Example

$$\dot{x}(t) = \begin{bmatrix} 2.2 & 1 \\ 3 & 6.5 \end{bmatrix} x(t) + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [2\ 1]\, x(t)$$

Solution

We use the MATLAB command: [num,den] = ss2tf(A,B,C,D,iu)

We enter **A= [2.2 1; 3 6.5]; B=[3;1]; C=[2 1];**

We must enter a D matrix with size of 'outputs by inputs' – in this example 1x1:

D=[0];

We only have one input so 'iu' will be 1. **[num,den] = ss2tf(A,B,C,D,1)**

MATLAB result:

```
num =
        0    7.0000  -30.2000
den =
        1.0000   -8.7000   11.3000
```

$$Y(S) = C(SI - A)^{-1}.B.U(s) + C(SI - A)^{-1}.X(0)$$

$$A = \begin{bmatrix} 2.2 & 1 \\ 3 & 6.5 \end{bmatrix}$$

$$B = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad , \quad C = [2\ 1]$$

$$SI - A = S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2.2 & 1 \\ 3 & 6.5 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S - 2.2 & -1 \\ -3 & S - 6.5 \end{bmatrix}$$

$$\det(SI - A) = (S - 2.2)(S - 6.5) - 3$$

$$\det(SI - A) = S^2 - 8.7S + 11.3$$

$$SI - A = \begin{bmatrix} S - 2.2 & -1 \\ -3 & S - 6.5 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{1}{\det(\mathrm{SI} - \mathrm{A})}\mathrm{adj}(\mathrm{SI} - \mathrm{A})$$

$$(SI - A)^{-1} = \frac{1}{S^2 - 8.7S + 11.3}\begin{bmatrix} S - 6.5 & 1 \\ 3 & S - 2.2 \end{bmatrix}$$

$$(SI - A)^{-1} = \begin{bmatrix} \frac{S - 6.5}{S^2 - 8.7S + 11.3} & \frac{1}{S^2 - 8.7S + 11.3} \\ \frac{3}{S^2 - 8.7S + 11.3} & \frac{S - 2.2}{S^2 - 8.7S + 11.3} \end{bmatrix}$$

$$C(SI - A)^{-1}(B) = [2\ 1]\begin{bmatrix} \frac{S - 6.5}{S^2 - 8.7S + 11.3} & \frac{1}{S^2 - 8.7S + 11.3} \\ \frac{3}{S^2 - 8.7S + 11.3} & \frac{S - 2.2}{S^2 - 8.7S + 11.3} \end{bmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \left[\frac{6S - 30}{S^2 - 8.7S + 11.3} + \frac{S - 0.2}{S^2 - 8.7S + 11.3}\right]$$

$$= \left[\frac{7S - 30.2}{S^2 - 8.7S + 11.3}\right]$$

Example The following transfer function represents a 4^{th} order system

$$G(s) = \frac{3}{s^4 + 2s^3 + 10s^2 + 6s + 3}$$

(i) Write down an equivalent state space representation.

(ii) Enter this model in MATLAB as an A,B,C,D representation.

(iii) Use the MATLAB command tf2ss to create a different state variable model.

Solution

(i)

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \dot{x}_4(t)\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -3 & -6 & -10 & -2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\\ x_4(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}u(t)$$

$$y(t) = [3\ 0\ 0\ 0]\ \mathbf{x}(t)$$

(ii)

```
A1= [0 1 0 0 ; 0 0 1 0; 0 0 0 1; -3 -6 -10 -2]; B1= [0;0;0;1]; C1=[3 0 0 0]; D1=[0];
system1= ss(A1,B1,C1,D1);
```

(iii)

```
num=[3]; den=[1 2 10 6 3];

[A2,B2,C2,D2]=tf2ss([3],[1 2 10 6 3])
```

MATLAB gives the following results

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} -2 & -10 & -6 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [0\ 0\ 0\ 3]\ \mathbf{x}(t)$$