

Structural Mechanics (1)

Lecture No-07

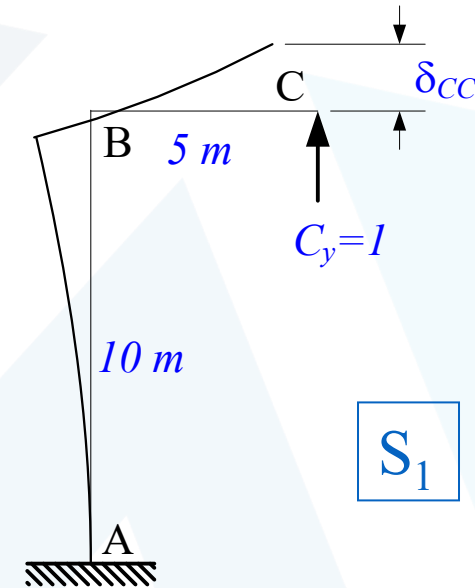
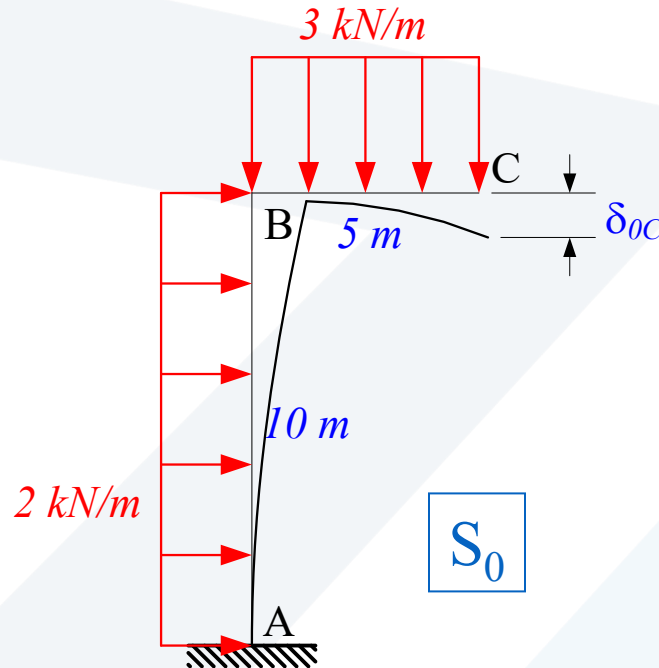
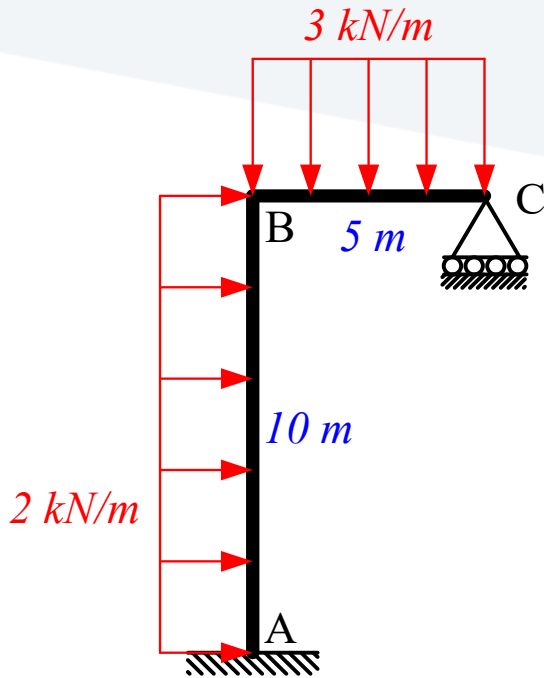
Part-01

Analysis of Indeterminate Structures - Force Method

- Indeterminate Structures vs. Determinate Structures
- Analysis of Indeterminate Structures.
- Structures with single Degree of Indeterminacy (Beams & Frames)
- Structures with single Degree of Indeterminacy (Trusses: Int. & Ext.)
- Structures with multiple Degrees of Indeterminacy
- Support Settlements
- Three-Moment Equation for Continuous Beams

Force method: Frame with a single Degree of Indeterminacy

Compute the support reactions in the frame. $E = 200 \text{ GPa}$, $I_c = 10^6 \text{ mm}^4$ for the column & $I_b = 2I_c$ for the beam. Then draw the BM & SF diagrams



SOLUTION: The frame is statically indeterminate to degree one. Select C_y as the redundant. Draw the two determinate frames (S_0) & (S_1)

The compatibility equation is

$$\delta_{0C} + C_y \delta_{CC} = 0$$

Force method: Frame with a single Degree of Indeterminacy

Segment	FBD in S_0	$M_0(x)$	FBD in S_1	$m_C(x)$
CB $0 \leq x \leq 5$ $2EI$ is constant		$-3x^2/2$		x
AB $0 \leq x \leq 10$ EI is constant		$-x^2 + 20x - 137.5$		5

$$\delta_{0C} = \frac{1}{2EI} \int_0^5 \left(-\frac{3x^2}{2} \right) (x) dx + \frac{1}{EI} \int_0^{10} (-x^2 + 20x - 137.5)(5) dx = -\frac{234.375}{2EI} - \frac{3541.67}{EI} = -\frac{3658.85}{EI}$$

$$\delta_{CC} = \frac{1}{2EI} \int_0^5 x^2 dx + \frac{1}{EI} \int_0^{10} 25 dx = \frac{270.833}{EI} \Rightarrow C_y = 13.5 \text{ kN}$$

Force method: Frame with a single Degree of Indeterminacy

Computing the reactions

$$\uparrow + \sum F_y = 0 \Rightarrow A_y + 13.5 - 15 = 0 \Rightarrow A_y = 1.5 \text{ kN}$$

$$\rightarrow + \sum F_x = 0 \Rightarrow A_x + 20 = 0 \Rightarrow A_x = -20 \text{ kN} = 20 \text{ kN}(\leftarrow)$$

$$(\downarrow +) \sum M_A = 0 \Rightarrow M_A - (5)20 - (2.5)15 + (5)13.5 = 0 \Rightarrow M_A = 70 \text{ kN.m}$$

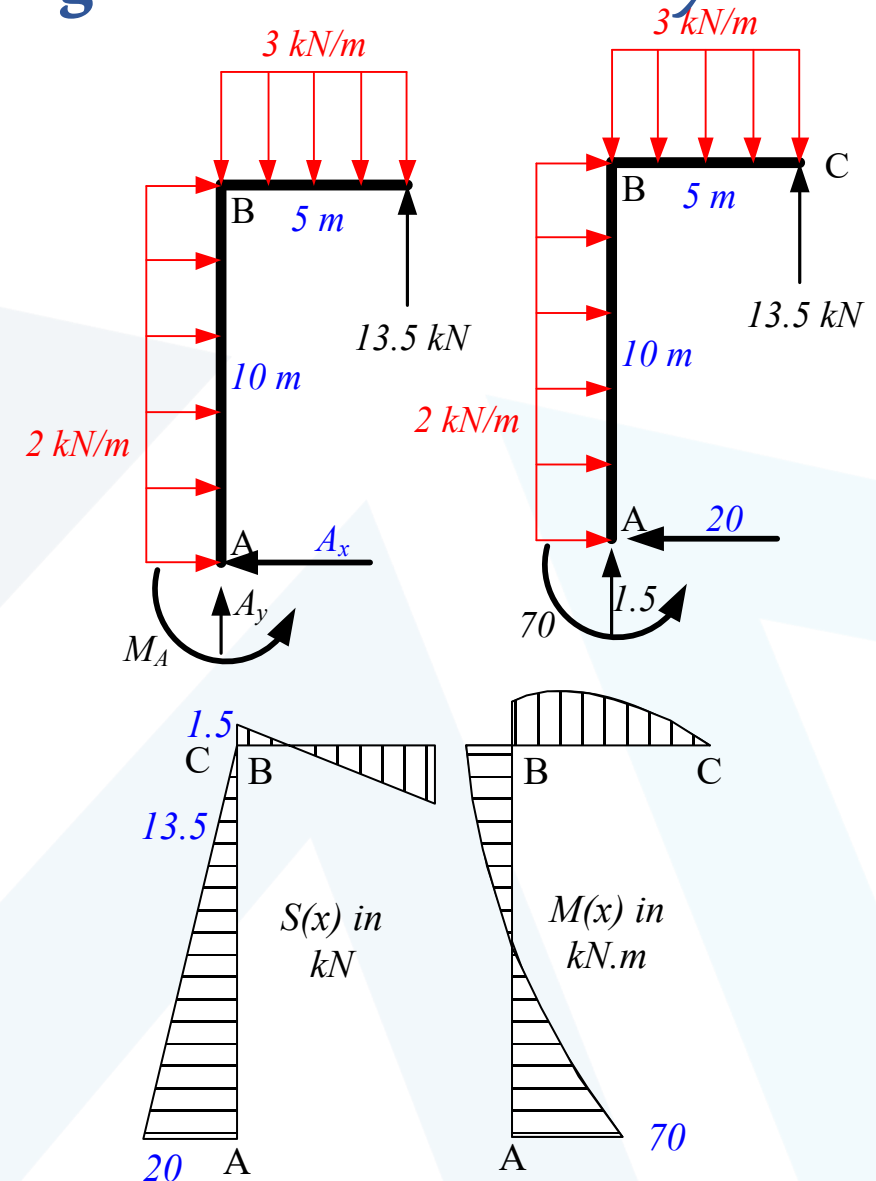
Drawing the BM & SF diagrams

In the segment AB: $0 < x < 10$,

$$M(x) = -x^2 + 20x - 70 \text{ and } S(x) = -2x + 20$$

In Segment CB: $0 < x < 5$,

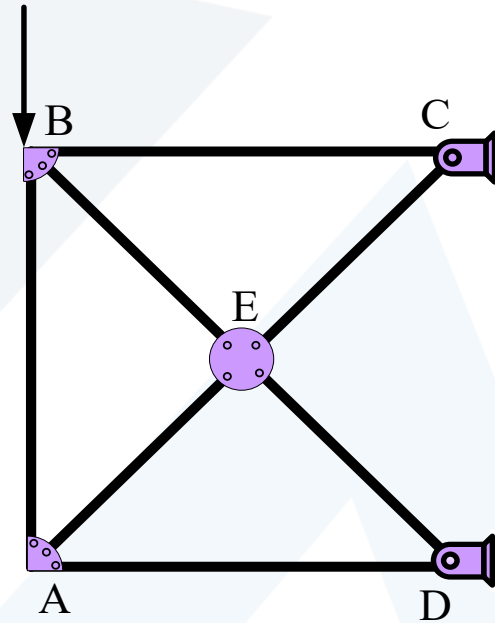
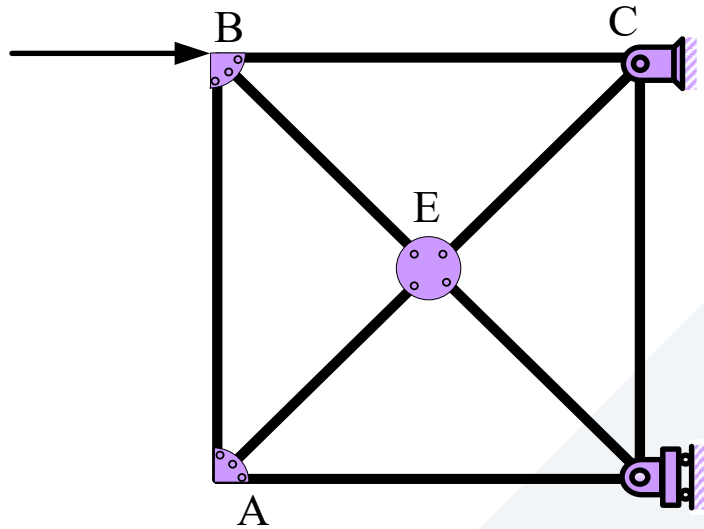
$$M(x) = -(3/2)x^2 + (27/2)x \text{ and } S(x) = 3x - 13.5$$



Force Method for Trusses that are statically indeterminate to degree one

Trusses can be statically indeterminate due to a variety of reasons:

- redundant support reactions (externally indeterminate),
- redundant members (internally indeterminate),
- or a combination of both.

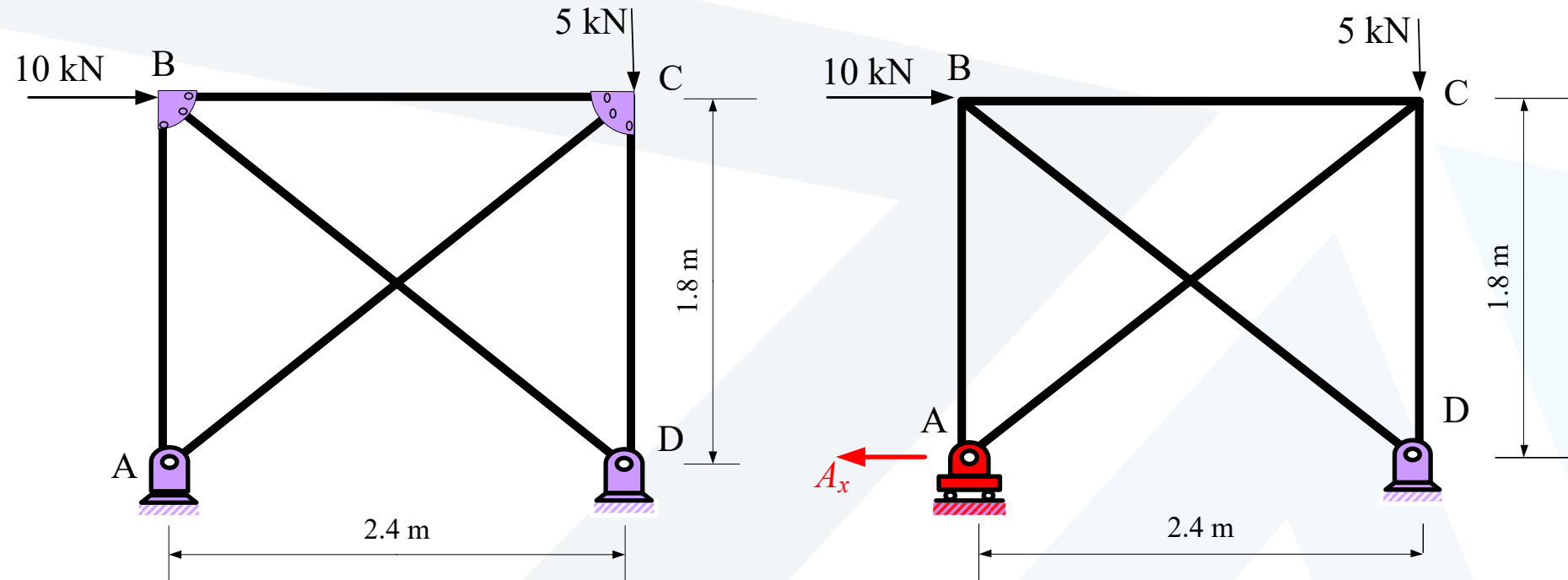


At the right, the truss is externally indeterminate $(m + r) - 2j = 7 + 4 - 2(5) = 1$.

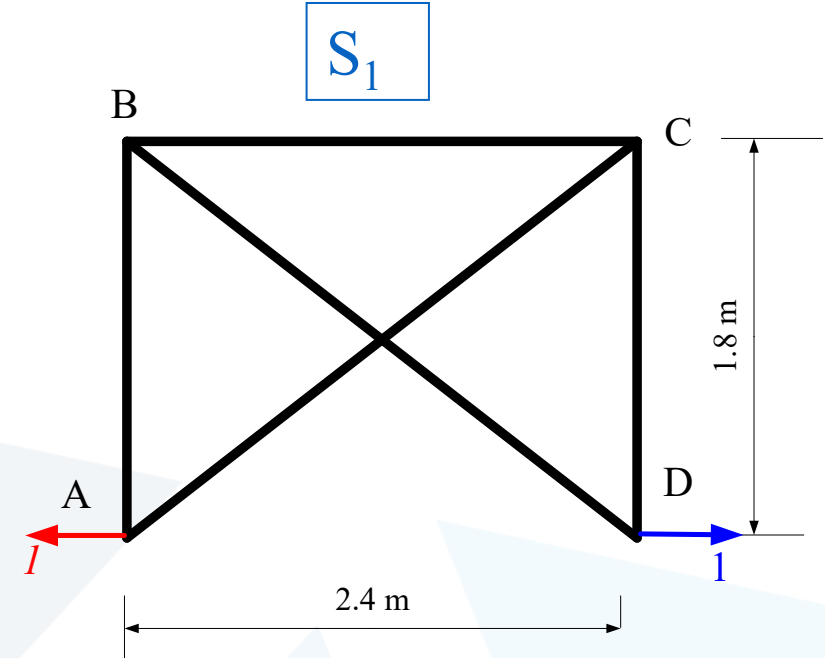
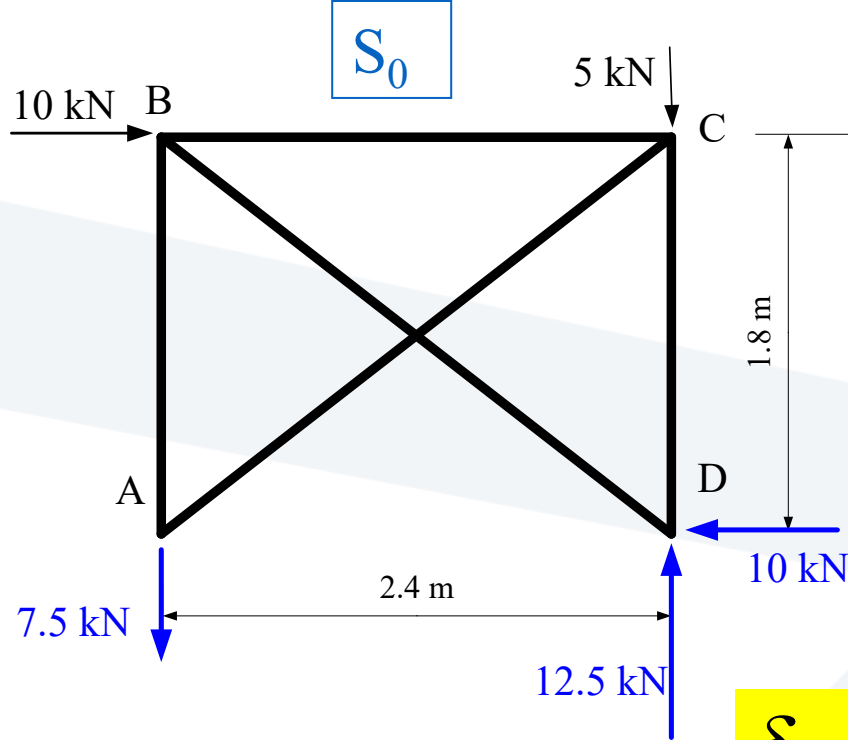
At the left, the truss is internally indeterminate $(m + r) - 2j = 8 + 3 - 2(5) = 1$.

Example: Externally Indeterminate Planar Truss

Compute the support reactions and the member forces for the truss in the next figure. Take $E = 200 \text{ E}+6 \text{ kN/m}^2$ and $A = 400\text{E}-6 \text{ m}^2$. for all the members.



A word about the crossing members: If we do not label the joint, we mean that the members do not intersect and move independently



$$\delta_{0A} + A_x \delta_{AA} = 0$$

$$\delta_{0A} = \sum \frac{NnL}{EA} \quad \& \quad \delta_{AA} = \sum \frac{n^2L}{EA}$$

Eqm. of joint A $\Rightarrow N_{AC}=0$ & $N_{AB}= + 7.5$ kN

Eqm. of joint C $\Rightarrow N_{BC}=0$ & $N_{CD}= - 5$ kN

Eqm. of joint B $\Rightarrow N_{BD}= - 12.5$ kN

Eqm. of joint A $\Rightarrow n_{AC}=+1.25$ & $n_{AB}= - 0.75$

Eqm. of joint C $\Rightarrow n_{BC}= -1$ & $n_{CD}= - 0.75$

Eqm. of joint B $\Rightarrow n_{BD}= 1.25$

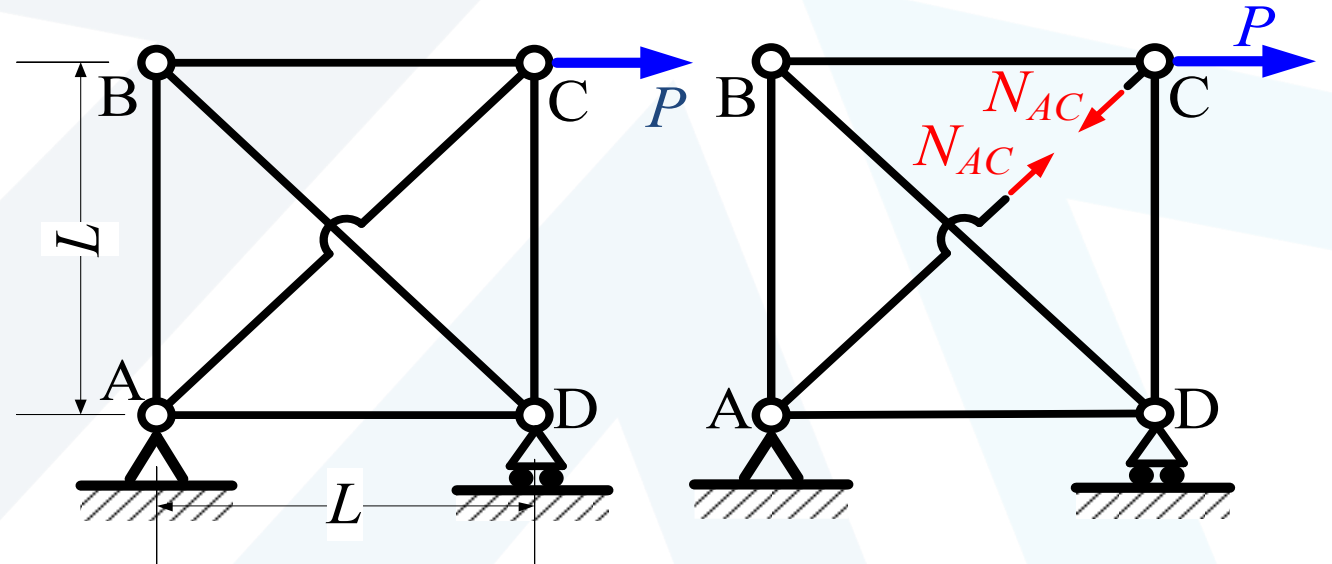
Computing Reactions and Axial Forces in the Indeterminate Truss								
Member	L [m]	A [m ²]	N [kN]	n [--]	L/A [1/m]	NnL/A [kn/m]	n^2L/A [1/m]	$F = N + A_x n$ [kN]
AB	1.8	4.00E-04	7.5	-0.750	4.50E+03	-25313	2531	4.7693
AC	3.0	4.00E-04	0.0	1.250	7.50E+03	0	11719	4.5513
BC	2.4	4.00E-04	0.0	-1.000	6.00E+03	0	6000	-3.6410
BD	3.0	4.00E-04	-12.5	1.250	7.50E+03	-117188	11719	-7.9488
CD	1.8	4.00E-04	-5.0	-0.750	4.50E+03	16875	2531	-7.7308
Sum						-125625	34500	
						$A_x = 125625/34500 = 3.64 \text{ kN}$		
Reaction			R_N [kN]	R_n [--]				$R = R_N + A_x R_n$ [kN]
A_x			0	-1				-3.6410
A_y			-7.5	0.0				-7.5000
D_x			-10	1				-6.3590
D_y			12.5	0.0				12.5000

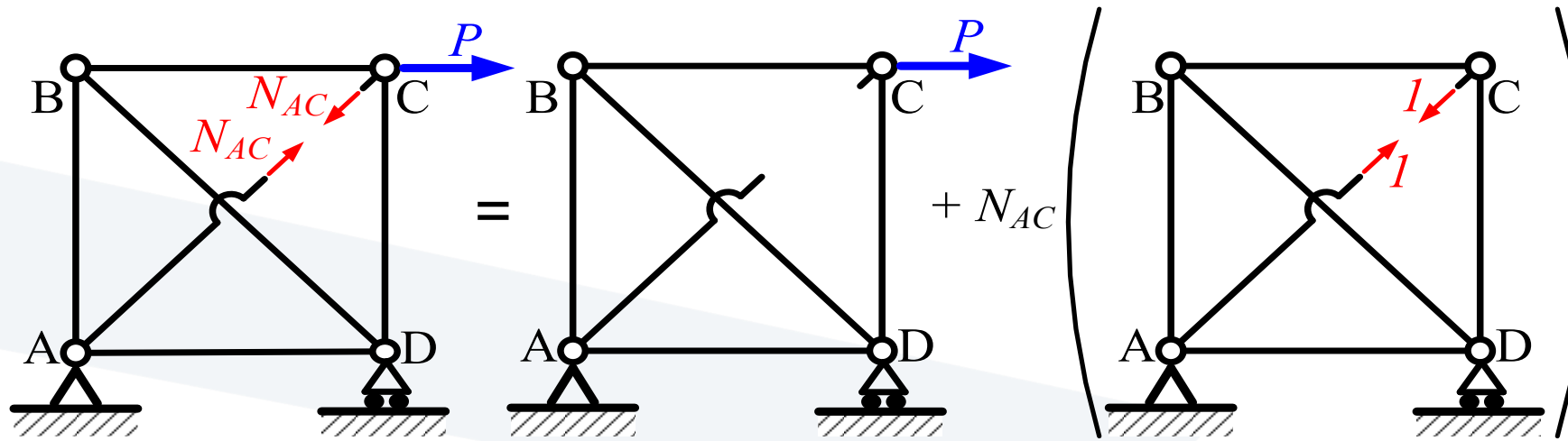
Example: Internally Indeterminate Planar Truss

Determine the forces in the members of the truss shown in Fig. The cross-sectional area, A , and, Young modulus, E , are the same for all members.

Introduction:

- There is no intersection between AC and BD.
- There are 6 members, 3 reactions and 4 joints.
- The truss is internally indeterminate to the first order.
- There is one redundant member, like AC. The truss and the corresponding compatibility equation are





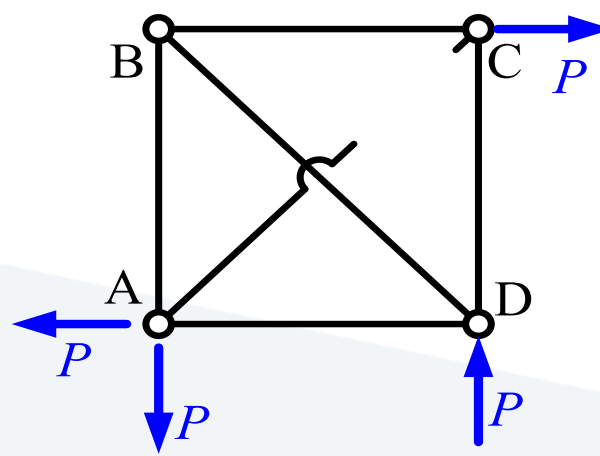
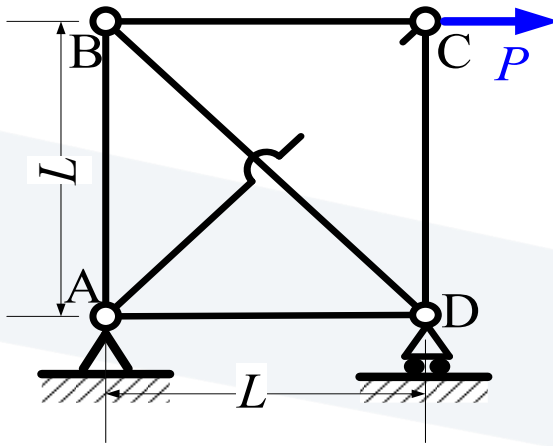
$$\Delta_{AC} = \delta_{AC}^0 + N_{AC} (\delta_{AC}^1)$$

$$\Delta_{AC} = -\frac{N_{AC} L_{AC}}{E_{AC} A_{AC}}, \quad \delta_{AC}^0 = \sum_{m=1}^{M-1} \frac{N_m^0 n_m L_m}{E_m A_m}, \quad \delta_{AC}^1 = \sum_{m=1}^{M-1} \frac{n_m n_m L_m}{E_m A_m}$$

As $N_{AC}^0 = 0$, and as: $n_{AC} = 1$, $\Delta_{AC} = -\frac{N_{AC} L_{AC}}{E_{AC} A_{AC}} = -N_{AC} \left(\frac{n_{AC} n_{AC} L_{AC}}{E_{AC} A_{AC}} \right)$

The Compatibility equation becomes

$$\left(\sum_{m=1}^M \frac{N_m^0 n_m L_m}{E_m A_m} \right) + N_{AC} \left(\sum_{m=1}^M \frac{n_m n_m L_m}{E_m A_m} \right) = 0$$



Analyzing S_0 :

Eq. Eqs. of S_0 :

$$\Sigma F_x = 0 \Rightarrow A_x = P(\leftarrow)$$

$$\Sigma M_A = 0 \Rightarrow D_y = P(\uparrow)$$

$$\Sigma M_D = 0 \Rightarrow A_y = P(\downarrow)$$

Eq. Eqs. of C:

$$\Sigma F_x = 0 \Rightarrow N_{CB} = P(T)$$

$$\Sigma F_y = 0 \Rightarrow N_{CD} = 0$$

Eq. Eqs. of B:

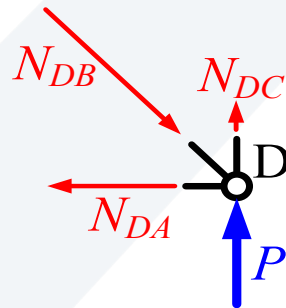
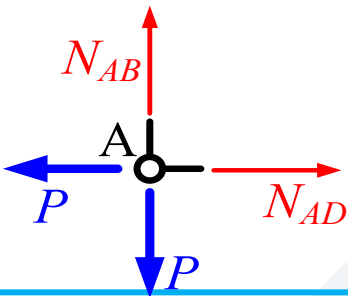
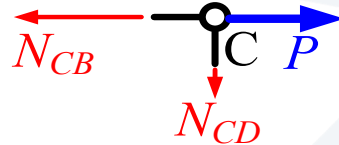
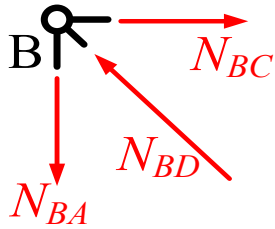
$$\Sigma F_x = 0 \Rightarrow N_{BD} = P\sqrt{2} \text{ (C)}$$

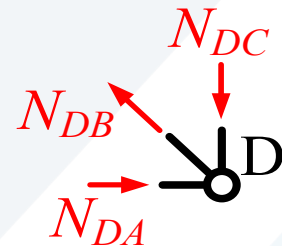
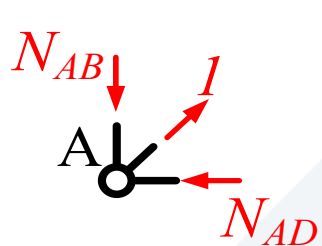
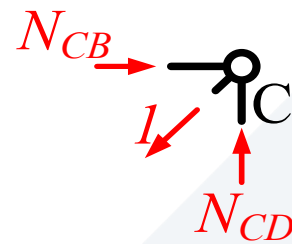
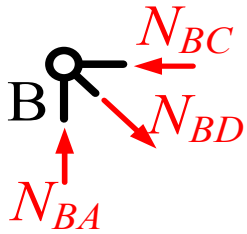
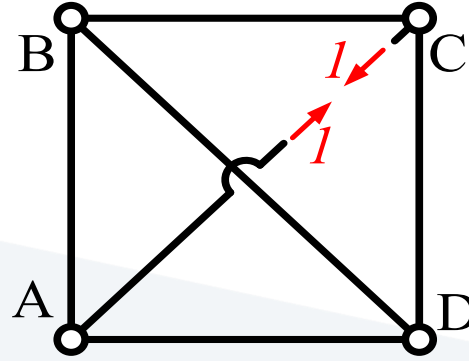
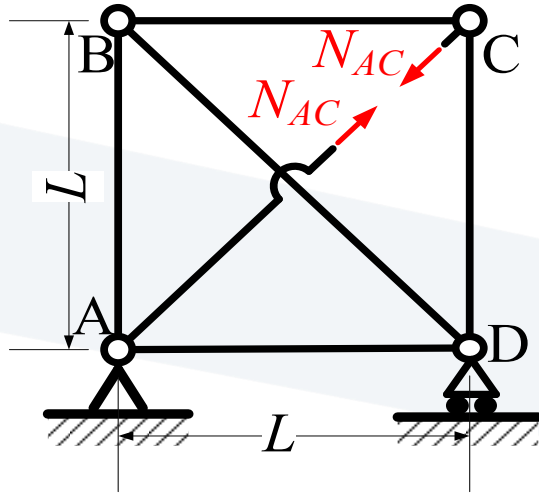
$$\Sigma F_y = 0 \Rightarrow N_{BA} = P(T)$$

Eq. Eqs. of A:

$$\Sigma F_x = 0 \Rightarrow N_{AD} = P(T)$$

$$\Sigma F_y = 0 \Rightarrow N_{AB} = P(T)$$





Analyzing S_1 :

Eq. Eqs. of S^1 :

$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

$$\Sigma M_A = 0 \Rightarrow D_y = 0$$

$$\Sigma M_D = 0 \Rightarrow A_y = 0$$

Eq. Eqs. of C:

$$\Sigma F_x = 0 \Rightarrow n_{CB} = 0.707 \text{ (C)}$$

$$\Sigma F_y = 0 \Rightarrow n_{CD} = 0.707 \text{ (C)}$$

Eq. Eqs. of A:

$$\Sigma F_x = 0 \Rightarrow n_{AB} = 0.707 \text{ (C)}$$

$$\Sigma F_y = 0 \Rightarrow n_{AD} = 0.707 \text{ (C)}$$

Eq. Eqs. of B:

$$\Sigma F_x = 0 \Rightarrow n_{BD} = 1 \text{ (T)}$$

$$\Sigma F_y = 0 \Rightarrow n_{BA} = 0.707 \text{ (C)}$$

Example: Internally Indeterminate Planar Truss

Computing Axial Forces in the Indetrminate Truss							
Member	$Length$	$area$	N^0	n	$N^0 n L / A$	$n^2 L / A$	$N = N^0 + N_{AC} n$
AB	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4 P$
BC	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4 P$
AD	L	A	P	$-1/\sqrt{2}$	$-PL/A\sqrt{2}$	$L/2A$	$0.4 P$
DC	L	A	0	$-1/\sqrt{2}$	0	$L/2A$	$-0.6 P$
DB	$L\sqrt{2}$	A	$-P/2$	$+1$	$-2PL/A$	$L\sqrt{2}/A$	$-0.56 P$
AC	$L\sqrt{2}$	A	0	$+1$	0	$L\sqrt{2}/A$	$0.85 P$
Σ					$-4.121 PL/A$	$4.828 L/A$	
$N_{AC} = 4.121 P / 4.828 = 0.85 P$							