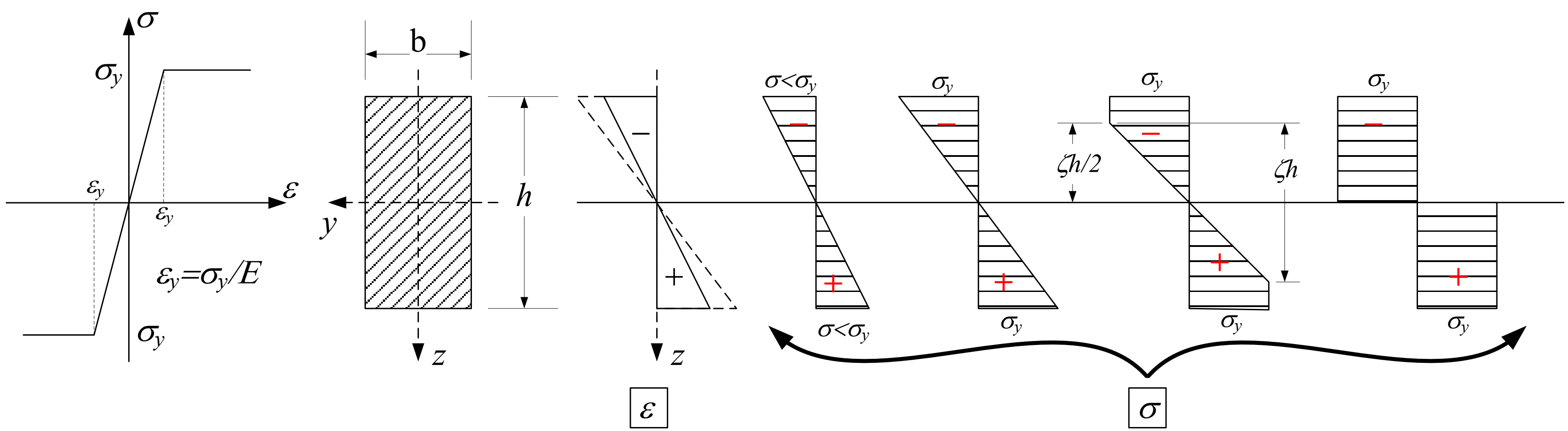
3. Plastic Bending

$$\varepsilon = z\kappa \qquad \kappa = 1/\rho \qquad M = \int_A z\,\sigma(z)\,dA$$



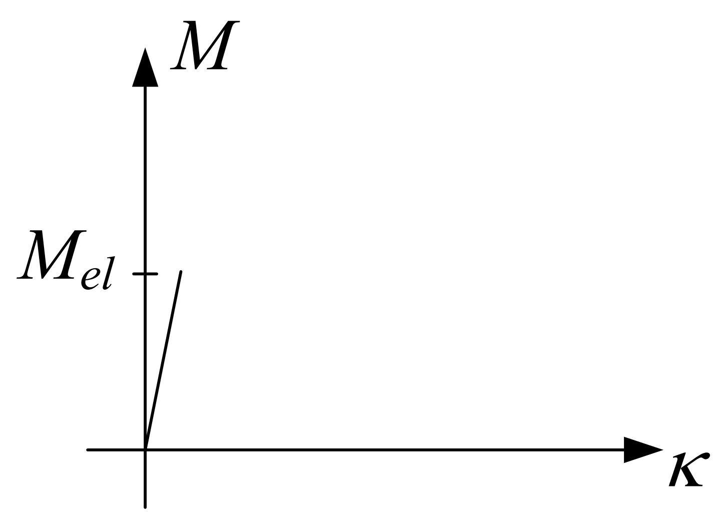
$$\varepsilon_{\max} \le \sigma_y / E$$

$$M = \int_A z\,\sigma(z)\,dA = \int_A zE\kappa z\,dA = E\kappa\int_A z^2 dA = EI\kappa$$

$$\kappa_{el} = \frac{\varepsilon_y}{z_{\max}} = \frac{\sigma_y / E}{h/2} = \frac{2\sigma_y}{Eh}$$

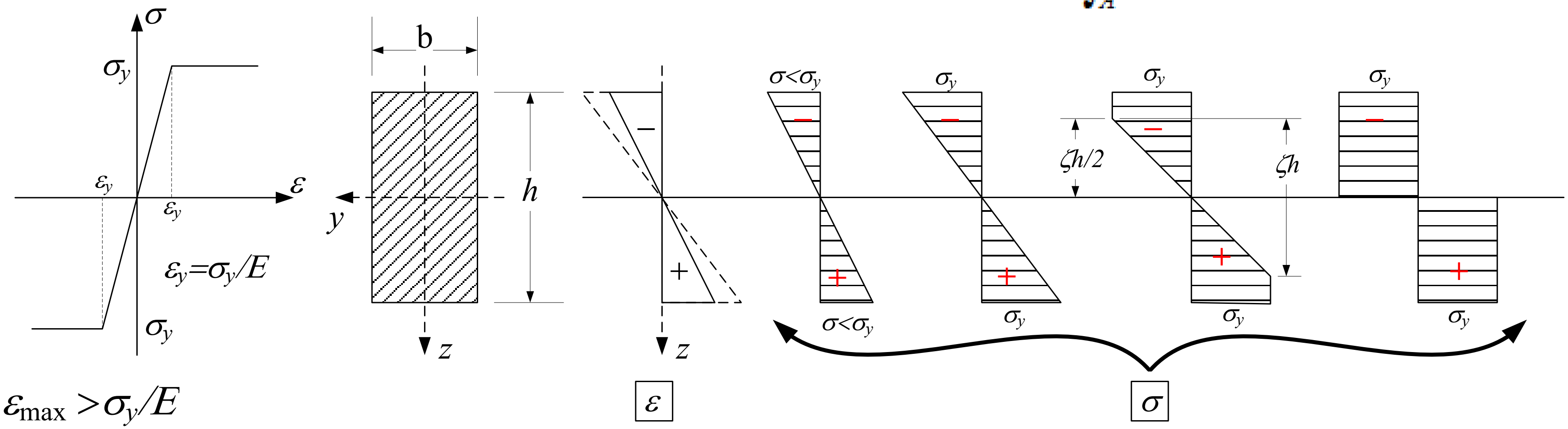
$$M_{el} = EI\kappa_{el} = E\left(\frac{1}{12}bh^3\right)\frac{2\sigma_y}{Eh} = \frac{bh^2}{6}\sigma_y$$

$$W_{el} = \frac{I}{|z_{\max}|} = \frac{bh^2}{6}$$



2. Plastic Bending

$\varepsilon = z\kappa \qquad \kappa = 1/\rho \qquad M = \int_A z\,\sigma(z)\,dA$



$\varepsilon_{\max} > \sigma_y / E$

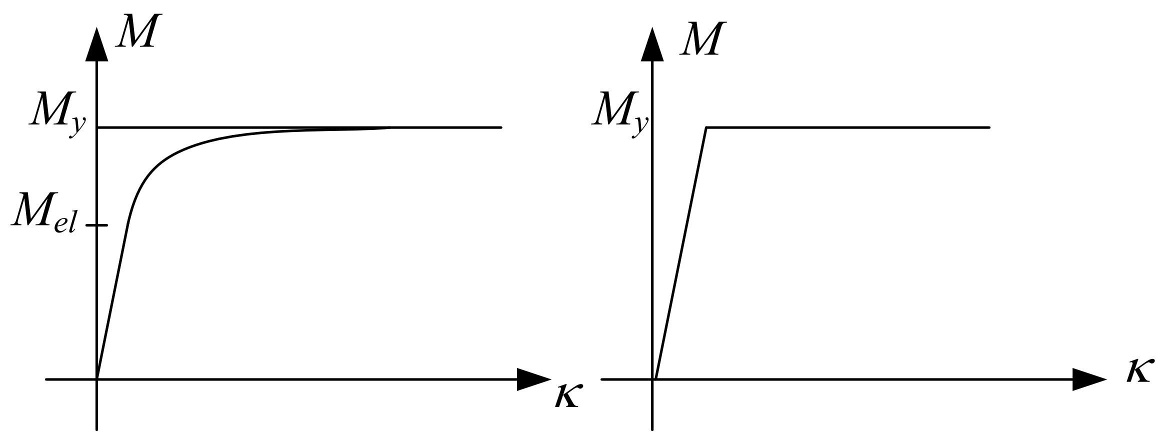
$$M = 2\sigma_y b\left[\tfrac{1}{2}\left(\frac{\zeta h}{2}\right)\left(\tfrac{2}{3}\frac{\zeta h}{2}\right)\right] + 2\sigma_y b\left[\tfrac{1}{2}\left(\frac{h}{2}-\frac{\zeta h}{2}\right)\left(\frac{h}{2}+\frac{\zeta h}{2}\right)\right]$$

$$= \left(\frac{bh^2}{4}\sigma_y\right)\left(1-\frac{\zeta^2}{3}\right) = M_y\left(1-\frac{\zeta^2}{3}\right)$$

$$M_y = \frac{bh^2}{4}\sigma_y = W_y\sigma_y$$

at $z = \zeta h/2$: $\varepsilon = \sigma_y/E = \kappa\zeta h/2$

$\Rightarrow \zeta = 2\sigma_y/Eh\kappa = \kappa_{el}/\kappa$



$$M(\kappa) = M_y\left(1-\frac{\kappa_{el}^2}{3\kappa^2}\right)$$

Shape factor $= W_y / W_{el} = 1.5$

Plastic hinge

- When the section is completely yielded, the section is fully plastic
- A fully plastic section behaves like a hinge: Plastic hinge

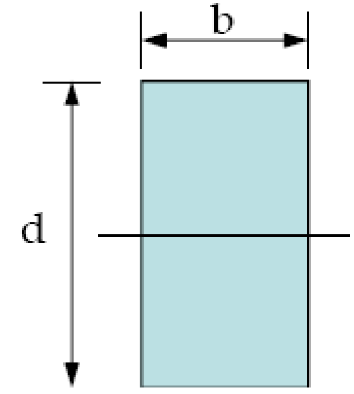
Plastic hinge is defined as an yielded zone due to bending in a structural member, at which large rotations can occur at constant plastic or yielding moment: M_p or M_y ,or M_0.

Mechanical hinge	Plastic hinge
Reality	Concept
Resists zero moment	Resists a constant moment M_P

Mechanical Hinge = Plastic Hinge with $M_P = 0$

Shape factor for various cross-sections

Rectangular cross-section:



Section modulus

$$W_{el} = \frac{I}{z_{\max}} = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}$$

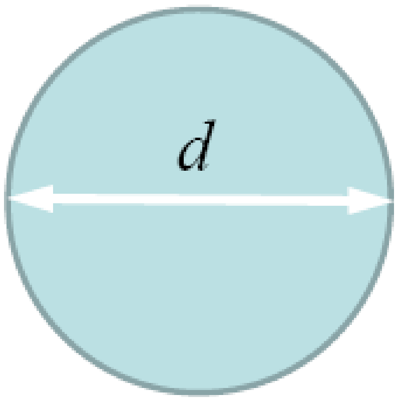
Plastic Modulus

$$W_y = \frac{A}{2}(z_t + z_c) = \frac{bd}{2}\left(\frac{d}{4} + \frac{d}{4}\right) = \frac{bd^2}{4}$$

Shape Factor

$$S = W_y / W_{el} = \left(\frac{bd^2}{4}\right) / \left(\frac{bd^2}{6}\right) = 1.5$$

Circular section



$$W_y = \frac{A}{2}(y_c + y_t)$$

$$= \left(\frac{\pi d^2}{8}\right)\left(\frac{2d}{3\pi} + \frac{2d}{3\pi}\right) = \frac{d^3}{6}$$

$$W_{el} = \frac{(\pi d^4/64)}{d/2} = \frac{\pi d^3}{32}$$

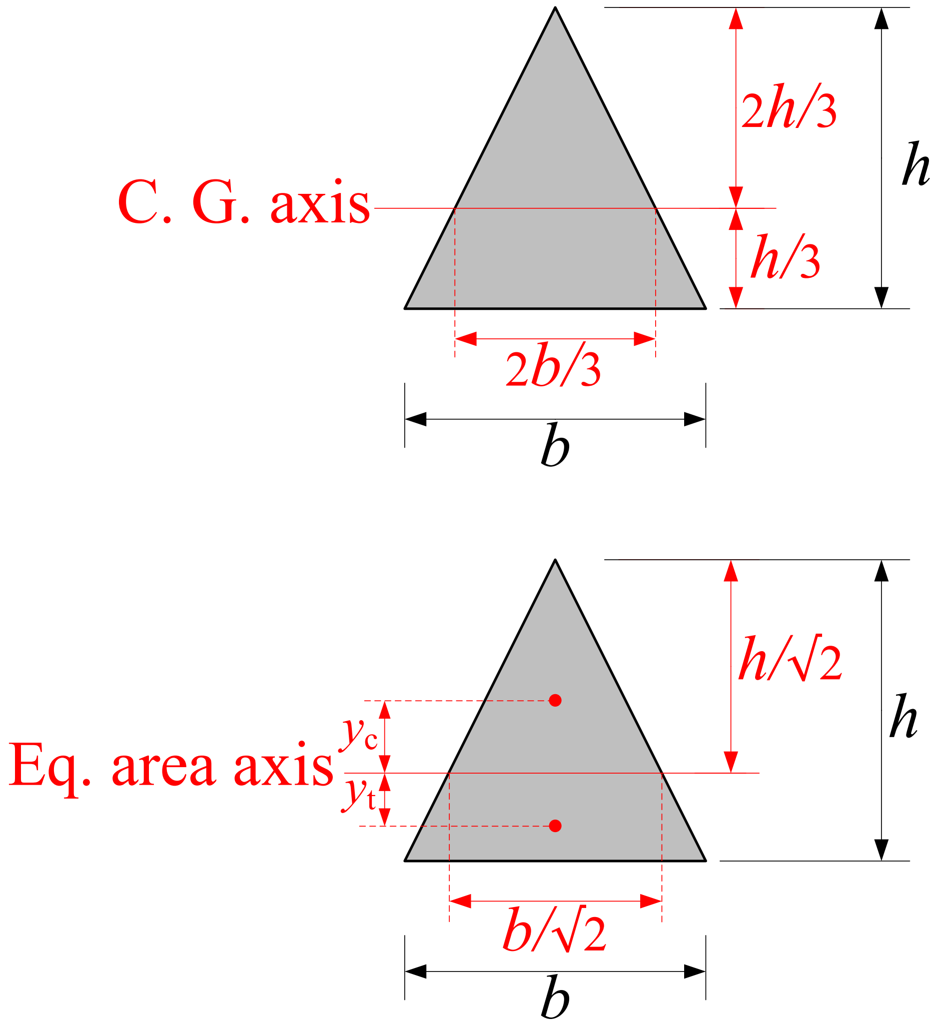
$$S = \frac{W_y}{W_{el}} = 1.7$$

Triangular section

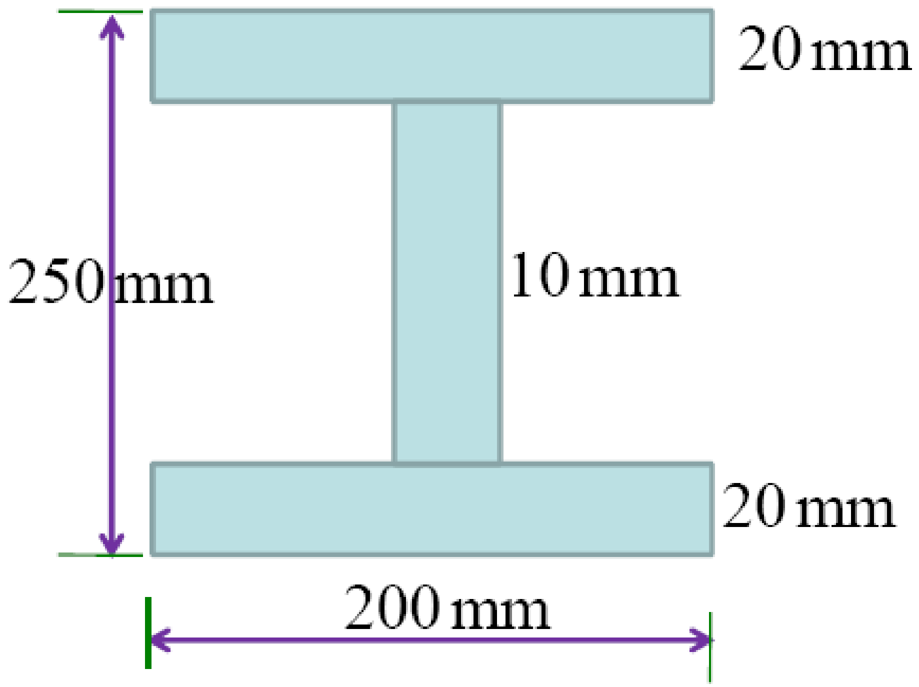
$$W_{el} = \frac{\left(\frac{bh^3}{36}\right)}{2h/3} = \frac{bh^2}{24}$$

$$W_y = \frac{A}{2}\left(y_c + y_t\right)$$

S = 2.346

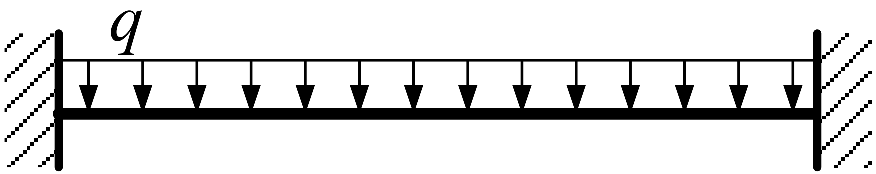


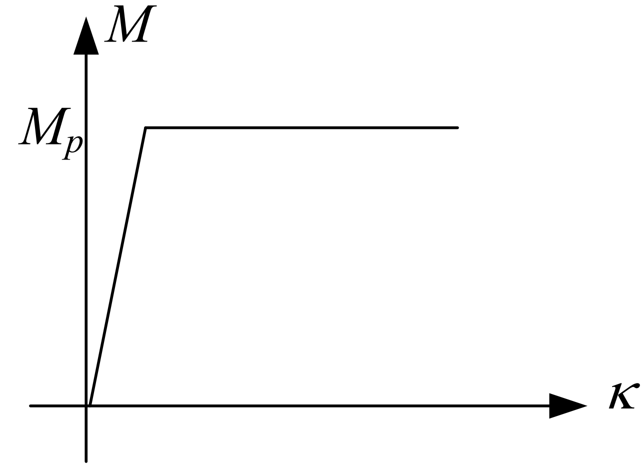
I section

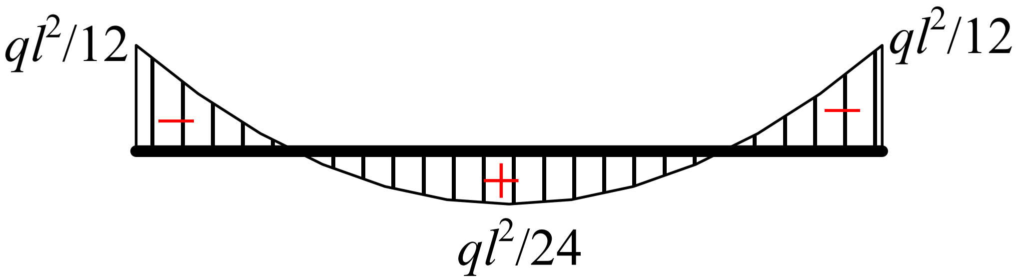


$$S = 1.132$$

The Elastic-Plastic behavior of Statically Indeterminate Beam

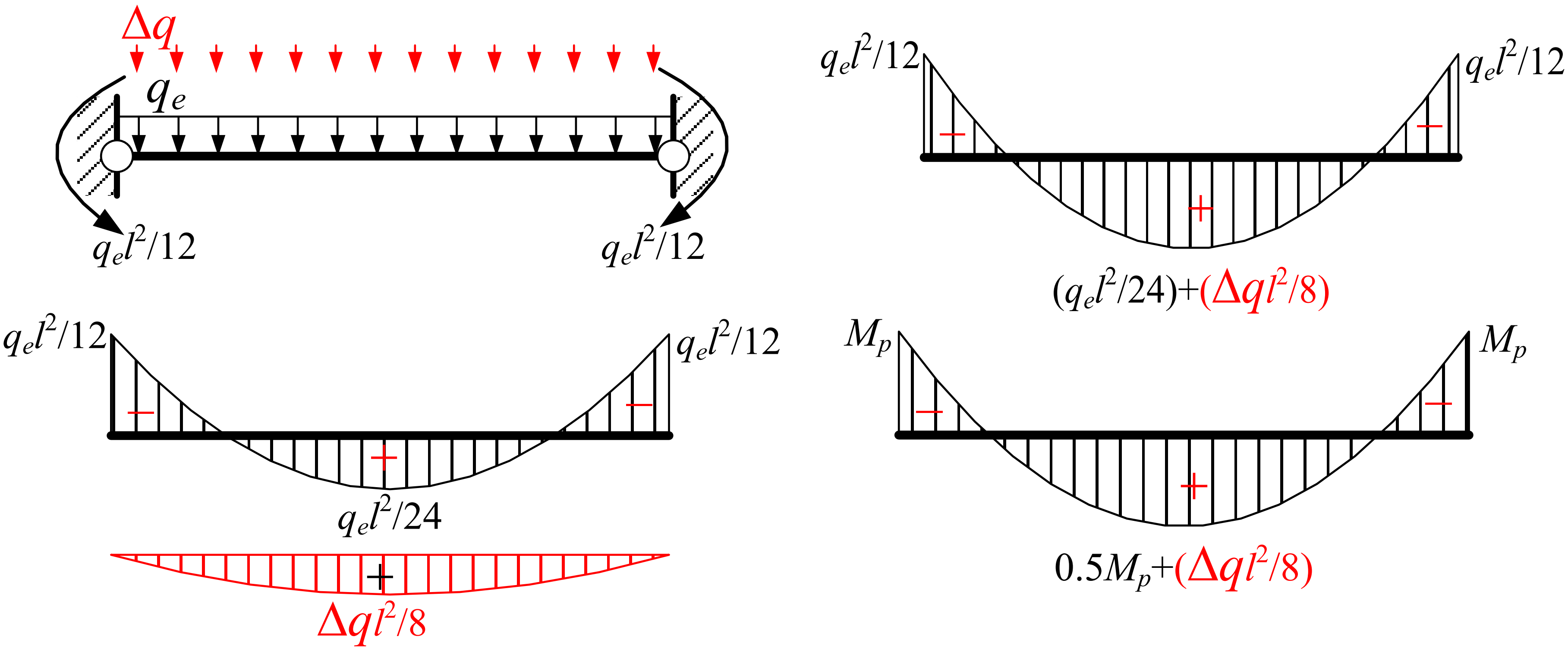






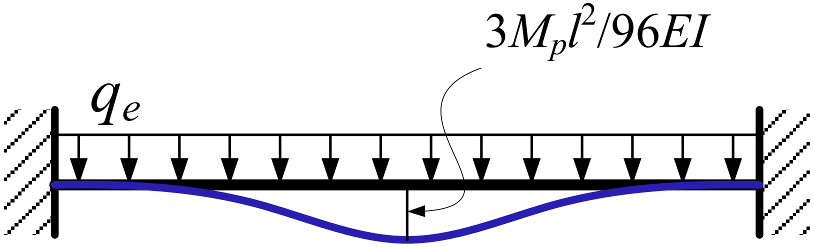
$$\frac{q_e l^2}{12} = M_p \Rightarrow q_e = \frac{12 M_p}{l^2}$$

The Elastic-Plastic behavior of Statically Indeterminate Beam

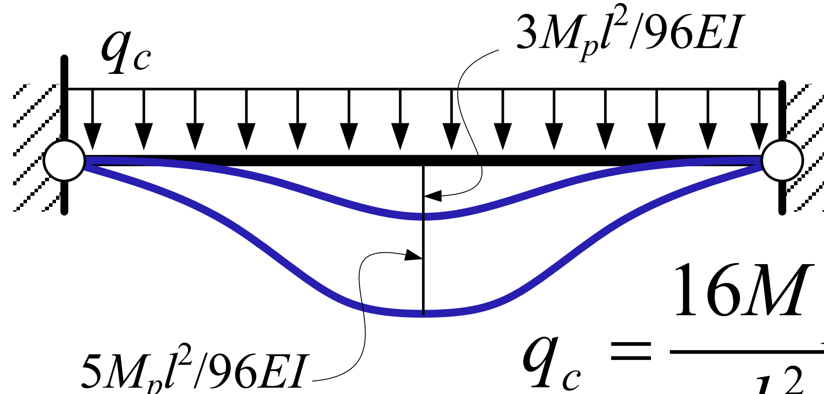


$$0.5M_p + \frac{\Delta q_c l^2}{8} = M_p \Rightarrow q_c = \frac{16M_p}{l^2}$$

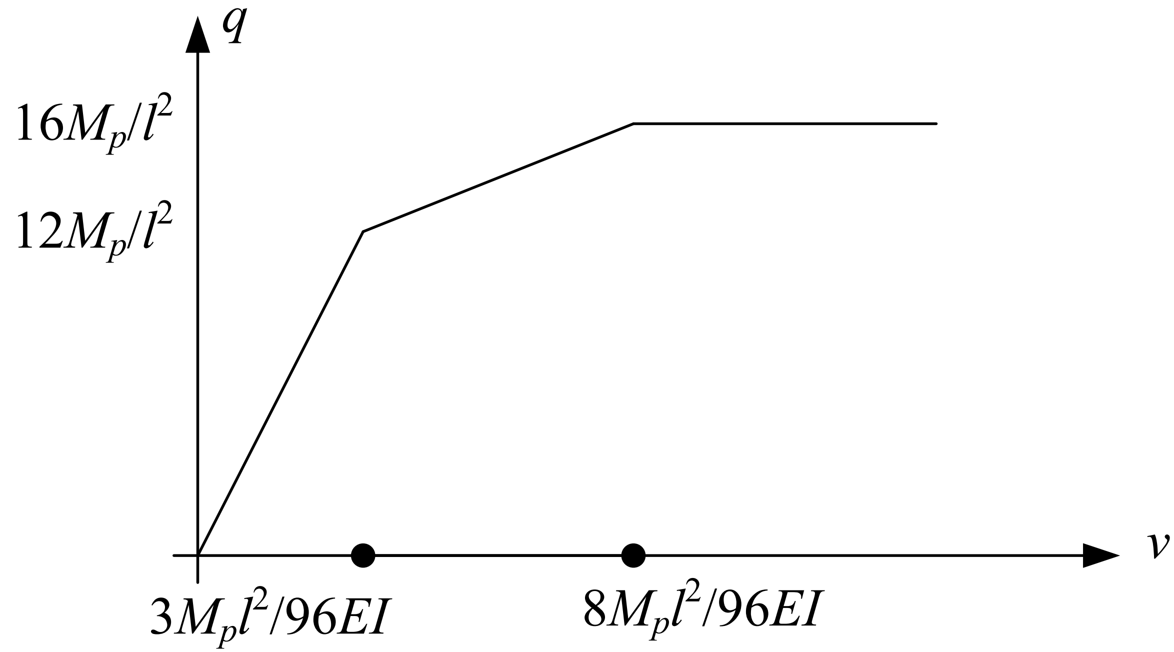
The Elastic-Plastic behavior of Statically Indeterminate Beam



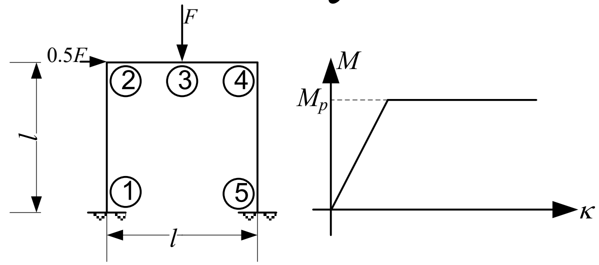
$$q_e = \frac{12M_p}{l^2}$$

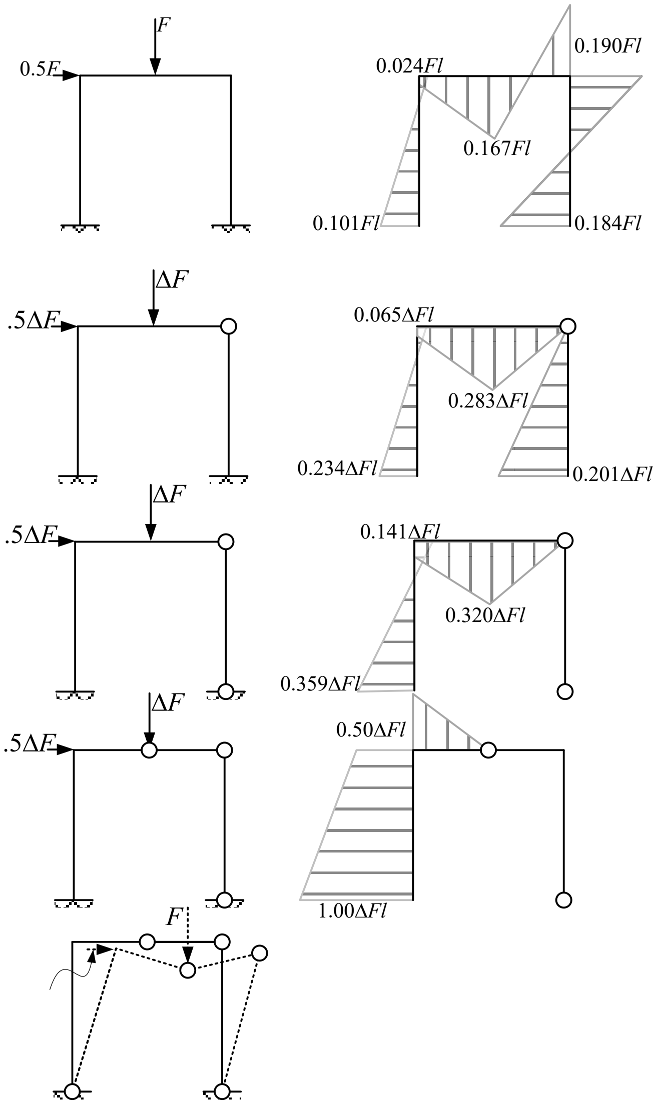


$$q_c = \frac{16M_p}{l^2}$$



The Elastic-Plastic behavior of Statically Indeterminate Frame





① $0.101Fl$
② $0.024Fl$
③ $0.167Fl$
④ $0.190Fl$
⑤ $0.184Fl$

④ $0.190Fl=M_p \Rightarrow F_1=5.263M_p/l$

① $0.532M_p$
② $0.126M_p$
③ $0.879M_p$
④ M_p
⑤ $0.968M_p$

① $-0.234\Delta Fl-0.532M_p=-M_p \Rightarrow \Delta F=2M_p/l$
② $0.065\Delta Fl+0.126M_p=M_p \Rightarrow \Delta F=13.4M_p/l$
③ $0.283\Delta Fl+0.879M_p=M_p \Rightarrow \Delta F=0.428M_p/l$
⑤ $0.201\Delta Fl+0.968M_p=M_p \Rightarrow \Delta F=\underline{0.159M_p/l}$

$F_2=F_1+\mathrm{Min}(\Delta F) \Rightarrow F_2=(5.263+0.159)M_p/l=5.422M_p/l$

① $0.569M_p$
② $0.136M_p$
③ $0.924M_p$
⑤ M_p

① $-0.359\Delta Fl-0.569M_p=-M_p \Rightarrow \Delta F=1.20M_p/l$
② $0.141\Delta Fl+0.136M_p=M_p \Rightarrow \Delta F=6.13M_p/l$
③ $0.320\Delta Fl+0.924M_p=M_p \Rightarrow \Delta F=\underline{0.237M_p/l}$

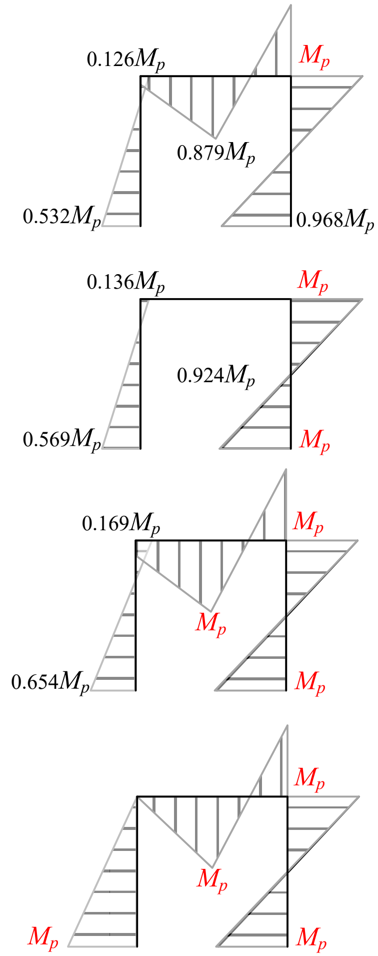
$F_3=F_2+\mathrm{Min}(\Delta F) \Rightarrow F_3=(5.422+0.237)M_p/l=5.659M_p/l$

① $0.654M_p$
② $0.169M_p$
③ M_p

① $-1.00\Delta Fl-0.654M_p=-M_p \Rightarrow \Delta F=\underline{0.346M_p/l}$
② $-0.500\Delta Fl+0.169M_p=-M_p \Rightarrow \Delta F=2.338M_p/l$

$F_4=F_3+\mathrm{Min}(\Delta F) \Rightarrow F_4=(5.659+0.346)M_p/l=6.00M_p/l$

① M_p
② ≈0



The Elastic-Plastic behavior of Statically Indeterminate Beam