Manara University
Faculty of Engineering
Department of IT

Introduction to Artificial Intelligence

PL,FOL
Exercises

Prof Dr. Eng. Mariam M. Saii

PL Exercises

Exercise1

Use the truth tables method to determine whether $(p \to q) \vee (p \to \neg q)$ is valid.

p	q	$p \to q$	$\neg q$	$p \to \neg q$	$(p \to q) \vee (p \to \neg q)$
T	T	T	F	F	**T**
T	F	F	T	T	**T**
F	T	T	F	T	**T**
F	F	T	T	T	**T**

Exercise2

Let's consider a propositional language where

- p *means* "Paola is happy",

- q *means* "Paola paints a picture",

- r *means* "Renzo is happy".

Formalize the following sentences:

1. "if Paola is happy and paints a picture then Renzo isn't happy"

2. "if Paola is happy, then she paints a picture"

3. "Paola is happy only if she paints a picture"

Solution.

1. $p \wedge q \to \neg r$

2. $p \to q$

3. $\neg(p \wedge \neg q)$ ..*which is equivalent to* $p \to q$

Let A = "Aldo is Italian" *and* B = "Bob is English".

Formalize the following sentences:

1. "Aldo isn't Italian"

2. "Aldo is Italian while Bob is English"

3. "If Aldo is Italian then Bob is not English"

4. "Aldo is Italian or if Aldo isn't Italian then Bob is English"

5. "Either Aldo is Italian and Bob is English, or neither Aldo is Italian nor Bob is English"

Solution.

1. $\neg A$

2. $A \wedge B$

3. $A \rightarrow \neg B$

4. $A \vee (\neg A \rightarrow B)$ *logically equivalent to $A \vee B$*

5. $(A \wedge B) \vee (\neg A \wedge \neg B)$ *logically equivalent to $A \leftrightarrow B$*

FOL Exercises

Define an appropriate language and formalize the following sentences using FOL formulas.

1. All Students are smart.
2. There exists a student.
3. There exists a smart student.
4. Every student loves student.
5. Every student loves some other student.
6. There is a student who is loved by every other student.
7. Bill is a student.
8. Bill takes either Analysis or Geometry (but not both).
9. Bill takes Analysis and Geometry.
10. Bill doesn't take Analysis.
11. No students love Bill

1. $\forall x.(Student(x) \to Smart(x))$
2. $\exists x.Student(x)$
3. $\exists x.(Student(x) \land Smart(x))$
4. $\forall x.(Student(x) \to \exists y.(Student(y) \land Loves(x, y)))$
5. $\forall x.(Student(x) \to \exists y.(Student(y) \land \neg(x = y) \land Loves(x, y)))$
6. $\exists x.(Student(x) \land \forall y.(Student(y) \land \neg(x = y) \to Loves(y, x)))$
7. $Student(Bill)$
8. $Takes(Bill, Analysis) \leftrightarrow \neg Takes(Bill, Geometry)$
9. $Takes(Bill, Analysis) \land Takes(Bill, Geometry)$
10. $\neg Takes(Bill, Analysis)$
11. $\neg \exists x.(Student(x) \land Loves(x, Bill))$

١. حول الصيغة التالية إلى شكل العطف النظامي CNF :

$$(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$$

<u>الحل :</u>

١ـ التخلص من الاقتضاءات :

$$\sim (\sim P \cup (\sim Q \cup R)) \cup (\sim(\sim P \cup Q) \cup (\sim P \cup R))$$
$$(P \cap \sim(\sim Q \cup R)) \cup ((P \cap \sim Q) \cup (\sim P \cup R))$$
$$(P \cap (Q \cap \sim R)) \cup ((P \cap \sim Q) \cup (\sim P \cup R))$$

٢ـ توزيع العمليات " ليصبح الشكل الأخير عبارة عن مجموعات بين ذراتها اجتماع وبين المجموعات تقاطعات " :

$$(P \cap Q \cap \sim R) \cup ((\sim P \cup R \cup P) \cap (\sim P \cup R \cup \sim Q))$$

٣ـ نحذف المجموعة الوسطى لاحتوائها على ذرة ونفيها فهي محققة دائماً وحسب قواعد الجبر يبقى :

$$(P \cap Q \cap \sim R) \cup (\sim P \cup R \cup \sim Q)$$

$$(P \cup \sim P \cup R \cup \sim Q) \cap (Q \cup \sim P \cup R \cup \sim Q) \cap (\sim R \cup \sim P \cup R \cup \sim Q)$$

logically equivalent??????

- $((P \wedge Q) \Rightarrow R)$ $(P \Rightarrow R) \wedge (Q \Rightarrow R)$

- $P \Leftrightarrow Q$ $\neg P \Leftrightarrow \neg Q$

Exercise 2

Define an appropriate language and formalize the following sentences using FOL formulas.

1. Bill has at least one sister.

2. Bill has no sister.

3. Bill has at most one sister.

4. Bill has (exactly) one sister.

5. Bill has at least two sisters.

6. Every student takes at least one course.

7. Only one student failed Geometry.

8. No student failed Geometry but at least one student failed Analysis.

9. Every student who takes Analysis also takes Geometry.

Solution

1. $\exists x. SisterOf(x, Bill)$

2. $\neg \exists x. SisterOf(x, Bill)$

3. $\forall x \forall y. (SisterOf(x, Bill) \wedge SisterOf(y, Bill) \rightarrow x = y)$

4. $\exists x. (SisterOf(x, Bill) \wedge \forall y. (SisterOf(y, Bill) \rightarrow x = y))$

5. $\exists x \exists y. (SisterOf(x, Bill) \wedge SisterOf(y, Bill) \wedge \neg(x = y))$

6. $\forall x. (Student(x) \rightarrow \exists y. (Course(y) \wedge Takes(x, y)))$

7. $\exists x. (Student(x) \wedge Failed(x, Geometry) \wedge \forall y. (Student(y) \wedge Failed(y, Geometry) \rightarrow x = y))$

8. $\neg \exists x. (Student(x) \wedge Failed(x, Geometry)) \wedge \exists x. (Student(x) \wedge Failed(x, Analysis))$

9. $\forall x. (Student(x) \wedge Takes(x, Analysis) \rightarrow Takes(x, Geometry))$

- كل إنسان فان

- سقراط إنسان

- أثبت أن سقراط فان

نلاحظ من معطيات المسألة أن سقراط ثابت (*const*) :

$H(X)$ تعبر عن الانسان .

$H(X) \rightarrow true\ or\ false$

$D(X)$ تعبر عن الموت.

$D(X) \rightarrow true\ or\ false$

1. $\forall X \; H(X) \Rightarrow D(X)$ أي أن كل إنساس فان

2. $H(sct)$ سقراط إنسان

Goal :

$D(sct)$ سقراط فان

- بعض المرضى تحب كل الأطباء .

- لايوجد مريض يحب الدجالين .

برهن لايوجد طبيب دجال ؟

<u>نمذجة المسألة:</u>

- $predicate$: $P(x)$ مريض

    $D(x)$ دكتور

    $\theta(x)$ دجال

    $L(x, y)$ x $Like$ y

- $Rulse$:

    $\exists x \quad P(x) \cap (\forall y \ D(y) \Rightarrow L(x, y))$

أي أنه يوجد بعض المرضى أيا كان الطبيب فإنه محبوب بالنسبة لهم .

$\neg \ \exists x \ \exists y \ P(x) \cap \theta(y) \cap L(x, y)$

أي أنه لايوجد مريض يحب الدكتور الدجال .

- $Goal$:

    $\neg \exists x \ D(x) \cap \theta(x)$

FOL Exercise

Exercise 1 : FOL

- Tony, Shi-Kuo and Ellen belong to the Hoofers Club. Every member of the Hoofers Club is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Ellen dislikes whatever Tony likes and likes whatever Tony dislikes. Tony likes rain and snow.

- **Who is the member that is a mountain climber but not a skier ??**

solution

- S(x): x is skier

- M(x): x is mountain climber

- Like(x,y): x likes y

- Every member of the Hoofers Club is either a skier or a mountain climber or both

- No mountain climber

$$\forall x \; S(x) \lor M(x)$$

$$\neg \exists x \; M(x) \land like(x, rain)$$

- all skiers like snow

$$\forall x\, S(x) \Rightarrow like(x, snow)$$

- Ellen dislikes whatever Tony likes

$$\forall x\, like(Tony, x) \Rightarrow \neg like(Ellen, x)$$

- Ellen likes whatever Tony dislikes

$$\forall x\, \neg like(Tony, x) \Rightarrow like(Ellen, x)$$

- Tony likes rain

$$like(Tony, rain)$$

- Tony likes snow

$$like(Tony, snow)$$