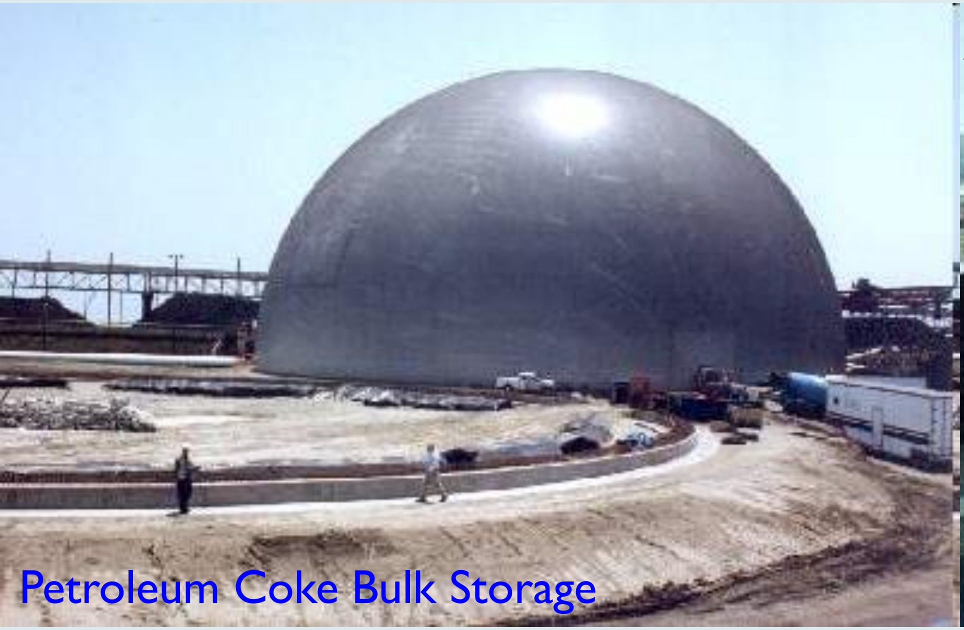
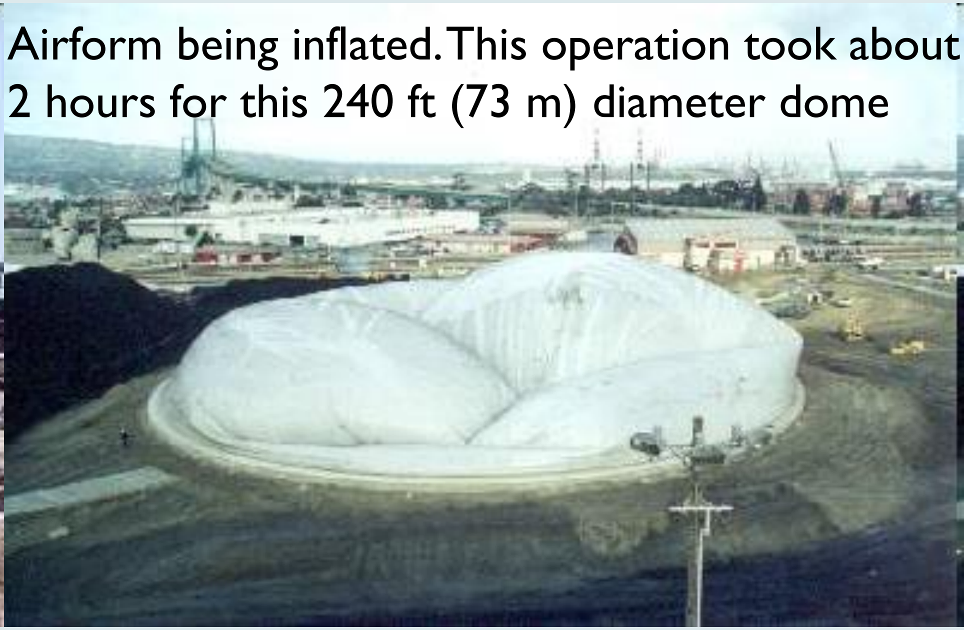
The Paraboloid Dome under Internal Pressure

Determine the membrane forces in a paraboloid dome under a uniform internal pressure, p per unit area. The dome is presumed to be simply supported, see the Figs., so that membrane theory will be applicable to the complete shell surface.

Uniform internal pressure loads are encountered in practice either when shells are used for the storage or when flexible inflated structures are subjected to their initial prestressing.



Petroleum Coke Bulk Storage

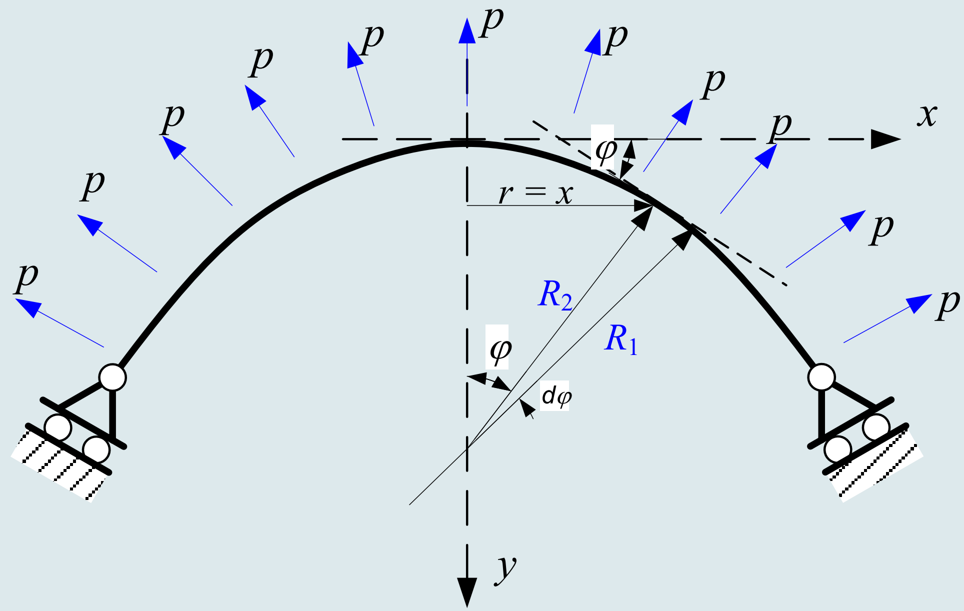


Airform being inflated. This operation took about 2 hours for this 240 ft (73 m) diameter dome






It is assumed that the shell is generated from a parabolic curve of the form: $2ay = x^2$ where a is a constant.



$$y = \frac{a}{2}\tan^2\phi$$

Since it is convenient to work in terms of φ, the Cartesian coordinates, x and y, and the principal radii of curvature R_1 & R_2 must first be expressed in terms of φ.

$$\frac{dy}{dx} = \tan\phi = \frac{1}{2a}2x = \frac{x}{a}$$

$$x = a\tan\phi$$

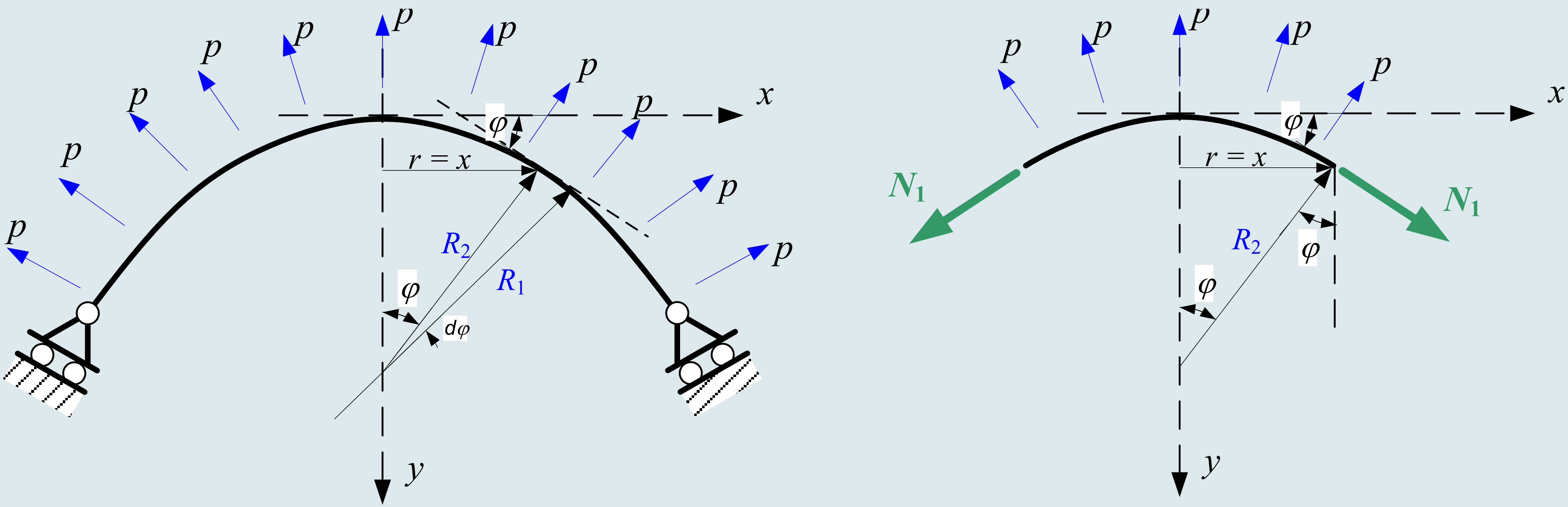
$$r = x = a\tan\phi$$

$$R_2 = \frac{x}{\sin\phi} = \frac{a}{\cos\phi}$$

$$R_1 = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{1}{a}$$

$$R_1 = \frac{(1+\tan^2\phi)^{3/2}}{1/a} = \frac{a}{\cos^3\phi}$$



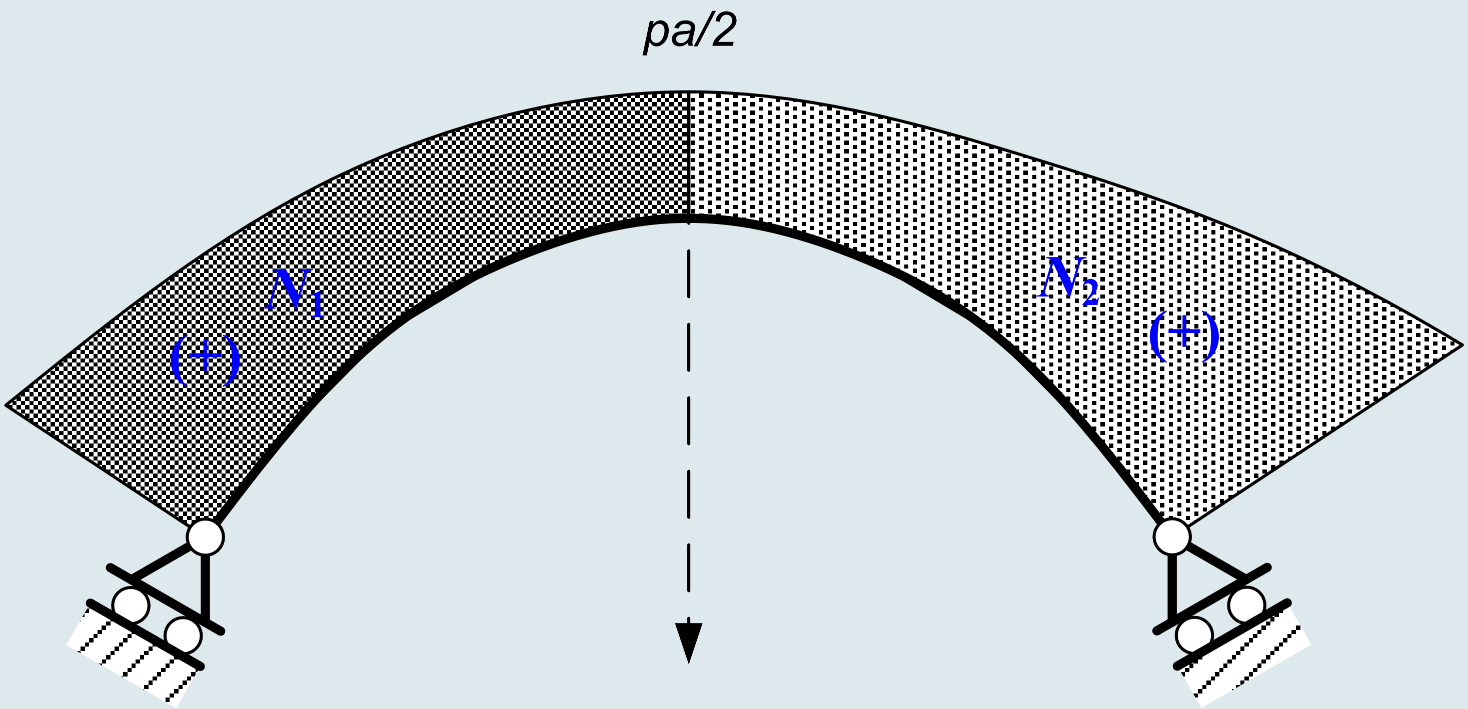
With the above geometric relationships, the membrane forces are determined as previously. By the overall vertical Eq. Eq.:

$$2\pi r \sin\varphi N_1 - p(\pi r^2) = 0 \Rightarrow 2\sin\varphi N_1 - pr = 0 \Rightarrow N_1 = pa/2\cos\varphi$$

Substitution in the normal Eq. Eq.

$$\frac{ap\cos^3\phi}{2\cos\phi a} + \frac{N_2\cos\phi}{a} - p = 0 \quad \Rightarrow \quad N_2 = pa(1+\sin^2\varphi)/2\cos\varphi$$

The entire dome is in tension if: $0 < \varphi < 90^o$

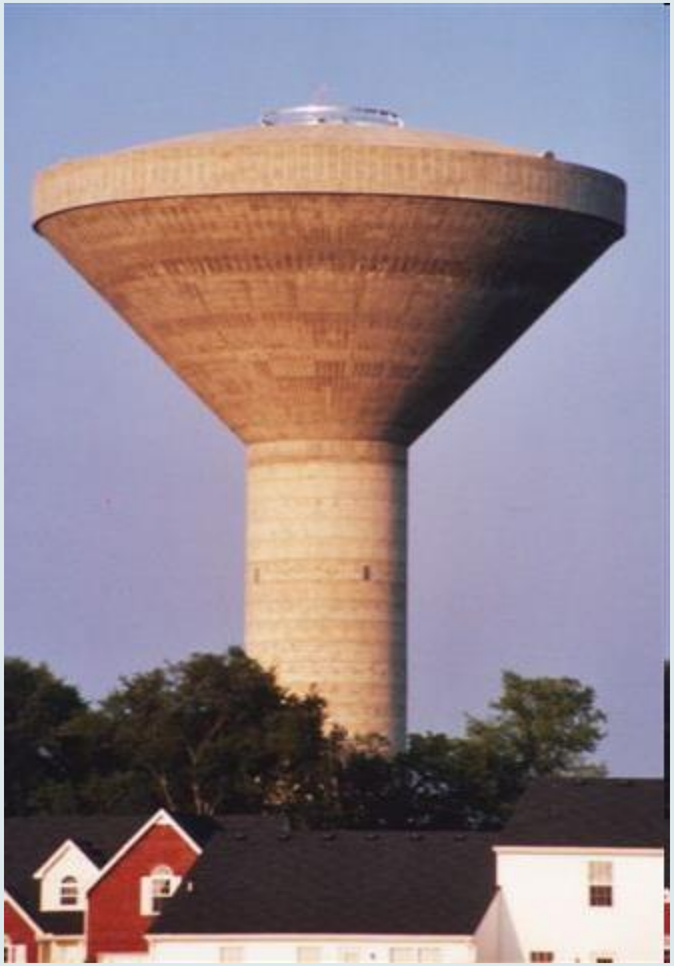


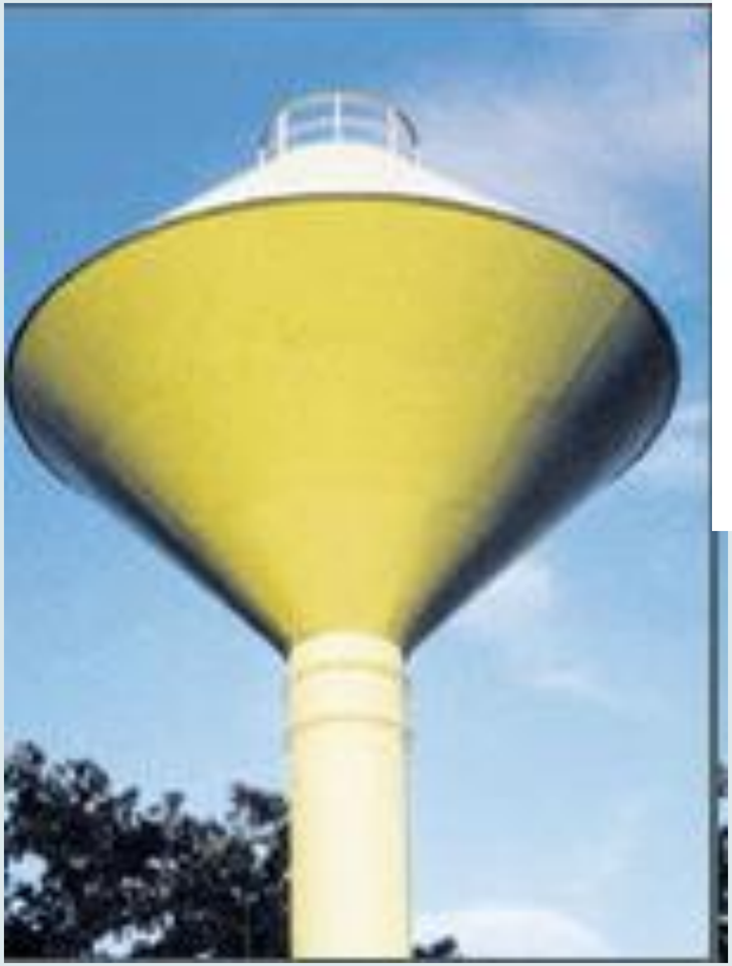
Diagrams of Membrane Forces

$N_1 = pa/2\cos\varphi$

$N_2 = pa\,(1+\sin^2\varphi)\,/\,2\cos\varphi$

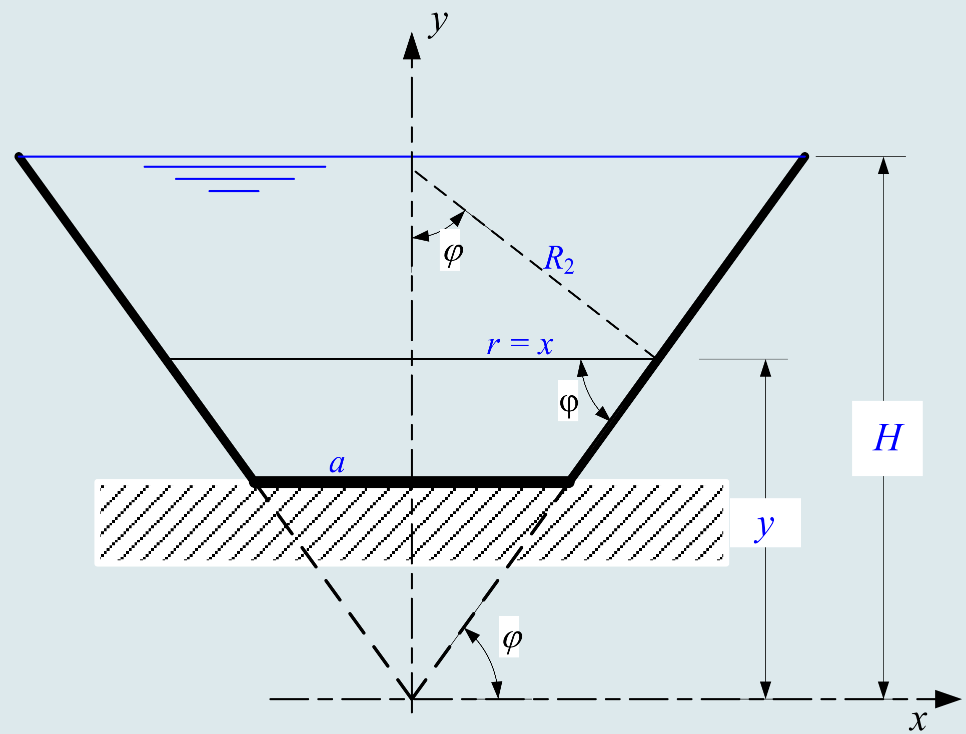
conical shell under fluid pressure





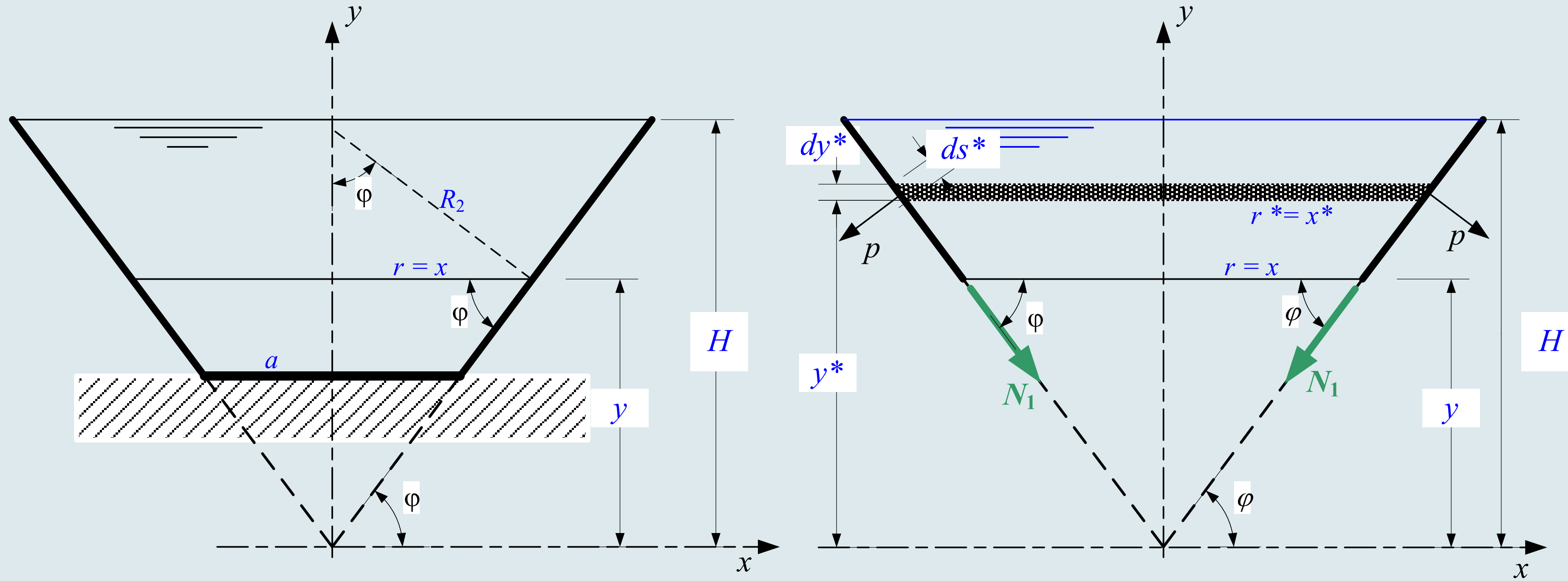
The conical tank shown in the Fig. is presumed to be full of fluid, and it is required to determine the distribution of the membrane forces in the tank walls due to the fluid pressure.

The meridian has (0) curvature, while the radius of the second principal curve is R_2. The radius of the parallel circle is denoted by r.
In this typical case, the angle φ is a constant and can no longer serve as a coordinate on the meridian. Instead, we introduce y the vertical coordinate.



The wall is generated from a straight line given by:

$$x = r,\ \ y = x\tan\varphi,\ \ R_2 = x/\sin\varphi\ \ \&\ \ R_1 \to \infty$$



With the above geometric relationships, the membrane forces are determined as previously. By the overall vertical Eq. Eq.:

$$2\pi r \sin\varphi N_1 = -\int_{y^*=y}^{y^*=H} p\cos\varphi(2\pi x^*)\,ds^*$$

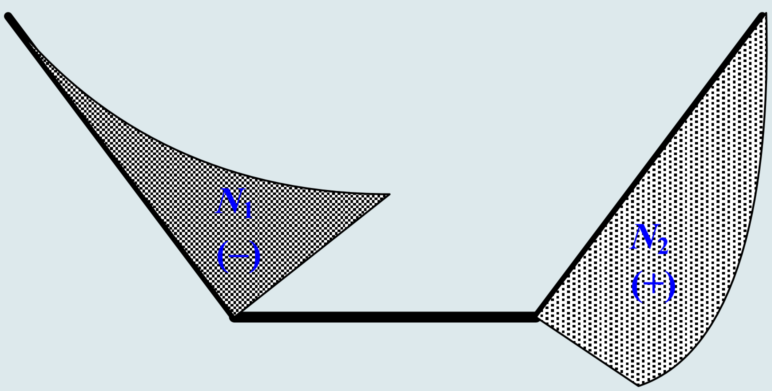
$$N_1 = \frac{-\gamma}{y\sin\varphi\tan\varphi}\left(\frac{H^3}{6} - \frac{Hy^2}{2} + \frac{y^3}{3}\right)$$

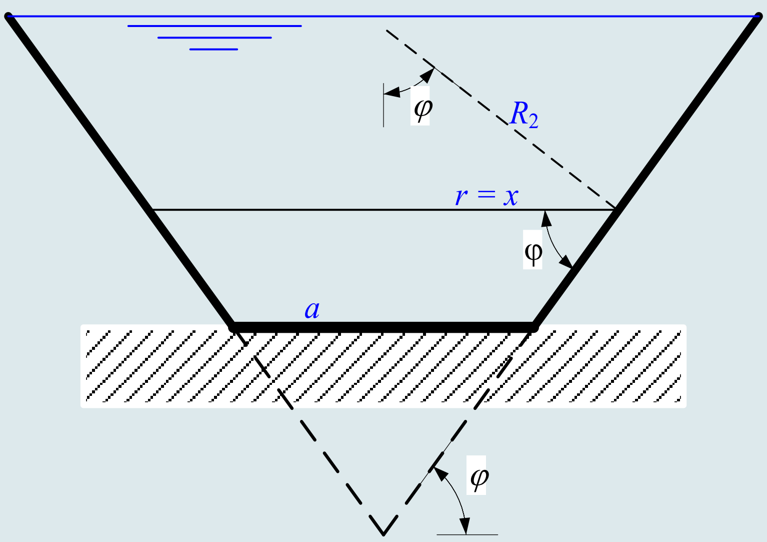
Substitution in the normal Eq. Eq. with $R_1 \to \infty$ & $(N_1/R_1) \to 0$

$$N_2 = R_2 p = \frac{xp}{\sin\varphi} = \frac{yp}{\sin\varphi\tan\varphi} = \frac{\gamma y(H-y)}{\sin\varphi\tan\varphi}$$

Then, the meridional & tangential membrane forces are compressive & tensile respectively throughout the shell.

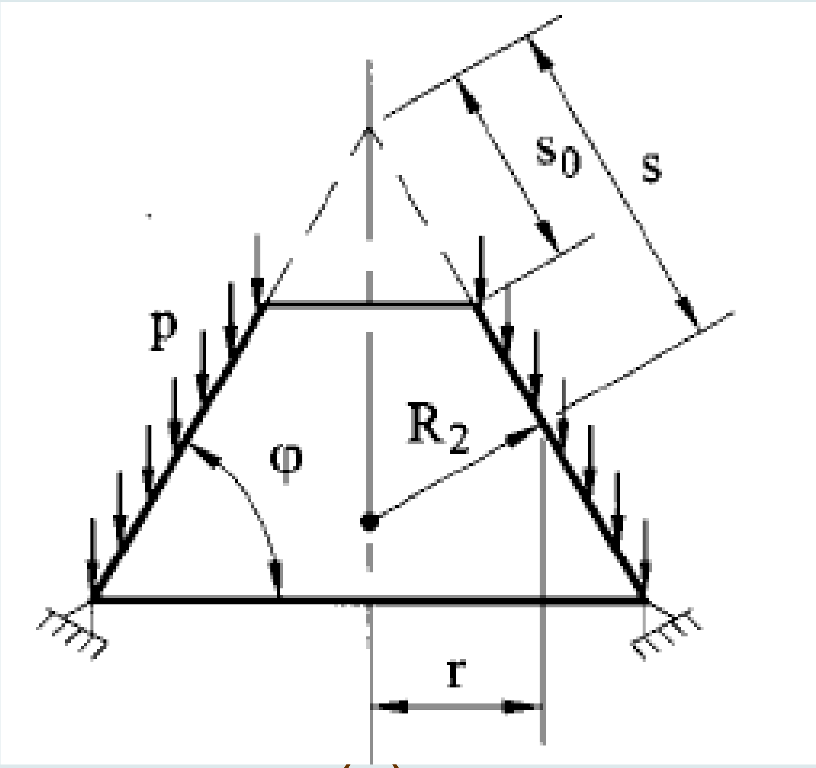
Differentiation of the two expressions further shows that the maximum meridional force occurs at the base of the tank but that the maximum tangential force occurs when: $y = H/2$.



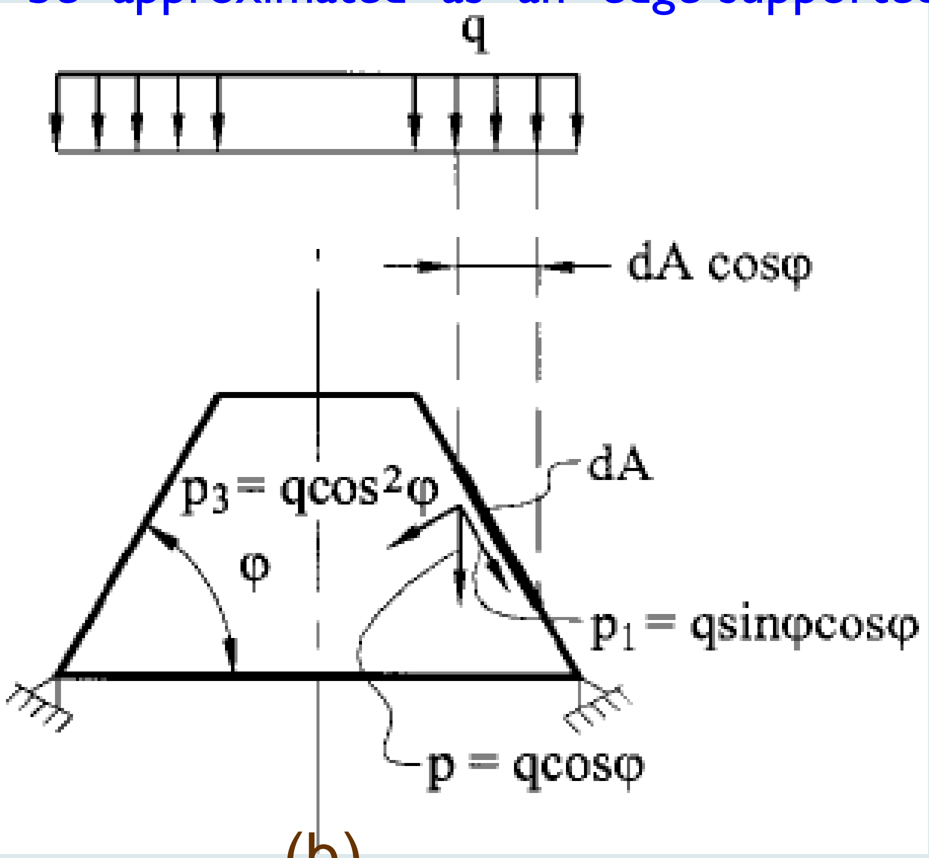


Conic roofs: axisymmetric loading

Consider a planetarium dome that may be approximated as an edge-supported truncated cone.



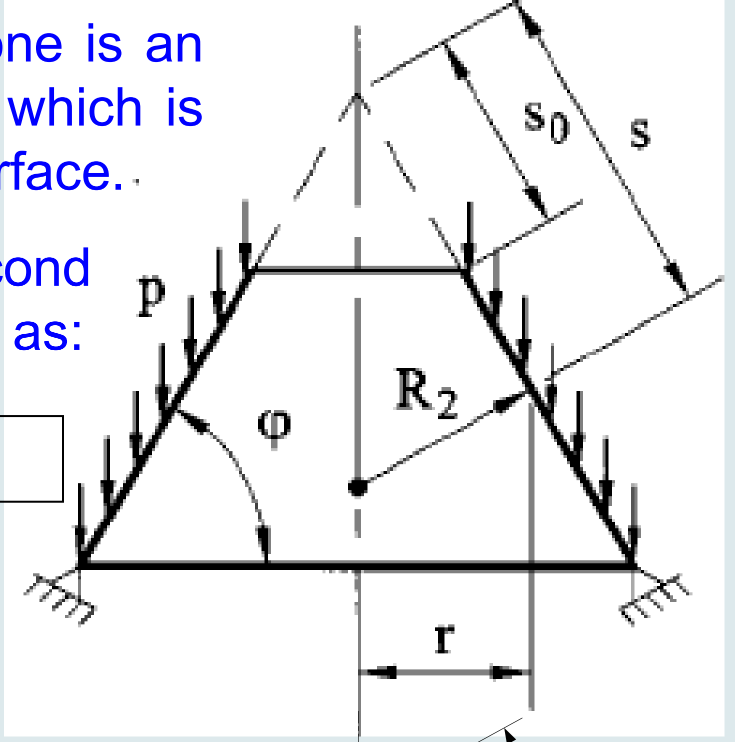
(a)



(b)

We derive expressions for the circumferential and meridional forces for two conditions of loading: (a) self-weight of the shell per unit area of its middle surface (Fig. 14.5a) and (b) snow load uniformly distributed over the shell plan (Fig.14.5b). Both types of loading are axisymmetric.

The inclined abscise s from the vertex of the cone is an alternative coordinate to the meridional angle φ which is constant in the case of the conic axisymmetric surface.



From the figure we can compute R_2, the second radius of curvature and *r*, the parallel circle radius as:

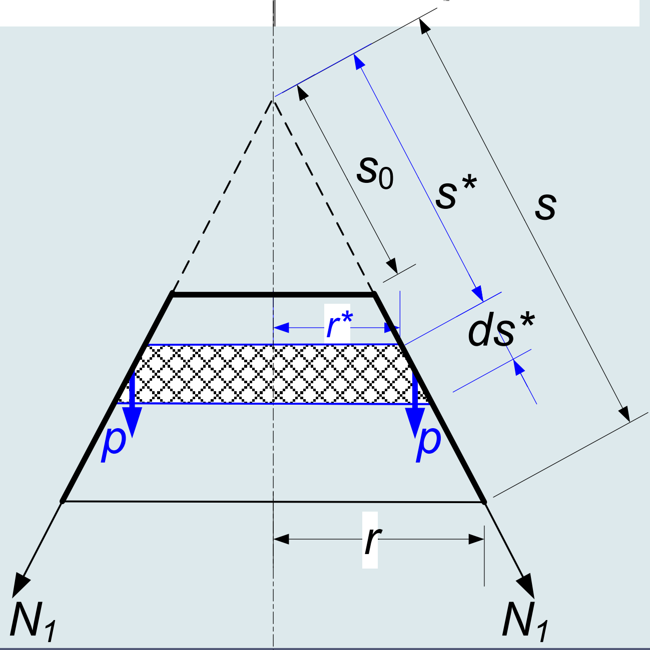
$R_2 = s / \tan\varphi$	$r = R_2 \sin\varphi = s \sin\varphi / \tan\varphi = s \cos\varphi$

With the above geometric relationships, the membrane forces are determined as previously.
By the overall vertical Eq. Eq.:

$$(2\pi r \sin\varphi) N_1 = -\int_{s_0}^{s} p(2\pi r^*) ds^*$$

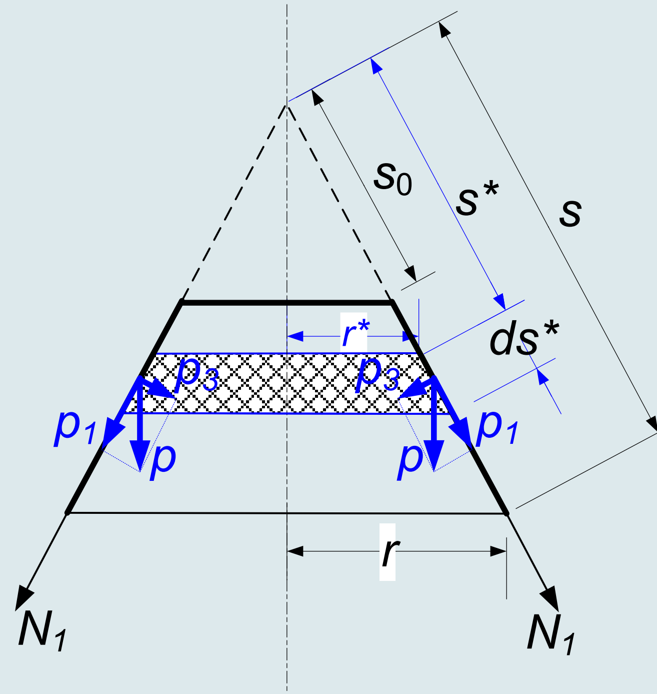
$$(2\pi r \sin\varphi) N_1 = -\int_{s_0}^{s} p(2\pi s^* \cos\varphi) ds^*$$

$$N_1 = -\frac{p}{2\sin\varphi}\left(\frac{s^2 - s_0^2}{s}\right)$$



Substitution in the normal Eq. Eq. with

$R_1 \rightarrow \infty$ & $(N_1/R_1) \rightarrow 0$



$$N_2 = R_2 p_3 = -p \frac{\cos^2 \varphi}{\sin \varphi} s$$

The entire conic roof is in compression

$$N_1 = -\frac{q}{2\tan\varphi}\left(\frac{s^2 - s_0^2}{s}\right)$$

$$N_2 = R_2 p_3 = -q\frac{\cos^3\varphi}{\sin\varphi}s$$