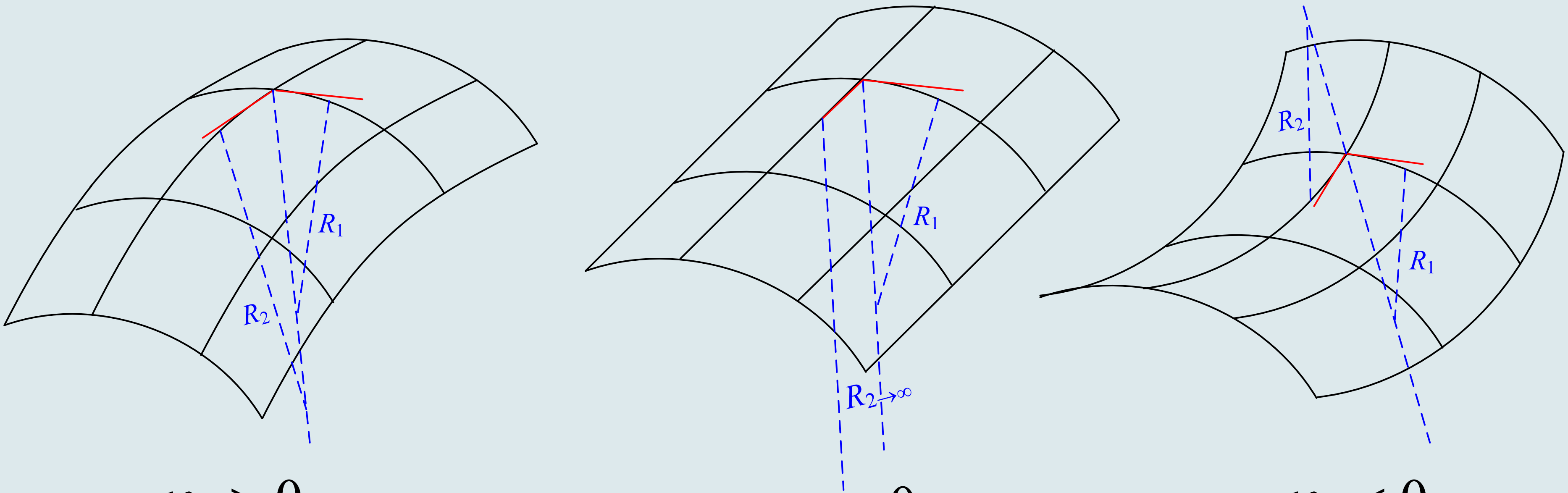
Membrane Theory for Shells

Coordinate System & Gaussian Curvature



$\kappa_G > 0$

$\kappa_G = 0$

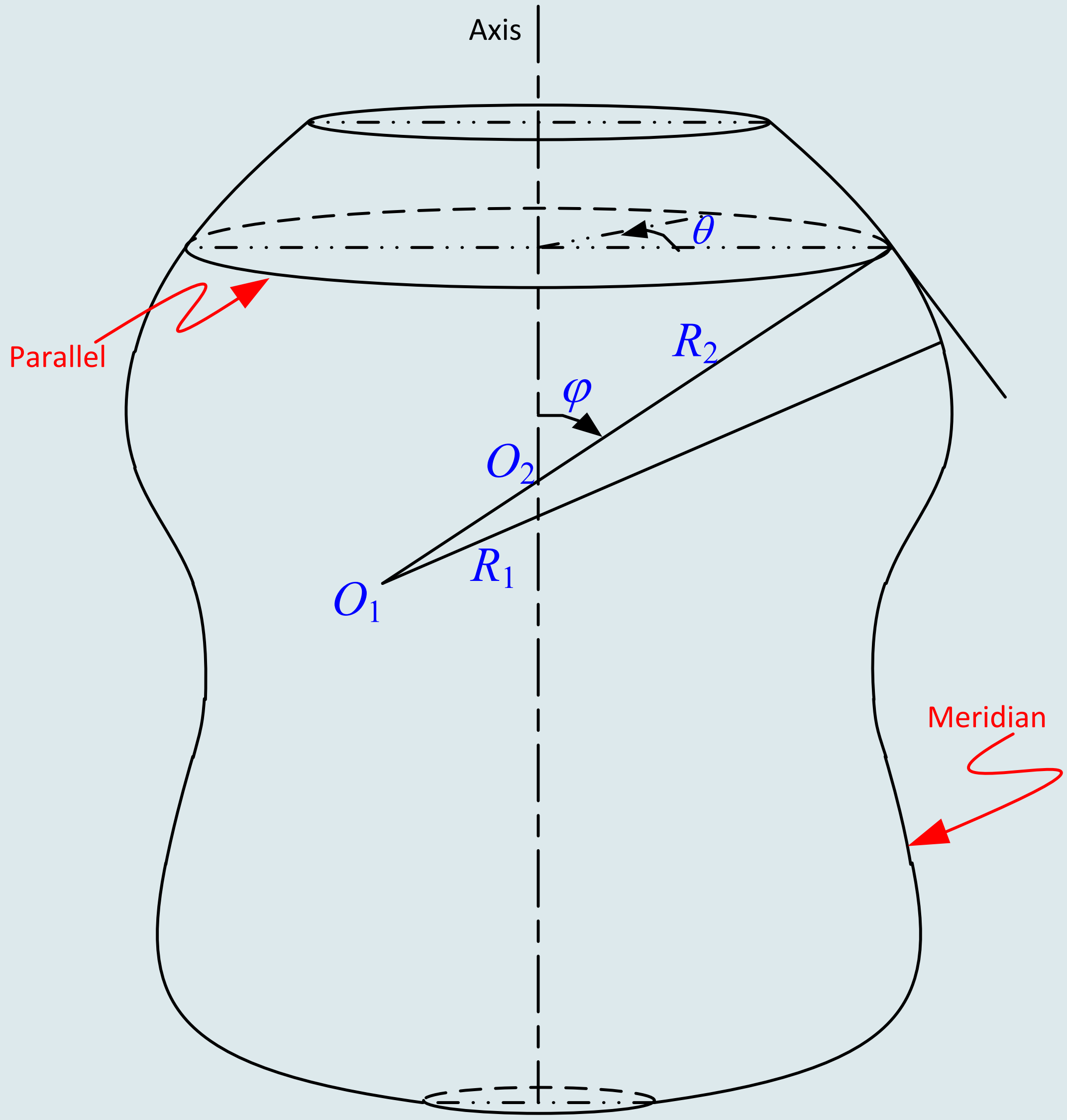
$\kappa_G < 0$

$$\kappa_G = \chi_1 \chi_2 = (1/R_1)(1/R_2)$$

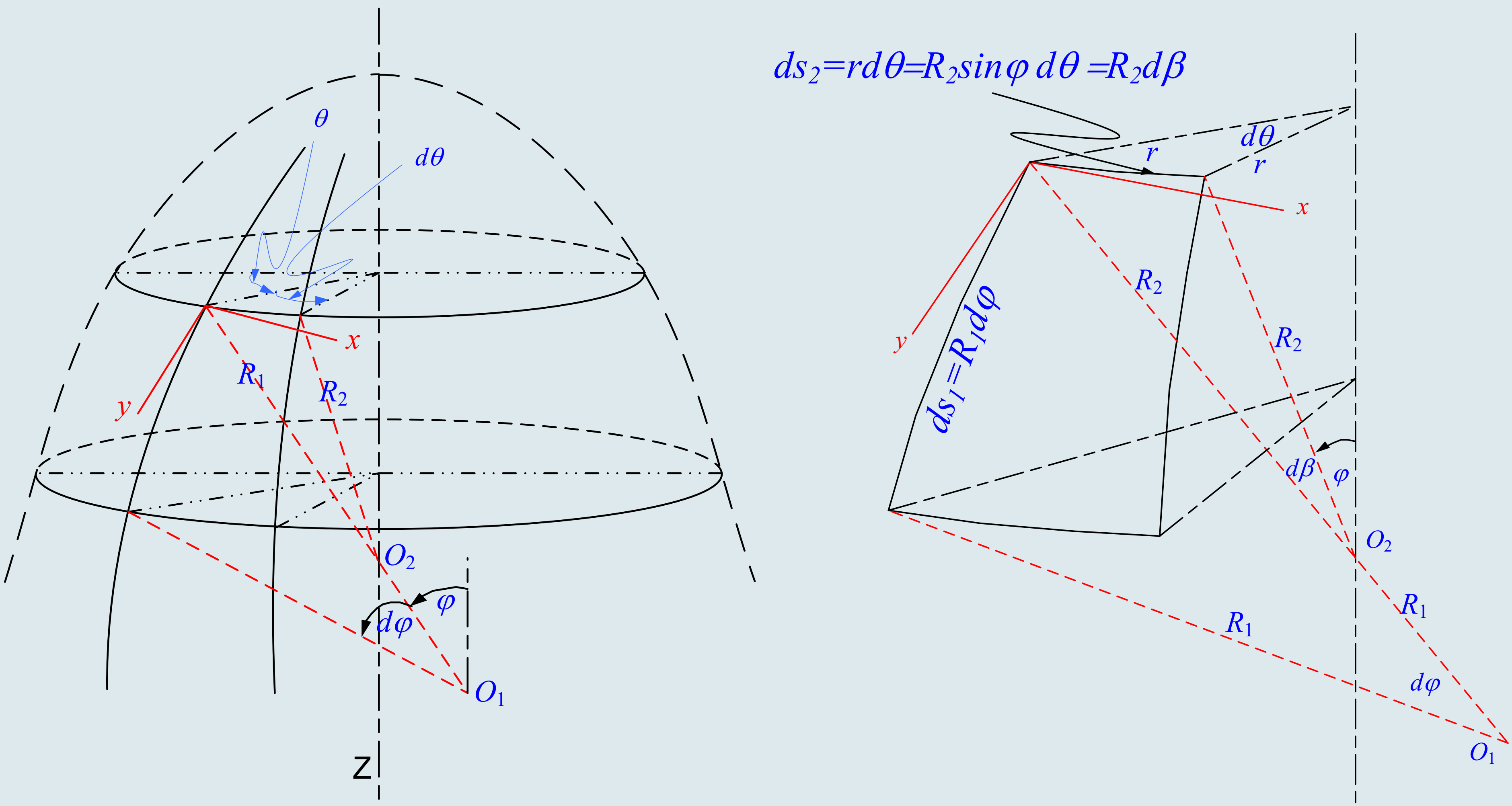
Good membrane behavior
& less sensitive to bending

More sensitive to bending

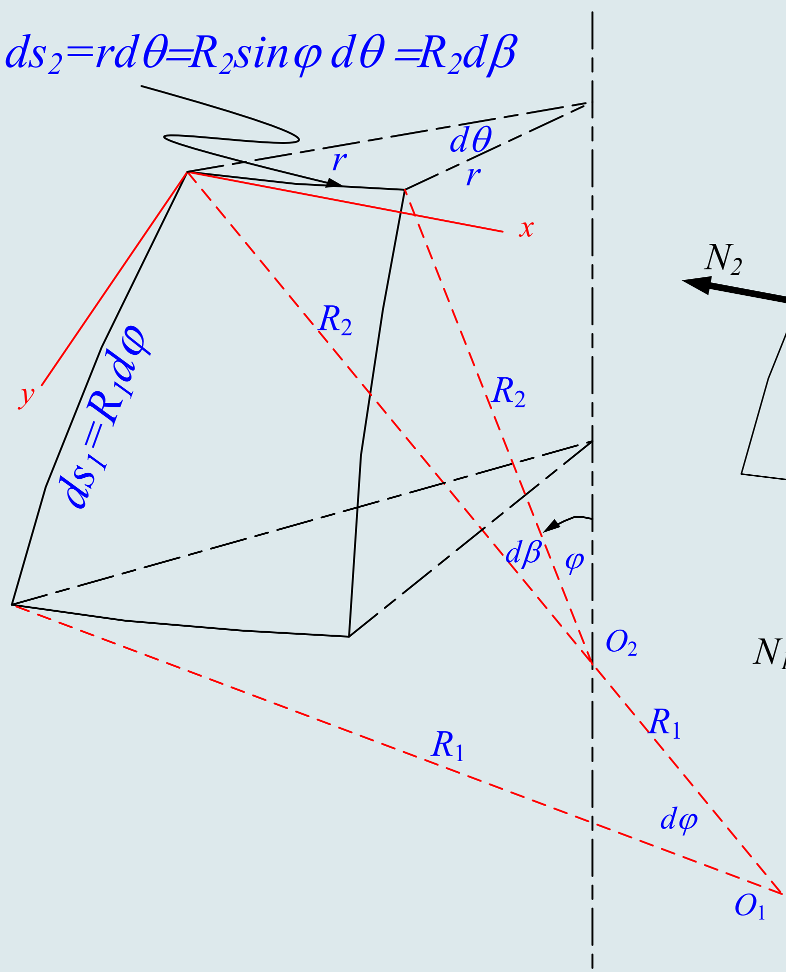
Membrane Theory for Shells of Revolution

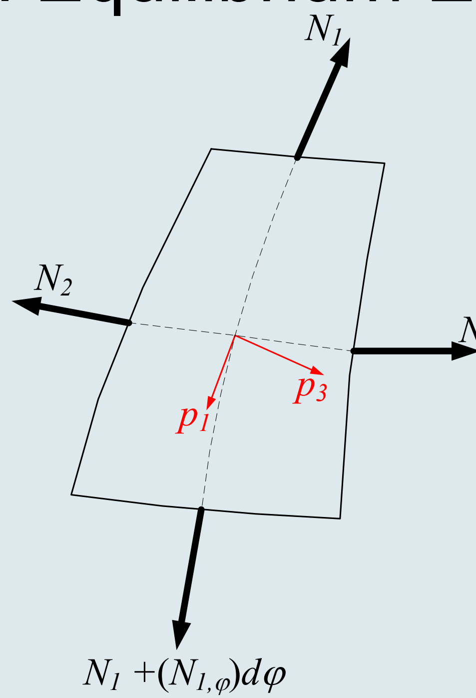


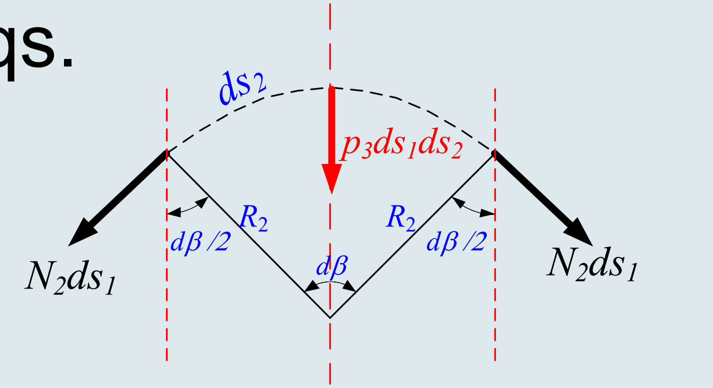
INTERNAL FORCES IN SYMMETRICALLY LOADED SHELLS OF REVOLUTION

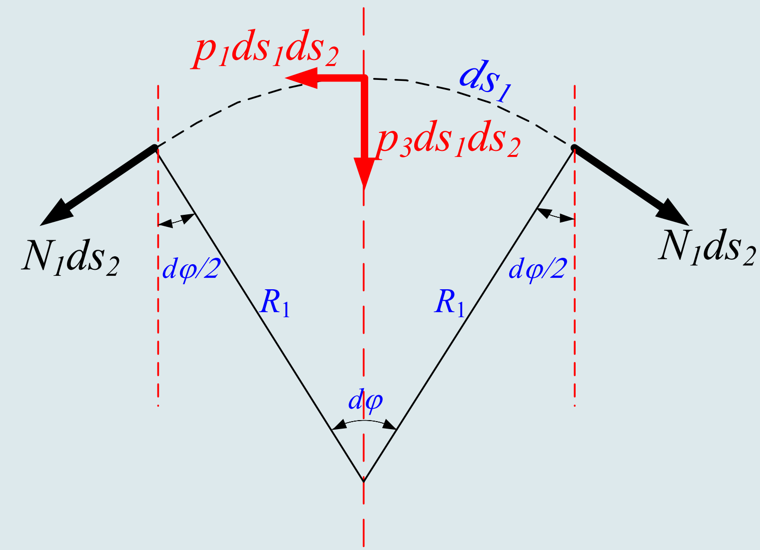


Normal & Meridional Equilibrium Eqs.









N_1 the meridional force

N_2 the tangential force

$$\frac{N_1}{R_1}+\frac{N_2}{R_2}+p_3=0$$

Normal Eq. Eq.

$$\frac{1}{R_1}N_{1,\varphi}+\frac{N_1-N_2}{R_2}\cot\varphi+p_1=0$$

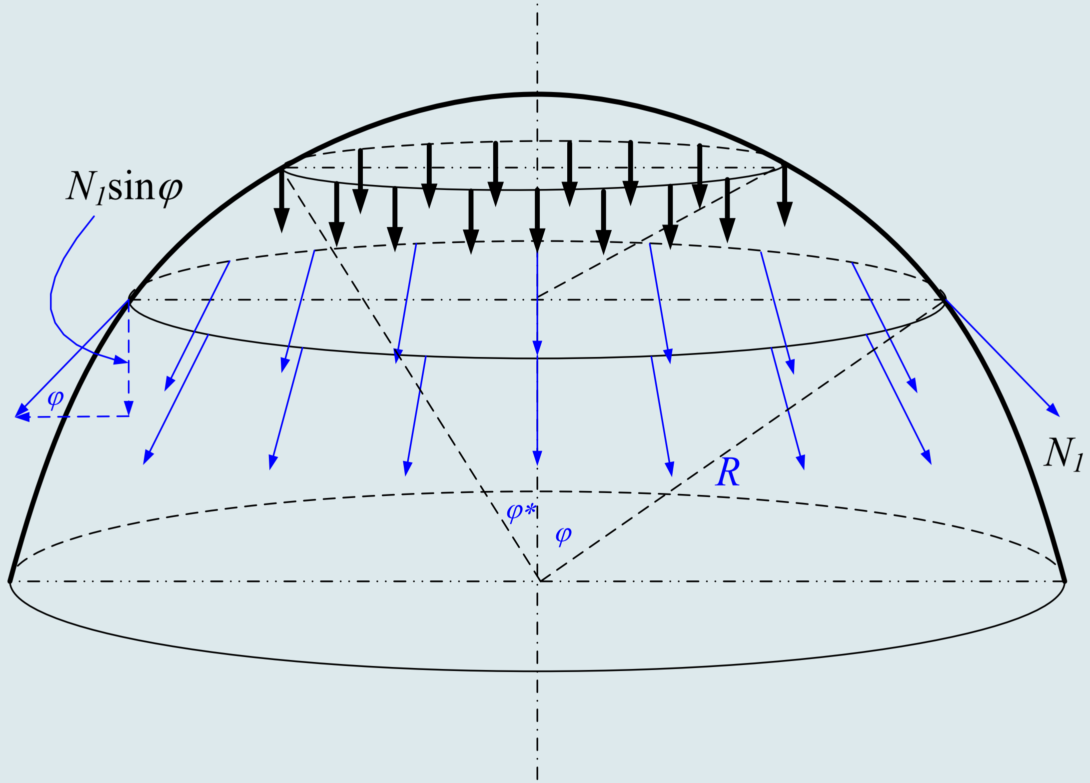
Meridional Eq. Eq. not used

Application of the Membrane Theory to the Analysis of Shell Structures

1. Axisymmetrically loaded dome roofs

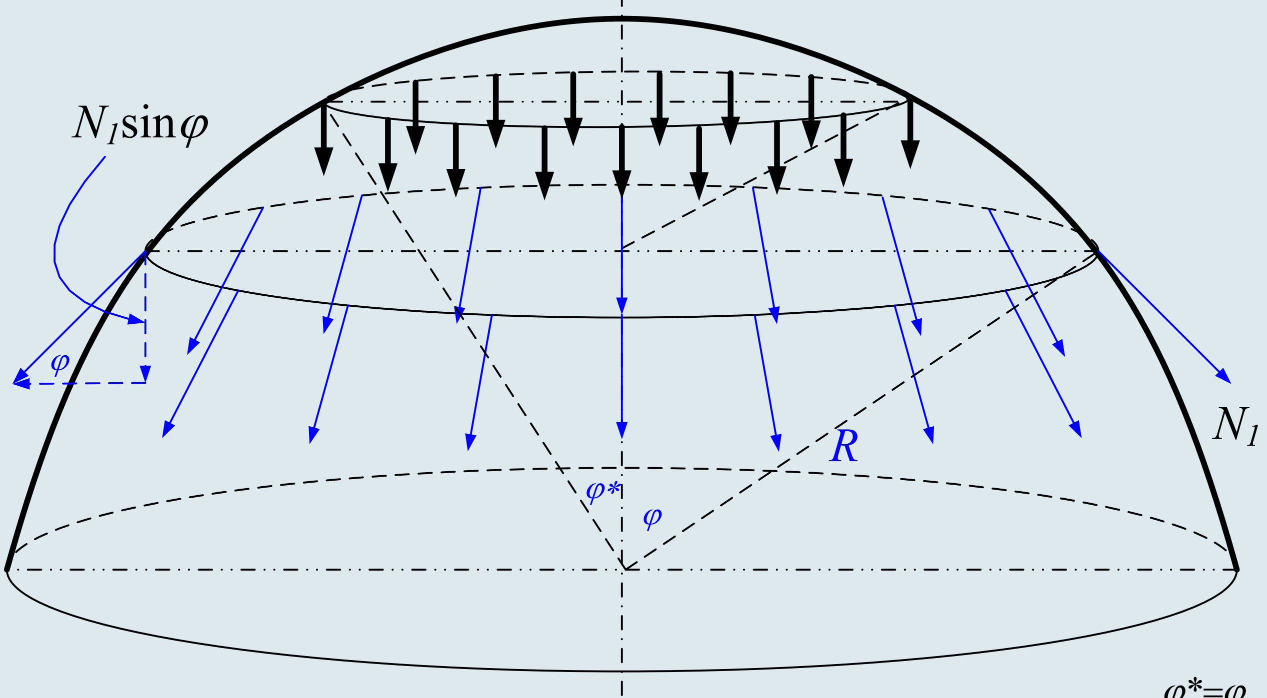
(a) Spherical domes under self-weight

First, we consider a dome with radius *R* closed at its apex and subjected to a dead load of *p* per unit area of the middle surface (e.g., own weight, weight of cladding, etc.).



Resolve the DL *p*, into $p_3 = p\cos\varphi$ & $p_1 = p\sin\varphi$

From geometry: $R_1 = R_2 = R$



Normal Eq. Eq.

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} + p_3 = 0$$

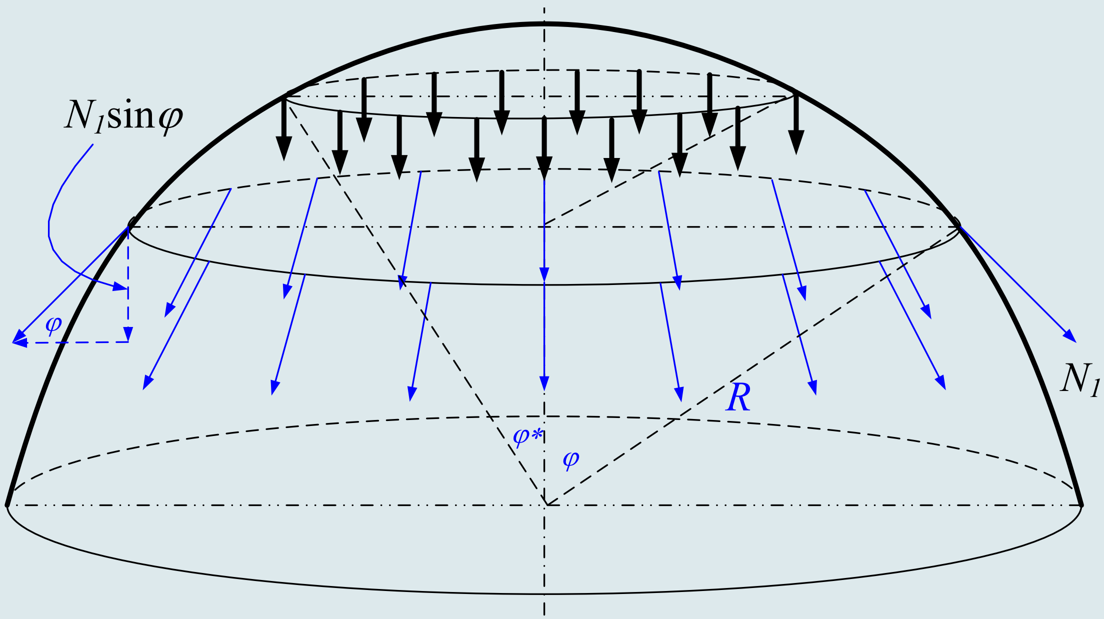
Overall Vertical Eq. Eq.: $N_1 \sin\varphi(2\pi R \sin\varphi) + \int_{\varphi^*=0}^{\varphi^*=\varphi} p(2\pi R \sin\varphi^*) R d\varphi^* = 0$

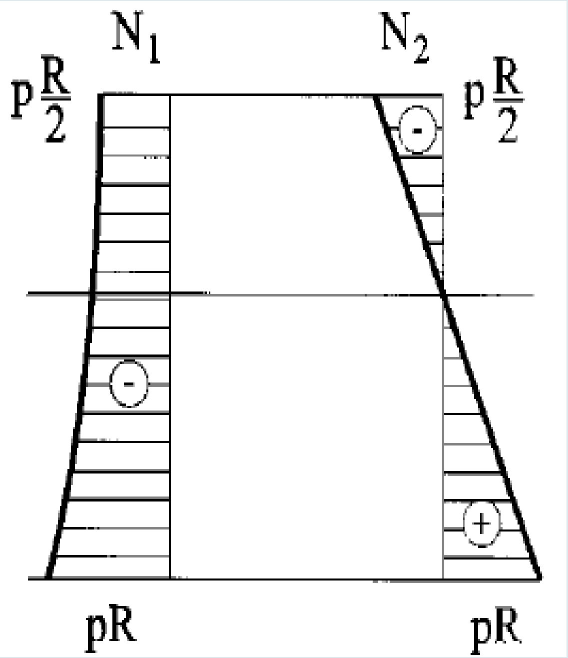
$$N_1 = -\frac{pR}{\sin^2\varphi} \int_{\varphi^*=0}^{\varphi^*=\varphi} \sin\varphi^* d\varphi^* = -\frac{pR(1-\cos\varphi)}{\sin^2\varphi} = -\frac{pR}{1+\cos\varphi}$$

Substituting in the normal Eq. Eq.

$$N_2 = -pR\cos\varphi + \frac{pR}{1+\cos\varphi} = -pR(\cos\varphi - \frac{1}{1+\cos\varphi})$$

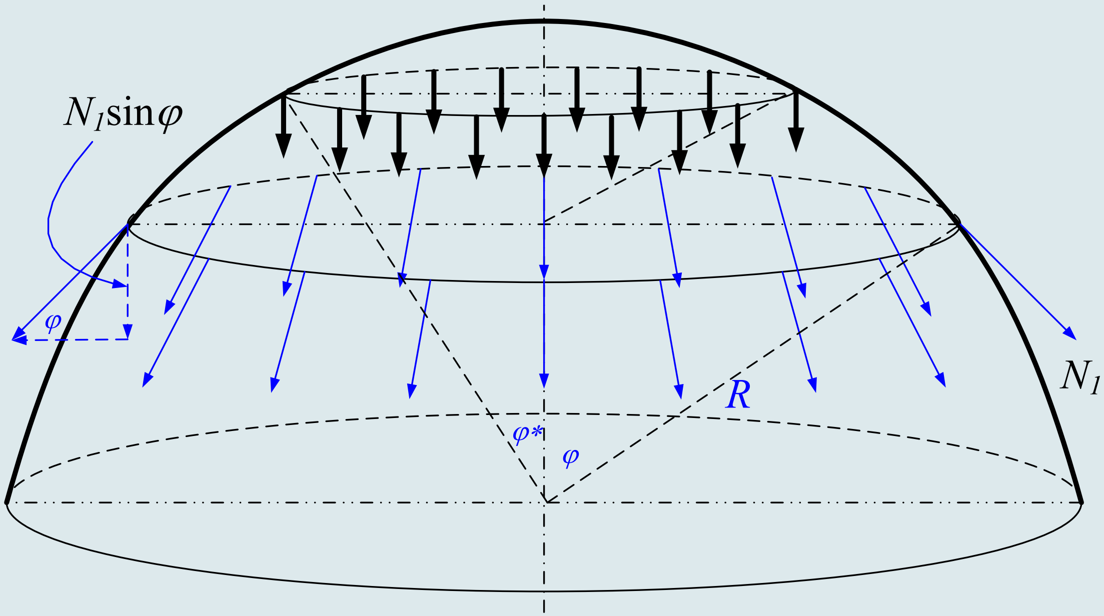
At the crown of the dome, where φ = 0, we have : $N_1 = N_2 = -pR/2$

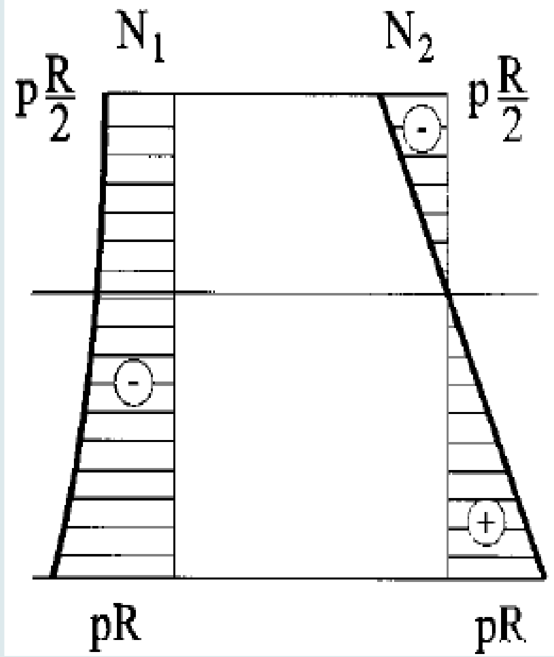




The meridional force, N_1, is compressive along the meridian of the dome, increasing from the apex to the bottom of the dome.

The circumferential compressive force N_2, near the crown, decreases gradually with φ and changes sign. At φ = 90, $N_1 = -pR$ & $N_2 = pR$



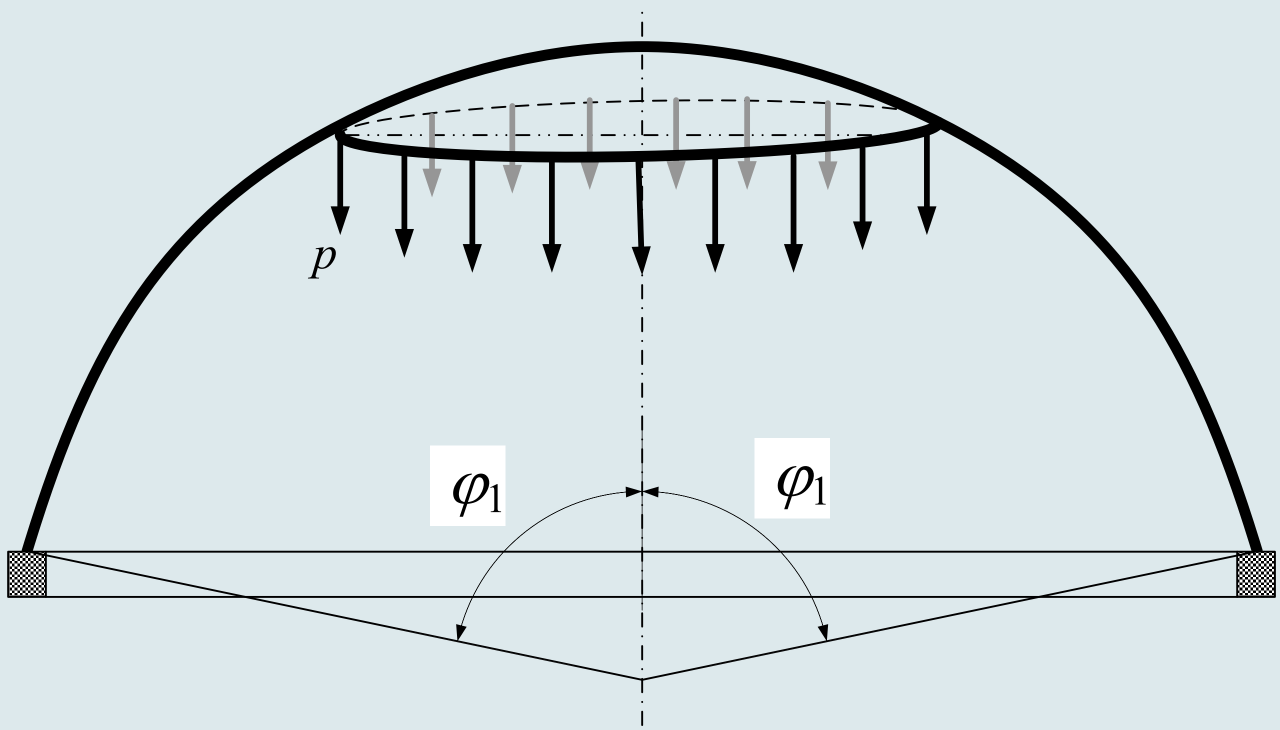


The Change in sign of the circumferential compressive force N_2, takes place when $N_2 = 0$

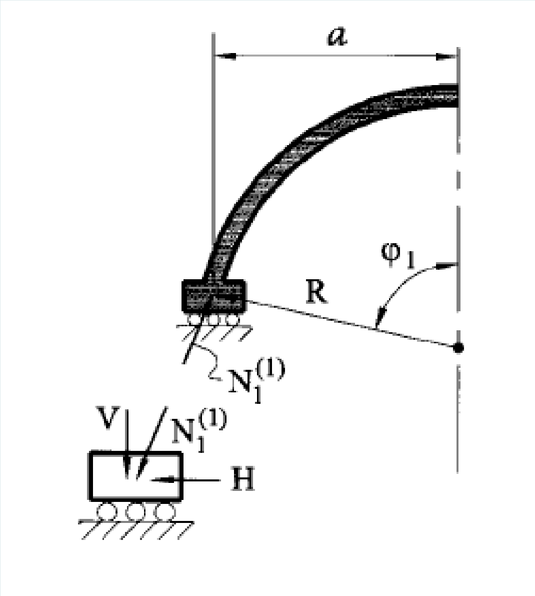
$$N_2 = -pR(\cos\varphi - \frac{1}{1+\cos\varphi}) = 0$$

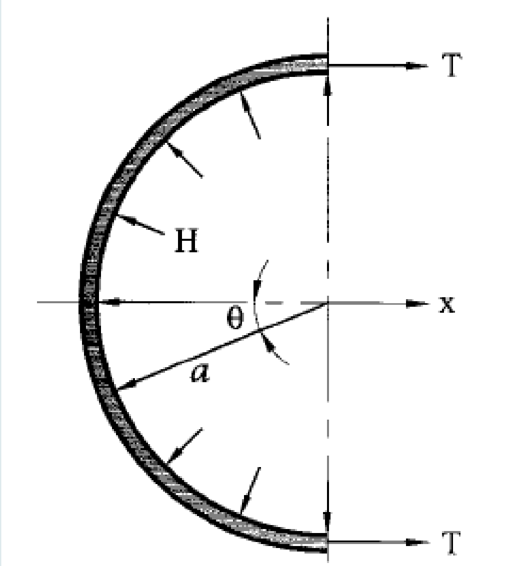
$$\cos\bar{\varphi} - \frac{1}{1+\cos\bar{\varphi}} = 0 \Rightarrow \cos^2\bar{\varphi} + \cos\bar{\varphi} - 1 = 0 \Rightarrow \bar{\varphi} = 51^0 49'$$

Spherical domes whose opening angle is less than [2(51°49‘)] are free from tensile stresses.



p
φ_1
φ_1





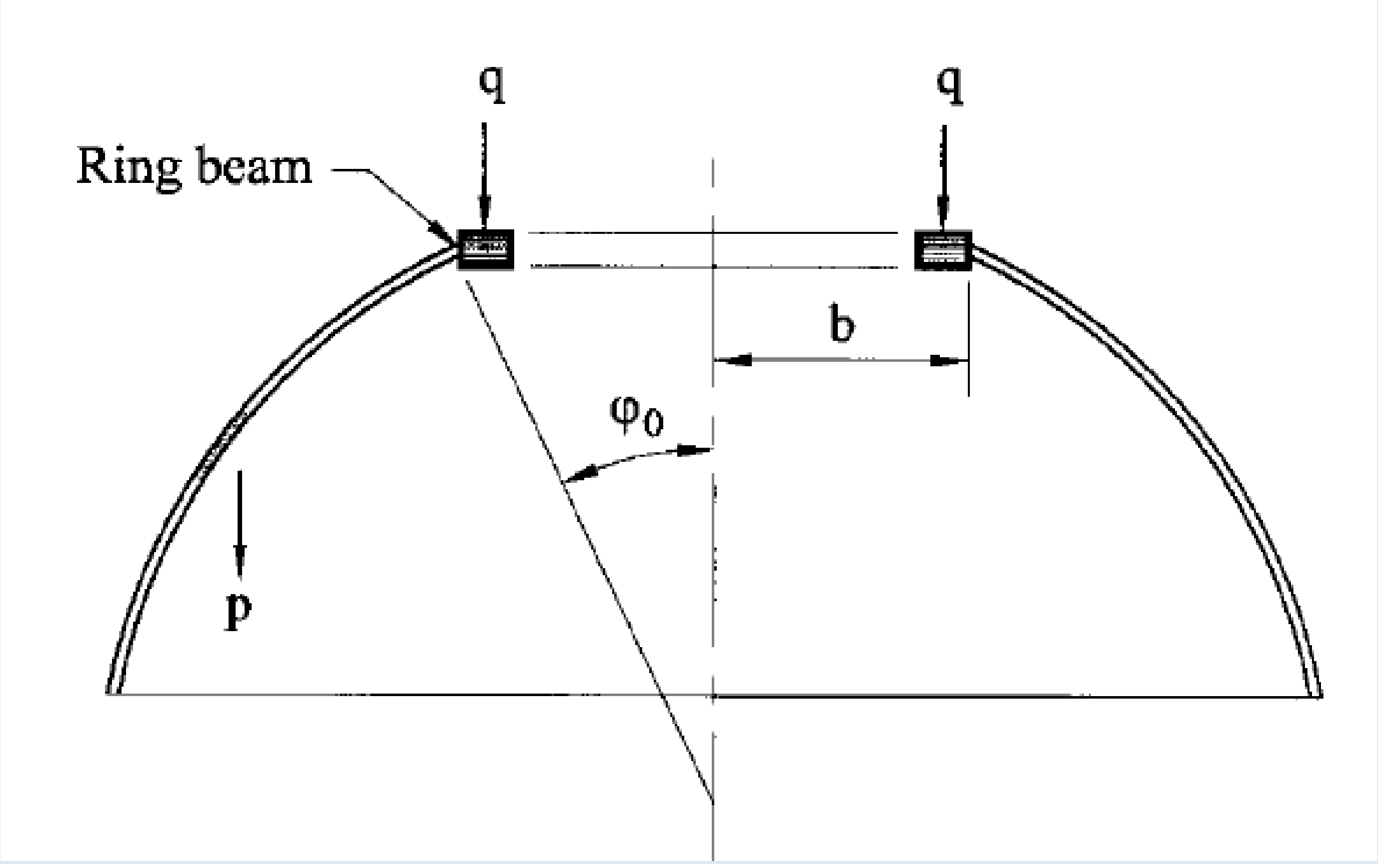
$$V = N_1^{(1)} \sin\varphi_1, \quad H = -N_1^{(1)} \cos\varphi_1.$$

$$2T = \int_{-\pi/2}^{\pi/2} -N_1^{(1)} \cos\varphi_1 \cos\theta (a\,\mathrm{d}\theta) \quad \text{or} \quad T = -N_1^{(1)} a \cos\varphi_1$$

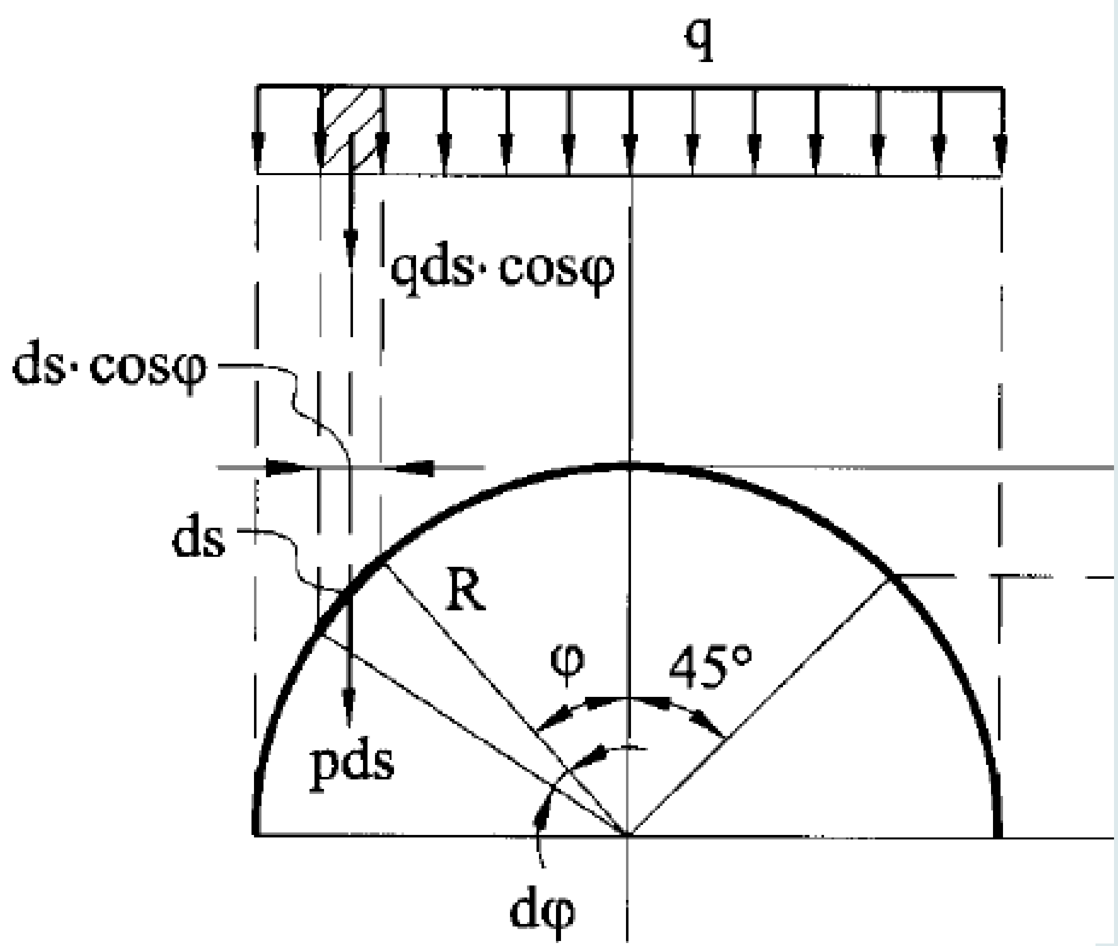
$$T = -N_1^{(1)} R \cos\varphi_1 \sin\varphi_1$$

$$T = pR^2 \frac{\cos\varphi_1 \sin\varphi_1}{1 + \cos\varphi_1}$$

Membrane forces distribution in a spherical dome under self-weight, which has a skylight at its top



(b) Spherical domes: live or snow loads



$$pR\mathrm{d}\varphi = (qR\mathrm{d}\varphi)\cos\varphi \quad \text{or} \quad p = q\cos\varphi$$

$$p_3 = q\cos^2\varphi, \qquad p_1 = q\sin\varphi\cos\varphi$$