

Structural Mechanics (2)

Lecture No-08

Part-01

Displacement Method for Beams and Frames or Slope-Deflection Method

Displacement Method for Beams and Frames (Slope – Deflection Method)

- Basic Concept of the Slope-Deflection Method and Slope-Deflection Equations.
- Analysis of Continuous Beams.
- Analysis of Frames without Sidesway.
- Analysis of Frames with Sidesway.

Beam Displacements

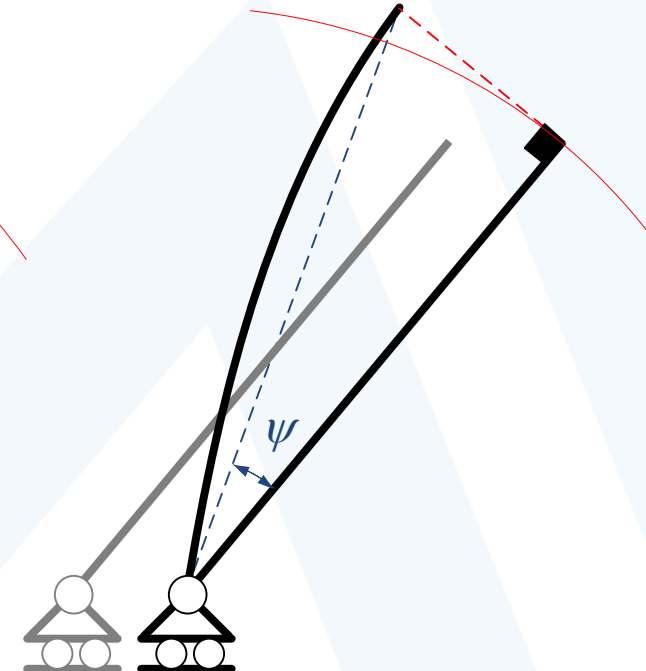
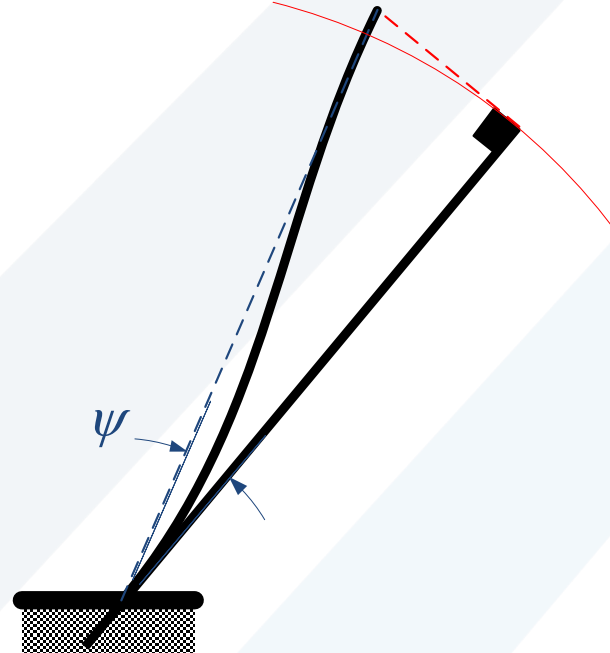
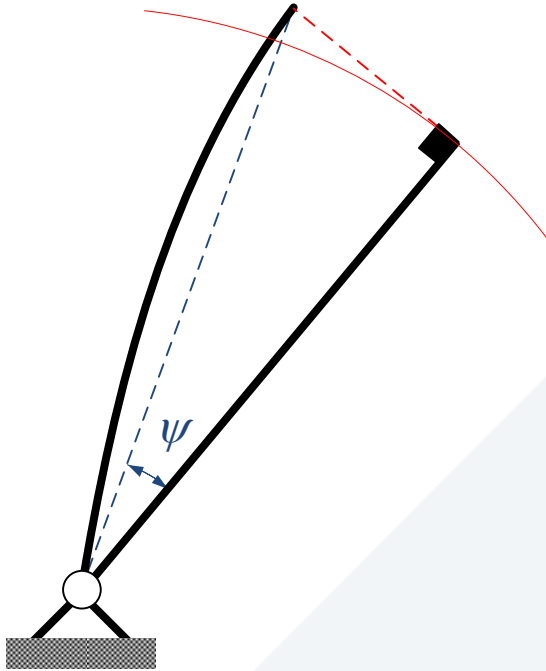
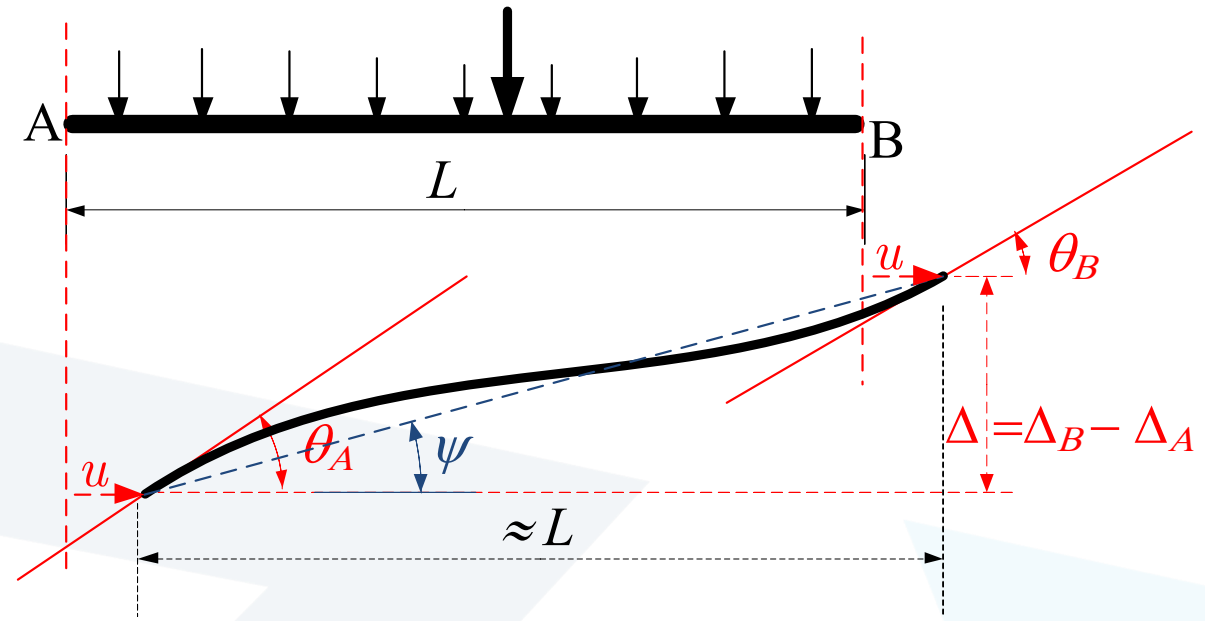
Δ Deflection

u Axial displacement

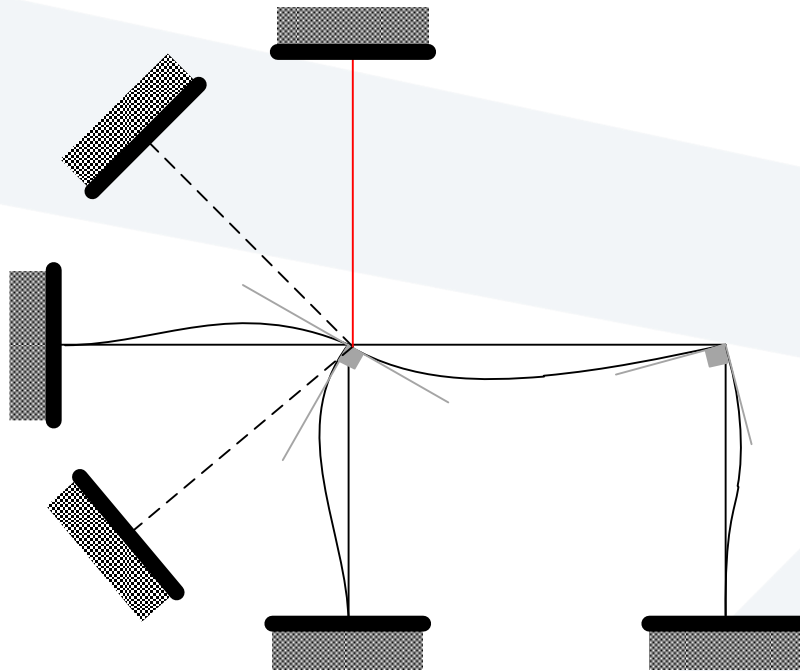
$\theta \ll 1, \sin \theta \approx \theta, \cos \theta \approx 1$

$\tan \psi \approx \sin \psi \approx \Delta/L$

$\psi \ll 1, \sin \psi \approx \psi, \cos \psi \approx 1$



Analysis of Frames with Sidesway



(a) Frame without Sidesway

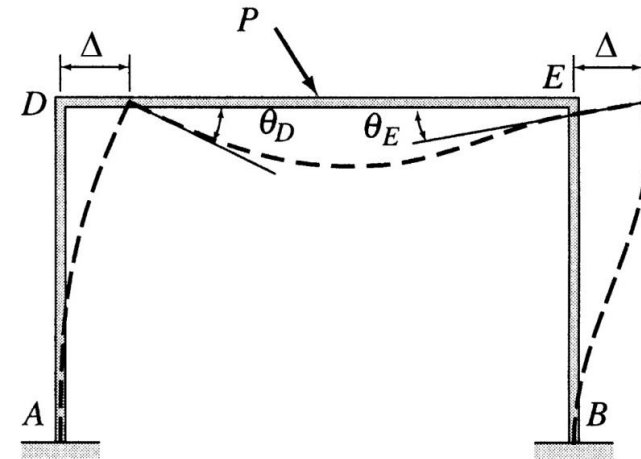
$$N_{ss} = 2j - [2(f+h) + r + m]$$

j : Number of joints f : Number of fixed supports

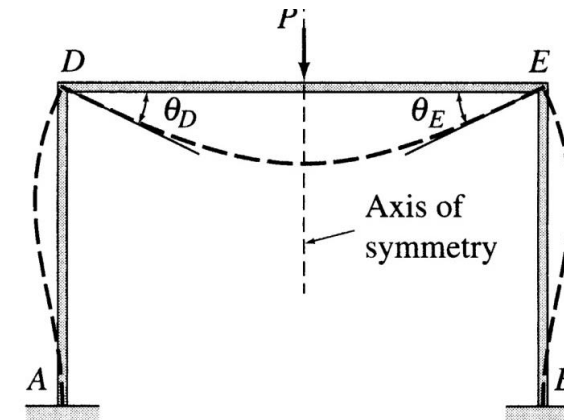
h : Number of hinged supports

r : Number of roller supports

m : Number of **effective** inextensible members



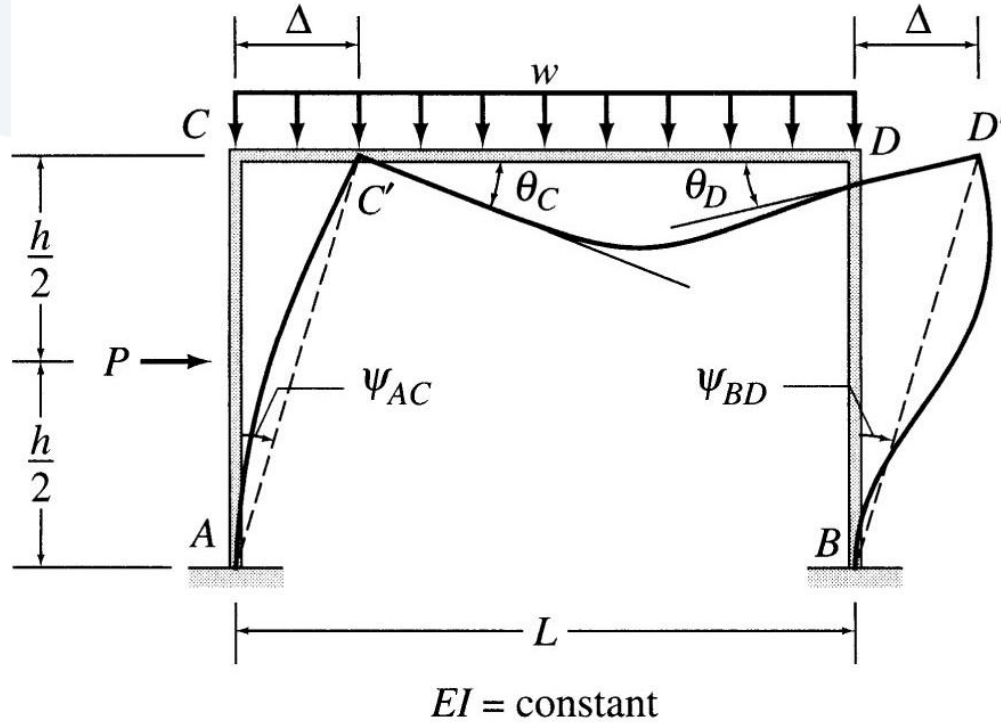
(b) Frame with Sidesway



$EI = \text{constant}$

(c) Symmetric Frame Subjected to Symmetric Loading — No Sidesway

Analysis of Frames with Sidesway



3 kinematic unknowns
 θ_C, θ_D & Δ .

$$M_{AC} = \frac{2EI}{h} \left(\theta_C + \frac{3\Delta}{h} \right) + FEM_{AC}$$

$$M_{CA} = \frac{2EI}{h} \left(2\theta_C + \frac{3\Delta}{h} \right) + FEM_{CA}$$

$$M_{BD} = \frac{2EI}{h} \left(\theta_D + \frac{3\Delta}{h} \right)$$

$$M_{DB} = \frac{2EI}{h} \left(2\theta_D + \frac{3\Delta}{h} \right)$$

$$M_{CD} = \frac{2EI}{L} (2\theta_C + \theta_D) + FEM_{CD}$$

$$M_{DC} = \frac{2EI}{L} (2\theta_D + \theta_C) + FEM_{DC}$$

Analysis of Frames with Sidesway

$$M_{CA} + M_{CD} = 0 \quad (1)$$

$$M_{DB} + M_{DC} = 0 \quad (2)$$

$$P - S_{AC} - S_{BD} = 0$$

$$\downarrow \uparrow + \sum M_C^{AC} = 0$$

$$M_{AC} - S_{AC}(h) + P(h/2) + M_{CA} = 0$$

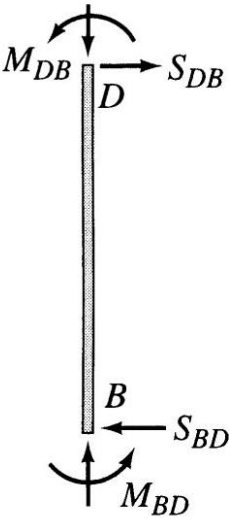
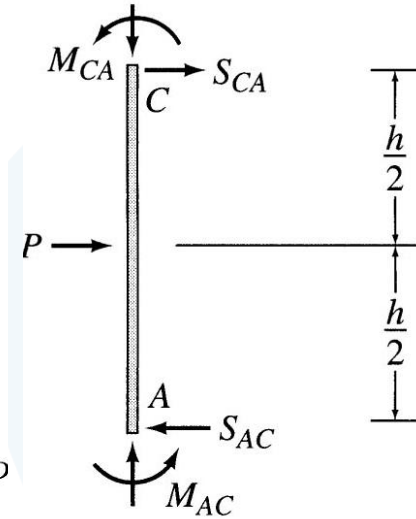
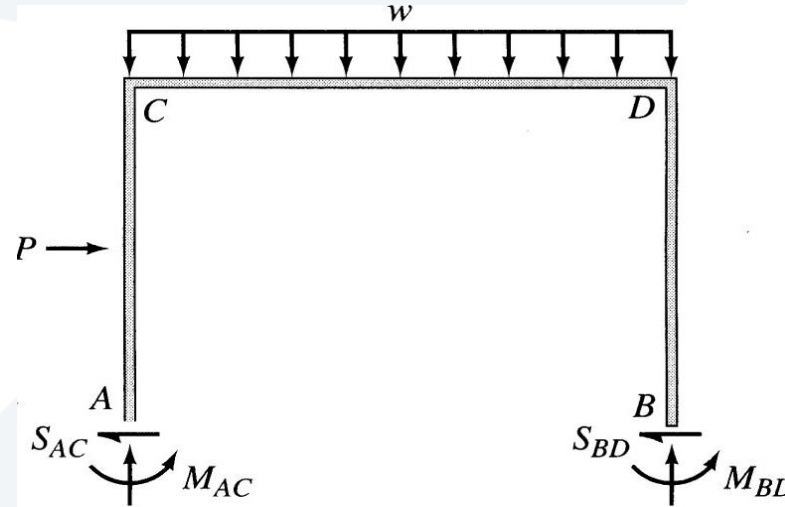
$$S_{AC} = \frac{M_{AC} + M_{CA}}{h} + \frac{P}{2}$$

$$\downarrow \uparrow + \sum M_D^{BD} = 0$$

$$M_{BD} - S_{BD}(h) + M_{DB} = 0$$

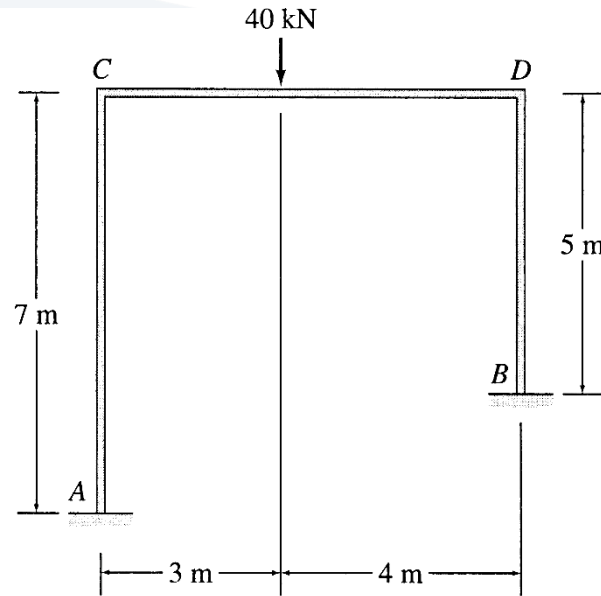
$$S_{BD} = \frac{M_{BD} + M_{DB}}{h}$$

$$M_{AC} + M_{CA} + M_{BD} + M_{DB} = ph/2 \quad (3)$$



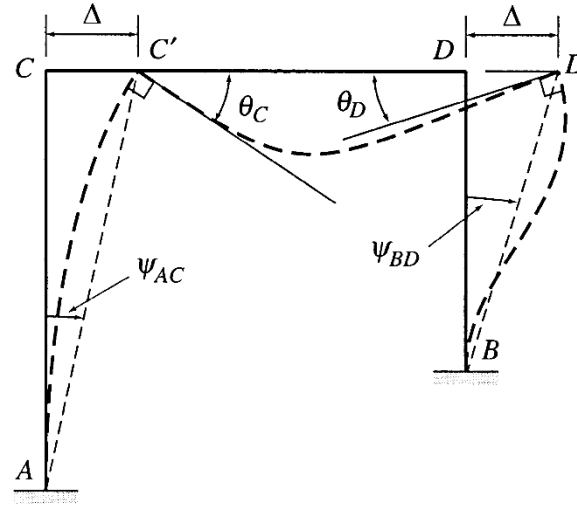
(1), (2) & (3) three equations with three unknowns θ_C, θ_D & Δ .

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



$EI = \text{constant}$

(a) Frame



(b) Qualitative Deflected Shape of the Frame

Degrees of freedom:

$$\theta_A = 0 \text{ \& } \theta_B = 0$$

$$\theta_C \neq 0, \theta_D \neq 0, \Delta \neq 0$$

Chord rotations:

$$\psi_{AC} = -\frac{\Delta}{7} \quad \psi_{BD} = -\frac{\Delta}{5}$$

$$\psi_{BD} = -\frac{\Delta}{5}$$

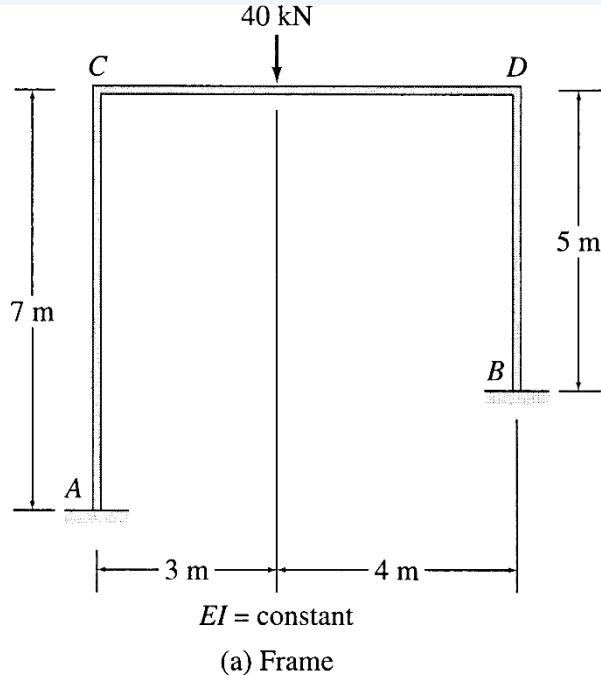
Fixed-End Moments:

$$FEM_{AC} = FEM_{CA} = FEM_{DB} = FEM_{BD} = 0$$

$$FEM_{CD} = \frac{40(3)(4)^2}{(7)^2} = 39.2 \text{ kN.m} \curvearrowright \text{ or } + 39.2 \text{ kN.m}$$

$$FEM_{DC} = -\frac{40(3)^2(4)}{(7)^2} = 29.4 \text{ kN.m} \curvearrowleft \text{ or } - 29.4 \text{ kN.m}$$

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



Slope Deflection Equations:

$$M_{AC} = \frac{2EI}{7} \left[\theta_C - 3 \left(-\frac{\Delta}{7} \right) \right] = 0.286EI\theta_C + 0.122EI\Delta$$

$$M_{CA} = \frac{2EI}{7} \left[2\theta_C - 3 \left(-\frac{\Delta}{7} \right) \right] = 0.571EI\theta_C + 0.122EI\Delta$$

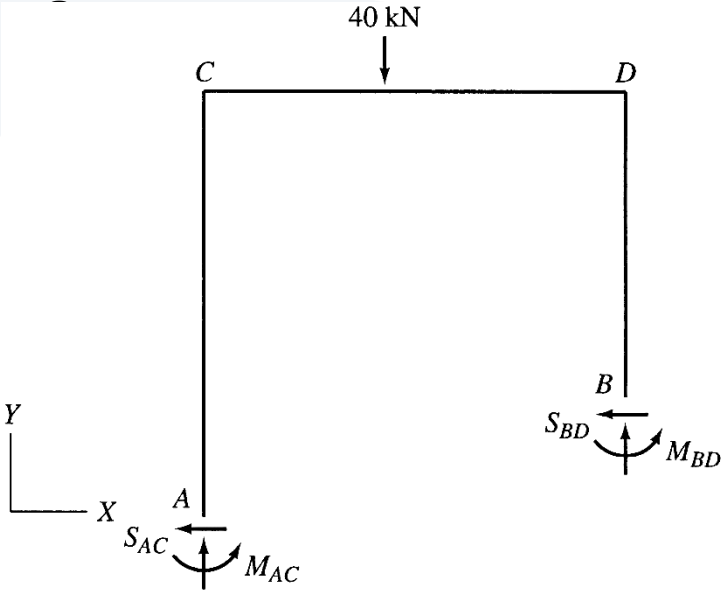
$$M_{BD} = \frac{2EI}{5} \left[\theta_D - 3 \left(-\frac{\Delta}{5} \right) \right] = 0.4EI\theta_D + 0.24EI\Delta$$

$$M_{DB} = \frac{2EI}{5} \left[2\theta_D - 3 \left(-\frac{\Delta}{5} \right) \right] = 0.8EI\theta_D + 0.24EI\Delta$$

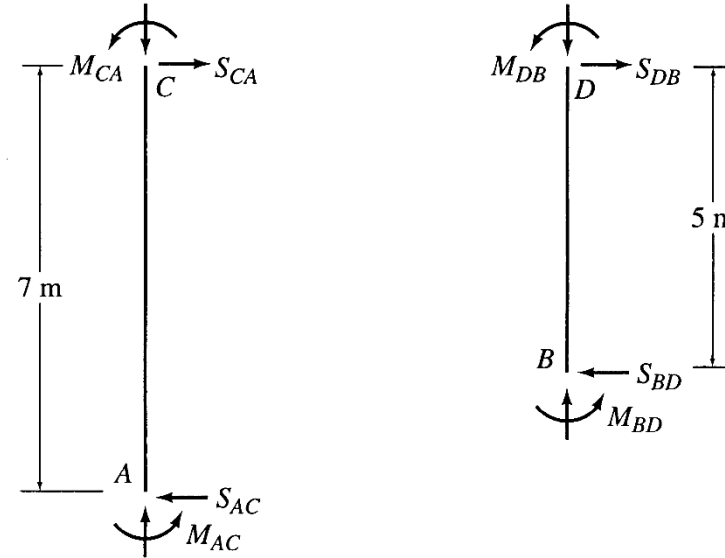
$$M_{CD} = \frac{2EI}{7} (2\theta_C + \theta_D) + 39.2 = 0.571EI\theta_C + 0.286EI\theta_D + 39.2$$

$$M_{DC} = \frac{2EI}{7} (\theta_C + 2\theta_D) - 29.4 = 0.286EI\theta_C + 0.571EI\theta_D - 29.4$$

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method



(c) Free-Body Diagram of the Entire Frame



(d) Free-Body Diagrams of Columns AC and BD

Equilibrium Equations:

$$M_{CA} + M_{CD} = 0$$

$$M_{DC} + M_{DB} = 0$$

$$S_{AC} + S_{BD} = 0$$

$$S_{AC} = \frac{M_{AC} + M_{CA}}{7} \quad \text{and} \quad S_{BD} = \frac{M_{BD} + M_{DB}}{5}$$

$$\frac{M_{AC} + M_{CA}}{7} + \frac{M_{BD} + M_{DB}}{5} = 0$$

$$5(M_{AC} + M_{CA}) + 7(M_{BD} + M_{DB}) = 0$$

Joint Displacements:

$$1.142EI\theta_C + 0.286EI\theta_D + 0.122EI\Delta = -39.2$$

$$0.286EI\theta_C + 1.371EI\theta_D + 0.24EI\Delta = 29.4$$

$$4.285EI\theta_C + 8.4EI\theta_D + 4.58EI\Delta = 0$$

$$EI\theta_C = -40.211 \text{ kN.m}^2$$

$$EI\theta_D = 34.24 \text{ kN.m}^2$$

$$EI\Delta = -25.177 \text{ kN.m}^2$$

Ex.1: Determine the member end moments and reactions for the frame shown using slope-deflection method

$$EI\theta_C = -40.211 \text{ kN.m}^2$$

$$EI\theta_D = 34.24 \text{ kN.m}^2$$

$$EI\Delta = -25.177 \text{ kN.m}^2$$

Member End Moments:

$$M_{AC} = -14.6 \text{ kN.m} \text{ (14.6 kN.m } \curvearrowright \text{)}$$

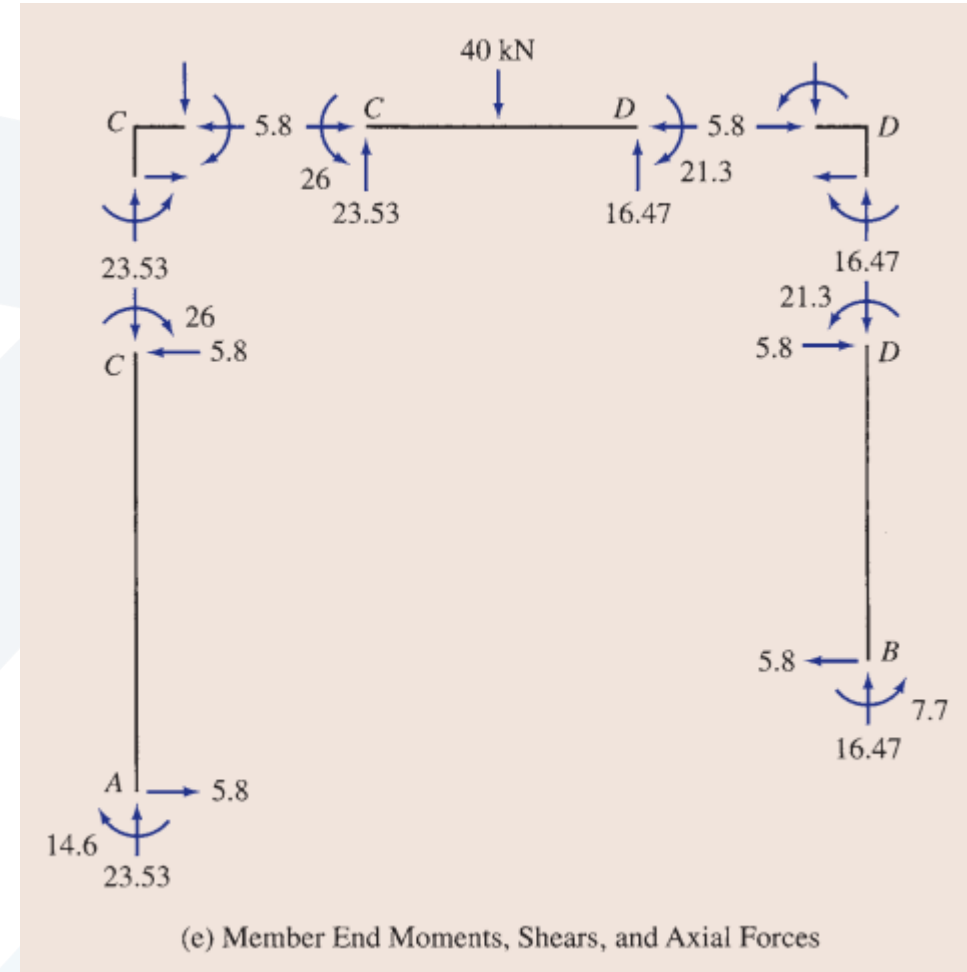
$$M_{CA} = -26 \text{ kN.m} \text{ (26 kN.m } \curvearrowright \text{)}$$

$$M_{BD} = 7.7 \text{ kN.m} \text{ (7.7 kN.m } \curvearrowright \text{)}$$

$$M_{DB} = 21.3 \text{ kN.m} \text{ (21.3 kN.m } \curvearrowright \text{)}$$

$$M_{CD} = 26 \text{ kN.m} \text{ (26 kN.m } \curvearrowright \text{)}$$

$$M_{DC} = -21.3 \text{ kN.m} \text{ (21.3 kN.m } \curvearrowright \text{)}$$



Structural Mechanics (2)

Week No-08

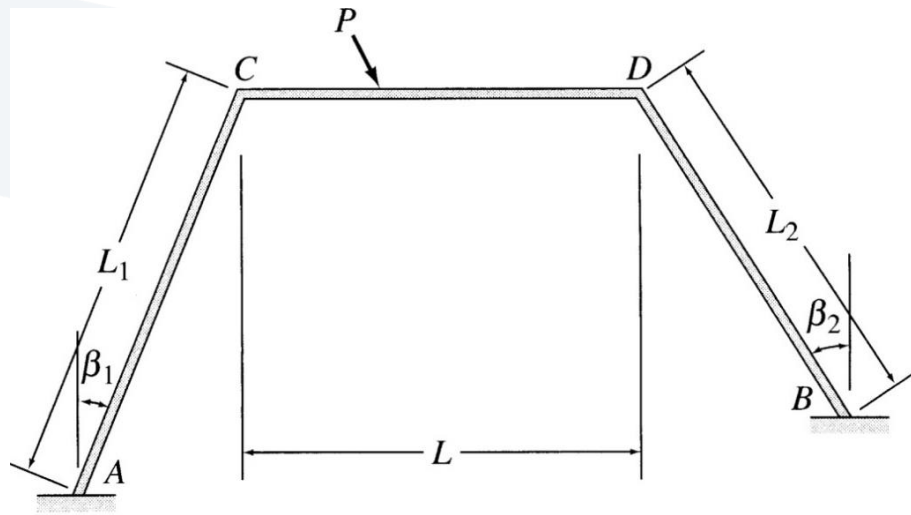
Part-02

Frames with one or more inclined legs

30/06/2025

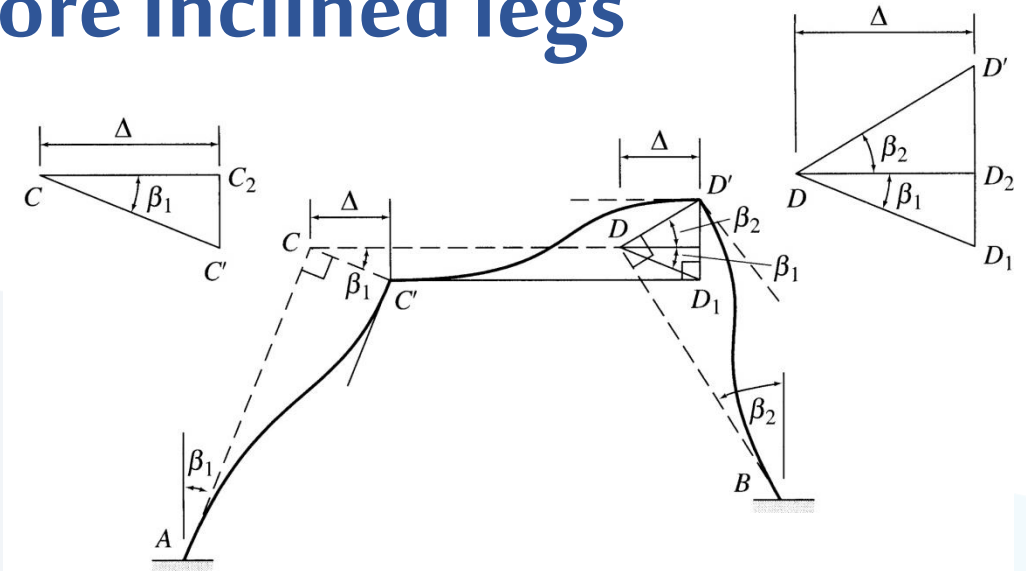
B. Haidar

Structural Mechanics (2)

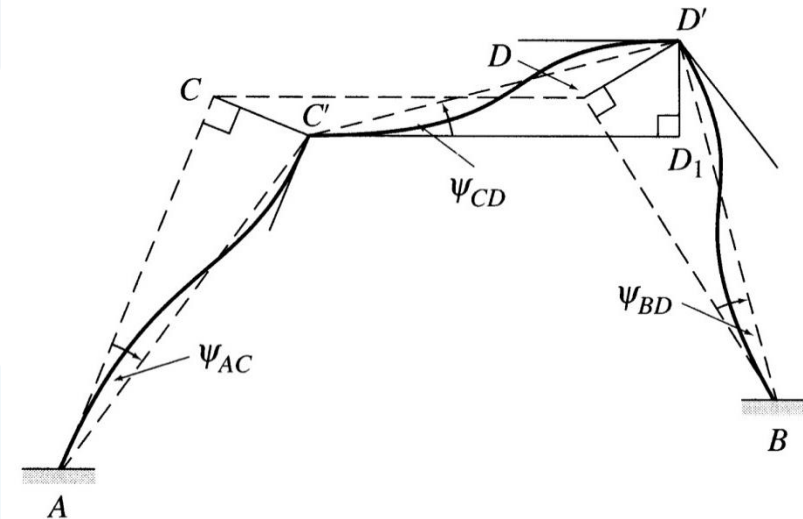


(a) Frame with Inclined Legs

3 K. Unknowns.: θ_C, θ_D & Δ .



(b) Deflected Shape of the Frame Due to Sidesway



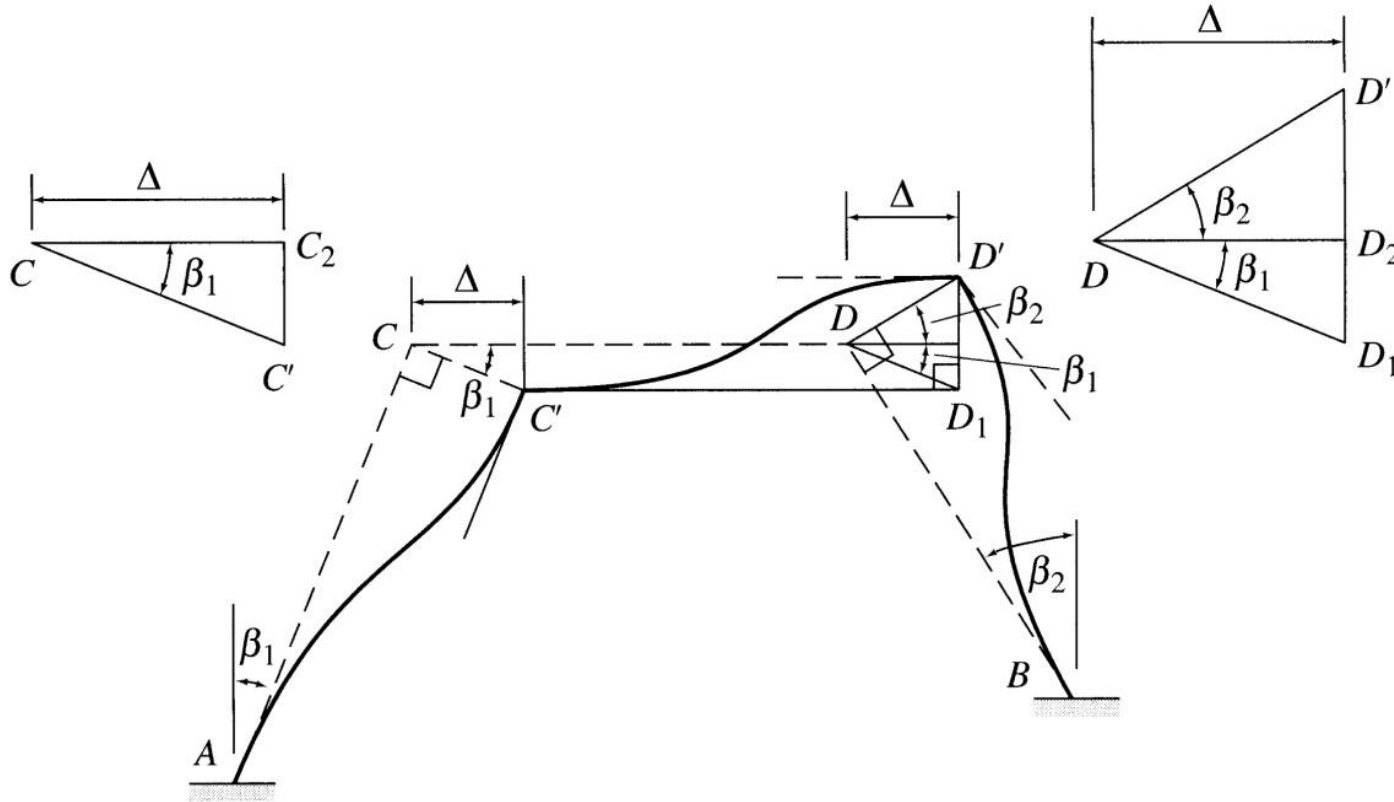
(c) Chord Rotations Due to Sidesway

Frames with one or more inclined legs

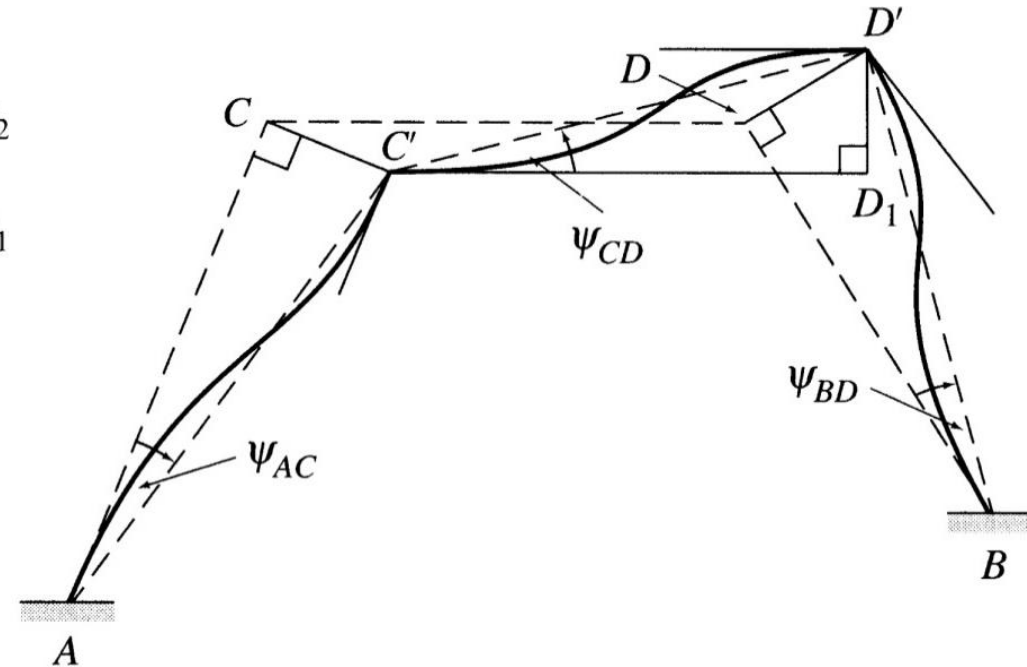
$$\psi_{AC} = -\frac{CC'}{L_1} = -\frac{\Delta}{L_1 \cos \beta_1}$$

$$\psi_{BD} = -\frac{DD'}{L_2} = -\frac{\Delta}{L_2 \cos \beta_2}$$

$$\psi_{CD} = \frac{D_1D'}{L} = \frac{\Delta}{L} (\tan \beta_1 + \tan \beta_2)$$

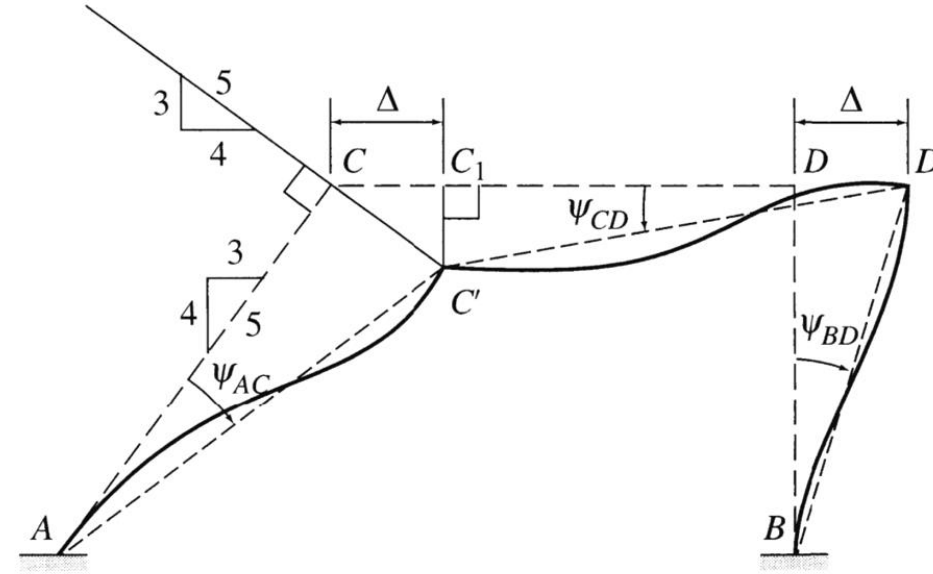
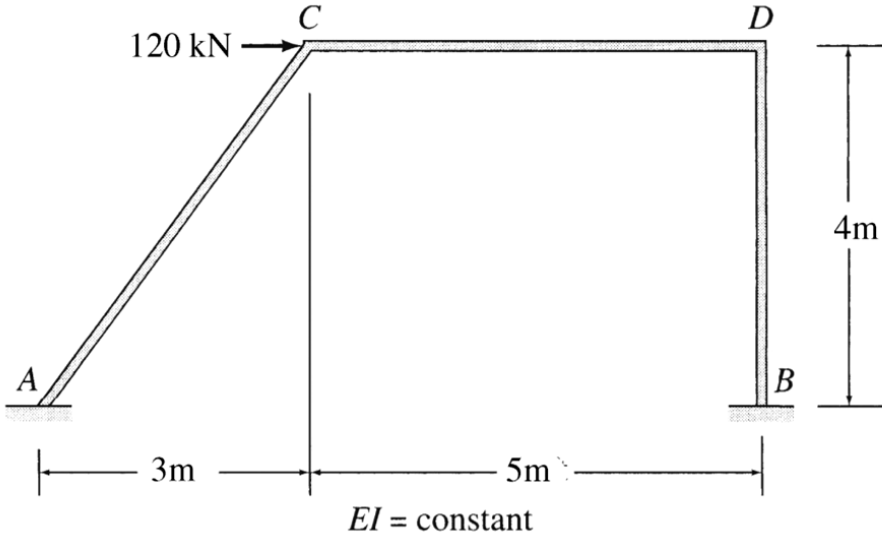


(b) Deflected Shape of the Frame Due to Sidesway



(c) Chord Rotations Due to Sidesway

Ex.2: Determine the member end moments and reactions for the frame shown using slope-deflection method



Chord rotations

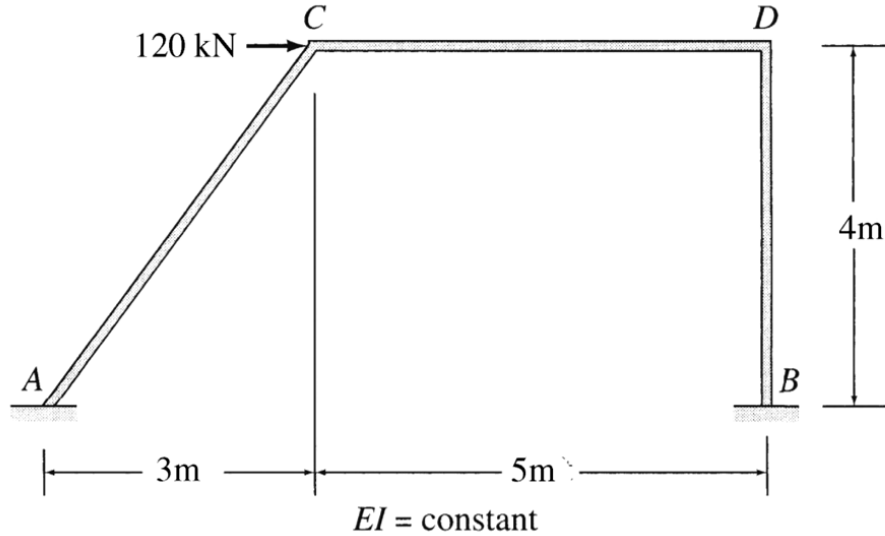
Degrees of Freedom θ_C, θ_D & Δ .

$$\psi_{AC} = -CC'/5 = [(-5/4)\Delta]/5 = -0.25 \Delta$$

$$\psi_{BD} = -DD'/4 = -\Delta/4 = -0.25 \Delta$$

$$\psi_{CD} = C'C_1/5 = [(3/4)\Delta]/5 = 0.15 \Delta$$

Ex.2: Determine the member end moments and reactions for the frame shown using slope-deflection method



Degrees of Freedom θ_C, θ_D & Δ .

$$\psi_{AC} = -0.25 \Delta \quad \psi_{BD} = -0.25 \Delta$$

$$\psi_{CD} = 0.15 \Delta$$

Slope Deflection Equations:

$$M_{AC} = \frac{2EI}{5} [\theta_C - 3(-0.25\Delta)] = 0.4EI\theta_C + 0.3EI\Delta$$

$$M_{CA} = \frac{2EI}{5} [2\theta_C - 3(-0.25\Delta)] = 0.8EI\theta_C + 0.3EI\Delta$$

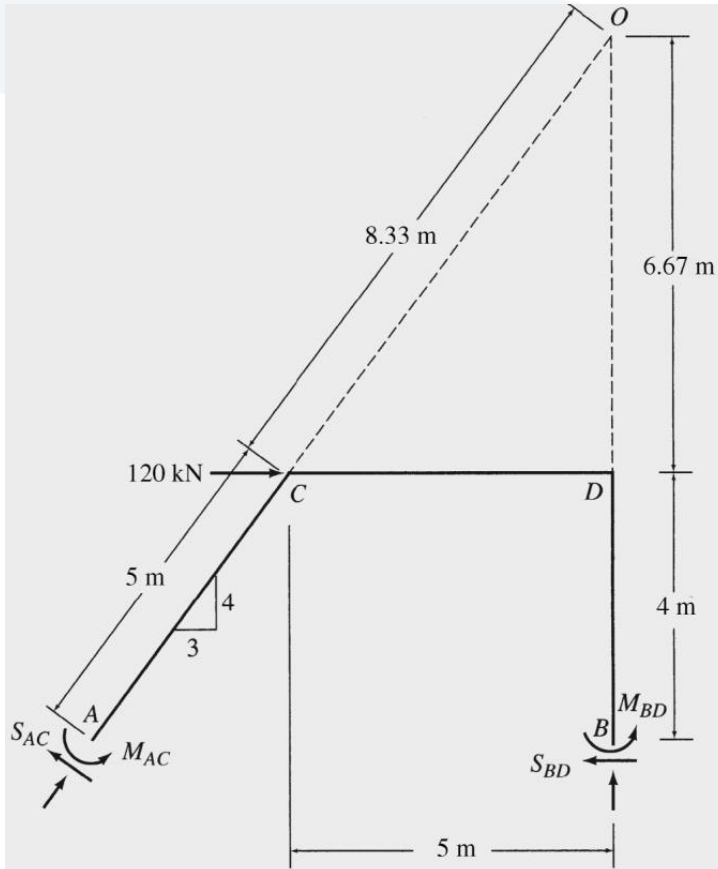
$$M_{BD} = \frac{2EI}{4} [\theta_D - 3(-0.25\Delta)] = 0.5EI\theta_D + 0.375EI\Delta$$

$$M_{DB} = \frac{2EI}{4} [2\theta_D - 3(-0.25\Delta)] = EI\theta_D + 0.375EI\Delta$$

$$M_{CD} = \frac{2EI}{5} [2\theta_C + \theta_D - 3(0.15\Delta)] = 0.8EI\theta_C + 0.4EI\theta_D - 0.18EI\Delta$$

$$M_{DC} = \frac{2EI}{5} [2\theta_D + \theta_C - 3(0.15\Delta)] = 0.8EI\theta_D + 0.4EI\theta_C - 0.18EI\Delta$$

Ex.2: Determine the member end moments and reactions for the frame shown using slope-deflection method



(c) Free-Body Diagram of the Entire Frame

Equilibrium Equations:

$$M_{CA} + M_{CD} = 0$$

$$M_{DC} + M_{DB} = 0$$

$$M_{AC} - S_{AC}(13.33) + M_{BD} - S_{BD}(10.67) + 120(6.67) = 0$$

$$S_{AC} = \frac{M_{AC} + M_{CA}}{5} \quad \& \quad S_{BD} = \frac{M_{BD} + M_{DB}}{4}$$

$$1.67M_{AC} + 2.67M_{CA} + 1.67M_{BD} + 2.67M_{DB} = 800$$

$$1.6EI\theta_C + 0.4EI\theta_D + 0.12EI\Delta = 0$$

$$0.4EI\theta_C + 1.8EI\theta_D + 0.195EI\Delta = 0$$

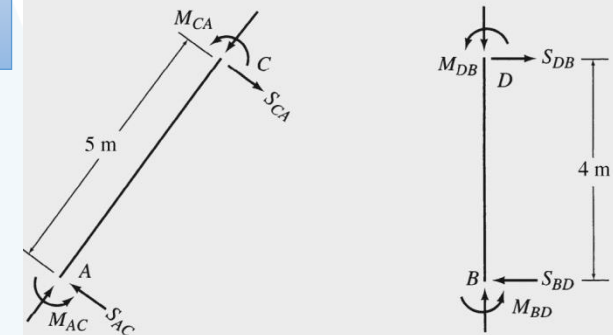
$$2.804EI\theta_C + 3.505EI\theta_D + 2.93EI\Delta = 800$$

Joint Displacements:

$$EI\theta_C = -16.59 \text{ kN.m}^2$$

$$EI\theta_D = -31.73 \text{ kN.m}^2$$

$$EI\Delta = 326.96 \text{ kN.m}^2$$



(d) Free-Body Diagrams of Columns AC and BD

Ex.2: Determine the member end moments and reactions for the frame shown using slope-deflection method

$$EI\theta_C = -16.59 \text{ kN.m}^2$$

$$EI\theta_D = -31.73 \text{ kN.m}^2$$

$$EI\Delta = 326.96 \text{ kN.m}^2$$

Member End Moments:

$$M_{AC} = 91.7 \text{ kN.m} \text{ (91.7 kN.m } \curvearrowright \text{)}$$

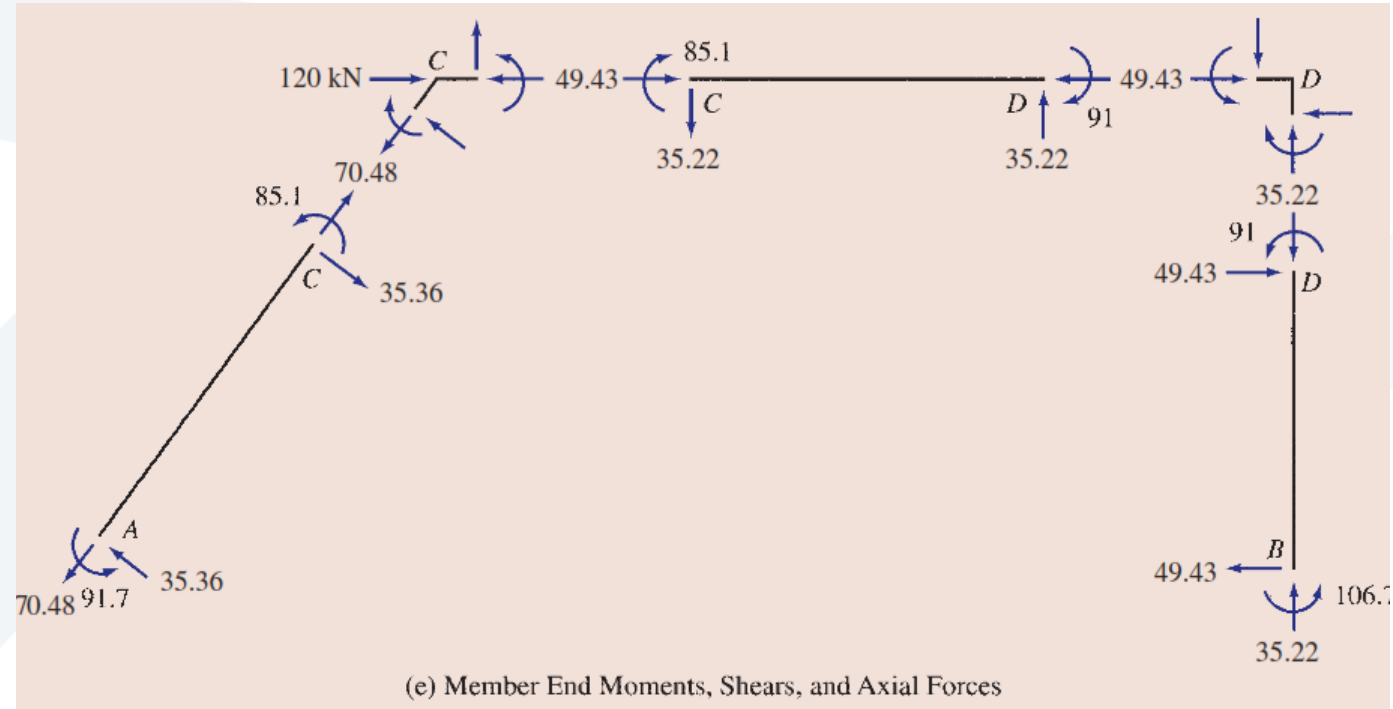
$$M_{CA} = 85.1 \text{ kN.m} \text{ (85.1 kN.m } \curvearrowright \text{)}$$

$$M_{BD} = 106.7 \text{ kN.m} \text{ (106.7 kN.m } \curvearrowright \text{)}$$

$$M_{DB} = 91 \text{ kN.m} \text{ (91 kN.m } \curvearrowright \text{)}$$

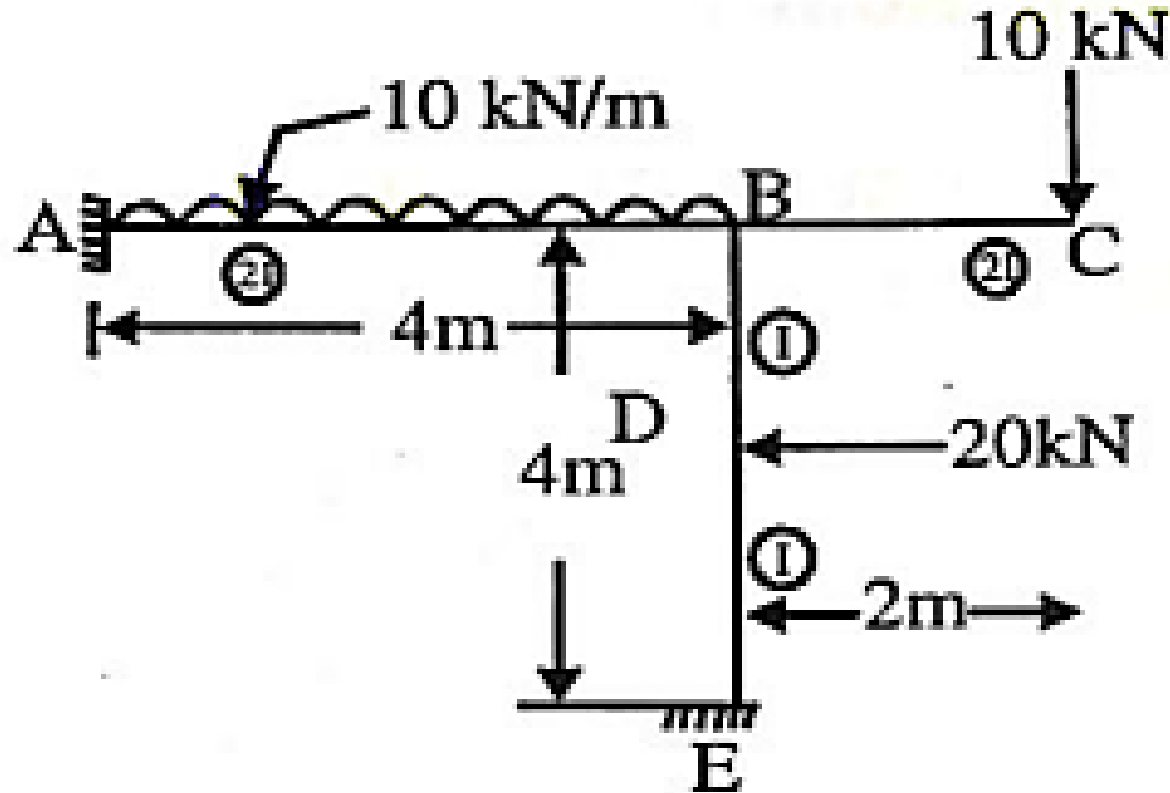
$$M_{CD} = -85.1 \text{ kN.m} \text{ (85.1 kN.m } \curvearrowright \text{)}$$

$$M_{DC} = -91 \text{ kN.m} \text{ (91 kN.m } \curvearrowright \text{)}$$



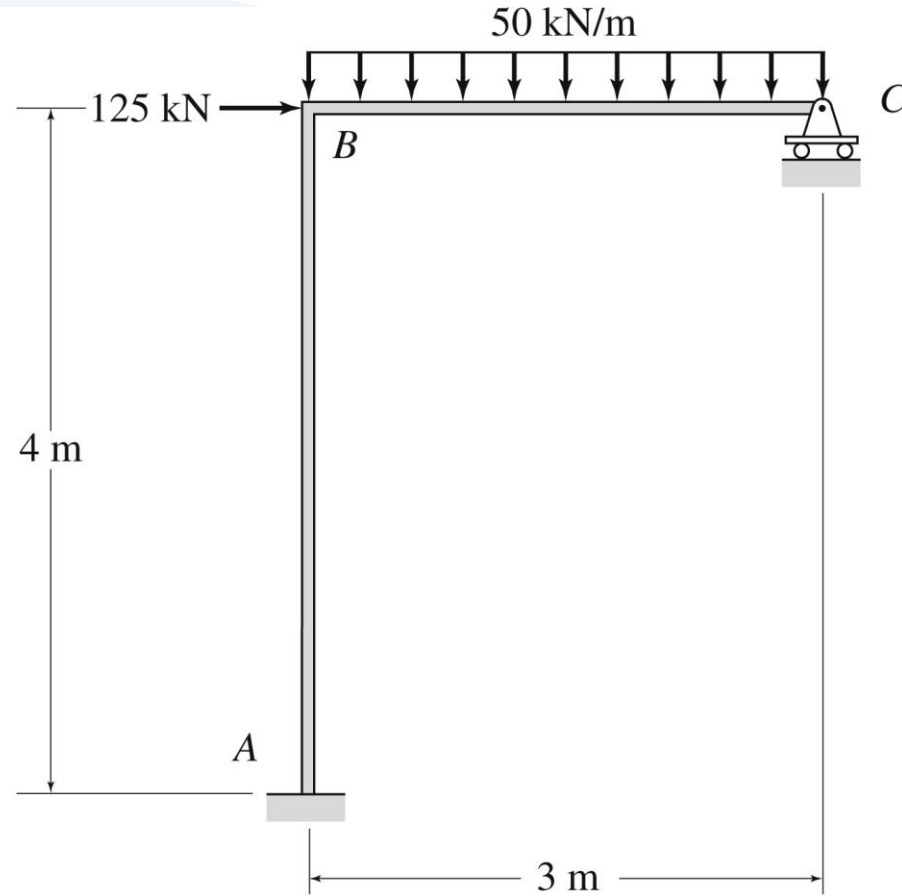
Homework

Pr-01: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams.



Homework

Pr-02: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams.



$EI = \text{constant}$

Homework

Pr-03: For the next frame under the given loads, by the slope-deflection method, calculate the reactions, and draw the bending moment, shear force & normal force diagrams. ($EI = \text{const.}$)

