Support Reactions المساند وردود الأفعال

1. Plane Structures الهياكل الحاملة (الإنشاءات أو الجمل الإنشائية) المستوية

1.1 Supports المساند

1.2 Statical Determinacy التقرير الستاتيكي

1.3 Determination of the Support Reactions حساب ردود الأفعال في المساند

Objectives:

In this chapter, the most common kinds of supports of simple structures and the different connecting elements of structures are introduced. We will discuss their characteristic features and how they can be classified, so that the students will be able to decide whether or not a structure is statically and kinematically determinate. Students will also learn from this chapter how the reactions (forces and couple moments) appearing at the supports and the connecting elements of a loaded structure can be determined. Here, the most important steps are the sketch of the free-body diagram and the correct application of the equilibrium conditions.

أنماط المساند. درجات الحرية والتقييد: الجمل الإنشائية (الهياكل الحاملة) المقررة ستاتيكيا وكينماتيكيا. ردود الأفعال. مخطط الجسم الحر.

1. Plane Structures الهياكل الحاملة (الإنشاءات أو الجمل الإنشائية) المستوية

1.1 Supports

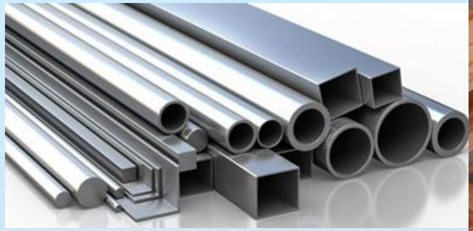
Geometrical Classification of Structures and Structural Elements:

التصنيف الجيومتري (حسب الشكل) للهياكل الحاملة وعناصرها الأساسية:

Slender structural elements (cross-sectional dimensions much smaller than its length) that are loaded solely in their axial direction (tension or compression) are called *bars* or *rods*

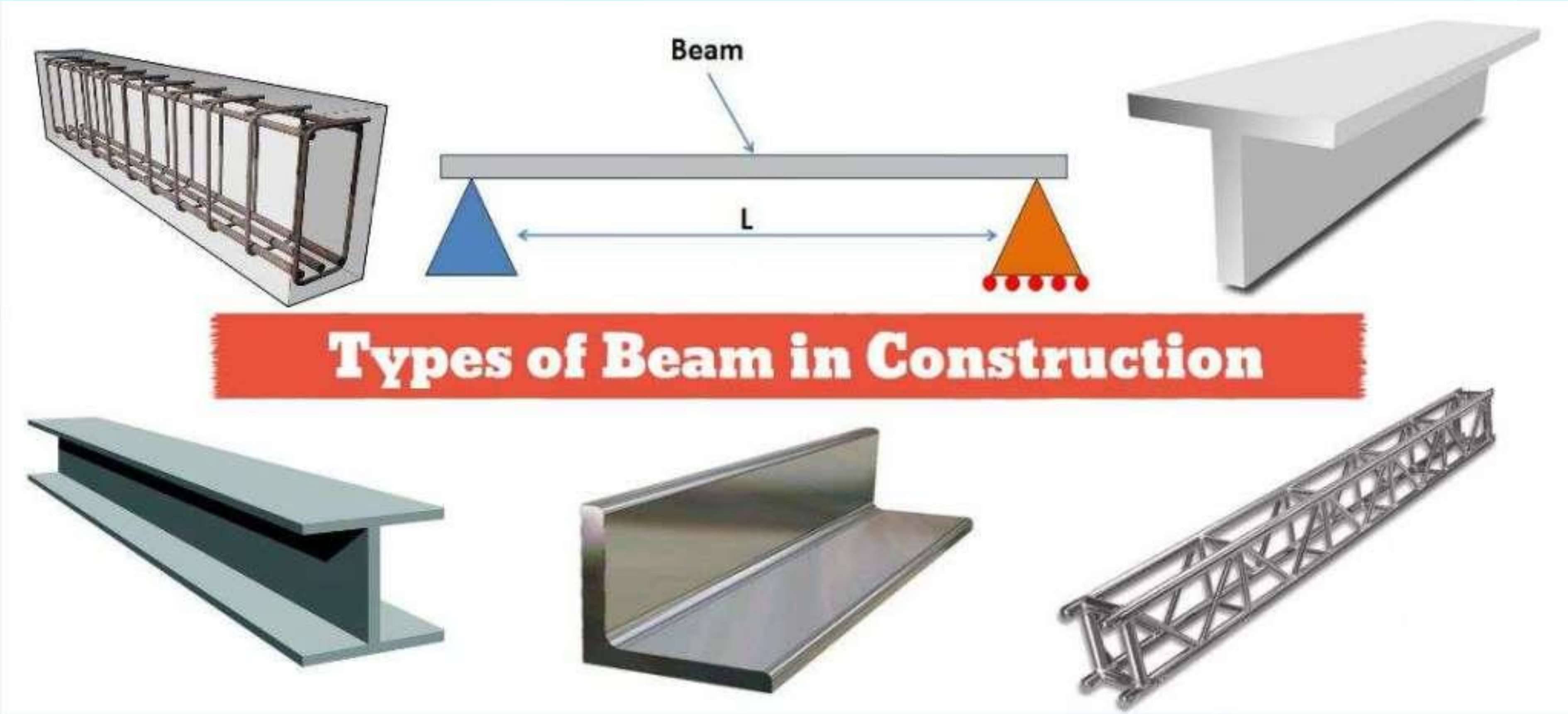
تُعتبر العناصر الطولانية، نحيلةً، إذا كان أبعاد مقاطعها العرضية أصغر بكثير من أطوالها. تُدعى هذه العناصر أذرع ، قضبان، عوارض، قوائم، أو هراوات...، إذا كانت محملة في اتجاه محورها الطولي فقط.





If these Elements are subjected to a load perpendicular to its axis, they are called *beams*.

وتدعى هذه العناصر النحيلة إذا ما تعرضت لحمولات عمودية على محورها الطولي جيزاناً ومفردها جائز.



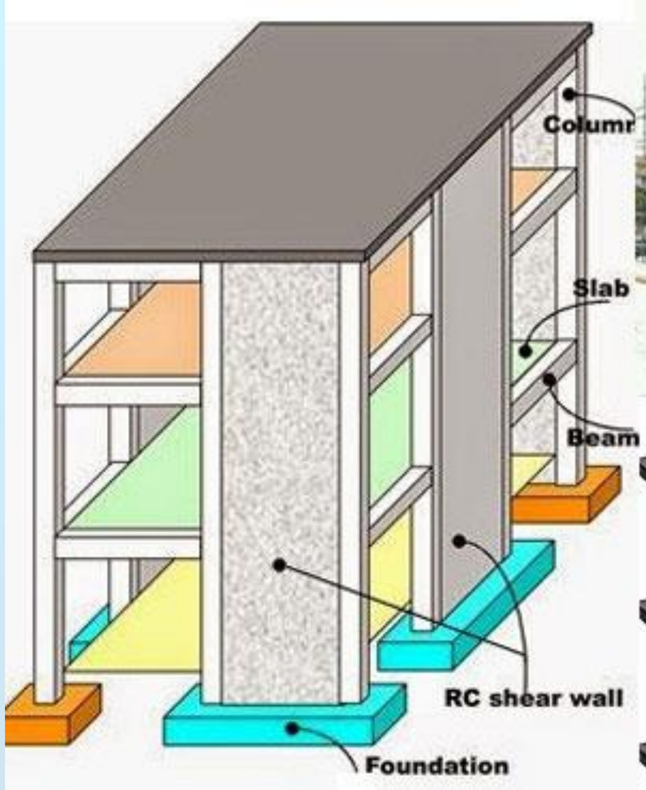
A curved beam is usually designated as an *arch*.

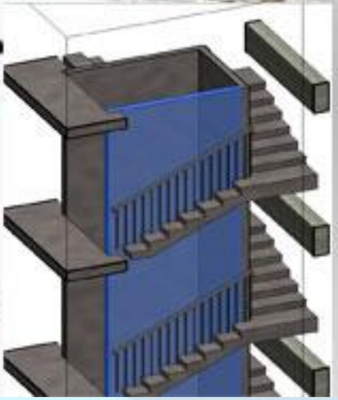
إذا كان العنصر النحيل منحنٍ يسمى قوساً أو قنطرةً.

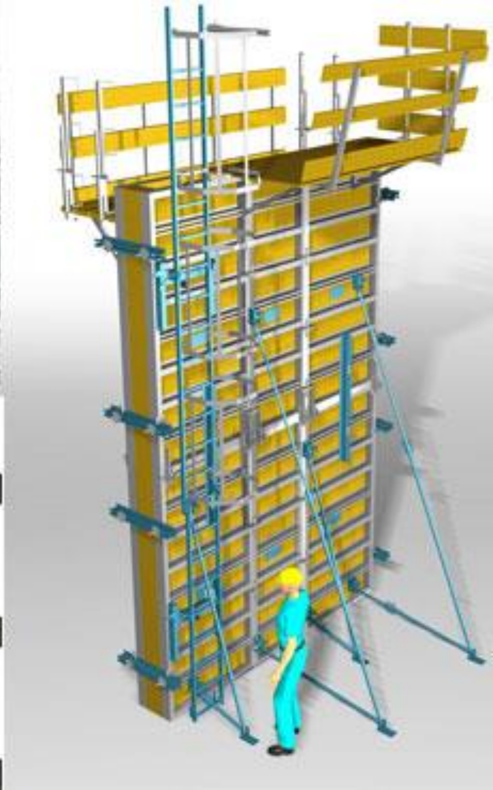
Structures consisting of orthogonal or inclined, rigidly joined beams are called frames.

تدعى الهياكل الحاملة المؤلفة من جيزان متعامدة أو مائلة ومتصلة بعقد صلبة كلياً أو جزئياً، إطاراتٍ ومفردها إطار.

A plane structural element with a thickness much smaller than its characteristic inplane length is called *disk, panel* (shear wall) if it is solely loaded by in-plane forces.



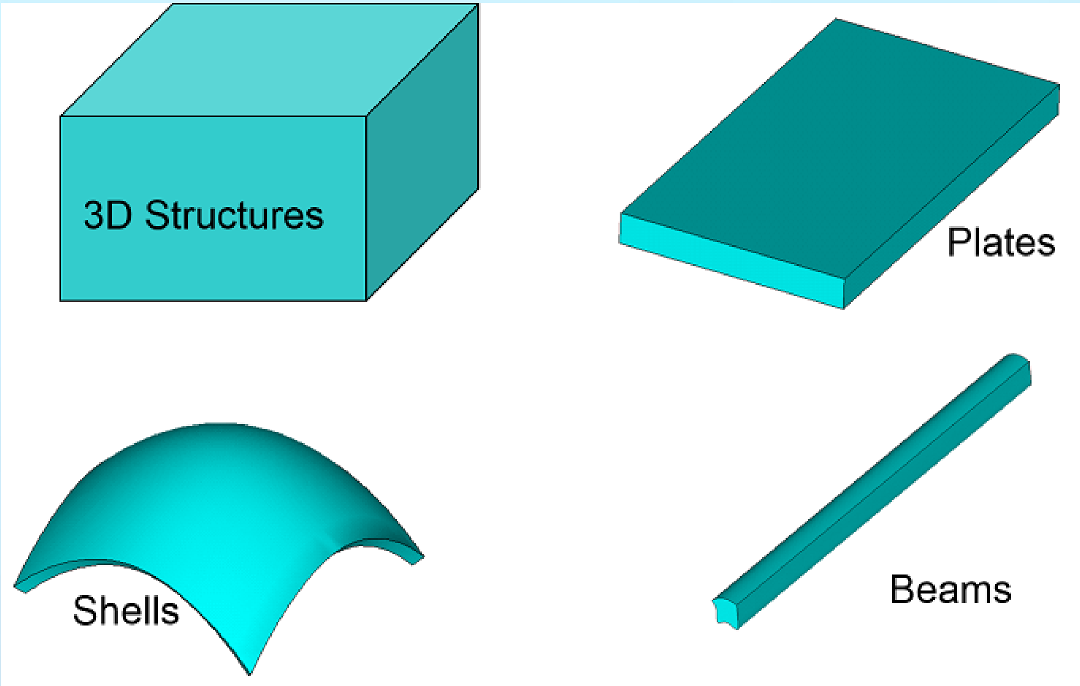


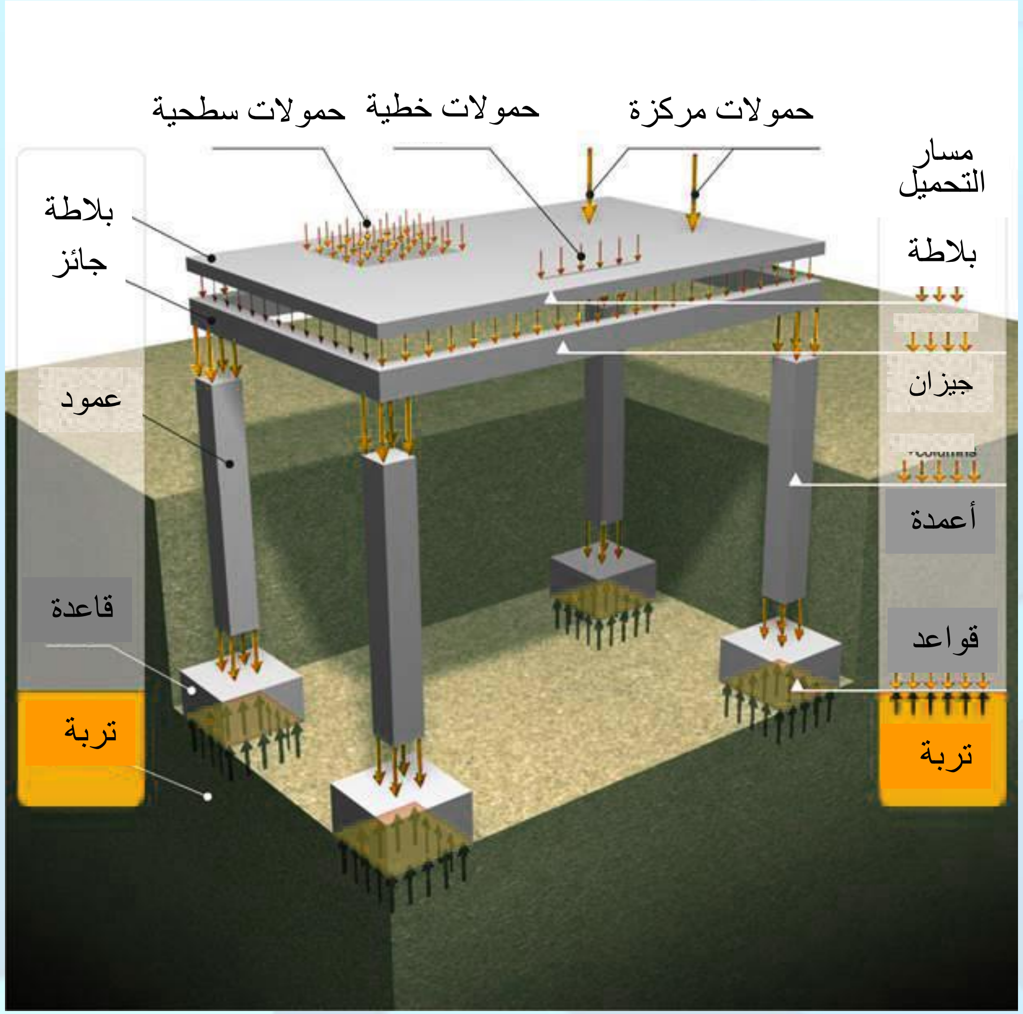


تحوي الهياكل الحاملة عناصر مستوية رقيقة سماكاتها أصغر بكثير من أبعادها الأخرى في مستويها. وإذا كانت هذه العناصر محملة في مستويها فقط دُعيت بالشرائح أو الألواح (ومنها جدران القص).

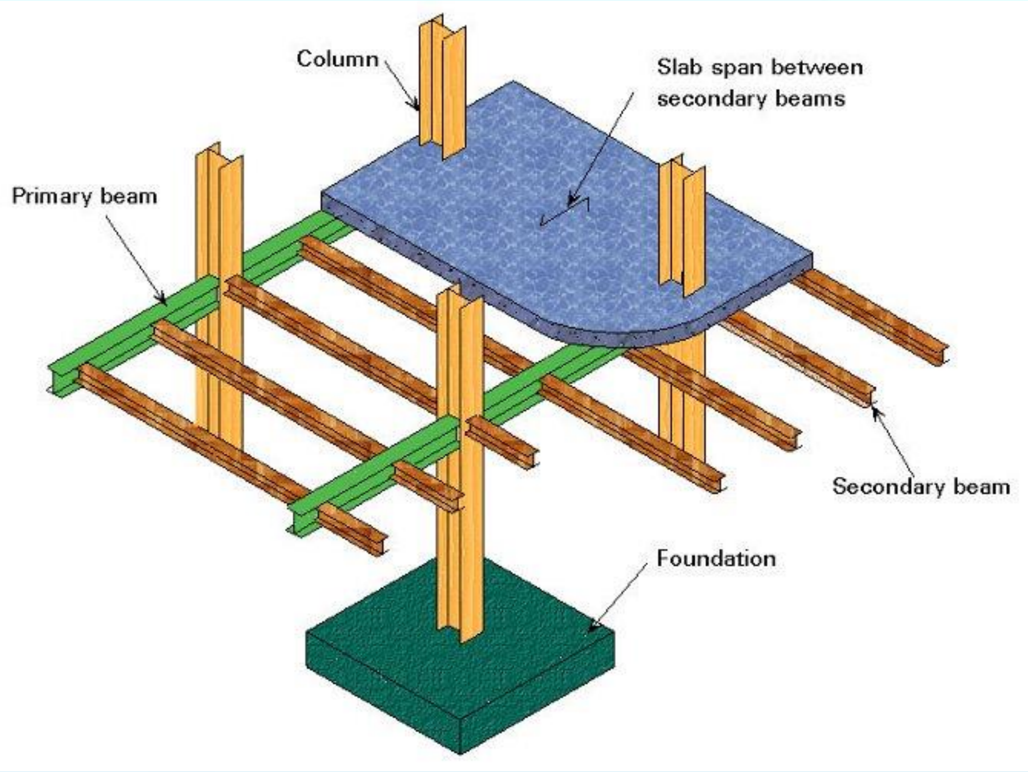
If the same geometrical structural Element is loaded perpendicularly to its midplane it is called a *plate or slab*. If such a structure is curved it is a *shell*.

أما إذا تلقت هذه العناصر المستوية الرقيقة حمولات متعامدة مع مستويها فتدعى بلاطة. وإذا كانت رقيقة لكن منحنية فتدعى قشرية.





Structures are connected to their surroundings by *supports* (مساند)whose purpose is to fix the structure in space in a specific position. *Supports* act against mortice forces.

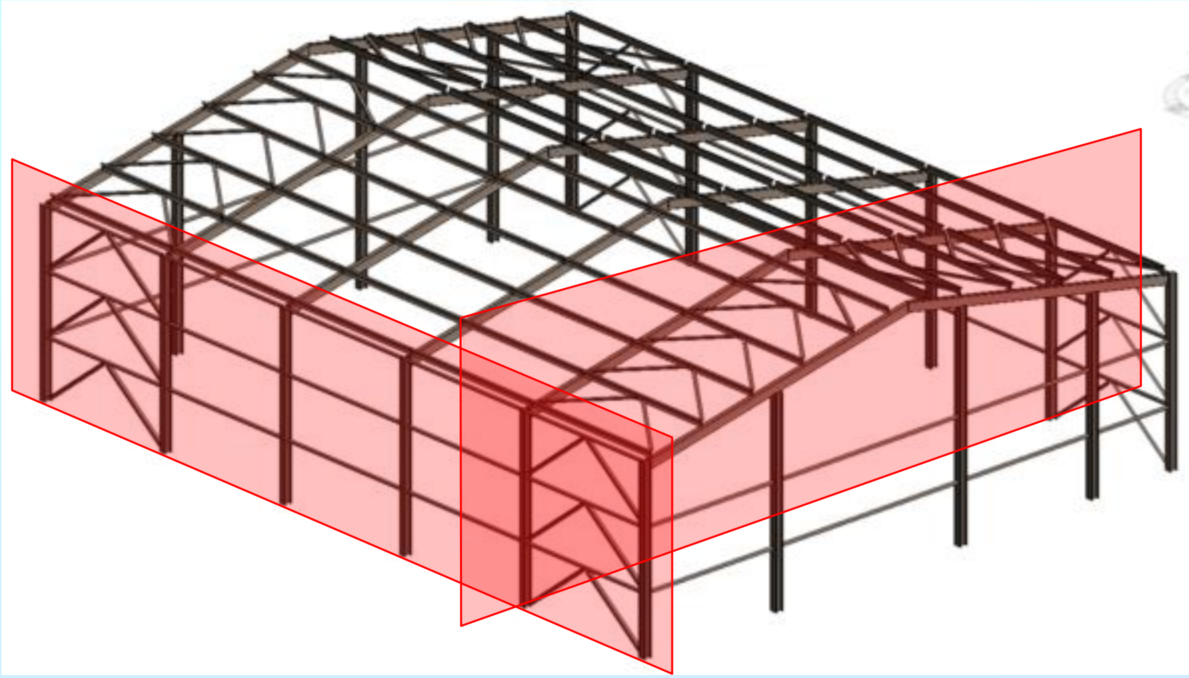


تكبح المساند القوى المحركة:
كقوى الجاذبية الشاقولية أو الدفع الجانبي لقوى الريح، ضغط التربة والمياه وأفعال الزلزال وعصف الانفجارات.

تصنف المساند حسب درجة كبحها للحركة كما سنرى.

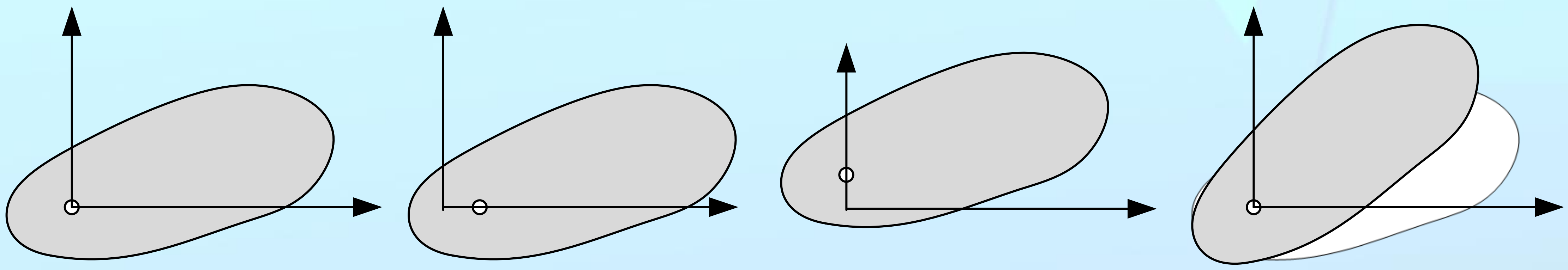
بعضها دائم وبعضها مؤقت.

This course is limited to a single part of a structure located and loaded in a plane.



Degrees of Freedom and Support Restraints

A free body in a plane with no restraints has three degrees of freedom.
It can be independently displaced by two translations in different directions and by one rotation about an axis perpendicular to the plane.



Supports (restraints) reduce the feasible displacements: each support reaction imposes a constraint. Let r be the number of support reactions.
Then the number f of degrees of freedom of a body in a plane is given by: $f=3-r$.

We will now consider different types of supports and classify them by the number of support reactions involved.

Supports that can transmit only one single reaction ($r = 1$). Examples of this type are the **roller or rocker** supports without friction, the simple support and the support by a **strut**.

المسند المتدحرج بدون احتكاك والرقاص هما مثالان عن المساند التي تعيق درجة حرية واحدة ($r = 1$). مثال آخر هو الدعامة وهي ذراع طويل خفيف الوزن متمفصل من الطرفين. في هذه الأمثلة المسند يسمح بانسحاب واحد ويسمح بالدوران في المستوي.

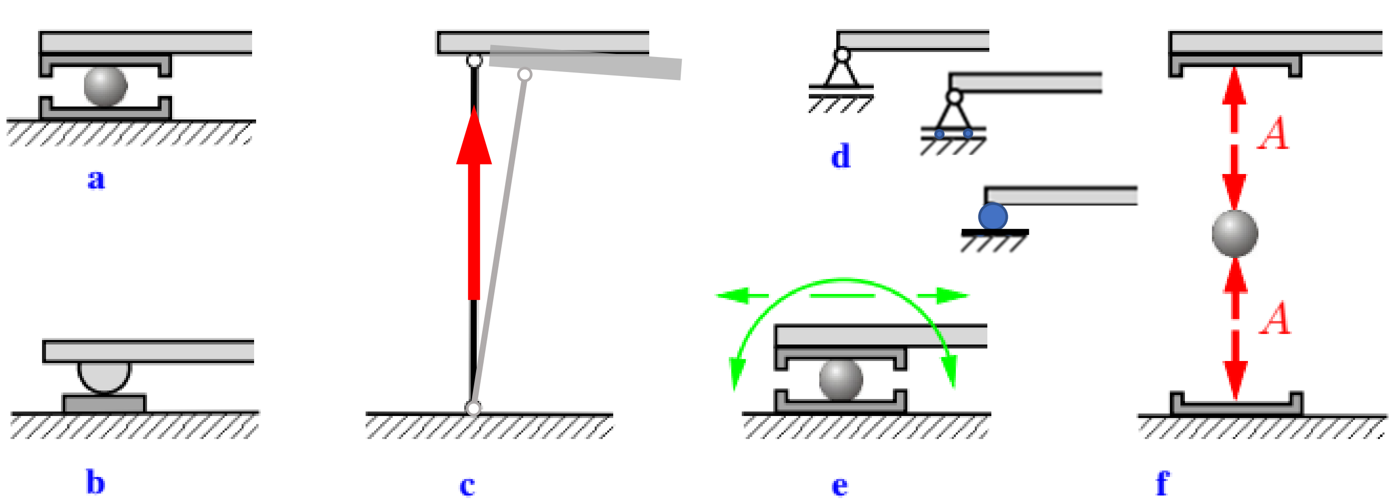
Roller مسند متدحرج

Roller مسند متدحرج

Rocker مسند رقاص

Strut دعامة

In this case, the direction of the reaction force is known, but its intensity is unknown.



في هذه الحالة، اتجاه قوة رد الفعل معلوم بينما شدتها مجهولة

One static unknown.

مجهول ستاتيكي واحد

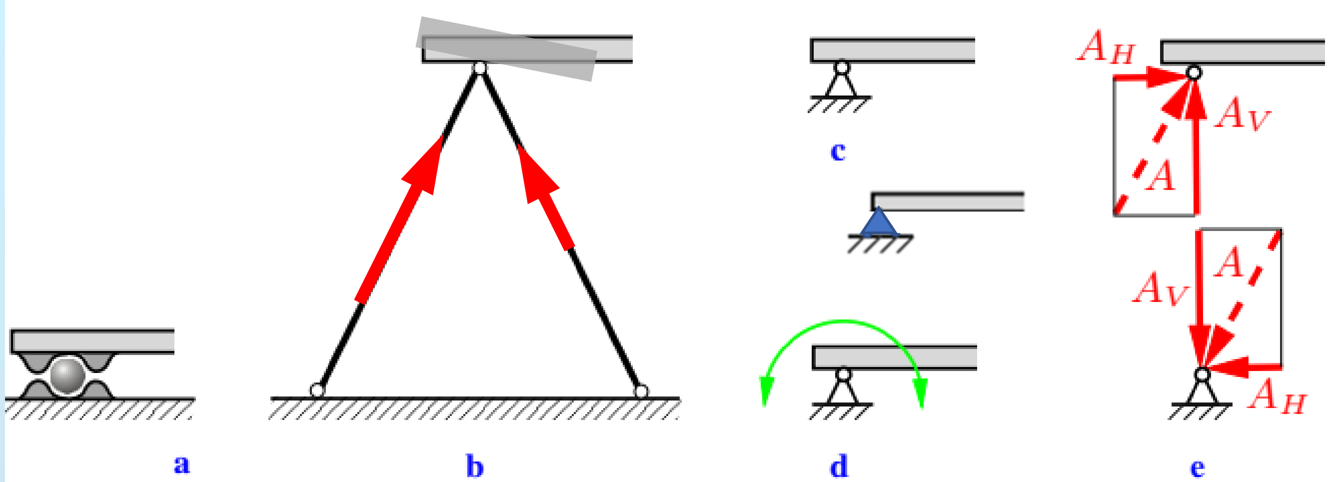
Supports that transmit two reactions ($r = 2$) . Examples of this type of support are the hinged support or pin.

المسند المفصلي واختصارا المفصل، يمنع الحركة الانسحابية في الاتجاهين ($r = 2$)، ويسمح بالدوران في المستوي.

يتكون المفصل في المنشآت الفولاذية والخشبية من فراغ اسطواني يخترق الجزء الثابت والجزء المراد تثبيته، ومن محور يملؤ هذا الفراغ يجمع الجزئين ويسمح لهما بالدوران حوله.

كما يُشكيل هذا المسند من جمع دعامتين تلتقيان في مفصل مشترك.

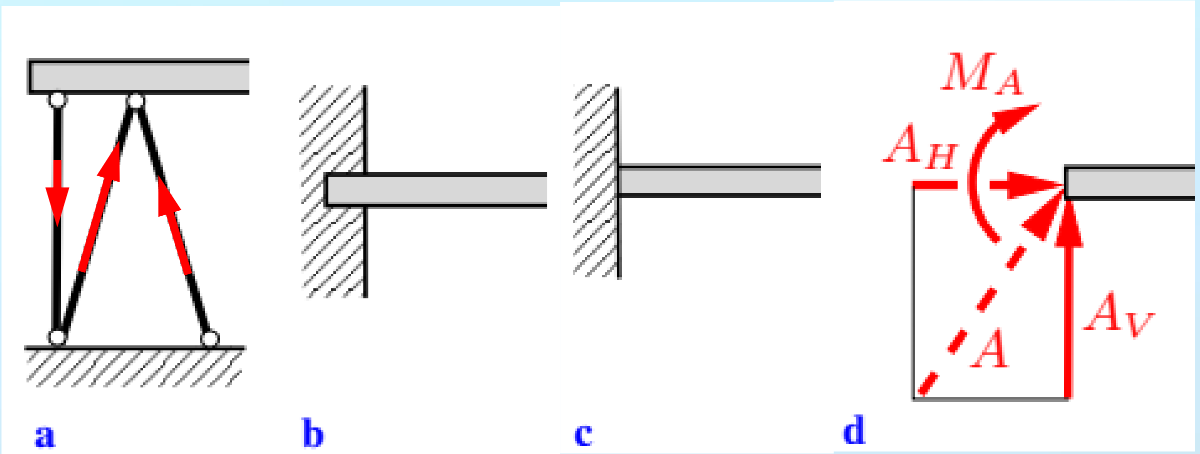
ويُمثل المفصل رمزياً بهما مصغرين كما في الشكل c.



يطبق هذا المفصل على العنصر المراد تثبيته قوة مجهولة الشدة والاتجاه أو مركبتين أي أن هناك مجهولين يجب تحديدهما.

The rotational degree of freedom disappears if a support by two struts is complemented by an additional, somewhat shifted, third strut figure a. The structure becomes immobile. In addition to the two force components, the support can now also transmit a couple moment, i.e., in total three reactions: $r = 3$. The same situation appears in the case of a clamped support (fixed support) according to figure b which symbolically is depicted in figure c.

يمكن منع الدوران إذا أُضيفت دعامة ثالثة إلى الخلف قليلاً من الدعامتين المشكلتين للمفصل كما في الشكل a. وبذلك تمنع درجات الحرية الثلاثة في المستوي. ويُسمى المسند عندئذٍ وثاقة. كما يمكن أن تُشكل الوثاقة بغرس العنصر المراد تثبيته إلى قاعدة متينة جداً أو عبر تثبيته إلى جدار قوي بواسطة صفيحة تلتصق بالجدار ببراغي تغرس فيه بعمق كاف. تُمثل الوثاقة رمزيا كما في الشكل c.

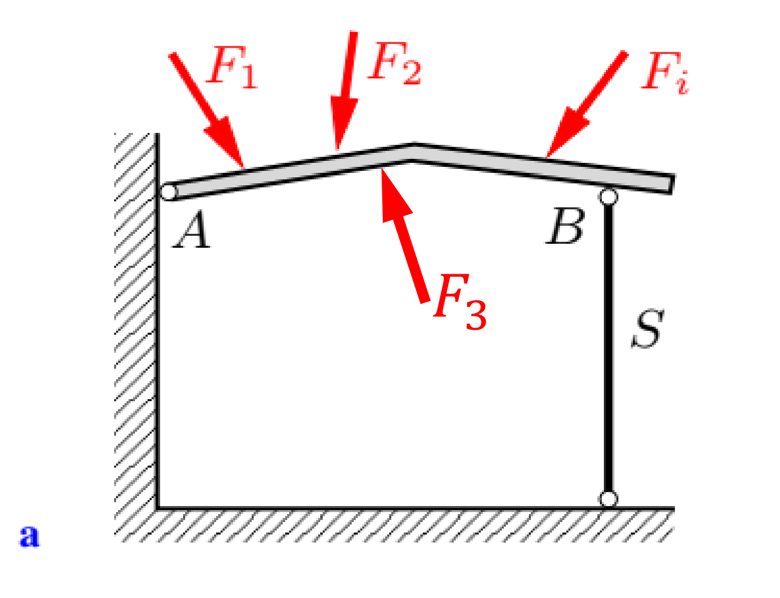


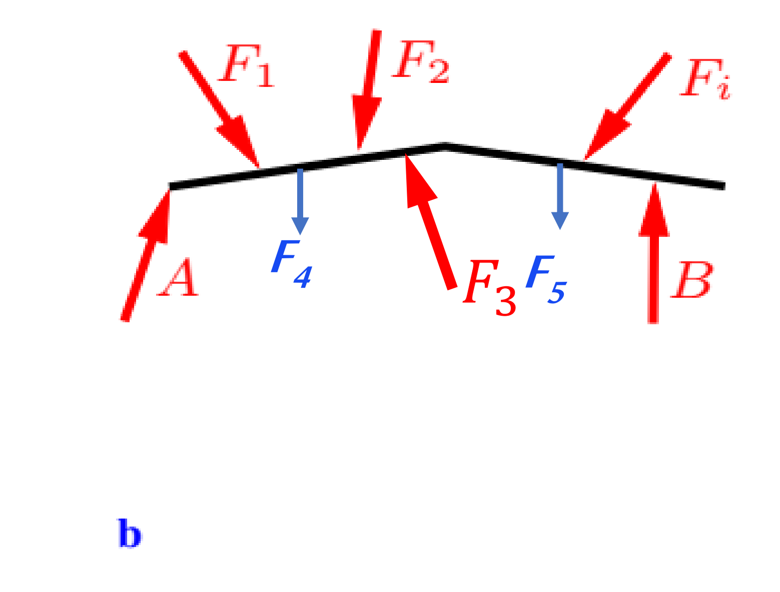
The free-body diagram in figure d shows that the clamped support can transmit a reaction force A of arbitrary magnitude & direction (or A_H and A_V) and a couple moment M_A (or A_M).

تستبدل الوثاقة في مخطط الجسم الحر للعنصر المراد تثبيه، بقوة A مجهولة الشدة والاتجاه أو بمركبتيها (A_H and A_V) بالإضافة إلى عزم M_A أو A_M.

As an idealized example, consider the "roof" in figure a, loaded by external forces F_i, joined at A to a vertical wall by a pin, and supported at B by the strut S. Forces are transmitted to the wall and the ground via the supports A & B. According to the law of action & reaction (action=reaction) the same forces are exerted in opposite directions from the wall & the ground onto the roof. These forces from the environment onto the structure are reaction forces, and are termed support reactions. They become visible in the free-body diagram (Figure b), where they are generally denoted by the same symbols as the supports, i.e. by A and B in this example.

كمثال توضيحي لنأخذ سقف المظلة المبين على الشكل a، محملة بالقوى الخارجية F_i، ومثبتة إلى الجدار الشاقولي بمفصل A، وتستند في على الدعامة S. تنتقل قوى الحمولات إلى الجدار والأرض عبر المسندين . وحسب مبدأ الفعل ورد الفعل يرد المسندان على السقف بقوى مساوية بالقيمة ومعاكسة بالاتجاه نسميها ردود أفعال المساند ونبينها على الشكل b، في مخطط الجسم الحر للسقف كقوة A مجهولة الشدة والاتجاه (مجهولان) يطبقها المسند المفصلي وكقوة B معلومة الاتجاه مجهولة الشدة (مجهول وحيد). وبالمحصلة يكون لدينا ثلاثة مجاهيل.

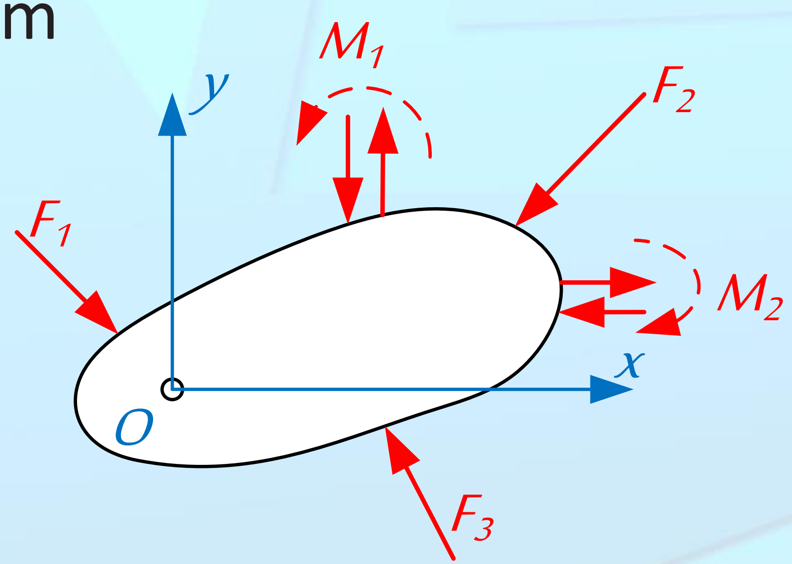




1.2 Statical Determinacy

A structure is called statically determinate if the support reactions can be calculated from the equilibrium conditions.

Hence, a rigid body under the action of a general system of coplanar forces is in equilibrium if the following equilibrium conditions are satisfied:



$$\sum F_{ix} = 0, \qquad \sum F_{iy} = 0, \qquad \sum M_{i/O} = 0.$$

The axes and/or the pivotal point are arbitrary

$$\sum F_{ix'} = 0, \qquad \sum F_{iy'} = 0, \qquad \sum M_{i/O'} = 0.$$

Since the number of unknowns must coincide with the number of equations, three unknown reactions (forces or couple moments) must exist at the supports: $r = 3$. *It will be explained later that this necessary condition may not be sufficient for the determination of the support reactions.*

The beam in figure a is supported by the hinged support *A* and the simple support *B*. Accordingly, the three unknown support reactions A_H , A_V & B exist. Therefore, with $r = 3$ it follows from (5.1) that the beam is immobile: $f = 3 - r = 0$; it is statically determinate.

The support reactions of the clamped beam in figure b consist of the two force components A_H , A_V & the couple moment M_A (or A_M) .

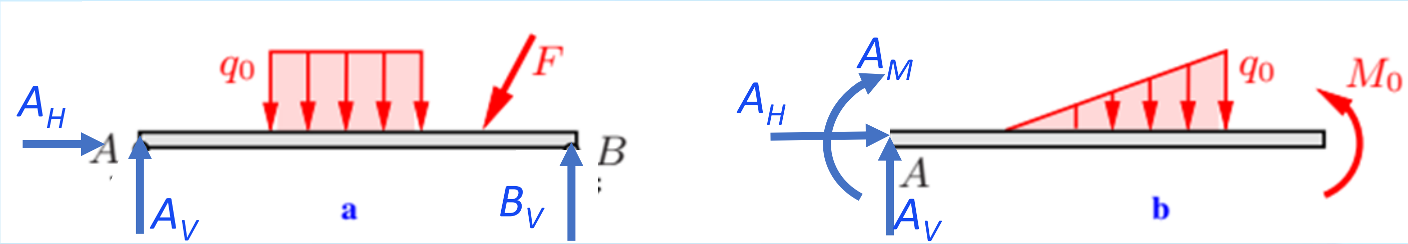
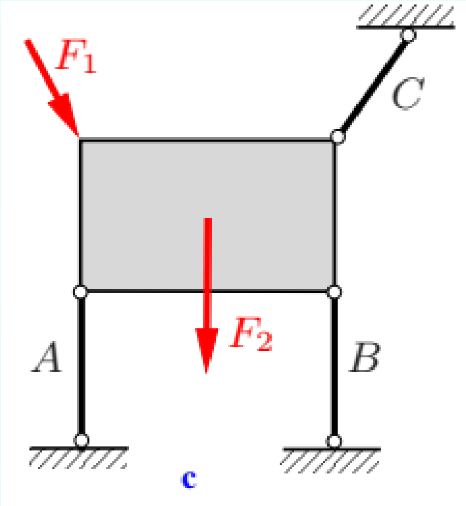
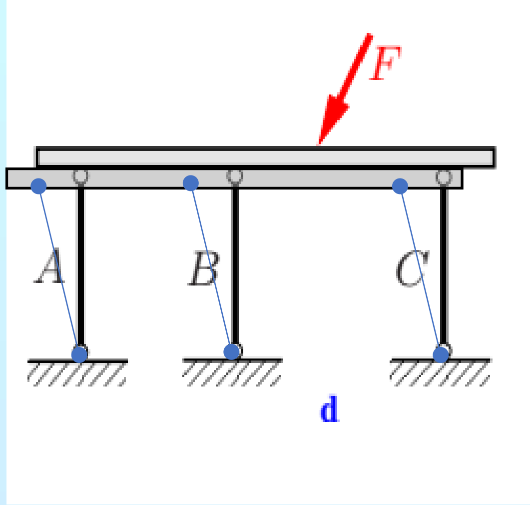
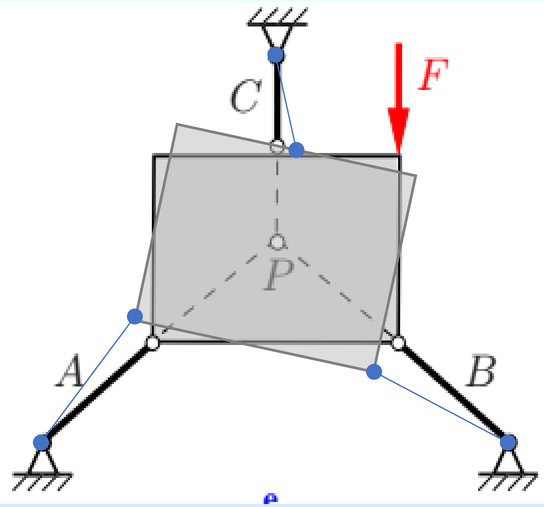


Figure c shows a panel supported by the three struts *A*, *B* & *C*, each transmitting one reaction. In both cases, with *r* = 3 and *f* = 0, the panel is statically determinate.

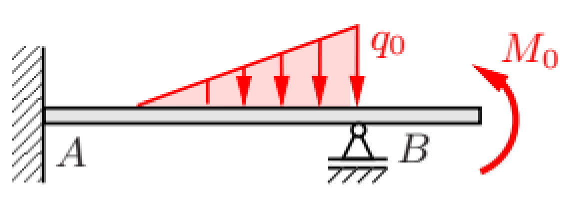


statically determinate





Kinematically indeterminate



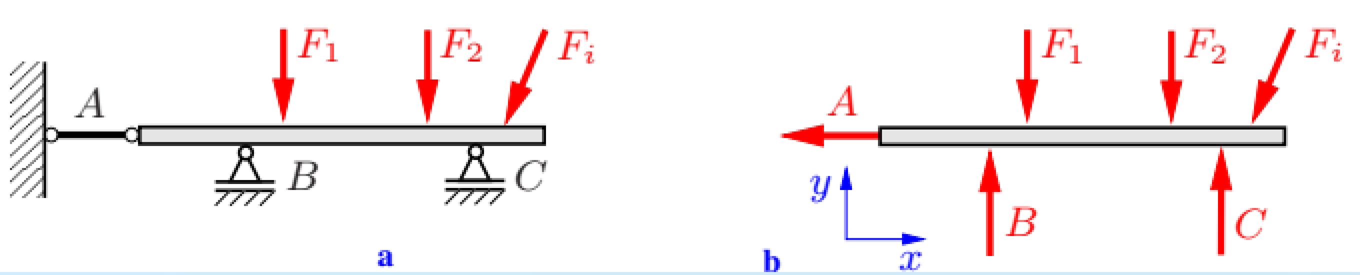
statically indeterminate

1.3 Determination of the Support Reactions

In order to determine the support reactions, the method of free body diagram is applied: تتبع تقنية مخطط الجسم الحر من أجل تحديد ردود الأفعال.

The body is freed from its supports and their action on the body is replaced by the unknown reactions.

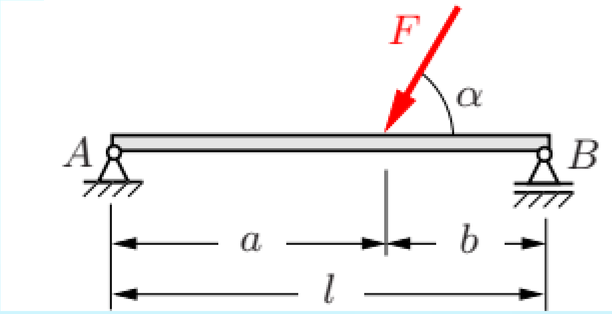
يحرر العنصر الحامل من مسانده التي تستبدل بردود الأفعال المجهولة.



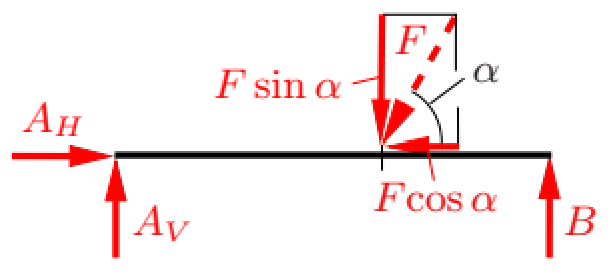
ثم تطبق معادلات (شروط) التوازن المناسبة بين الحمولات وردود الأفعال.

$$\sum F_{ix} = 0, \qquad \sum F_{iy} = 0, \qquad \sum M_{i/O} = 0.$$

Example 1 The beam shown in figure a is loaded by the force F which acts under an angle α. Determine the reaction forces at the supports A and B.

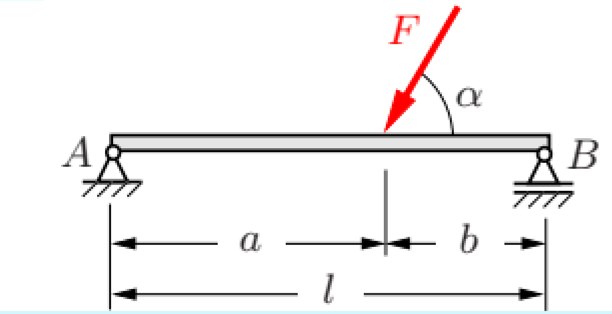


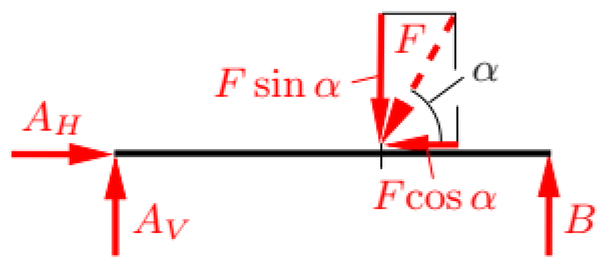
Solution: The beam is rigidly supported; the support A transmits two reactions and support B one reaction. In total, the three unknown reaction forces A_H, A_V & B exist, therefore, the beam is statically determinate.



We free the beam from its supports and make the reaction forces visible in the free-body diagram where we choose their senses of direction along the action lines freely. Hence, the equilibrium conditions are given by

$$\sum F_{ix} = 0, \qquad \sum F_{iy} = 0, \qquad \sum M_{i/O} = 0.$$





$$\sum F_{ix} = 0, \qquad \sum F_{iy} = 0, \qquad \sum M_{i/O} = 0.$$

$$\sum \vec{F}_{ix} = 0: \qquad A_H - F\cos\alpha = 0, \qquad (1)$$

$$\uparrow \sum F_{iy} = 0: \quad A_V + B - F\sin\alpha = 0, \qquad (2)$$

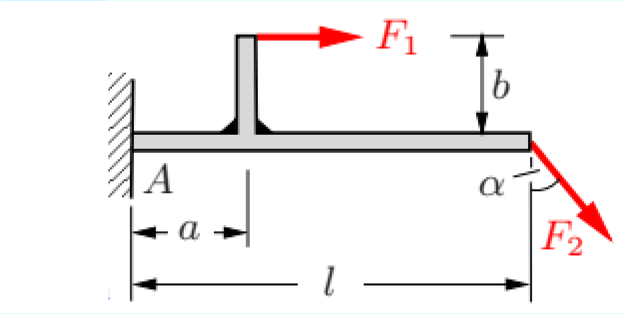
$$\downarrow\uparrow \sum M_{i/A} = 0: \qquad lB - aF\sin\alpha = 0, \qquad (3)$$

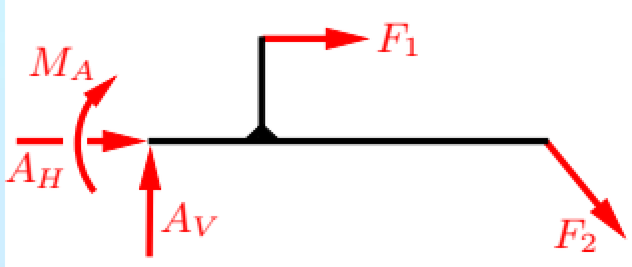
Solving (1) $\Rightarrow A_H = F\cos\alpha$

Solving (3) $\Rightarrow B = (a/l)F\sin\alpha$

Sub. in (2) $\Rightarrow A_V = (b/l)F\sin\alpha$

Example 2 The clamped beam shown in figure a is loaded by the two forces F_1 and F_2 .
Determine the reactions at the support.





Solution:

We free the beam from its supports and make the reaction forces visible in the free-body diagram where we choose their senses of direction along the action lines freely. Hence, the equilibrium conditions are given by

$$\sum F_{ix} = 0, \qquad \sum F_{iy} = 0, \qquad \sum M_{i/O} = 0.$$

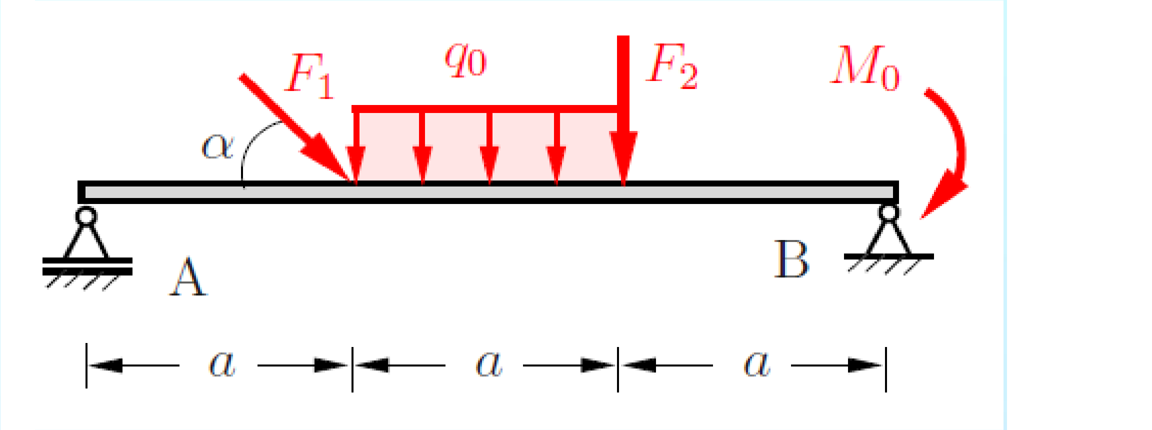
$$\overset{\rightarrow}{\sum} F_{ix} = 0: \qquad A_H + F_1 + F_2 \sin\alpha = 0 \Rightarrow \quad A_H = -(F_1 + F_2 \sin\alpha)$$

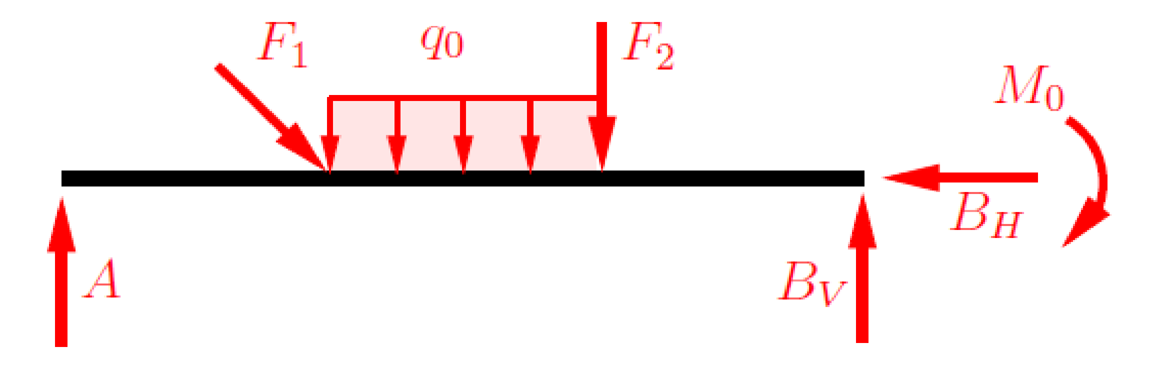
$$\uparrow \sum F_{iy} = 0: \quad A_V - F_2 \cos\alpha = 0 \Rightarrow A_V = F_2 \cos\alpha$$

$$\downarrow\uparrow \sum M_{i/A} = 0: -M_A - bF_1 - lF\cos\alpha = 0 \Rightarrow M_A = -(bF_1 + lF\cos\alpha)$$

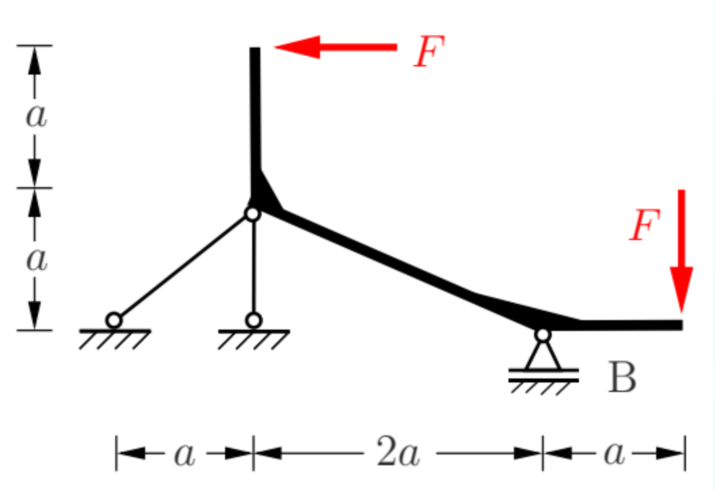
Problem 1. Determine the support reactions for the depicted system.

Given: F_1 = 2 kN, F_2 = 3 kN, a = 1m, M_0 = 4 kNm, q_0 = 5 kN/m, α = 45°.





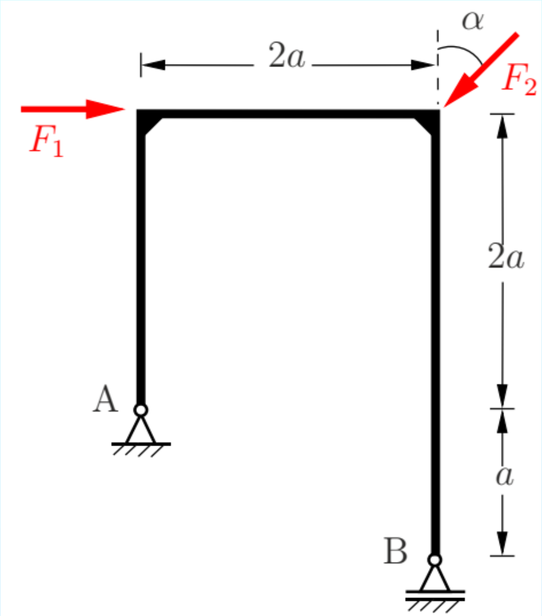
Problem 2. Determine the support reactions for the depicted systems



Problem 3.

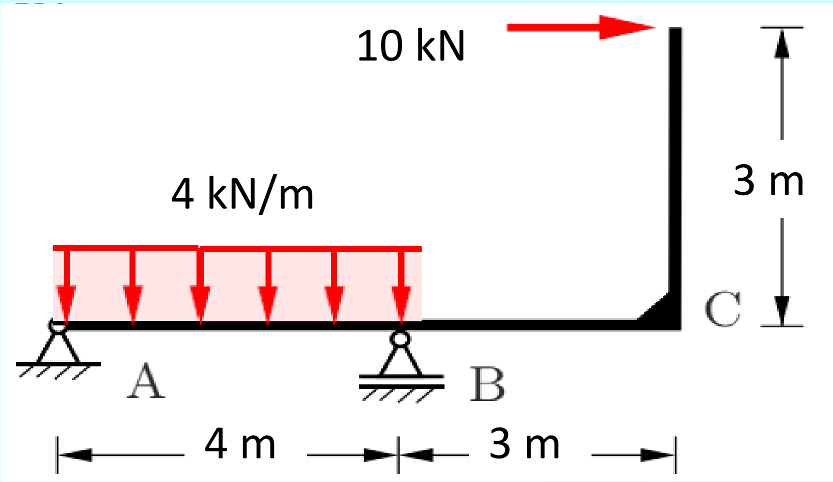
Determine the support reactions for the depicted frame.

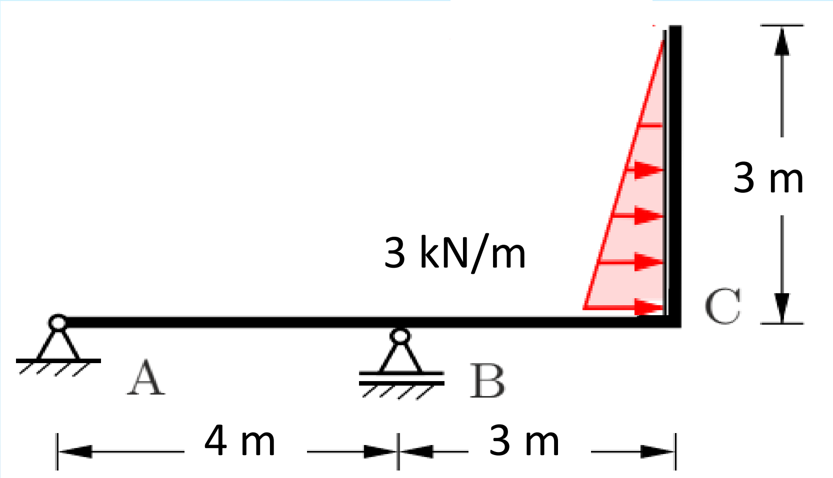
Given: F_1 = 2000 N, F_2 = 3000 N, α = 45°, a = 5 m.



Problem 4.

1. Draw the free Body Diagram of the shown Element
2. Find the reactions of the supports

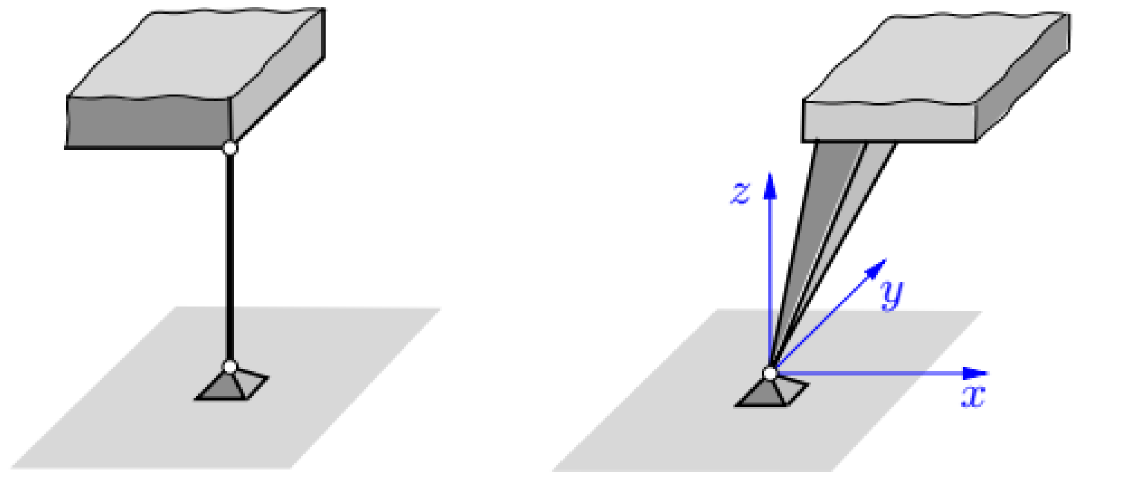




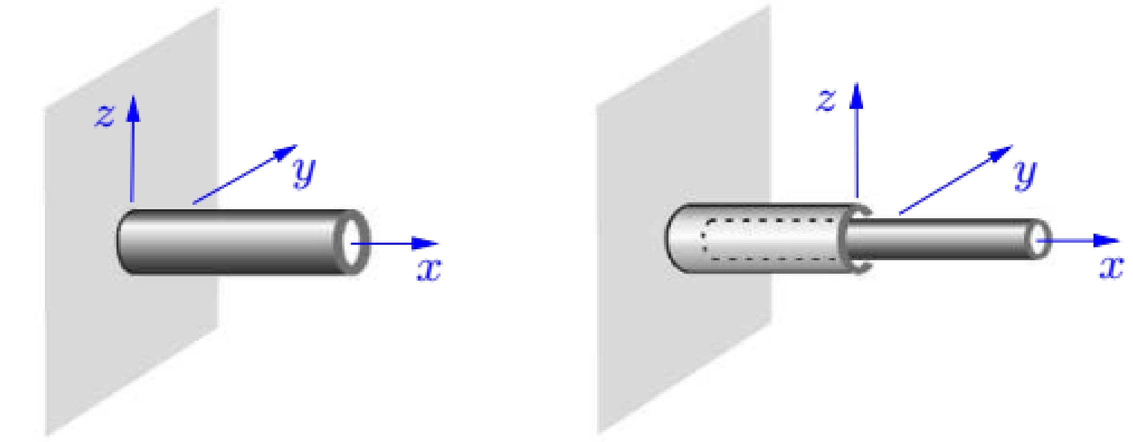
2. Spatial Structures

A body that can move freely in space has six degrees of freedom $f = 6$:
3 translations in x, y & z direction and 3 rotations about the three axes.

Supports constrain the possible displacements. As in the plane case, different types of support are classified by the number of transferable support reactions.



a $r = 1$ b $r = 3$



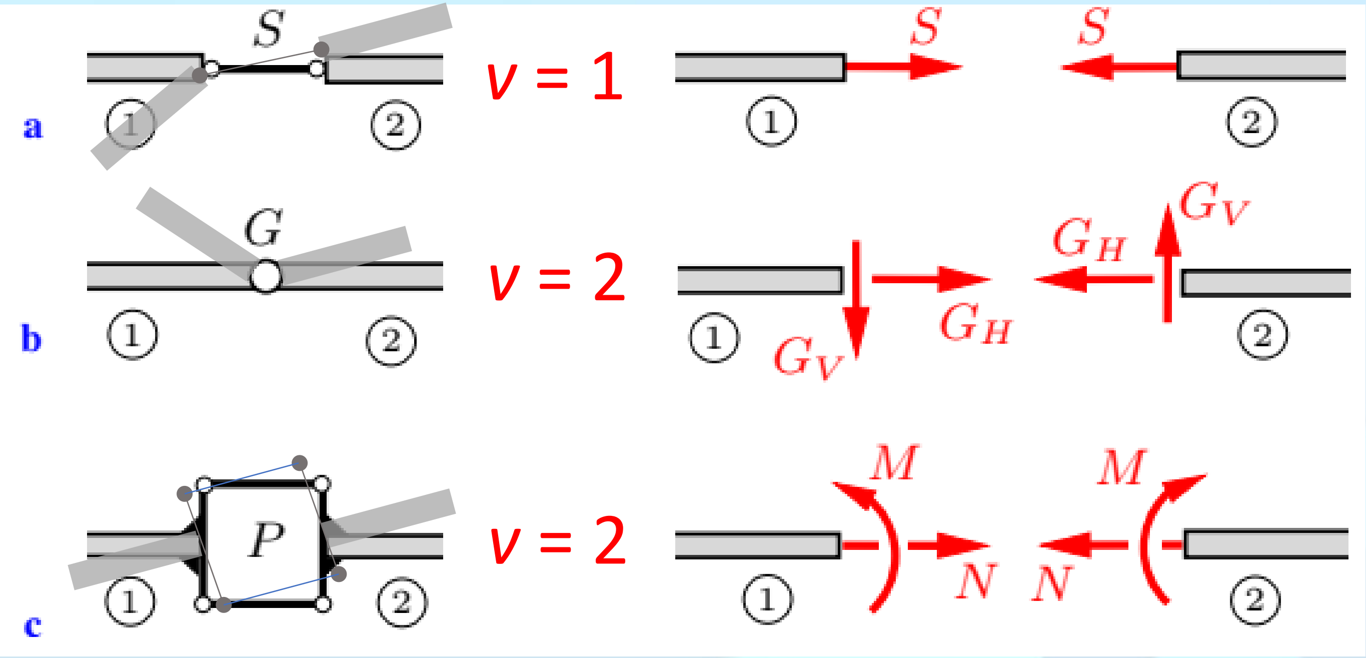
c $r = 6$ d $r = 4$

3. Multi-Part Structures

3.1. Statical Determinacy

Structures often consist not only of one single part but of a number of rigid bodies that are appropriately connected. The connecting members transfer forces and moments, respectively, which can be made visible by passing cuts through the connections. In the following the discussion is restricted to plane structures.

$r + v = 3\,n$

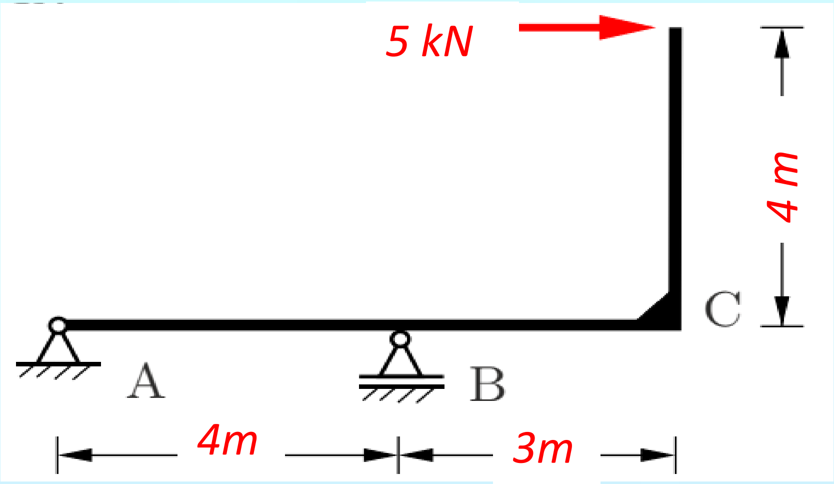


Strut

Hinge

Parallel motion

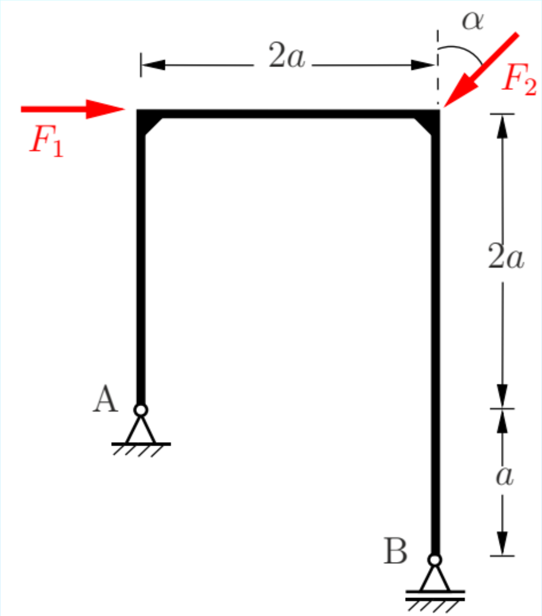
Problem 1. Determine the support reactions for the depicted systems



Problem 5.

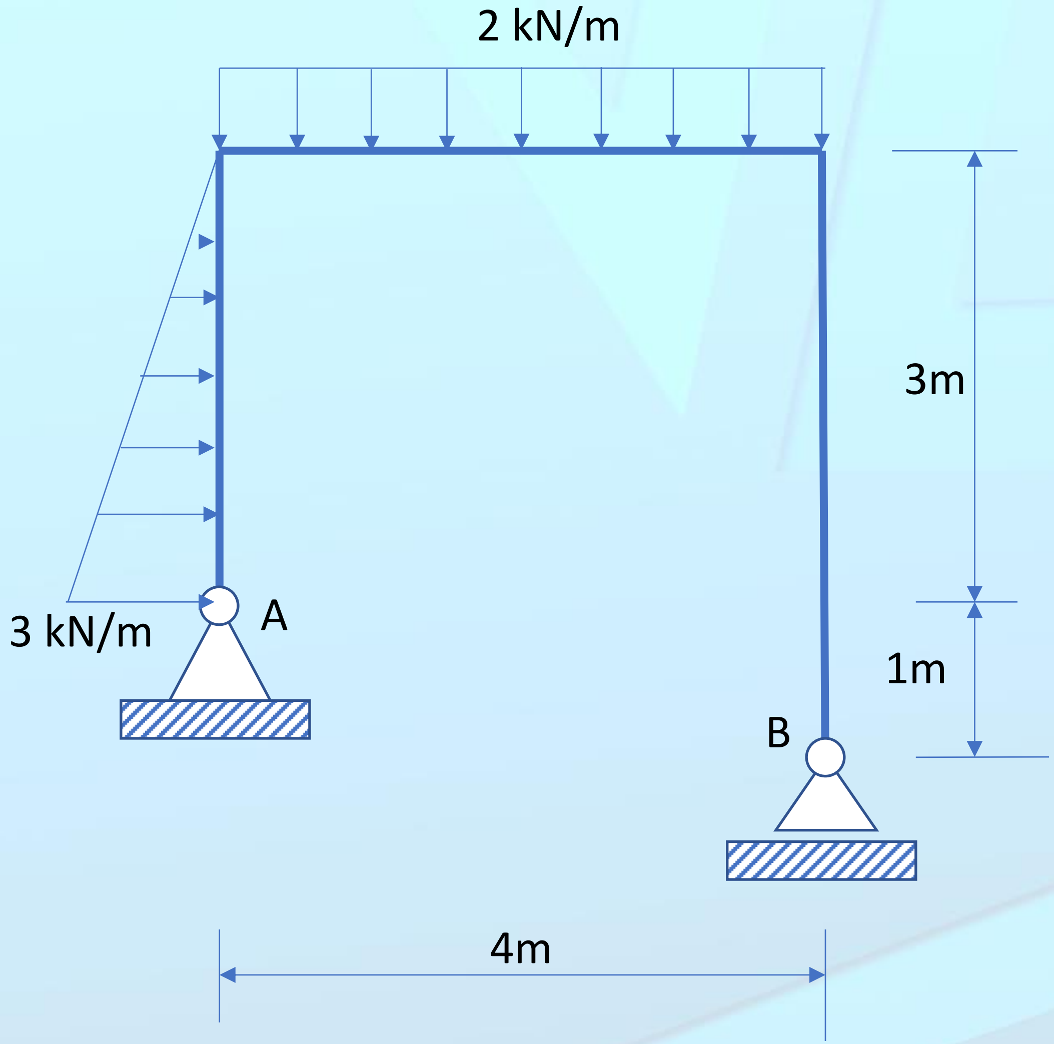
Determine the support reactions for the depicted frame.

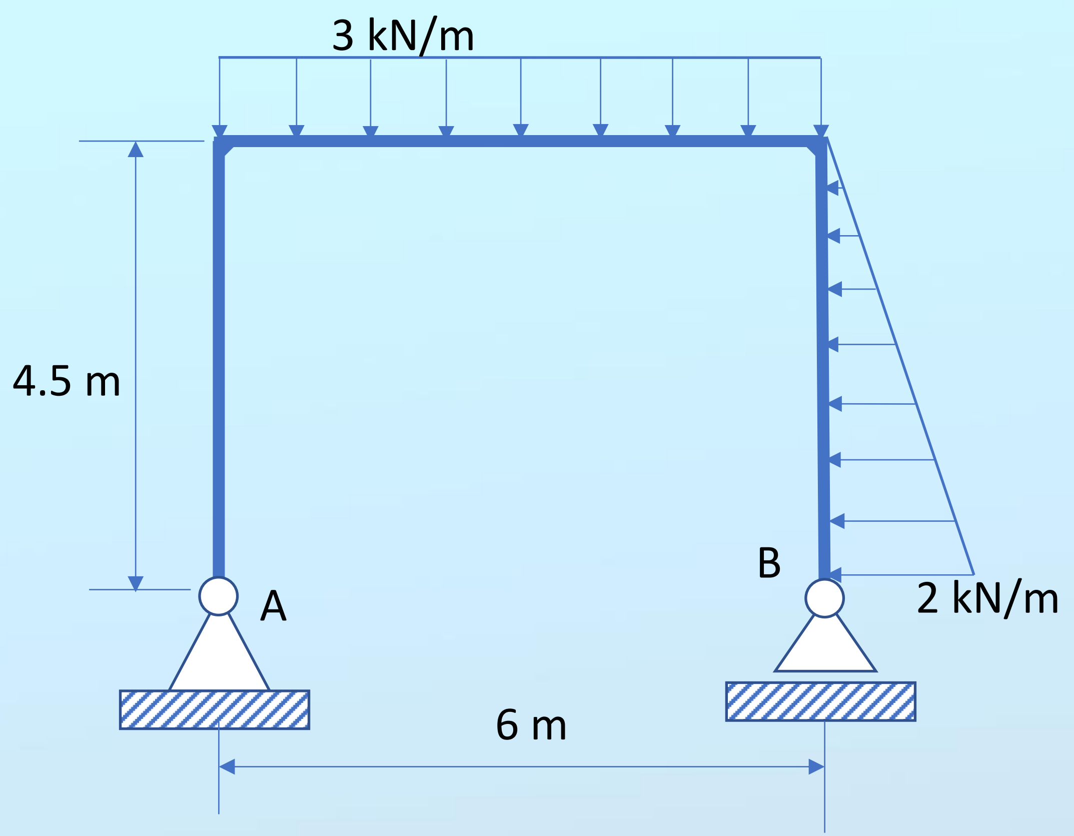
Given: F_1 = 3 kN, F_2 = 5 kN, α = 40°, a = 3 m.

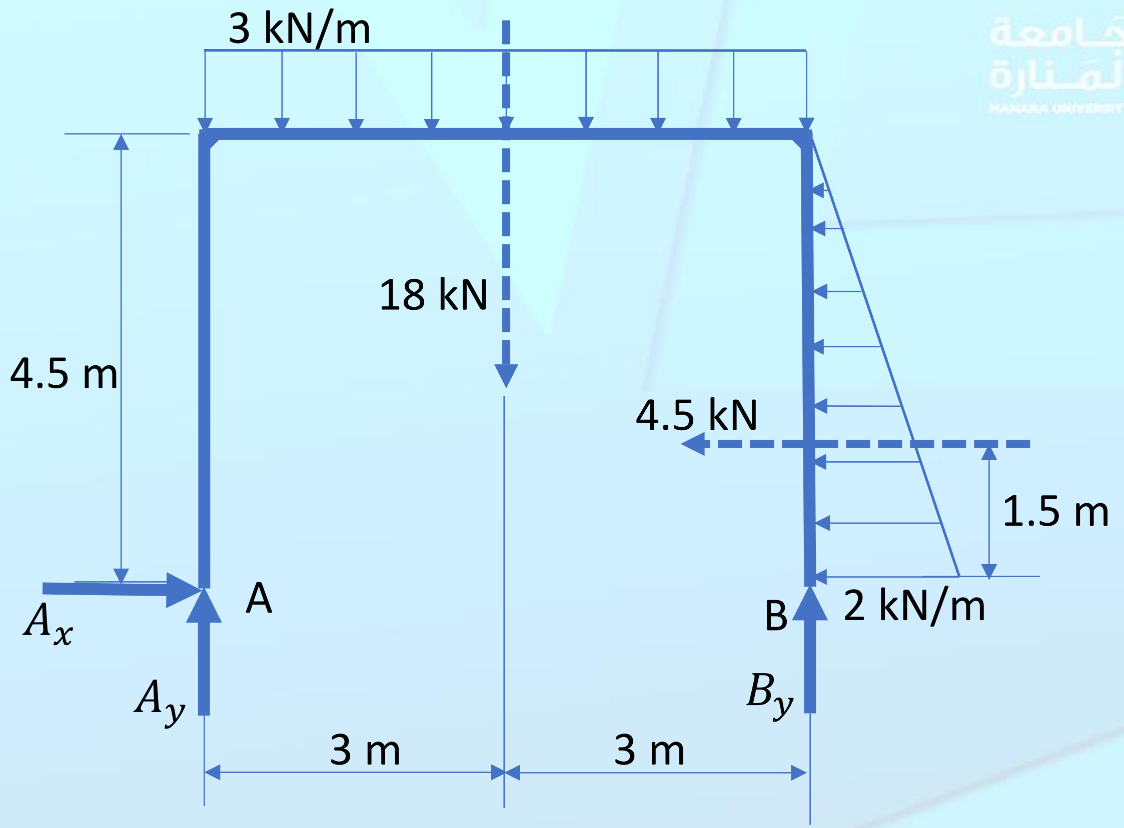


Problem 6.

Determine the support reactions for the depicted frame.




3 kN/m
4.5 m
A
B
2 kN/m
6 m


3 kN/m
18 kN
4.5 m
4.5 kN
1.5 m
A_x
A
A_y
B
B_y
2 kN/m
3 m
3 m