Plastic Limit State Analysis: الحالة الحدية اللدنة في الجيزان والإطارات

The limit load can also be determined directly from the upper-bound and lower-bound theorems of the theory of plasticity.

Plastic Hinge: Defined as a yielded zone due to bending in structural member. Rotation can take place but with a constant resisting moment M_p

المفصل اللدن: مقطع من جائز بلغت إجهادات الشد والضغط حد الخضوع في نقاطه كافة.
وكاي مفصل عادي تحصل على جانبيه دورانات ولكن مع عزم مقاوم مقداره M_p ،العزم اللدن
لهذا المقطع كما عرفناه سابقاً

Mechanism:

An n degree indeterminate structure developing n plastic hinges, becomes determinate.

Formation of an additional hinge will reduce the structure to mechanism, which has a rigid body displacement with one difference that the plastic hinge permits rotation offering a constant resisting moment M_p.

So for causing a rotation θ at a plastic hinge, work W has to be done.

This work equals θM_p .

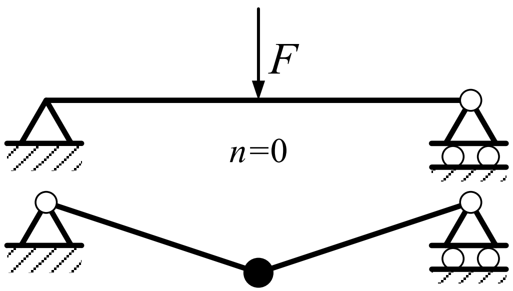
[Compare with Elastic work (1/2) θM].

A. Independent or Simple Mechanism:

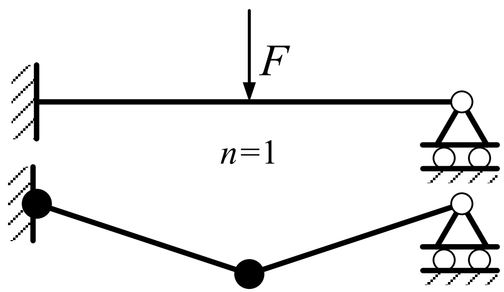
Mechanical (ideal) hinge is denoted by o. Plastic hinge is denoted by ●.

1. Beam mechanisms:

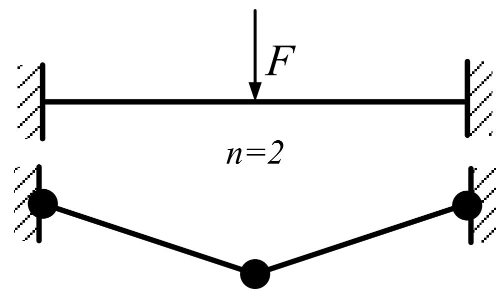
(a) simple beam.



(b) propped cantilever.

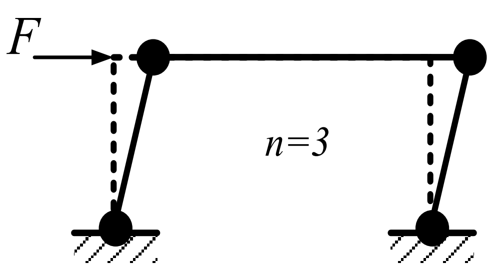


(c) fixed beam.

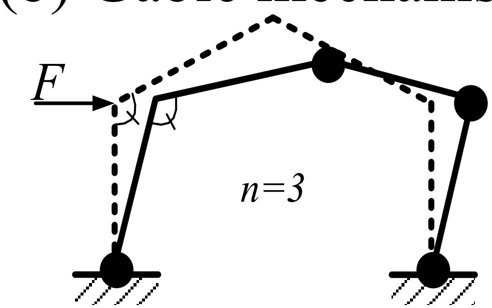


2. Frame mechanisms:

(a) Sway mechanism.



(b) Gable mechanism.



B. Composite or Combined Mechanism:

This is a combination of two or more of the above mechanism.

Methods For Plastic Limit State Analysis:

1. Static Method or Virtual Work Method:
   Based on the Lower-Bound theorem which states
   "A Load computed on the basis of an assumed equilibrium bending moment diagram in which the moments are not greater than M_p, is less than or equal to the true ultimate load".

2. Kinematic Method or Mechanism Method:
   Based on the Upper-Bound theorem which states:
   "A Load computed on the basis of an assumed mechanism, is always be greater than or equal to the true ultimate load".

Procedure for Mechanism Method:

1. Locate the points of possible plastic hinges:
   The plastic hinges are likely to be formed
   (a)under concentrated loads,
   (b) at supports,
   (c) at connections or joints,
   (d) or points of maximum bending moments.

2. Determine the number of possible independent mechanisms and the combined mechanisms.
   If n is the redundancy of the structures
   and m is the number of possible plastic hinges,
   then the number of independent or elementary mechanisms is:
   $e = m - n$.

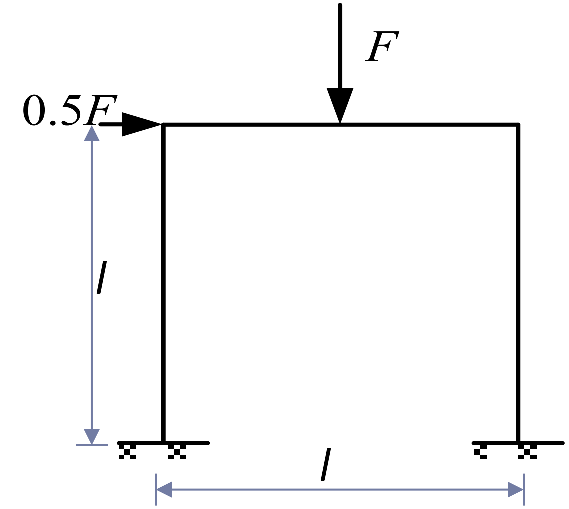
Procedure for Mechanism Method:

3. Give the mechanism a consistent virtual displacement and work out the consequent displacements and rotations at strategic points.
   The gravity or lateral loads produce an external work W_E.
   The plastic hinges offer a resistance to the mechanism rotations.
   The total work done at the plastic hinges is : $W_I = \sum \theta M_p$.
   If the loads are big enough so that $W_E > W_I$, collapse will occur.
   Hence $W_E = W_I$, represents the collapse condition.

4. This procedure has a draw back. If there are *e* possible mechanisms, we end up with *e* values of ultimate loads.
   The least of these values is the correct one and the corresponding mechanism is the likely one.
   In other words, the actual ultimate load cannot be more than the least value obtained by this method.
   Hence this is called the upper bound method.

Example 1.

Consider the portal frame of the next Figure.



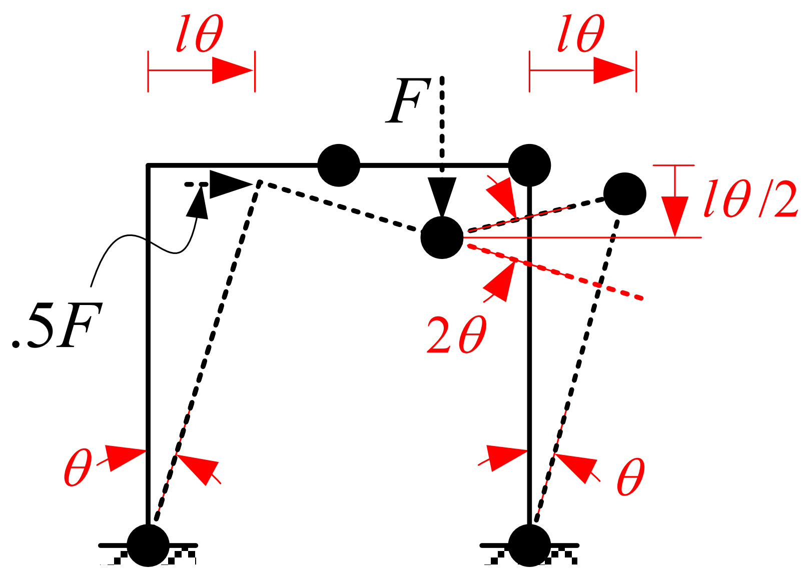
Solution:

From the elastic-plastic analysis in the previous lecture the collapse mechanism is known.

$W_{\mathrm{I}} = \theta M_p + 2\theta M_p + 2\theta M_p + \theta M_p = 6\theta M_p$

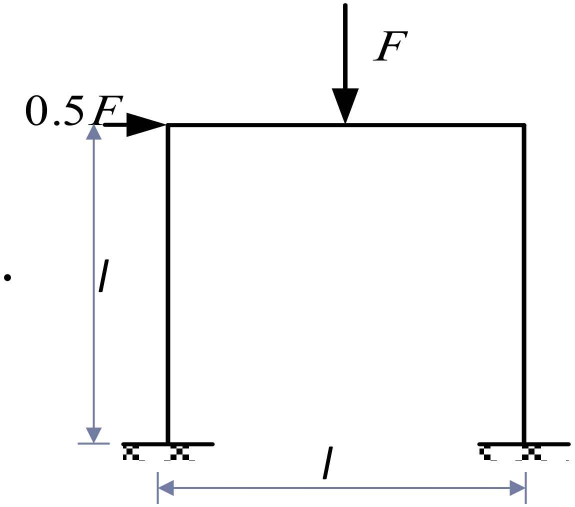
$W_{\mathrm{E}} = 0.5F\,(l\theta) + F(l\theta/2) = Fl\theta$

$W_{\mathrm{E}} = W_{\mathrm{I}},\ \Rightarrow F = 6M_p / l$



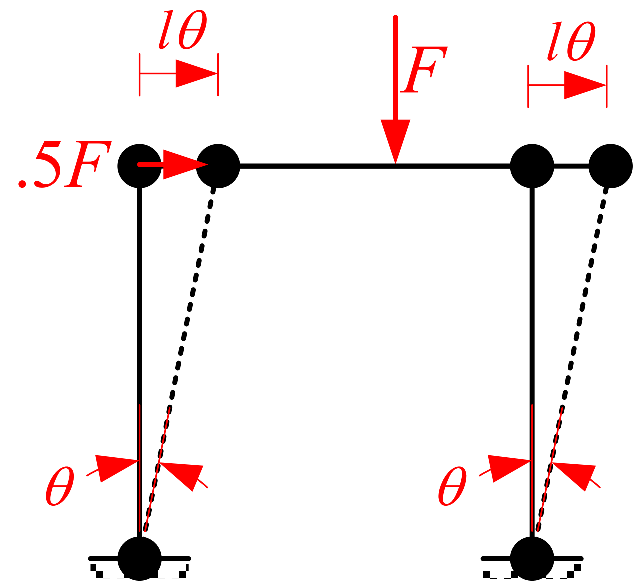
Example 1.

Consider the portal frame of the next Figure.



Solution:

Take another mechanism.



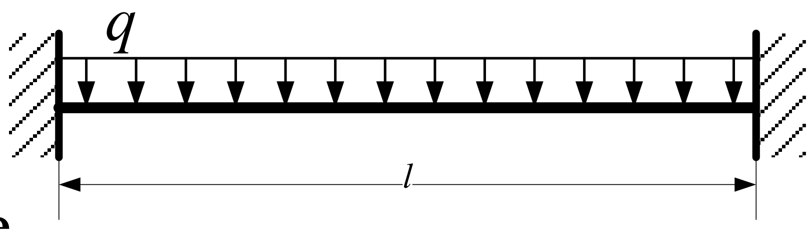
$W_I = \theta M_p + \theta M_p + \theta M_p + \theta M_p = 4\theta M_p$

$W_E = 0.5F\,(l\theta) = Fl\theta/2$

$W_E = W_I, \Rightarrow F = 8M_p / l$

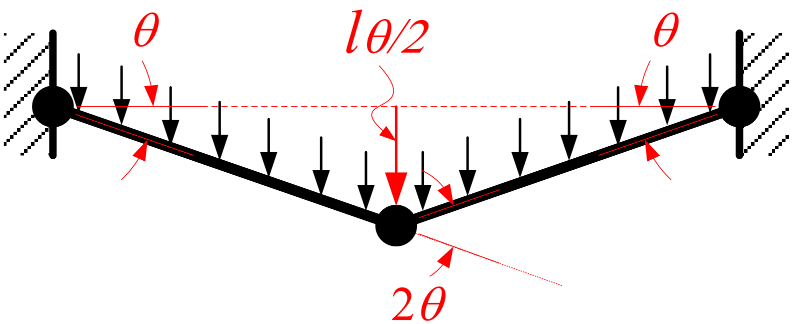
Example 2.

Consider the fixed beam of the next Figure.



Solution:

Take the mechanism.



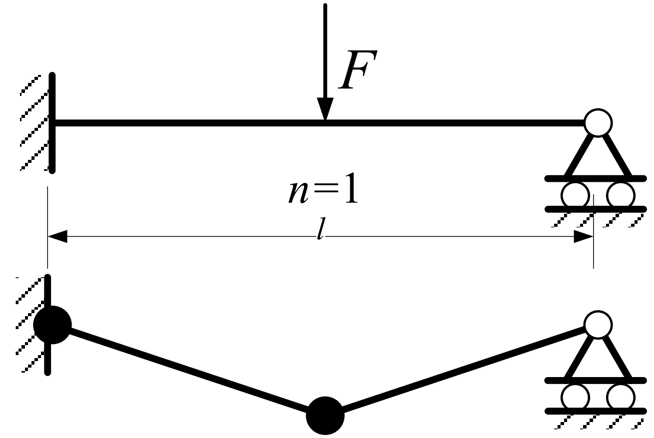
$W_{\mathrm{I}} = \theta M_p + 2\theta M_p + \theta M_p = 4\theta M_p$

$W_{\mathrm{E}} = q\,[0.5l\,(l\theta/2)] = \theta\, q\, l^2/4$

$W_{\mathrm{E}} = W_{\mathrm{I}}, \Rightarrow q = 16 M_p / l^2$

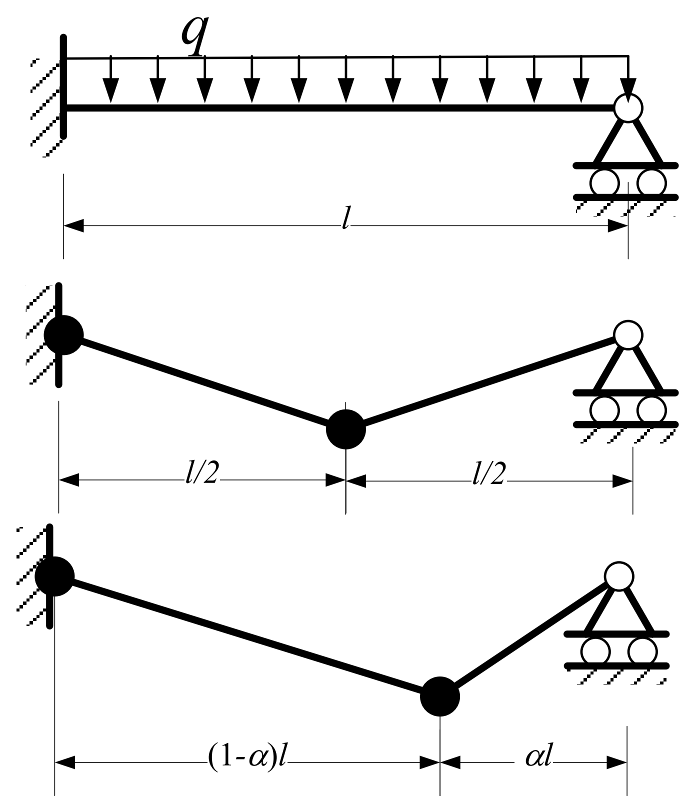
Example 3.

Consider the propped beam of the next Figure.



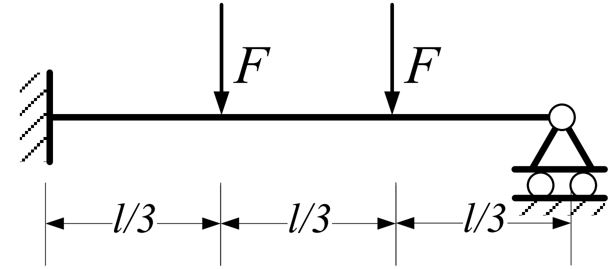
Example 4.

Consider the propped beam of the next Figure.



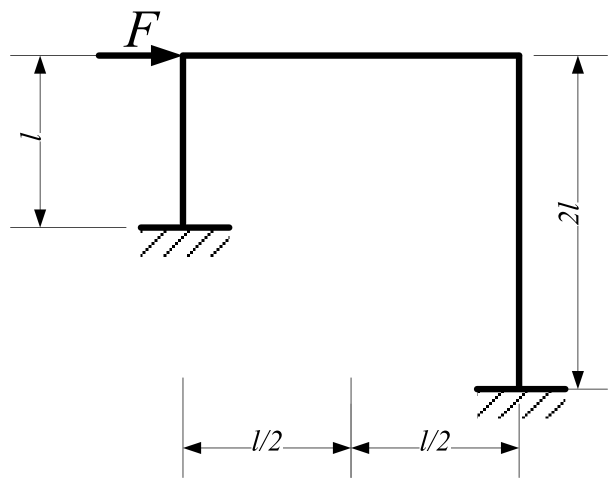
Example 5.

Consider the propped beam of the next Figure.



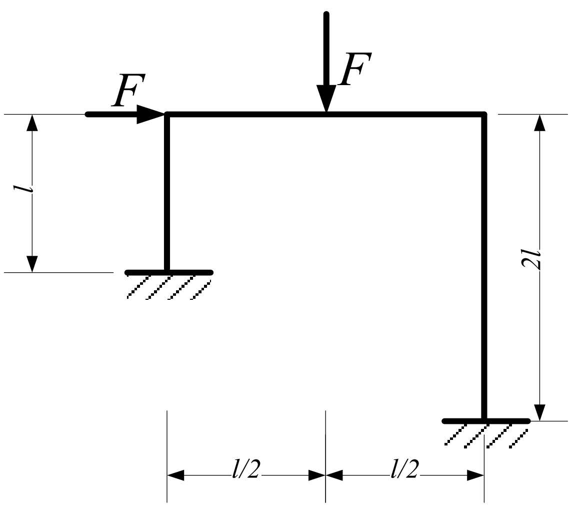
Example 6.

Consider the frame of the next Figure.

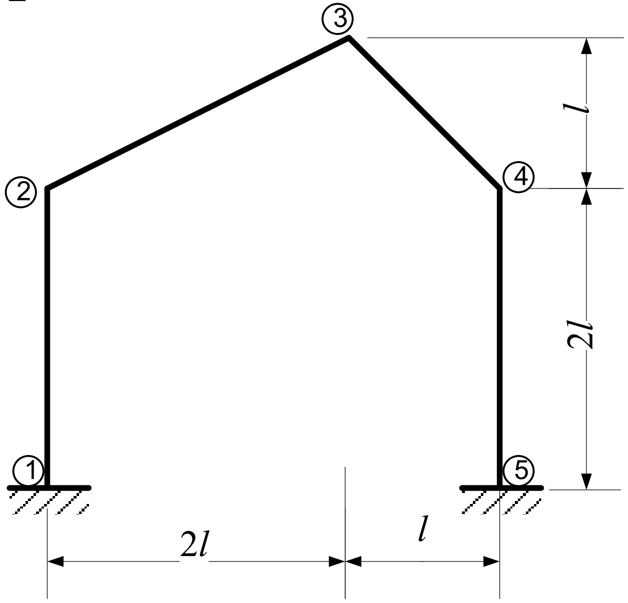


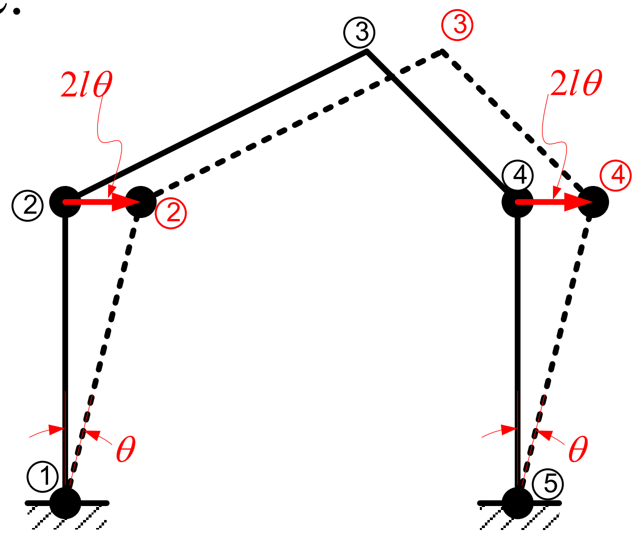
Example 7.

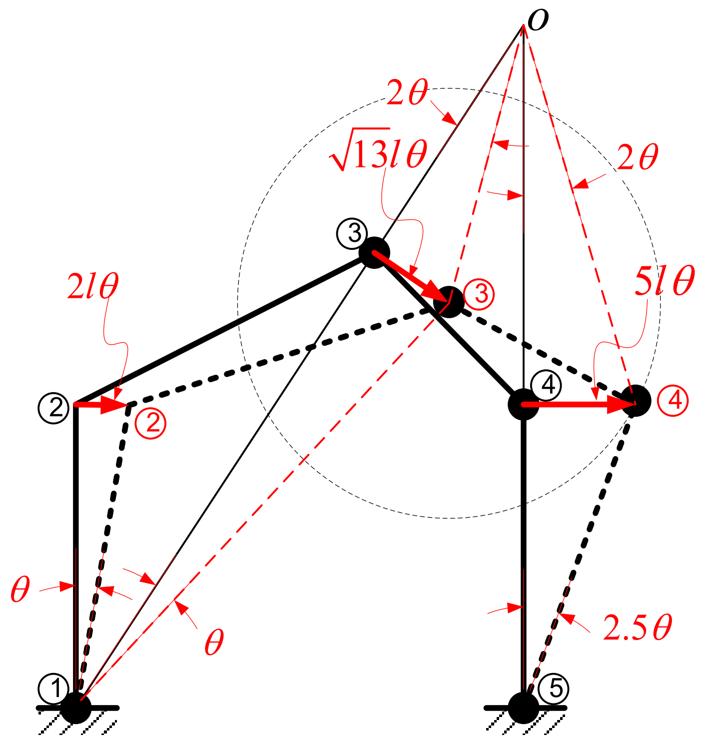
Consider the frame of the next Figure.



Example 8: Mechanisms for gable frame.





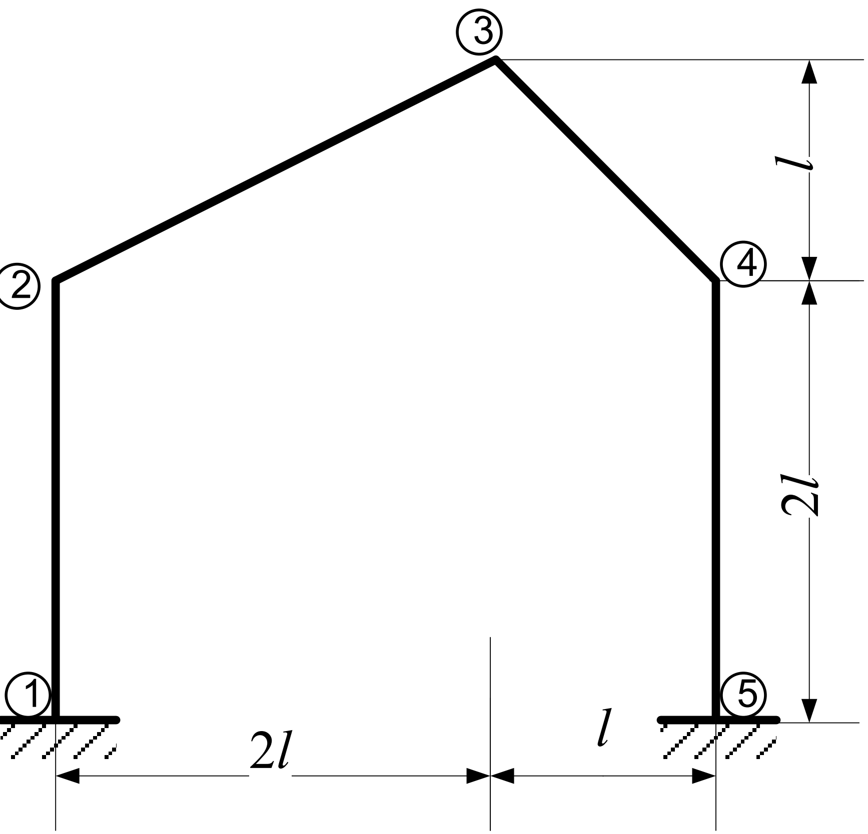


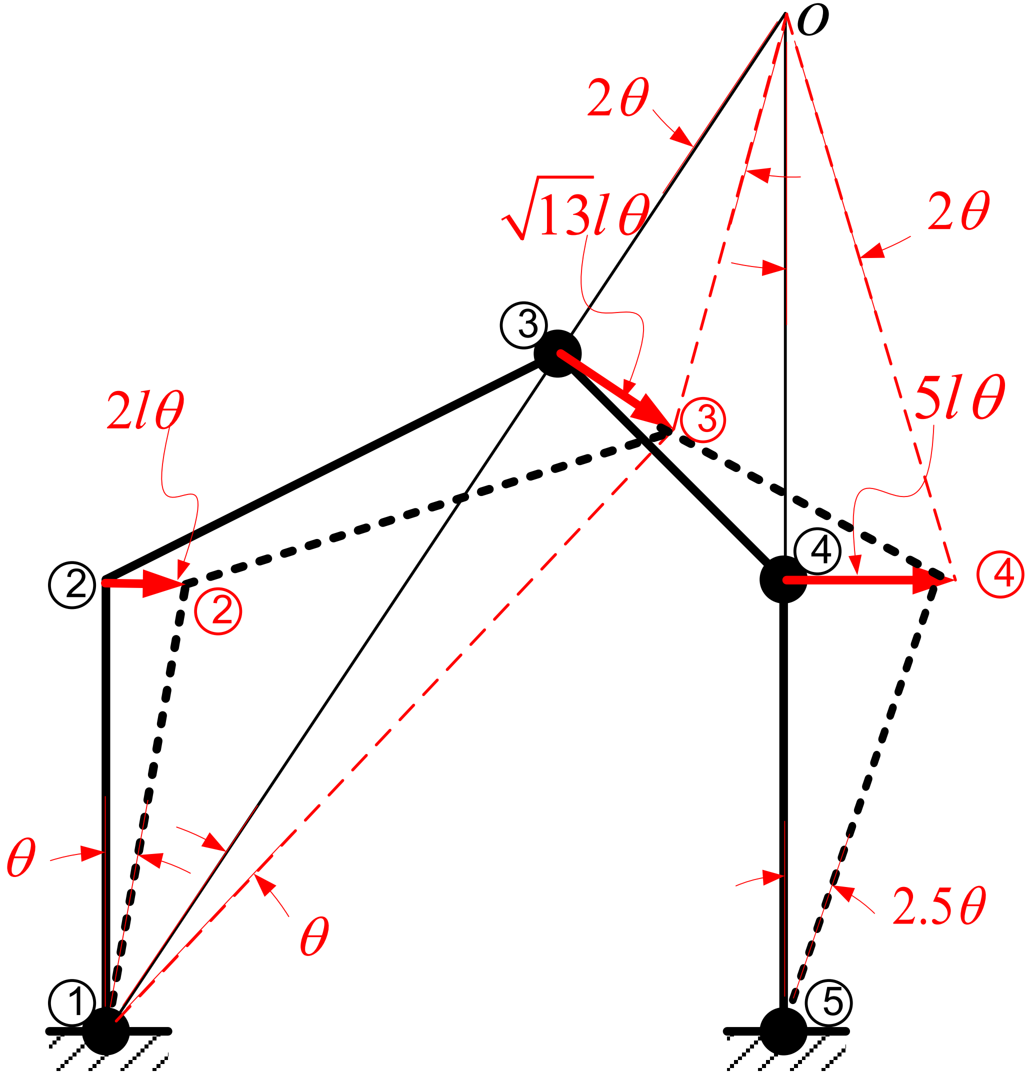
Example 8: Mechanisms for gable frame.

$$L_{13} = \sqrt{(2l)^2 + (3l)^2} = \sqrt{13}l$$

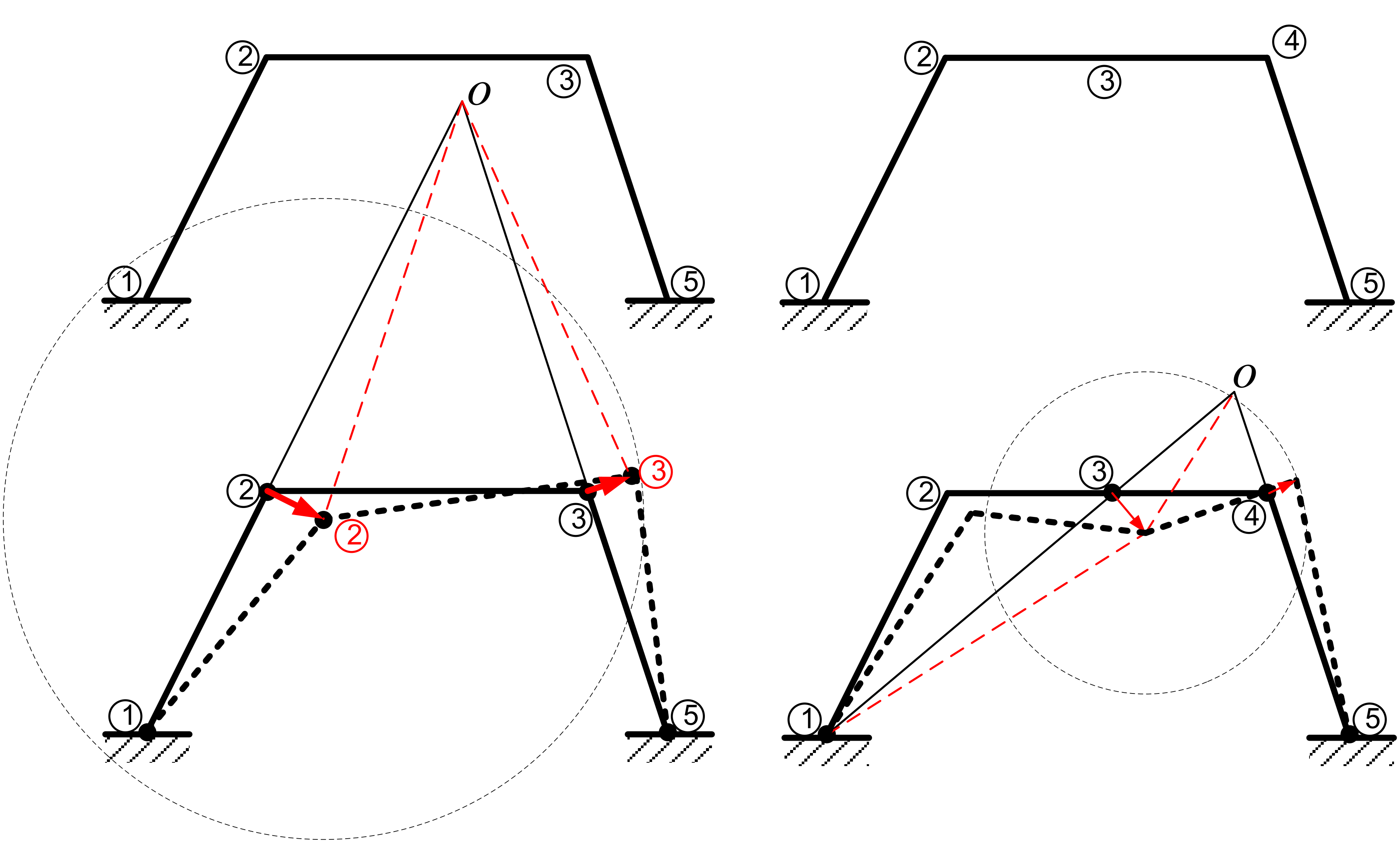
$$L_{O3} = \sqrt{(l)^2 + (1.5l)^2} = \frac{\sqrt{13}}{2}l$$

$$L_{O4} = l + 1.5l = 2.5l$$

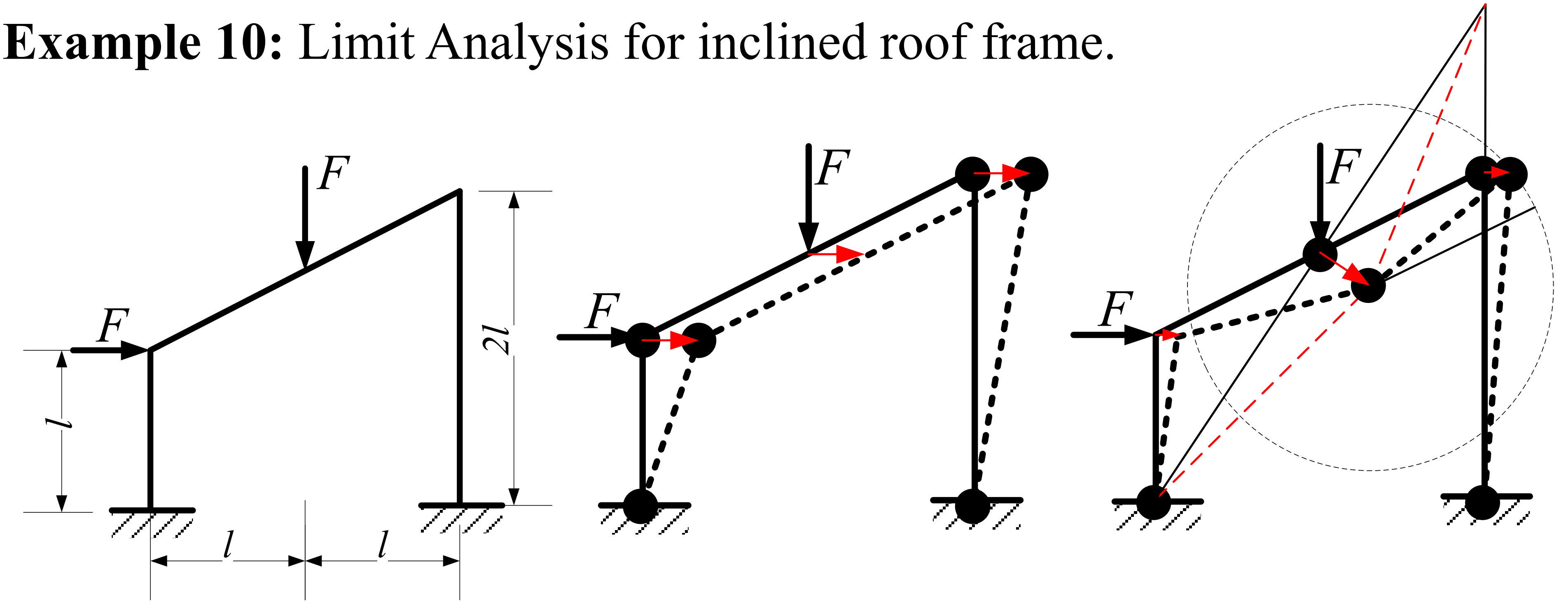




Example 9: Mechanisms for inclined legs frame.



Example 10: Limit Analysis for inclined roof frame.



Example 11: Limit Analysis for the two frames.
Use three mechanisms for each one.