

Introduction to Modeling and Simulation of Electromechanical Systems (DC MOTOR)





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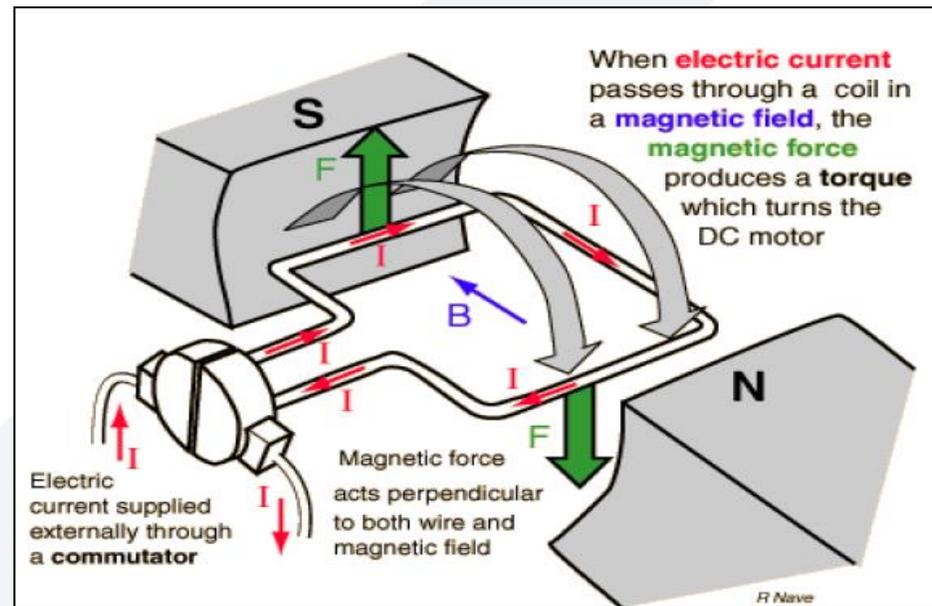
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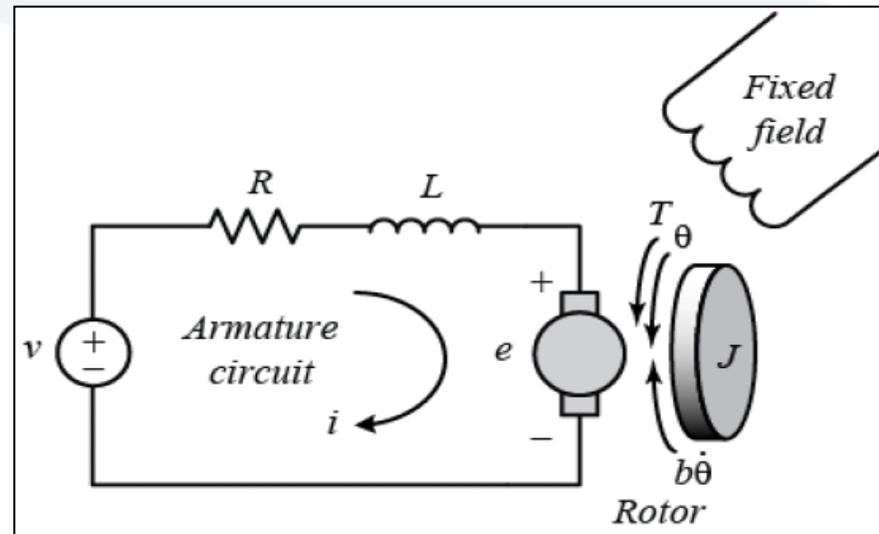
DC Motor Operating Principles

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion.

The commutator reverses the current each half revolution to keep the torque turning the coil in the same direction.



The electric circuit of the armature and the free-body diagram of the rotor are shown in the following figure:



The input to the system is the voltage applied to the motor's armature (V), while the output is the angular position of the shaft (θ).

In general, the torque generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In this example we will assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current i by a constant factor K_t as shown in the equation below.

This is referred to as an armature-controlled motor. $T = K_t i$ (1)

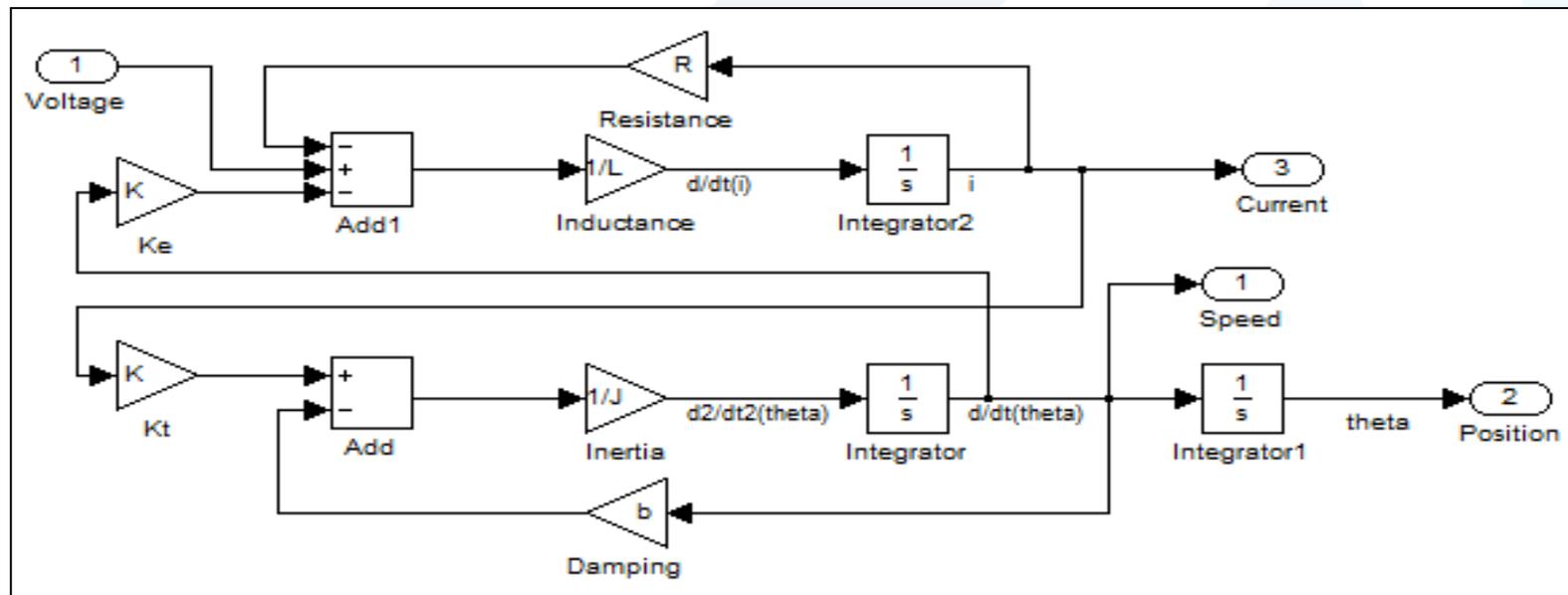
The back emf, e , is proportional to the angular velocity of the shaft by a constant factor K_e . $e = K_e \dot{\theta}$ (2)

In SI units, the motor torque and back emf constants are equal, that is, $K_t = K_e$; therefore, we will use K to represent both the motor torque constant and the back emf constant.

Building the Model using Simulink

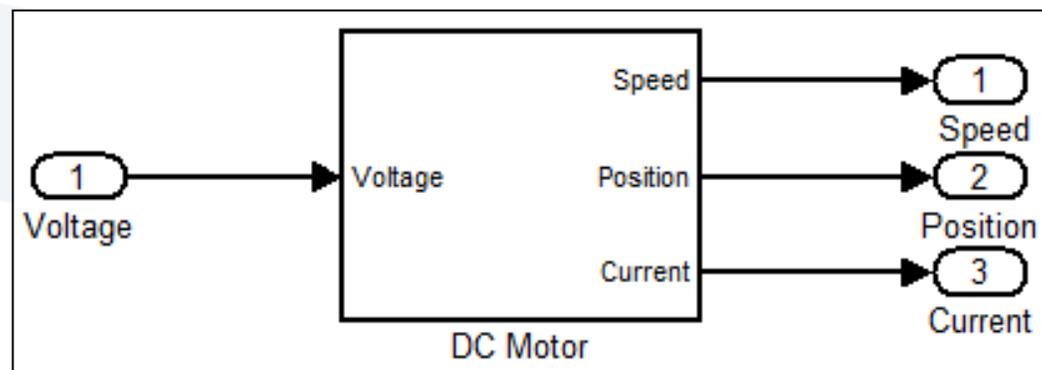
$$J \frac{d^2\theta}{dt^2} = T - b \frac{d\theta}{dt} \implies \frac{d^2\theta}{dt^2} = \frac{1}{J} (K_t i - b \frac{d\theta}{dt})$$

$$L \frac{di}{dt} = -Ri + V - e \implies \frac{di}{dt} = \frac{1}{L} (-Ri + V - K_e \frac{d\theta}{dt})$$





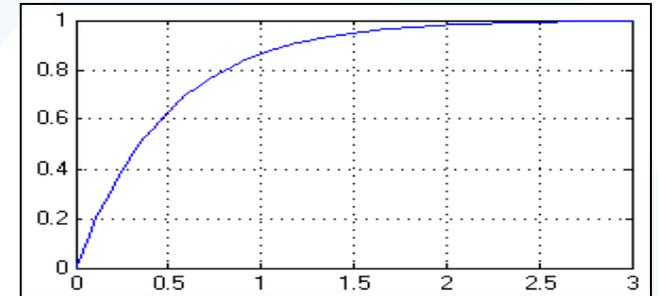
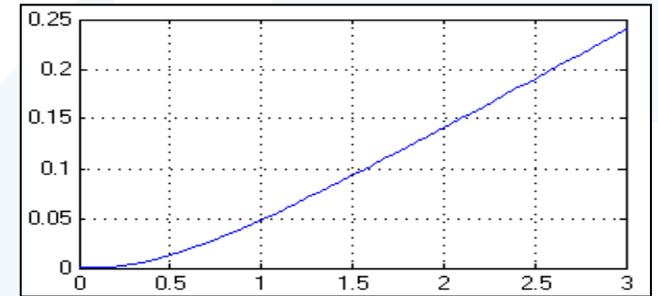
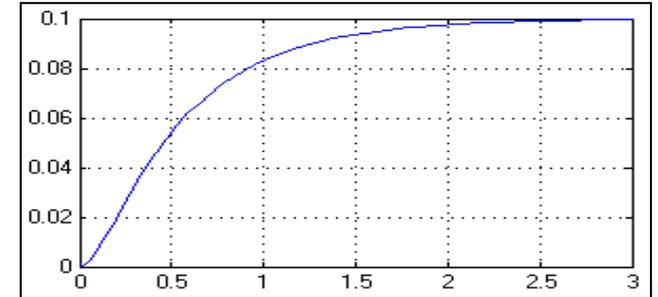
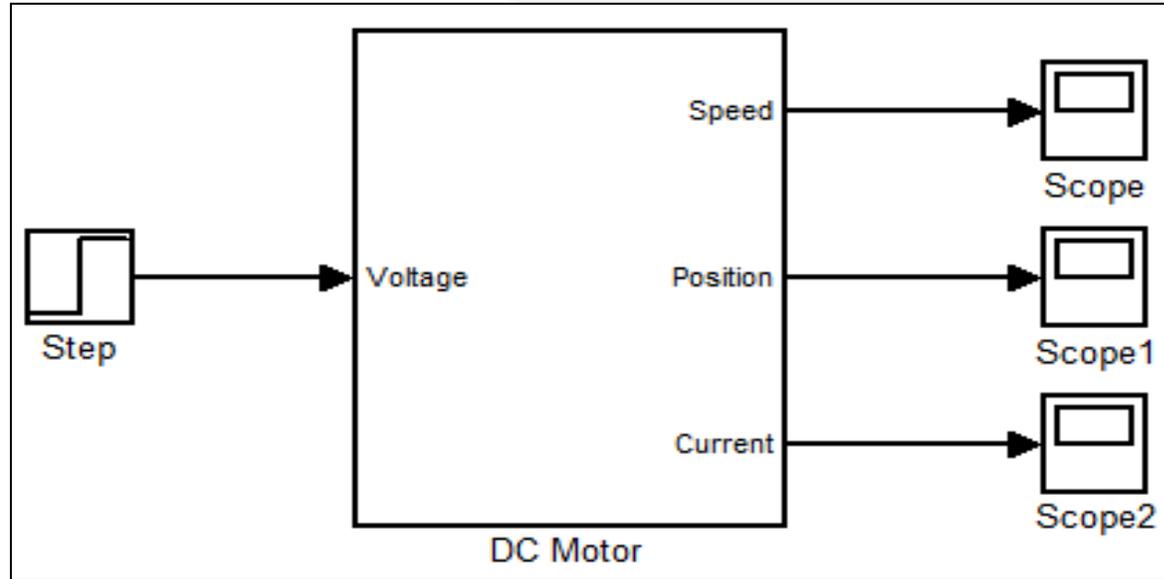
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We now need to define the parameters used in our simulation. Type the following commands at the prompt of the MATLAB command window.

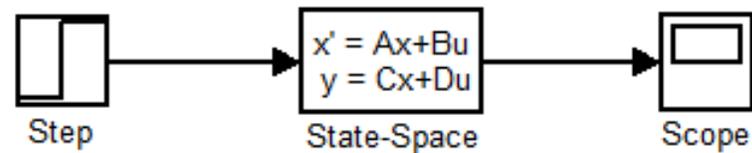
moment of inertia of the rotor	$J = 0.01 \text{ kg} \cdot \text{m}^2$
damping (friction) of the mechanical system	$b = 0.1 \text{ Nms}$
(back-)electromotive force constant	$K = 0.01 \text{ Nm/A}$
electric resistance	$R = 1 \Omega$
electric inductance	$L = 0.5 \text{ H}$

Open-Loop Response



Building the Model using State Space Equation

$$\begin{bmatrix} \theta' \\ \theta'' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b/J & K/J \\ 0 & -K/L & -R/L \end{bmatrix} \begin{bmatrix} \theta \\ \theta' \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix} v$$



Parameters

A:

[0 1 0; 0 -b/J K/J; 0 -K/L -R/L]

B:

[0 ; 0 ; 1/L]

C:

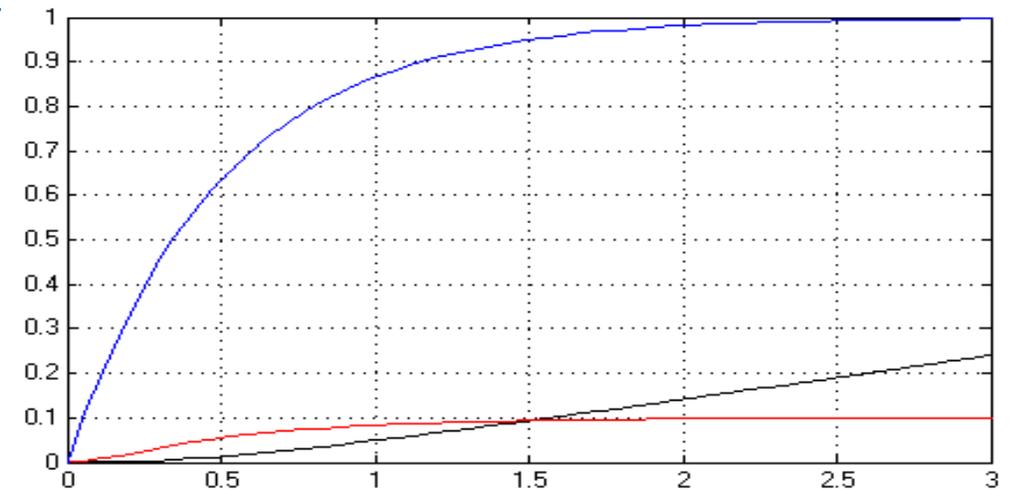
[1 0 0; 0 1 0; 0 0 1]

D:

[0; 0; 0]

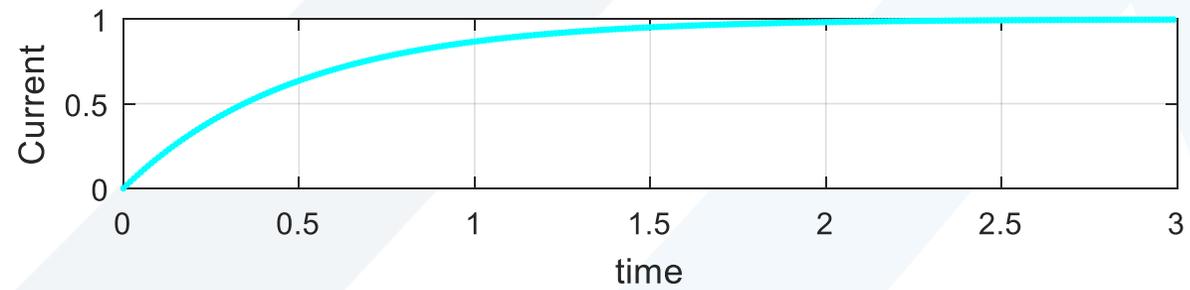
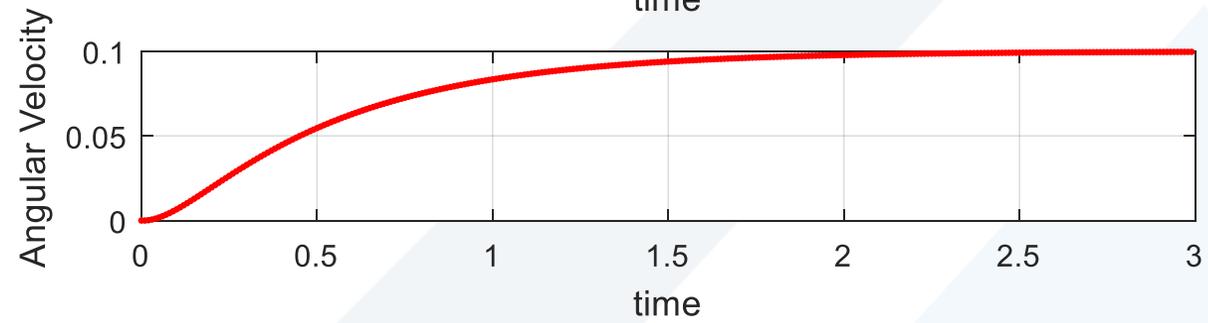
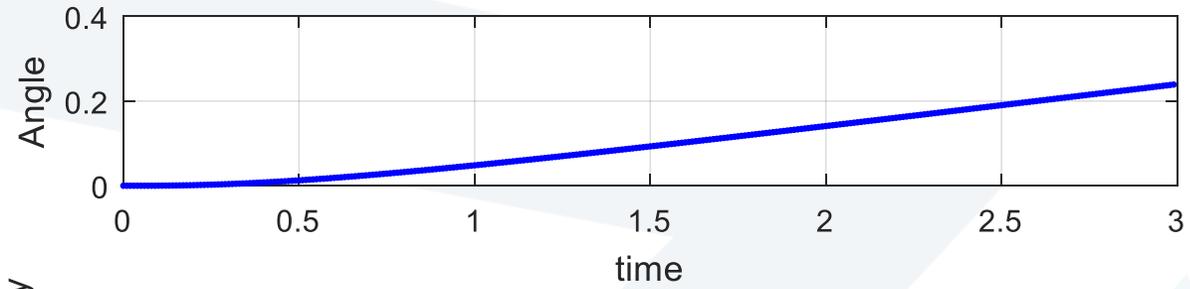
Initial conditions:

[0 0 0]



```
t=0; % Initial time
dt=0.01; % step size
tsim=3; % Simulation time
n=round( (tsim-t)/dt); % number of iterations
%system parameters
J=0.01;
b=0.1;
K=0.01;
R=1;
L=0.5;
A = [0 1 0; 0 -b/J K/J; 0 -K/L -R/L];
B = [0 ; 0 ; 1/L];
X=[0 0 0 ]';
u=1;
for i=1:n
    X1(i,:)=t,X';
    dx=A*X+B*u;
    X=X+dx*dt;
    t=t+dt;
end
```

```
subplot(3,1,1)
plot(X1(:,1),X1(:,2),'b.')
xlabel('time')
ylabel('Angle')
grid
subplot(3,1,2)
plot(X1(:,1),X1(:,3),'r.')
xlabel('time')
ylabel('Angular Velocity')
grid
subplot(3,1,3)
plot(X1(:,1),X1(:,4),'c.')
xlabel('time')
ylabel('Current')
grid
```



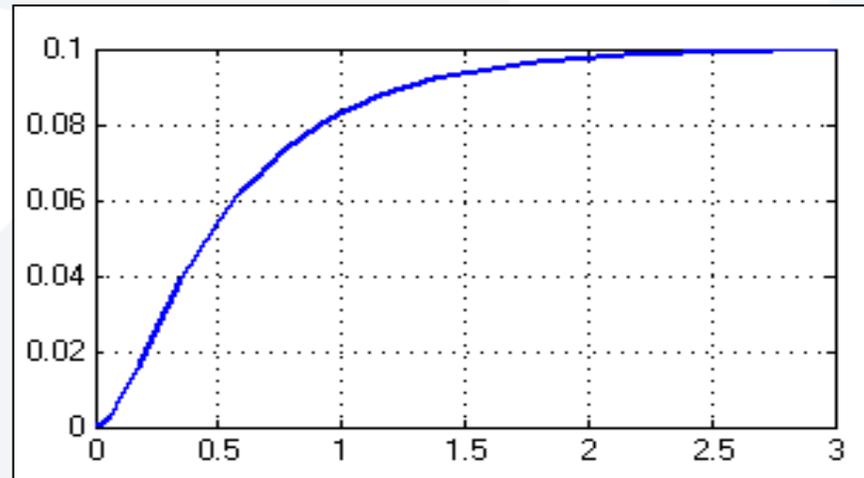
Mathematical techniques using state space model

```

t=0; % Initial time
dt=0.01; % step size
tsim=3; % Simulation time
n=round( (tsim-t)/dt); % number of iterations
%system parameters
J=0.01;
b=0.1;
K=0.01;
R=1;
L=0.5;
A=[0 1 0;0 -b/J K/J;0 -K/L -R/L];
B=[0;0;1/L];
X=[0 0 0]';
u=1;
for i=1:n
    X1(i,:)=t,X';
    if X1(i,3)>=0.08
        disp(X1(i,1))
        break
    end
    dx=A*X+B*u;
    X=X+dx*dt;
    t=t+dt;
end

```

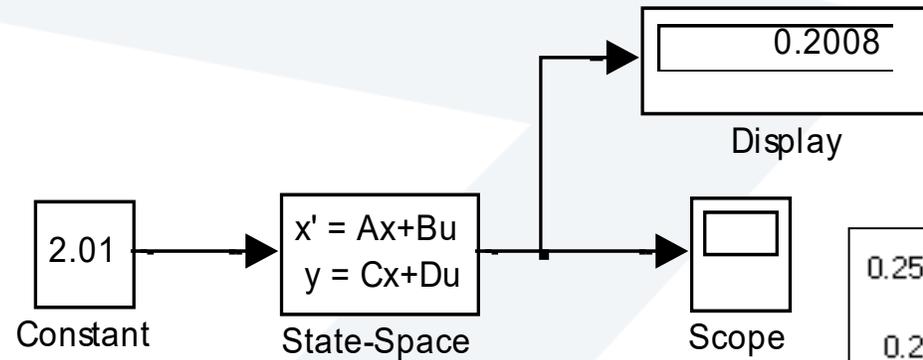
0.9100



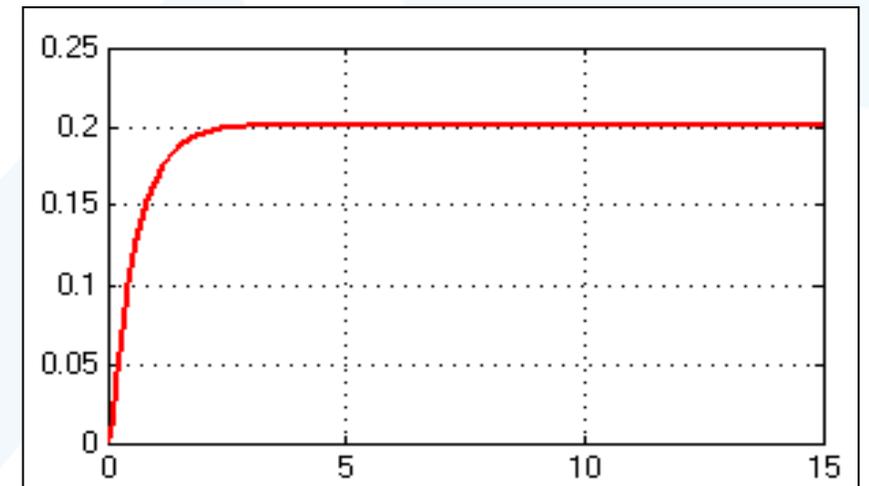
```

clc
clear
%system parameters
J=0.01;
b=0.1;
K=0.01;
R=1;
L=0.5;
A=[0 1 0;0 -b/J K/J;0 -K/L -R/L];
B=[0;0;1/L];
for u=1:0.01:5
    t=0; % Initial time
    dt=0.01; % step size
    tsim=15; % Simulation time
    n=round( (tsim-t)/dt); % number of iterations
    X=[0 0 0]';
    for i=1:n;
        X1(i,:)=t,X';
        dx=A*X+B*u;
        X=X+dx*dt;
        t=t+dt;
    end
    if X1(n,3)>=0.2
        disp(u)
        break
    end
end
end

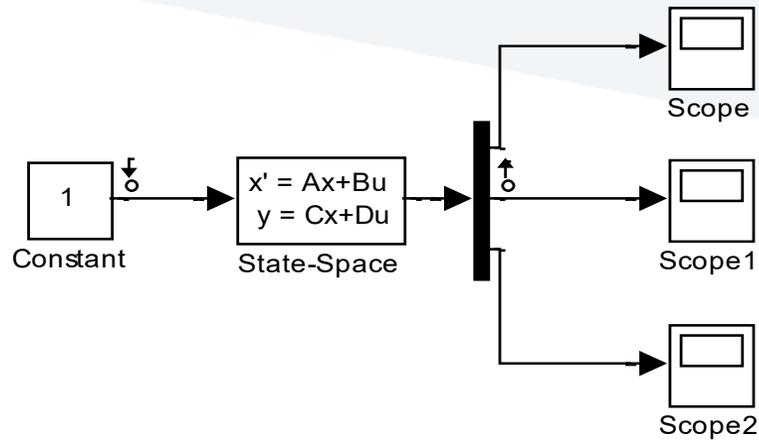
```



Verification using Simulink



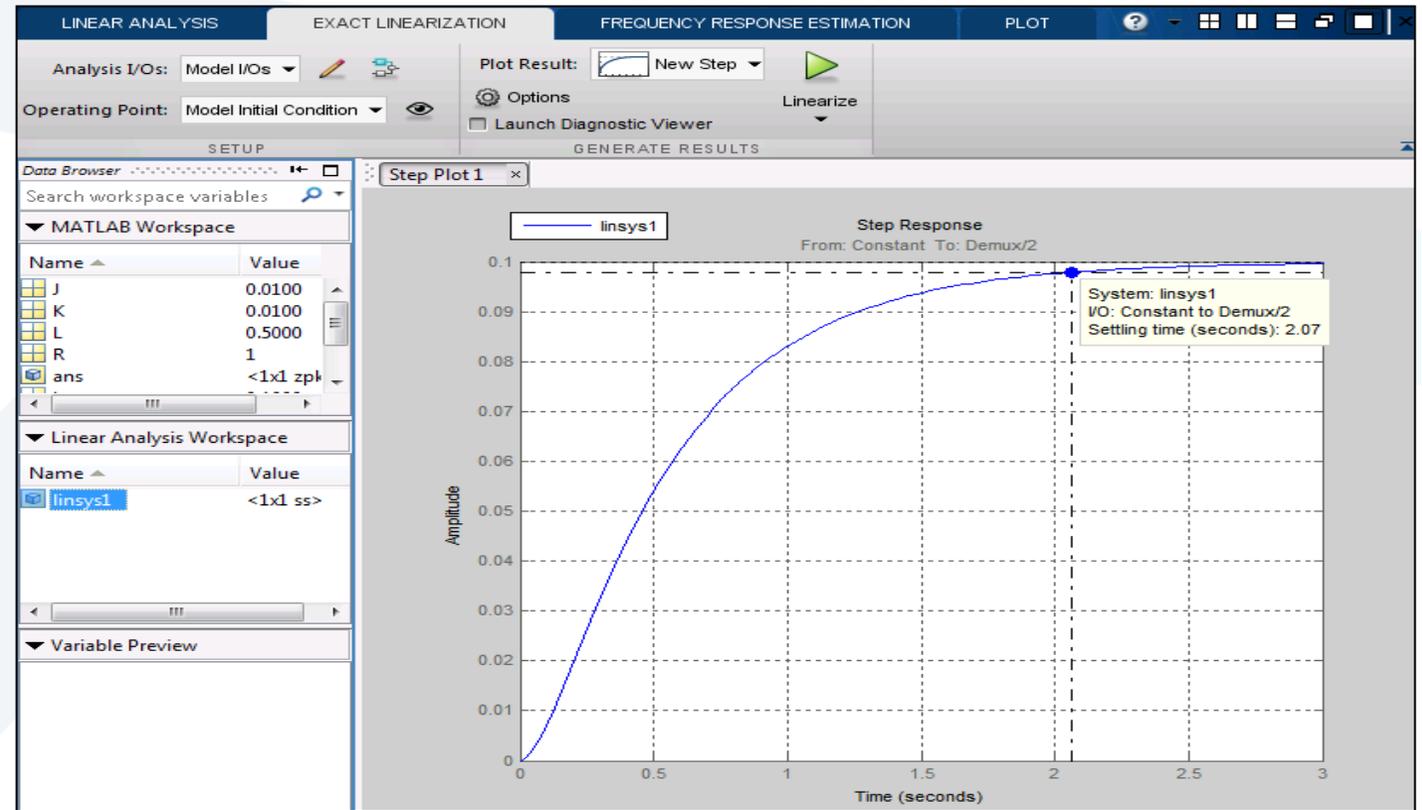
Extract transfer function model from state space model



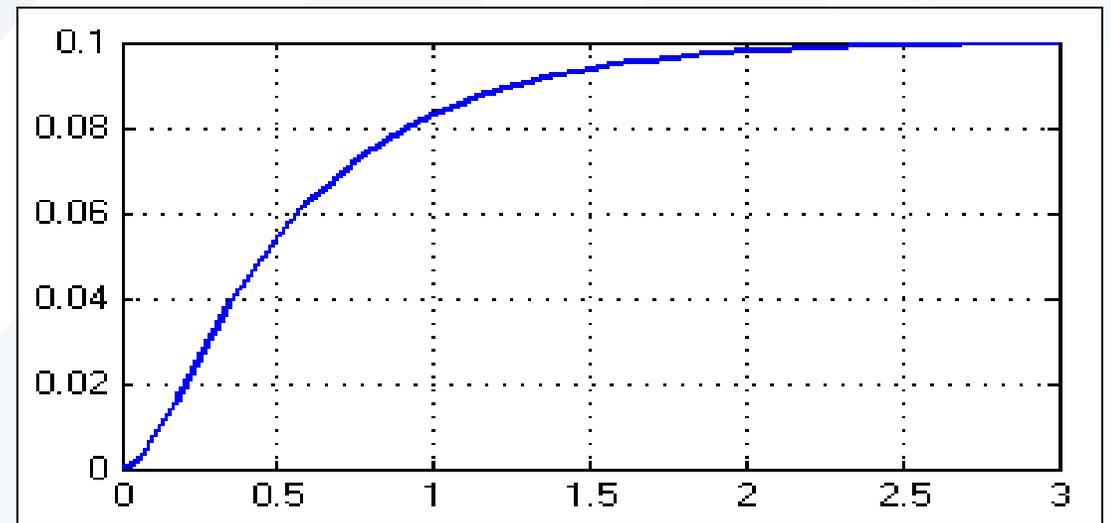
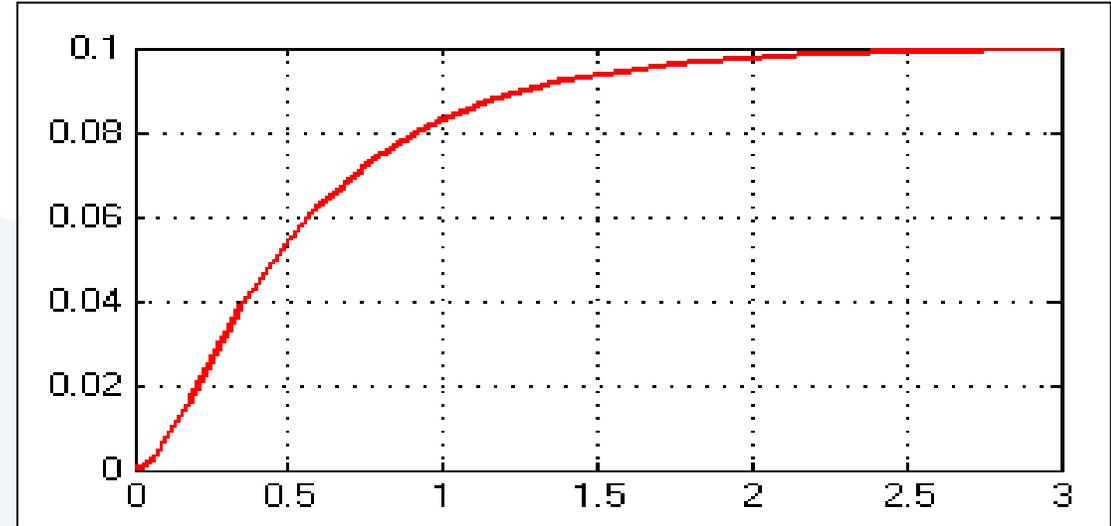
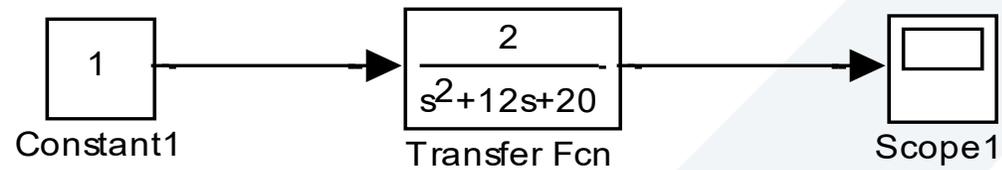
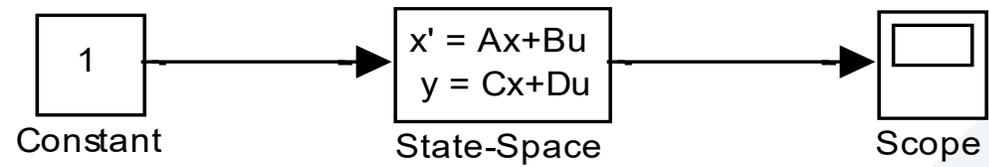
```
>> zpk(linsys1)

ans =

    From input "Constant" to output "Demux/2":
           2
    -----
(s+9.997) (s+2.003)
```



Verification process



System Modeling using Transfer Function

For this example, we will assume that the input of the system is the voltage source (V) applied to the motor's armature, while the output is the rotational speed of the shaft $d(\theta)/dt$. The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

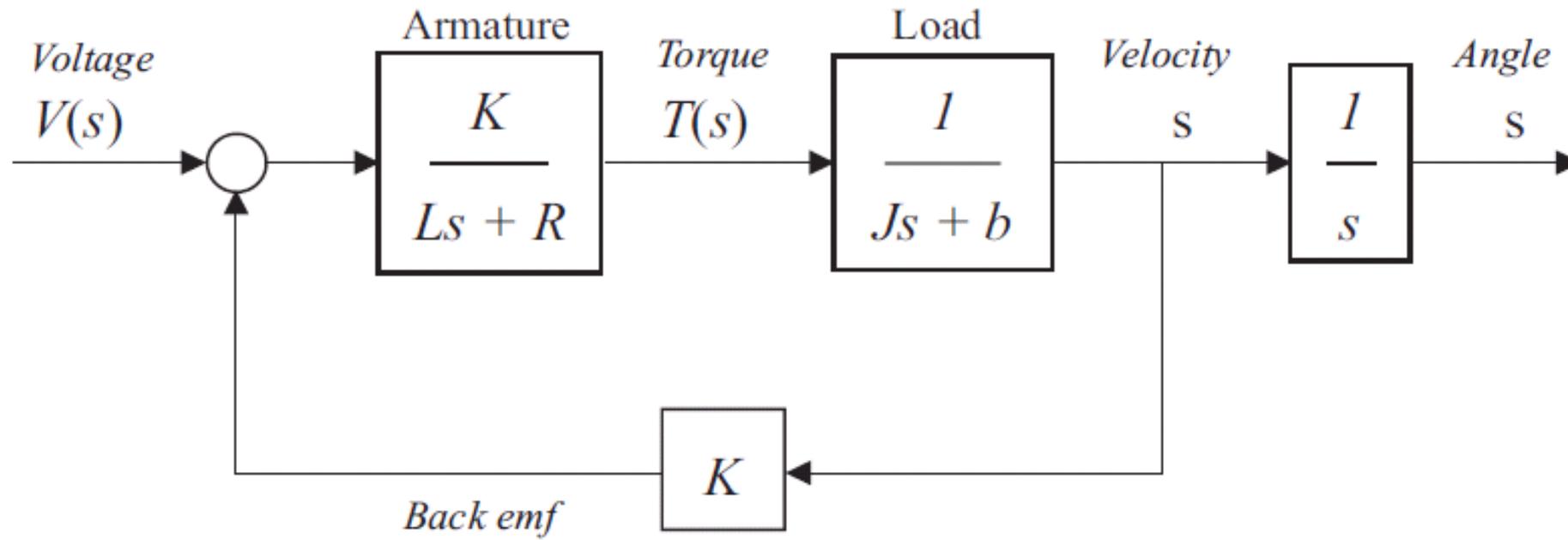
$$J \frac{d^2\theta}{dt^2} = T - b \frac{d\theta}{dt} \implies \frac{d^2\theta}{dt^2} = \frac{1}{J} (K_t i - b \frac{d\theta}{dt})$$

$$L \frac{di}{dt} = -Ri + V - e \implies \frac{di}{dt} = \frac{1}{L} (-Ri + V - K_e \frac{d\theta}{dt})$$

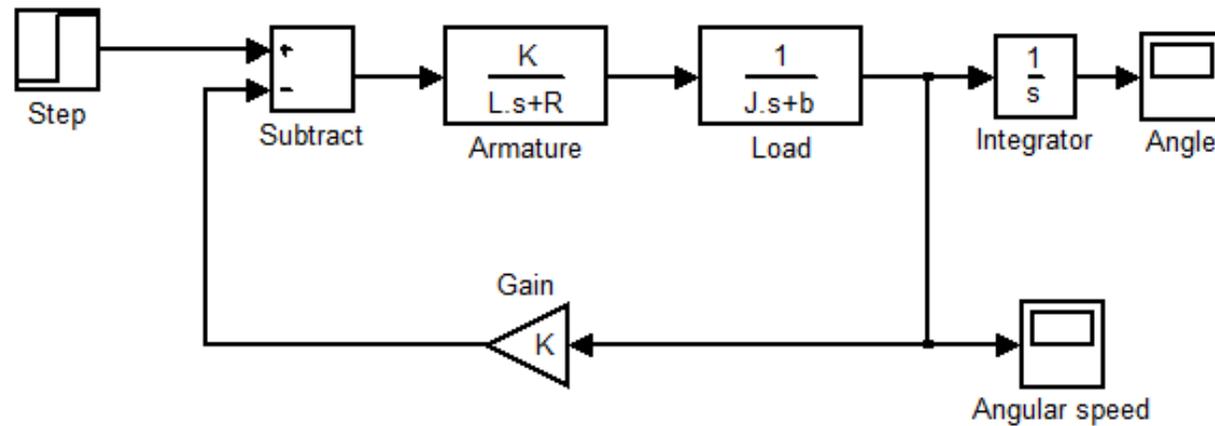
Applying the Laplace transform

$$I(s) = \frac{V(s) - K_s \theta(s)}{R + Ls}$$

$$Js^2\theta(s) + bs\theta(s) = K \frac{V(s) - K_s \theta(s)}{R + Ls}$$

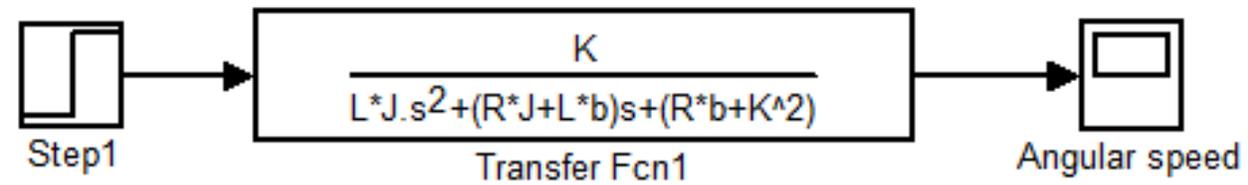
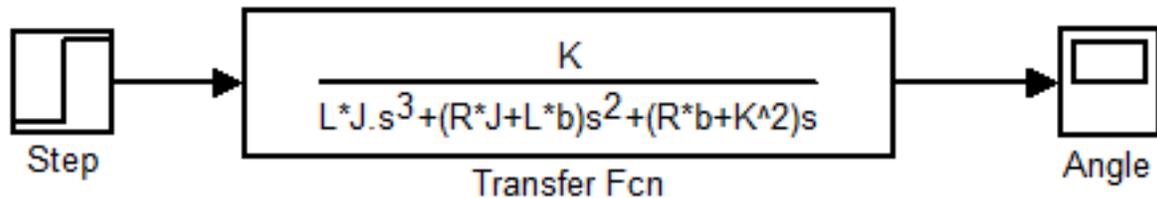


$$Js^2\theta(s) + bs\theta(s) = K \frac{V(s) - Ks\theta(s)}{R + Ls}$$



$$G_a(s) = \frac{\theta(s)}{V(s)} = \frac{K}{s[(R + Ls)(Js + b) + K^2]}$$

$$G_v(s) = \frac{\omega(s)}{V(s)} = \frac{K}{(R + Ls)(Js + b) + K^2}$$



As $G_a(s)$ can be expressed as $G_v(s) \cdot \frac{1}{s}$, we can enter these two transfer functions separately and combine them in series:

```
J=0.01;      aux = tf(K,conv([L R],[J b]));
b=0.1;       Gv = feedback(aux,K)
K=0.01;      Ga = tf(1,[1 0])*Gv
R=1;
L=0.5;
```

Gv =

$$\frac{0.01}{0.005 s^2 + 0.06 s + 0.1001}$$
Continuous-time transfer function.

Ga =

$$\frac{0.01}{0.005 s^3 + 0.06 s^2 + 0.1001 s}$$
Continuous-time transfer function.

Here, we made use of the function feedback to create a feedback connection of two transfer functions and the multiplication operator

Instead of using convolution, the first of the above three commands can be replaced by the product of two transfer functions:

```
aux = tf(K,[L R])*tf(1,[J b]);
```

Another possibility (perhaps the most convenient one) is to define the transfer function in a symbolic way. First introduce a system representing the Laplace operator s (differentiator) and then enter the transfer function as an algebraic expression:

```
s = tf([1 0],1);
Gv = K/((L*s + R)*(J*s + b) + K^2)
Ga = Gv/s
```

```
Gv =
      0.01
-----
0.005 s^2 + 0.06 s + 0.1001
Continuous-time transfer function.
```

```
Ga =
      0.01
-----
0.005 s^3 + 0.06 s^2 + 0.1001 s
Continuous-time transfer function.
```

It is convenient to label the inputs and outputs by the names of the physical variables they represent:

```
Gv.InputName = 'Voltage';  
Gv.OutputName = 'Velocity'  
Ga.InputName = 'Voltage';  
Ga.OutputName = 'Angle'
```

```
Gv =  
From input "Voltage" to output "Velocity":  
0.01
```

 $0.005 s^2 + 0.06 s + 0.1001$

Continuous-time transfer function.

```
Ga =  
From input "Voltage" to output "Angle":  
0.01
```

 $0.005 s^3 + 0.06 s^2 + 0.1001 s$

Continuous-time transfer function.

As we may want plot the responses for the velocity and angle in one figure, it convenient to group the two transfer functions into a single system with one input, the voltage, and two outputs, the velocity and the angle:

$G = [G_v; G_a]$ $G =$

From input "Voltage" to output...

0.01

Velocity: -----

$0.005 s^2 + 0.06 s + 0.1001$

0.01

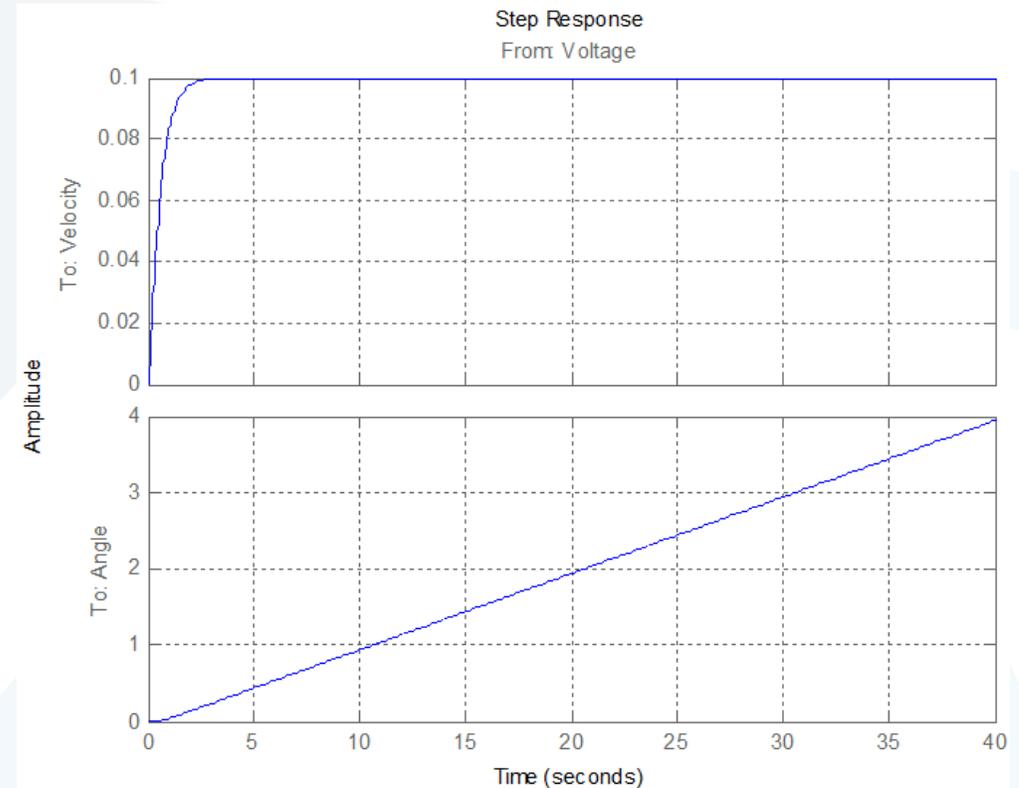
Angle: -----

$0.005 s^3 + 0.06 s^2 + 0.1001 s$

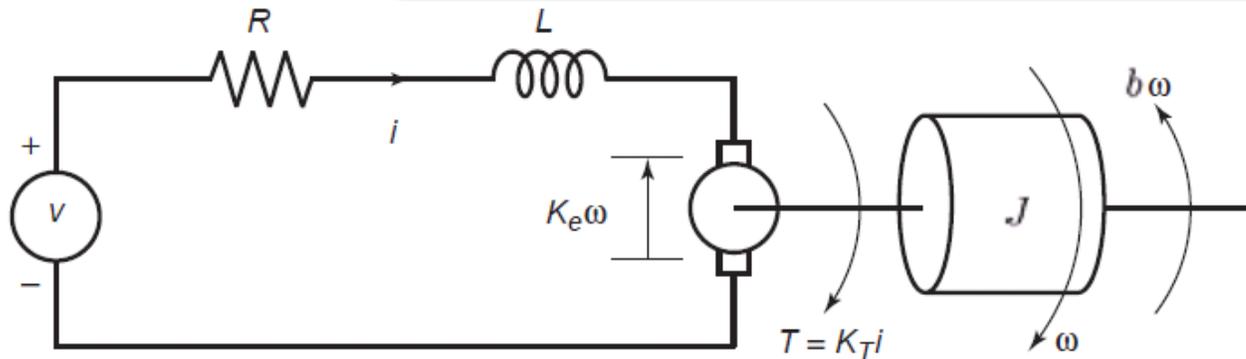
Continuous-time transfer function.

Now, we can plot the step of the motor model:

`step(G);`
`grid`



Model of a Relay-Controlled Motor



An armature-controlled dc motor

$$J \frac{d^2\theta}{dt^2} = T - b \frac{d\theta}{dt} - T_d(t) \implies \frac{d^2\theta}{dt^2} = \frac{1}{J} (K_t i - b \frac{d\theta}{dt} - T_d(t))$$

$$L \frac{di}{dt} = -Ri + V - e \implies \frac{di}{dt} = \frac{1}{L} (-Ri + V - K_e \frac{d\theta}{dt})$$

where the model now includes a torque $T_d(t)$ acting on the motor shaft due, for example, to some unwanted source such as Coulomb friction or wind gusts. Control system engineers call this a *disturbance*. These equations can be put into matrix form as follows, where $x_1 = i$ and $x_2 = \omega$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_e}{L} \\ \frac{K_t}{J} & -\frac{b}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} V(t) \\ T_d(t) \end{bmatrix}$$

Use the values $R = 0.6$, $L = 0.002$, $K_t = K_e = 0.04$, $b = 0.01$, and $J = 6 \times 10^{-5}$.

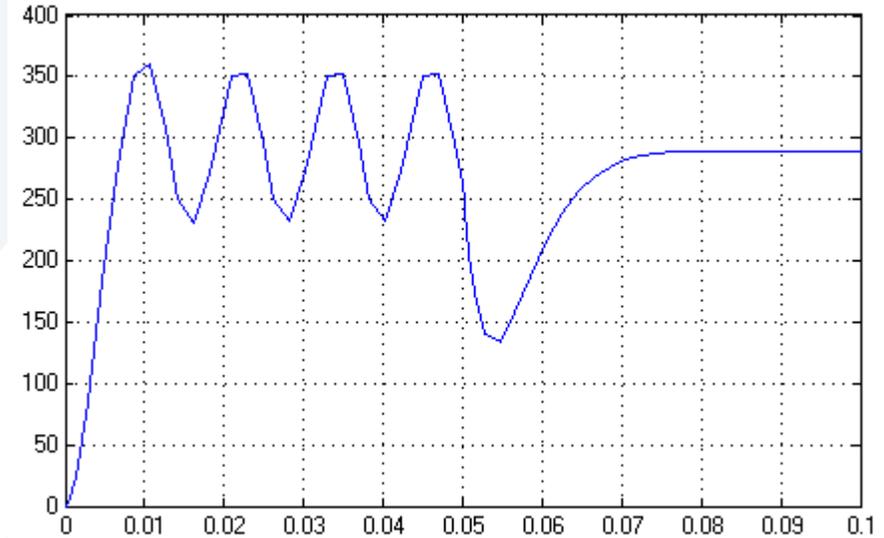
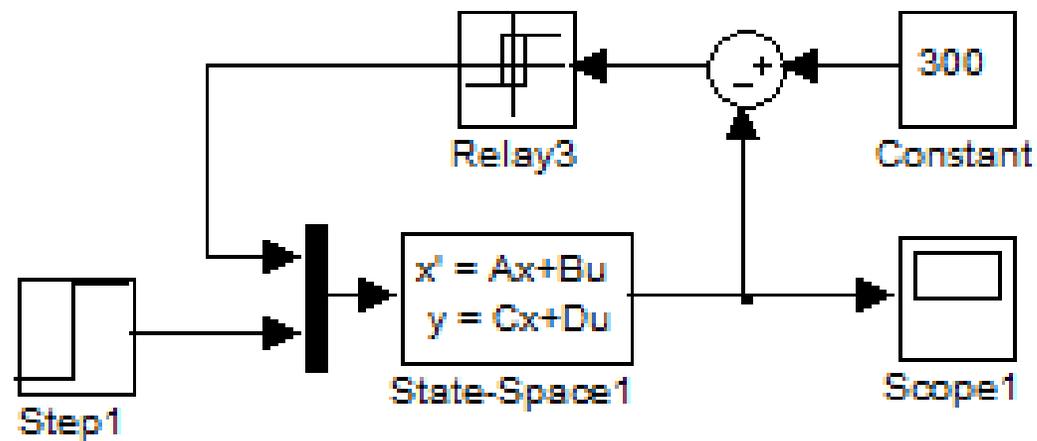
Suppose we have a sensor that measures the motor speed, and we use the sensor's signal to activate a relay to switch the applied voltage $v(t)$ between 0 and 100 V to keep the speed between 250 and 350 rad/s.

Investigate how well this scheme will work if the disturbance torque is a step function that increases from 0 to 3 N·m, starting at $t = 0.05$ s.

Assume that the system starts from rest with $\omega(0) = 0$ and $i(0) = 0$.

$$\mathbf{A} = \begin{bmatrix} -300 & -20 \\ 666.7 & -166.7 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 500 & 0 \\ 0 & -16667 \end{bmatrix}$$



The results show that the relay logic control scheme keeps the speed within the desired limits of 250 and 350 before the disturbance torque starts to act. The speed oscillates because when the applied voltage is zero, the speed decreases as a result of the back-emf and the viscous damping. The speed drops below 250 when the disturbance torque starts to act.

Note that the speed becomes constant, instead of oscillating. This is so because with $v = 100$, the system achieves a steady-state condition in which the motor torque equals the sum of the disturbance torque and the viscous damping torque.

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