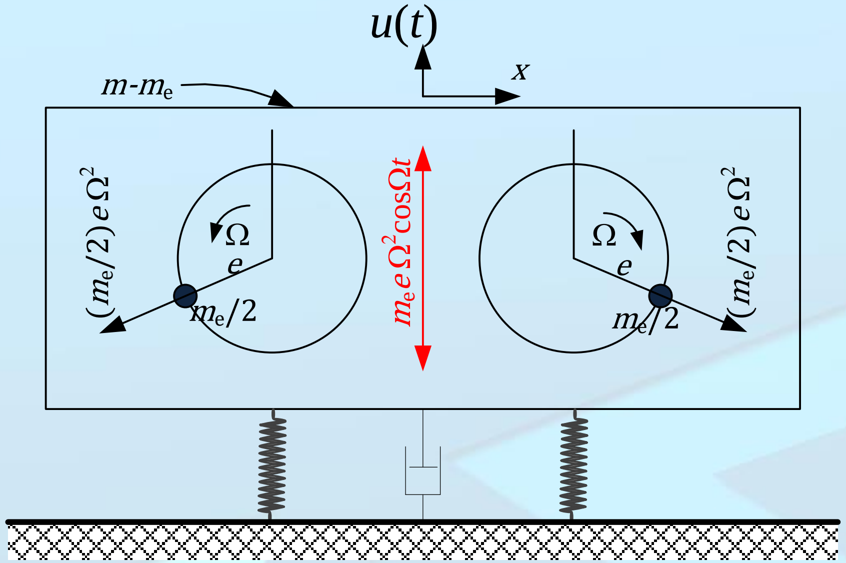
(2) at Ω_2=60.99 rad/s, P_{o2}= 9300 kN, u_{max2}= 3.32mm & φ_2=174.29°.

Compute the natural frequency ω_n & the damping ratio ξ for the structure

$$\text{DAF} = \frac{u_{max}}{u_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \qquad \tan\varphi = \frac{2\xi r}{1-r^2}$$

$$u_{max} = \frac{u_{st}}{(1-r^2)} \frac{1}{\sqrt{1+\left[(2\xi r)/(1-r^2)\right]^2}}$$

$$u_{max} = \frac{u_{st}\cos\varphi}{(1-r^2)} = \frac{u_{st}\,\omega_n^2\cos\varphi}{(\omega_n^2-\Omega^2)} = \frac{P_0\cos\varphi}{m(\omega_n^2-\Omega^2)}$$



Ex.6. An undamped spring-mass system having a mass of 4.5 kg and a spring of constant of k =3.5 N/mm is excited by a harmonic force with magnitude F_0=100 N and an operating frequency of 18 rad/sec. If the initial displacement is 15 mm and the initial velocity is 150 mm/sec, determine

(a) The frequency ratio

(b) The amplitude o the forced response

(c) The displacement of the mass at t =2 sec

Ex.7. A Structure having a mass of 100 kg and a translational stiffness of 40000 N/m is excited by a harmonic force with magnitude F_0=500 N and an operating frequency of 2.5 Hz. The damping ratio for the structure is 0.10. For the steady-state vibration determine

(a) The amplitude of the steady-state displacement

(b) Its phase with respect to the exciting force, and

(c) The maximum velocity of the response