Digital Image Processing

المحاضرة الثالثة

Relationships between pixels

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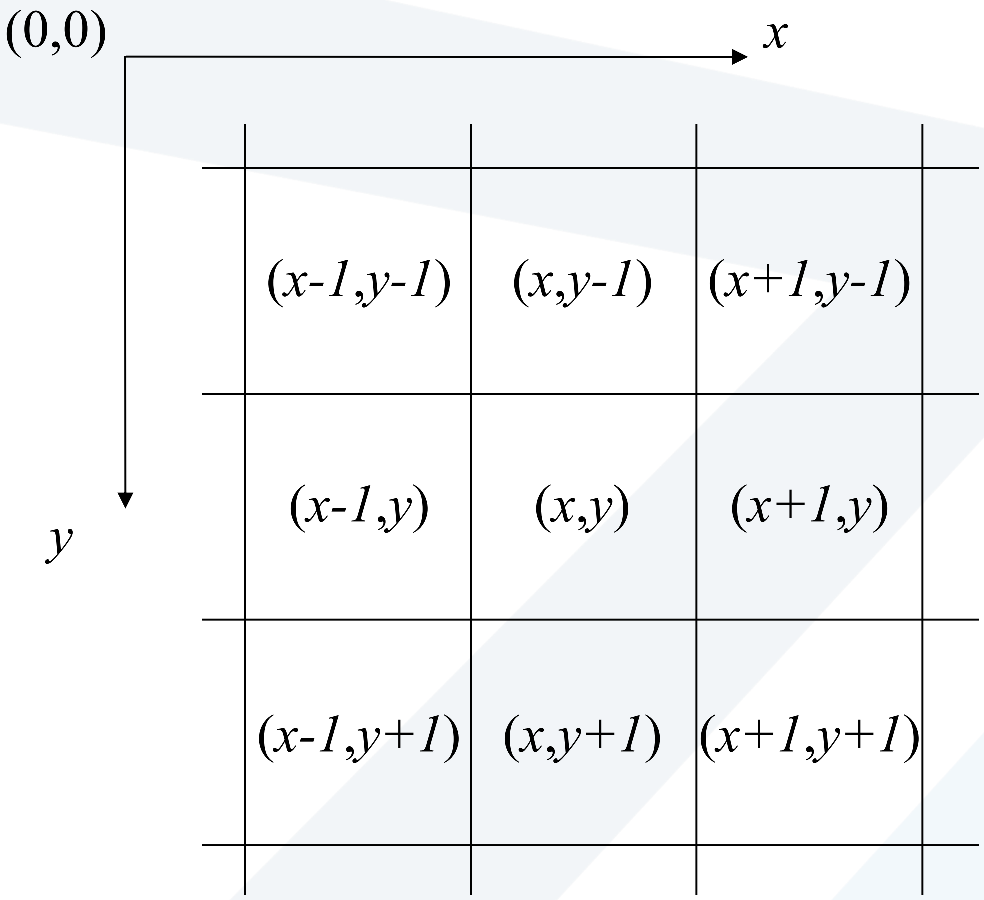
2025 الفصل الصيفي

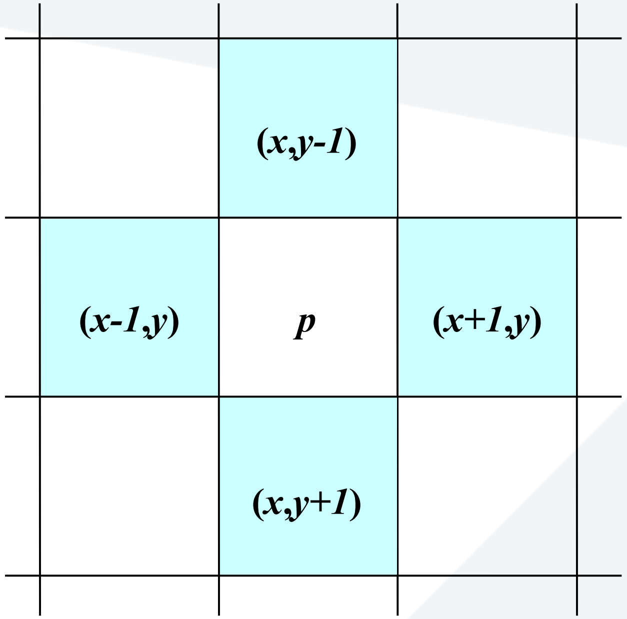
Basic Relationship of Pixels



Conventional indexing method

Neighbors of a Pixel

Neighborhood relation is used to tell adjacent pixels. It is useful for analyzing regions.



4-neighbors of *p*:

$$N_4(p) = \left\{ \begin{array}{c} (x-1,y) \\ (x+1,y) \\ (x,y-1) \\ (x,y+1) \end{array} \right\}$$

4-neighborhood relation considers only vertical and horizontal neighbors.

Note: $q \in N_4(p)$ implies $p \in N_4(q)$

Neighbors of a Pixel

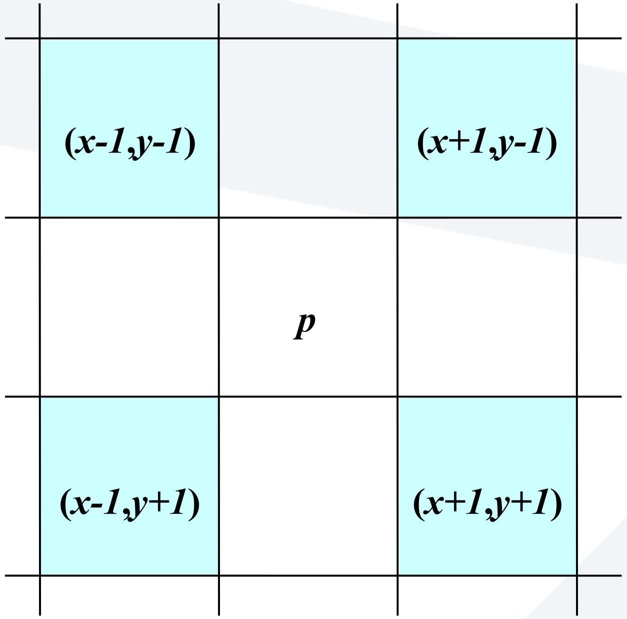
$(x-1,y-1)$	$(x,y-1)$	$(x+1,y-1)$
$(x-1,y)$	p	$(x+1,y)$
$(x-1,y+1)$	$(x,y+1)$	$(x+1,y+1)$

8-neighbors of *p*:

$$N_8(p) = \left\{ \begin{pmatrix} (x-1,y-1) \\ (x,y-1) \\ (x+1,y-1) \\ (x-1,y) \\ (x+1,y) \\ (x-1,y+1) \\ (x,y+1) \\ (x+1,y+1) \end{pmatrix} \right.$$

8-neighborhood relation considers all neighbor pixels.

Neighbors of a Pixel



Diagonal neighbors of *p*:

$$N_D(p) = \begin{Bmatrix} (x-1,y-1) \\ (x+1,y-1) \\ (x-1,y+1) \\ (x+1,y+1) \end{Bmatrix}$$

Diagonal -neighborhood relation considers only diagonal neighbor pixels.

Adjacency

- *Adjacency:* Two pixels are adjacent if they are neighbors and their intensity level 'V' satisfy some specific criteria of similarity.

  - For p and q from the same class
  - 4- adjacent : $q \in N_4(p)$
  - 8- adjacent : if $q \in N_8(p)$
  - mixed- adjacent (m- adjacent):

    $q \in N_4(p)$ or

    $q \in N_D(p)$ and $N_4(p) \cap N_4(q) = \emptyset$

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<u>*4-adjacency:*</u> Two pixels p and q with the values from set 'V' are 4-adjacent if q is in the set of $N_4(p)$.

e.g. V = { 0, 1}

1	1	2
1	1	0
1	0	1

p in **RED** color
q can be any value in **GREEN** color.

Adjacency

8-adjacency: Two pixels p and q with the values from set ‘V’ are 8-adjacent if q is in the set of $N_8(p)$.

e.g. V = { 1, 2}

0	1	1
0	2	0
0	0	1

p in **RED** color
q can be any value in **GREEN** color

Adjacency

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. V = { 1 }

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

Adjacency

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$

e.g. V = { 1 }

b & c

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 I

Soln: b & c are m-adjacent.

Adjacency

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$

e.g. V = { 1 }

b & e

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 I

Soln: b & e are m-adjacent.

Adjacency

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m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. V = { 1 }

e & i

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

Soln: e & i are m-adjacent.

Adjacency

m-adjacency: Two pixels p and q with the values from set 'V' are m-adjacent if

(i) q is in $N_4(p)$ OR

(ii) q is in $N_D(p)$ & the set $N_4(p) \cap N_4(q)$ have no pixels whose values are from 'V'.

e.g. V = { 1 }

(iv) e & c

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 I

Soln: e & c are NOT m-adjacent.

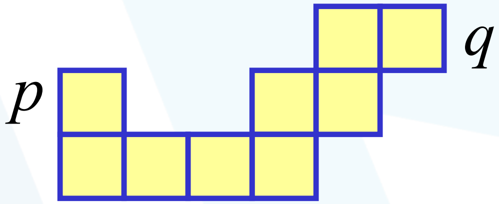
Path

A **path** from pixel p at (x,y) to pixel q at (s,t) is a sequence of distinct pixels:

$$(x_0,y_0), (x_1,y_1), (x_2,y_2),\ldots, (x_n,y_n)$$

such that: $(x_0,y_0) = (x,y)$ and $(x_n,y_n) = (s,t)$

and (x_i,y_i) is adjacent to (x_{i-1},y_{i-1}), $i = 1,\ldots,n$



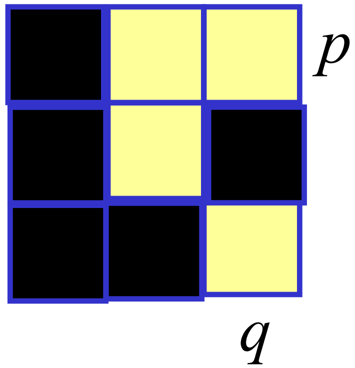
n is the length of the path

Closed path: $(x_0, y_0) = (x_n, y_n)$

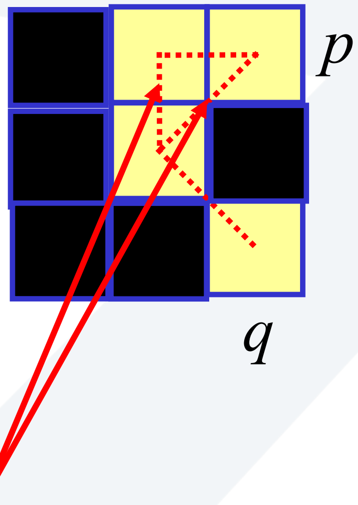
We can define type of path: 4-path, 8-path or m-path depending on type of adjacency.

Path

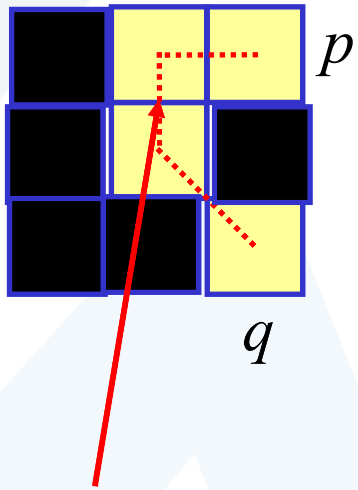
V={1}
black is zero
yellow is one



8-path



m-path



8-path from p to q
results in some ambiguity

m-path from p to q
solves this ambiguity

Paths

<u>Example # 1</u>: Consider the image segment shown in figure. Compute length of the **shortest-4, shortest-8 & shortest-m paths** between pixels p & q where,
V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

Paths

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1	2	3

Paths

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1 →	2	3

Paths

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1 →	2 ↑	3

Paths

Example # 1:

Shortest-4 path:

V = {1, 2}.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-4 path:

V = {1, 2}.

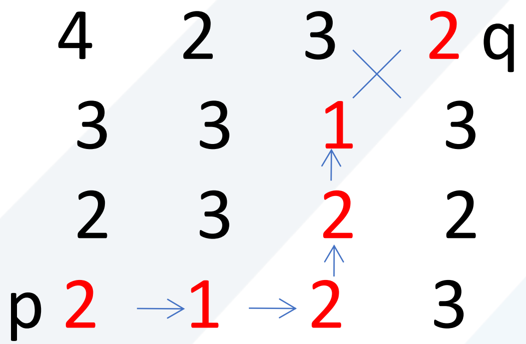
	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-4 path:

V = {1, 2}.



So, Path does not exist.

Paths

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2	1	2	3

Paths

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1	2	3

Paths

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	→ 1 ↗	2	3	

Paths

<u>Example # 1</u>:

<u>Shortest-8 path:</u>

V = {1, 2}.

	4	2	3	**2**	q
	3	3	**1**	3	
	2	3	**2**	2	
p	**2**	**1**	2	3	

Paths

<u>Example # 1</u>:

<u>Shortest-8 path:</u>

V = {1, 2}.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-8 path:

V = {1, 2}.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

So, shortest-8 path = 4

Paths

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p 2	1	2	3	

Paths

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
p	2 →	1	2	3

Paths

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2 →	1 →	2	3	

Paths

Example # 1:

Shortest-m path:

V = {1, 2}.

	4	2	3	2 q
	3	3	1	3
	2	3	2	2
			↑	
p	2 →	1 →	2	3

Paths

Example # 1:

Shortest-m path:

V = {1, 2}.

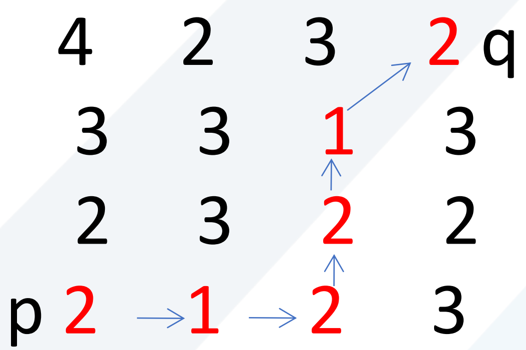
	4	2	3	2	q
	3	3	1	3	
	2	3	2	2	
p	2	1	2	3	

Paths

Example # 1:

Shortest-m path:

V = {1, 2}.

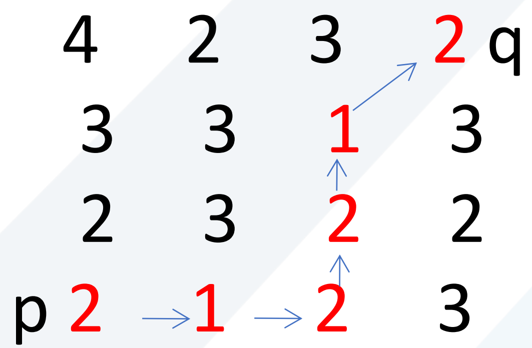


Paths

Example # 1:

Shortest-m path:

V = {1, 2}.



So, shortest-m path = 5

Distance

For pixel p, q, and z with coordinates (x,y), (s,t) and (u,v),

D is a **distance function** or **metric** if

- D(p,q) $\geq$ 0 (D(p,q) = 0 if and only if p = q) called reflexivity
- D(p,q) = D(q,p) called symmetry
- D(p,z) $\leq$ D(p,q) + D(q,z) called transmittivity

Example: Euclidean distance $D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$

Distance

D_4-distance (*city-block distance*): The D_4 distance between p & q is defined as

$$D_4(p,q) = |x - s| + |y - t|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with $D_4(p) = 1$ is 4-neighbors of p.

Distance

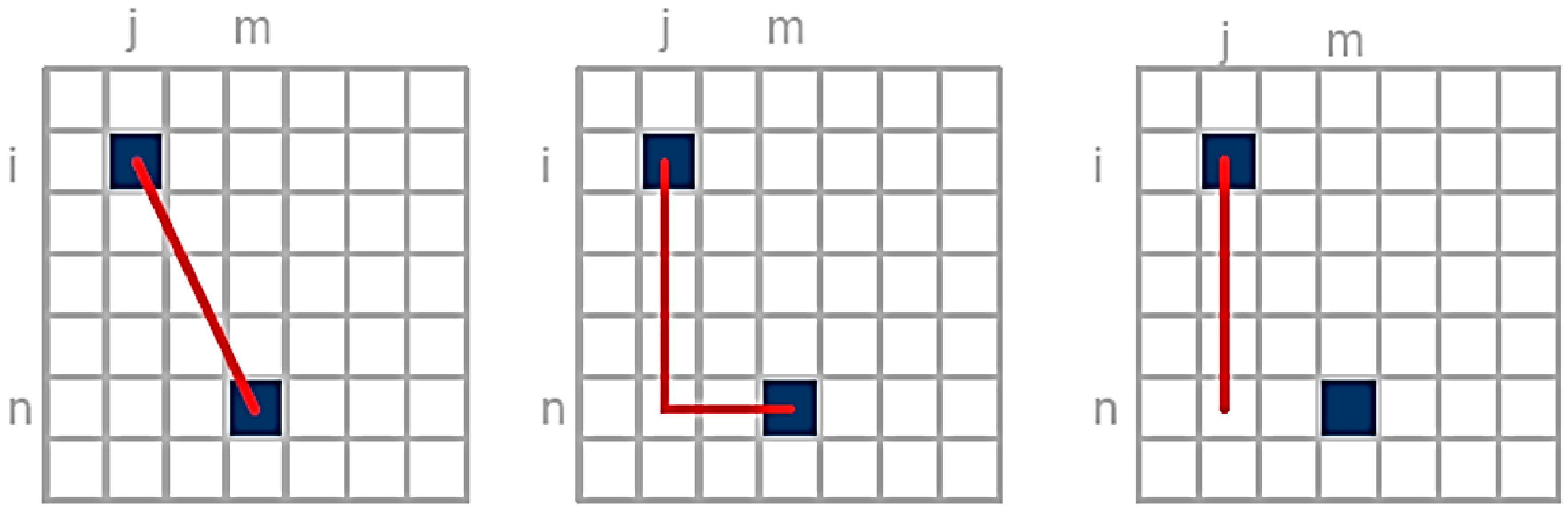
D_8-distance (*chessboard distance*): The D_8 distance between p & q is defined as:

$$D_8(p, q) = \max(|x - s|, |y - t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with $D_8(p) = 1$ is 8-neighbors of p.

Distance



Euclidean Distance

$$= \sqrt{(i-n)^{2} + (j-m)^{2}}$$

City Block Distance

$$= |i-n| + |j-m|$$

Chessboard Distance

$$= \max[\, |i-n|, |j-m| \,]$$

Distance

Compute the distance between the two pixels using the three distances:

q:(1,1)

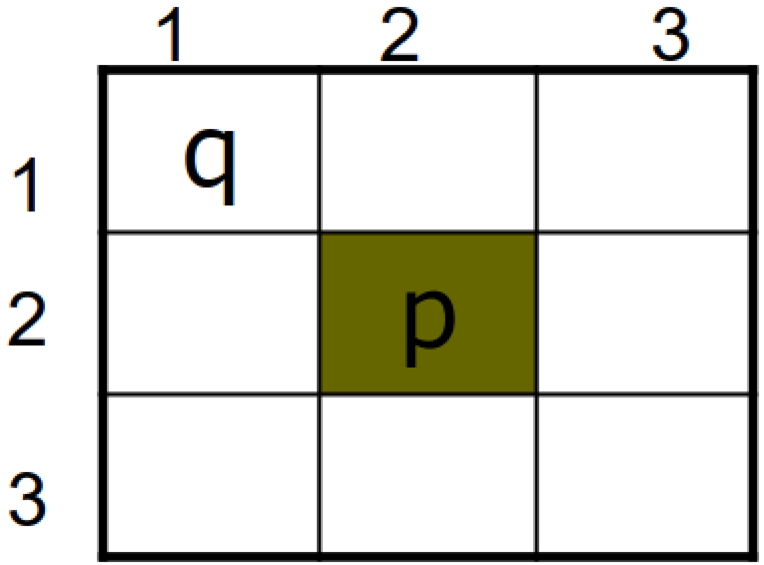
P: (2,2)

Euclidian distance : ((1-2)^2+(1-2)^2)^1/2 = sqrt(2).

D4(City Block distance): |1-2| +|1-2| =2

D8(chessboard distance) : max(|1-2|,|1-2|)= 1

(because it is one of the 8-neighbors)



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