Introduction to the General Shell Theory

1. SHELLS IN ENGINEERING STRUCTURES

Examples of shell structures in civil and architectural engineering are large-span roofs, liquid-retaining structures and water tanks, containment and cooling shells of nuclear power plants, concrete arch dams and concrete domes.









Concrete and metallic Silos

In mechanical engineering, shell forms are used in piping systems, turbine disks & pressure vessels technology. Aircrafts, missiles, rockets, ships, & submarines are examples of the use of shells in aeronautical & marine engineering.

Another application of shell engineering is in the field of biomechanics: shells are found in various biological forms, such as the skull, & plant & animal shapes. This is only a small list of shell forms in engineering and nature.

The wide application of shells is due to the following advantages:

- *Efficiency of load-carrying behavior.*
- *High degree of reserved strength and structural integrity.*
- *High strength / weight ratio.*

This criterion is commonly used to estimate a structural component efficiency: the larger this ratio, the more optimal is a structure. According to this criterion, shell structures are much superior to other structural systems having the same span and overall dimensions.

- *Very high stiffness.*
- *Containment of space.*

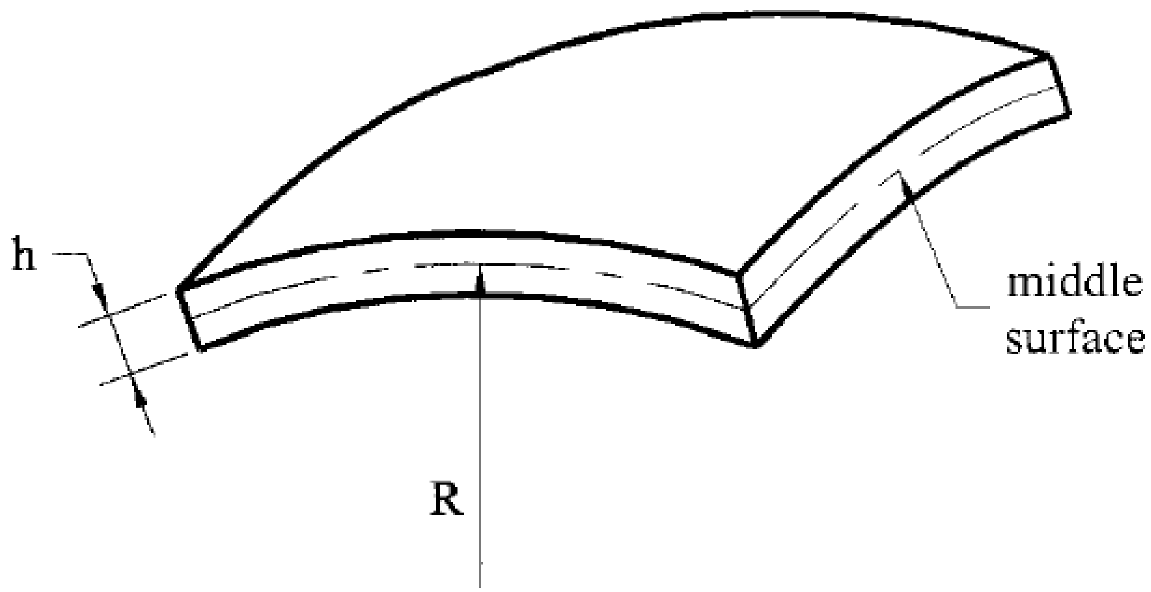
In addition to these mechanical advantages, shell structures enjoy the unique position of having extremely high aesthetic value in various architectural design

Contemporary engineers using scientifically justified methods of design tend to develop a structure that combines maximum strength, functional perfection, and economy during its lifetime. In addition, it is important that the best engineering solution ensues, other things being equal, at the expense of the selection of structural form and by not increasing the strength properties of the structure, e.g., by increasing its cross section. Note that the latter approach is easier.

Shell structures support applied external forces efficiently by virtue of their geometrical form, i.e., spatial curvatures; as a result, shells are much stronger and stiffer than other structural forms.

2. GENERAL DEFINITIONS & FUNDAMENTALS OF SHELLS
Definitions and principles in shell theory.

The term shell is applied to bodies bounded by two curved surfaces, where the distance between the surfaces is small in comparison with other dimensions



The locus of points that lie at equal distances from these two curved surfaces defines the middle surface of the shell. The length of the segment, which is perpendicular to the curved surfaces, is called the thickness of the shell and is denoted by h. The geometry of a shell is entirely defined by specifying the form of the middle surface and thickness of the shell at each point. In this book we consider mainly shells of a constant thickness.

Shells have all the characteristics of plates, with an additional one curvature.

The curvature could be chosen as the primary classifier of a shell because a shell’s behavior under an applied loading is primarily governed by curvature.

Depending on the curvature of the surface, shells are divided into cylindrical (noncircular and circular), conical, spherical, ellipsoidal, paraboloidal, toroidal, and hyperbolic paraboloidal shells.

Owing to the curvature of the surface, shells are more complicated than flat plates because their bending cannot, in general, be separated from their stretching. On the other hand, a plate may be considered as a special limiting case of a shell that has no curvature; consequently, shells are sometimes referred to as curved plates. This is the basis for the adoption of methods from the theory of plates, discussed in the first part of this course.

There are two different classes of shells: thick shells and thin shells. A shell is called thin if the maximum value of the ratio h/R (where R is the radius of curvature of the middle surface) can be neglected in comparison with unity. For an engineering accuracy, a shell may be regarded as thin if

$$\max\left(\frac{h}{R}\right) \leq \frac{1}{20}.$$

This inequality defines very roughly the boundary between thin and thick shells. In reality, it depends also upon other geometrical parameters of shells, the character of their boundary conditions, smoothness of a variation of external loads over the shell surface, etc.

Shells for which this inequality is violated are referred to as thick shells. For a large number of practical applications, the thickness of shells lies in the range

$$\frac{1}{1000} \leq \frac{h}{R} \leq \frac{1}{20}$$

3. BRIEF OUTLINE OF THE LINEAR SHELL THEORIES

The most common shell theories are those based on linear elasticity concepts. Linear shell theories predict adequately stresses and deformations for shells exhibiting small elastic deformations; i.e., deformations for which it is assumed that the equilibrium equation conditions for deformed shell surfaces are the same as if they were not deformed, and Hooke's law applies.

Shell theories of varying degrees of accuracy were derived, depending on the degree to which the elasticity equations were simplified. The approximations necessary for the development of an adequate theory of shells have been the subject of considerable discussions among investigators in the field. We present below a brief outline of elastic shell theories in an historical context.

4. LOADING-CARRYING MECHANISM OF SHELLS

The general theories of beams, arches, plates, and shells are usually based upon the unified set of assumptions. However, the load–resistance mechanisms of these members do not resemble one another. The load–resistance mechanism of flat plates was discussed in previous lectures. We can say that shells fall into a class of plates as arches relate to straight beams under the action of transverse loading. It is known that the efficiency of the arch form lies primarily in resisting the transverse load with a thrust N, thus minimizing the shear force V and bending moment M. It is possible to specify the arch shape and the manner of its loading in such a way that the arch does not experience bending at all. In this case, the arch is in the so-called momentless state of stress. For example, for a parabolic arch, bending will not be induced by a vertical load uniformly distributed over its chord. Thus, the ability of arches to support certain transverse loads without bending is the reason for their structural advantage over straight beams.

A shell mainly balances an applied transverse load, much like an arch, by means of tensile and compressive stresses, referred to as the membrane or direct stresses. These stresses are uniformly distributed over the shell thickness. Such a state of stress is called the momentless or membrane state of stress. Although the shear force and bending and twisting moments are still present in the general case of loading, the efficiency of the shell form rests with the presence of the membrane stresses, as the primary means of resistance with the bending stress resultants and couples are minimized. Thus, shells, like arches over beams, possess an analogous advantage over plates; however, with the following essential difference – while an arch of a given form will support only one completely determined load without bending, a shell of a given shape has, provided its edges are suitably supported, as a rule, the same property for a wide range of loads which satisfy only very general requirements.

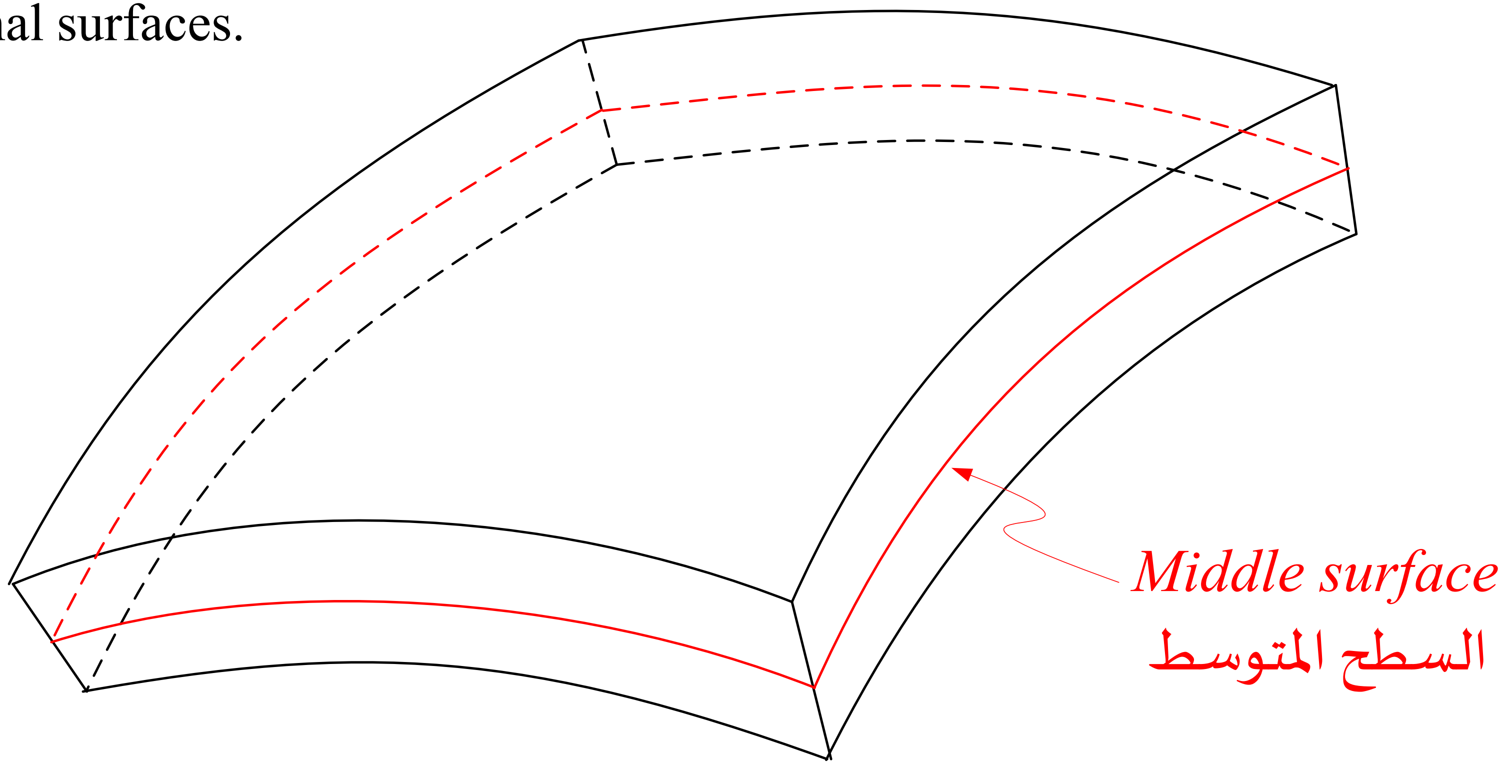
The membrane stress condition is an ideal state at which a designer should aim. It should be noted that structural materials are generally far more efficient in an extensional rather in a flexural mode because:

1. Strength properties of all materials can be used completely in tension (or compression), since all fibers over the cross section are equally strained and load-carrying capacity may simultaneously reach the limit for the whole section of the component.
2. The membrane stresses are always less than the corresponding bending stresses for thin shells under the same loading conditions.

Thus, the momentless or membrane stress conditions determine the basic advantages of shells compared with beams, plates, etc.

Thin shells. Introduction. Generation.

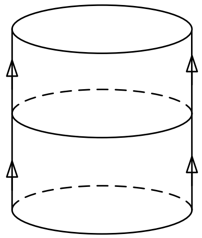
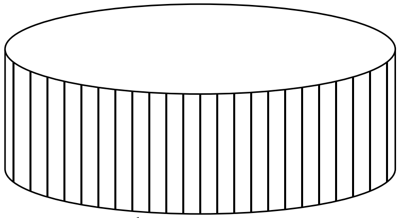
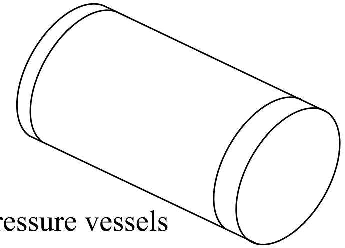
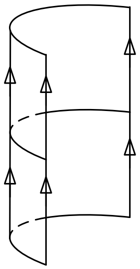
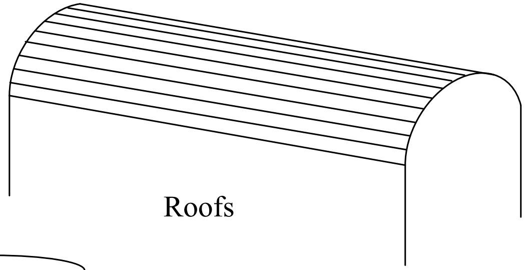
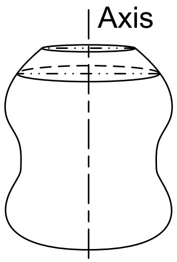
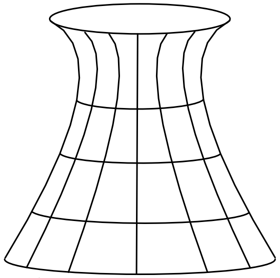
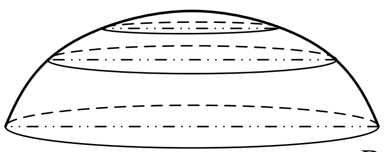
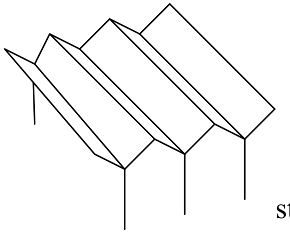
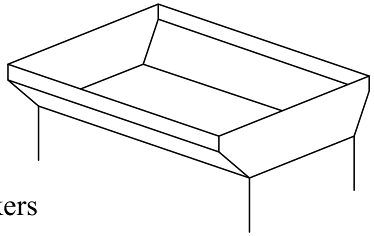
A thin shell is defined as the material contained between two closely separated 3-dimensional surfaces.



mid-thickness points form the middle surface

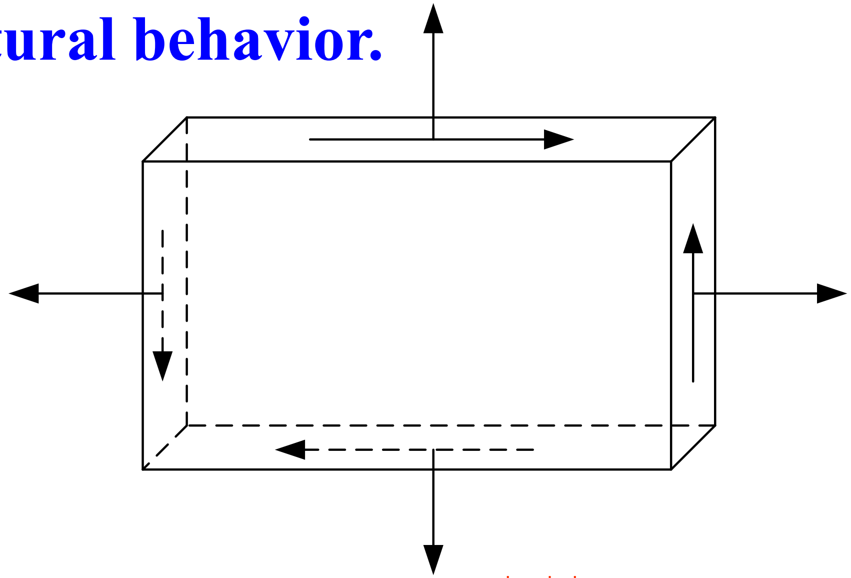
Only the behavior of the middle surface is considered in the following, because with reasonable assumptions this allows to determine the behavior of any point in the shell.

Thin shells. Introduction. Classification and application.

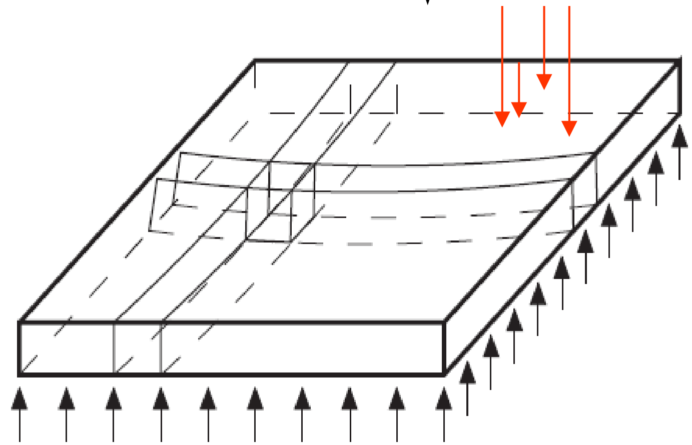
Type	Mode of Generation		Examples
Closed Cylindrical	Translation of a closed curve in its normal direction		 Water tanks  Pressure vessels
Open Cylindrical	Translation of a open curve in its normal direction		 Roofs
Axisymmetric (shell of rotation)	Rotation of a plane curve about an axis in its own plane		 Cooling towers  Domes
Folded plates	Interconnection of non-co-planer plates along their edges		  Roofs & storage tankers

Thin shells. Introduction. Structural behavior.

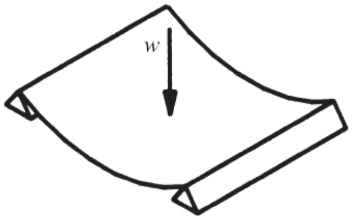
In general, shells resist loads by a combination of bending and in-plane actions. In the case of shells, in-plane action is characterized by the plane stress system of direct and shear stresses and is normally referred to as membrane action (sails, tents, balloons and inflated structures).

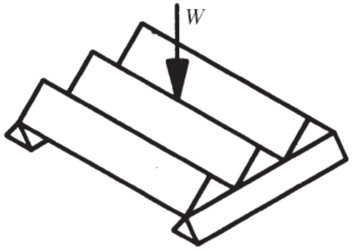


Plates represent a special case of shell and may be considered to be the antithesis of membranes in the sense that, when normally loaded, no membrane stresses exist and resistance is provided by bending alone.

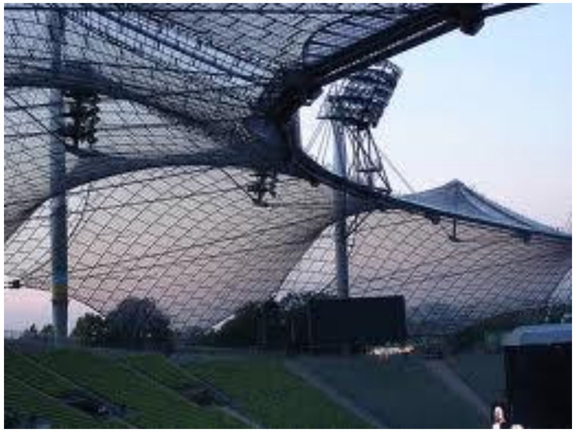


Membrane resistance may be given to a thin plate by folding it, and the effect of the folding is to dramatically increase the stiffness.









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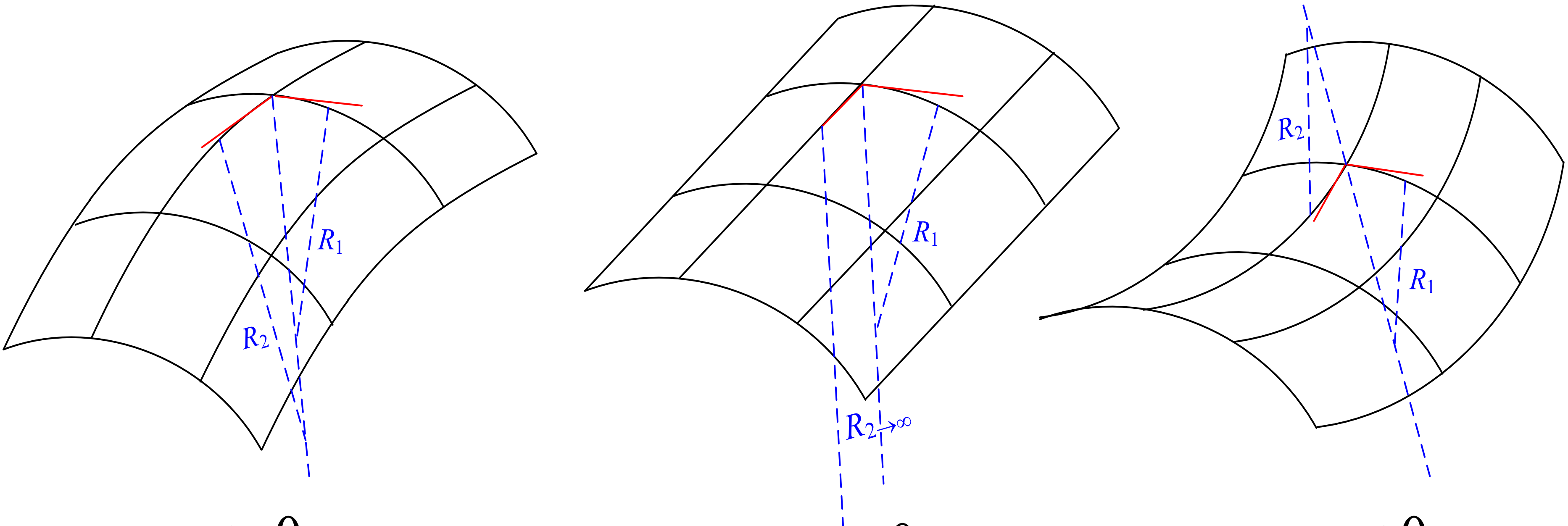
Membrane Theory for Shells

Coordinate System & Gaussian Curvature



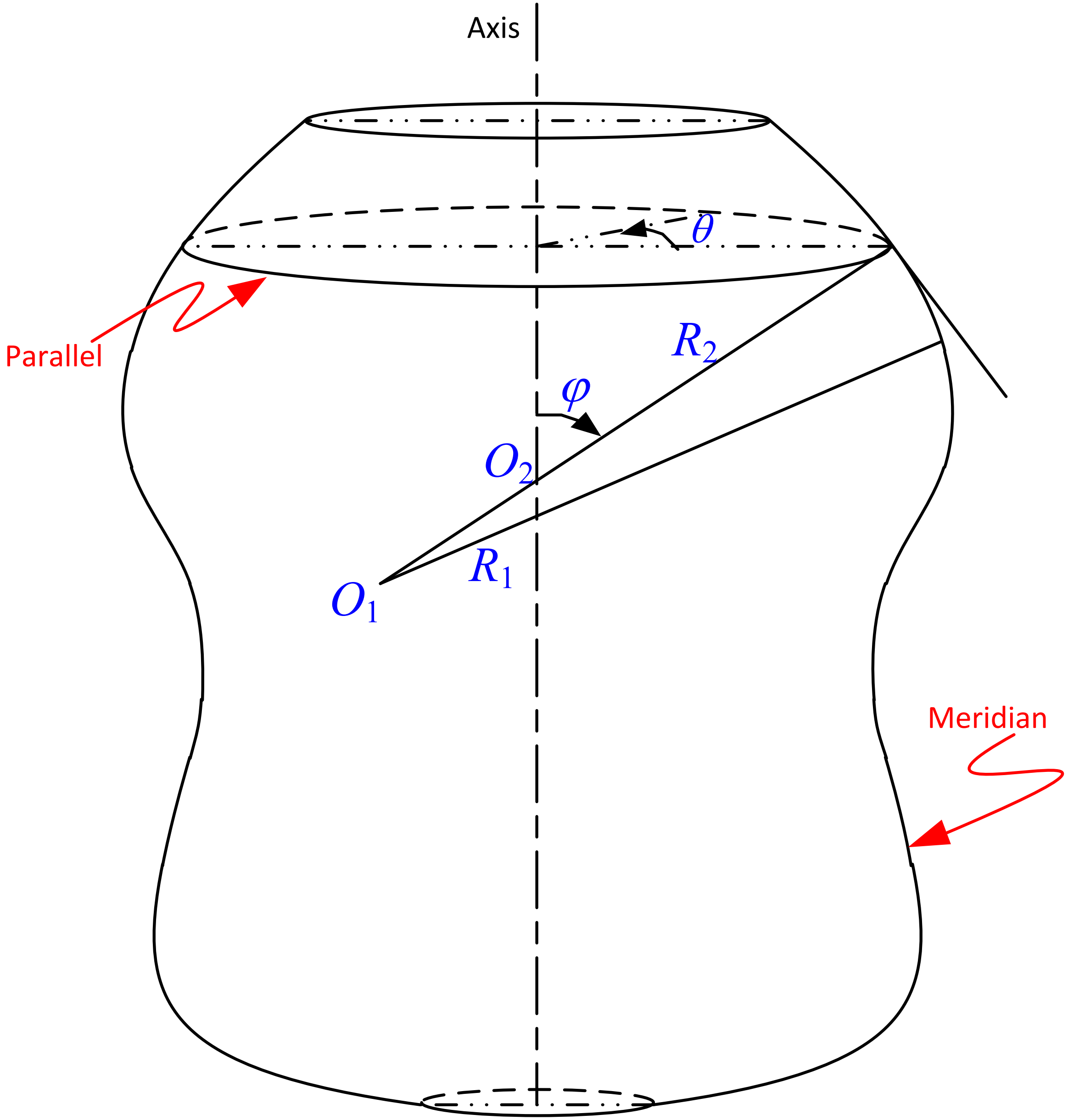
$\kappa_G > 0$ $\kappa_G = 0$ $\kappa_G < 0$

$$\kappa_G = \chi_1 \chi_2 = (1/R_1)(1/R_2)$$

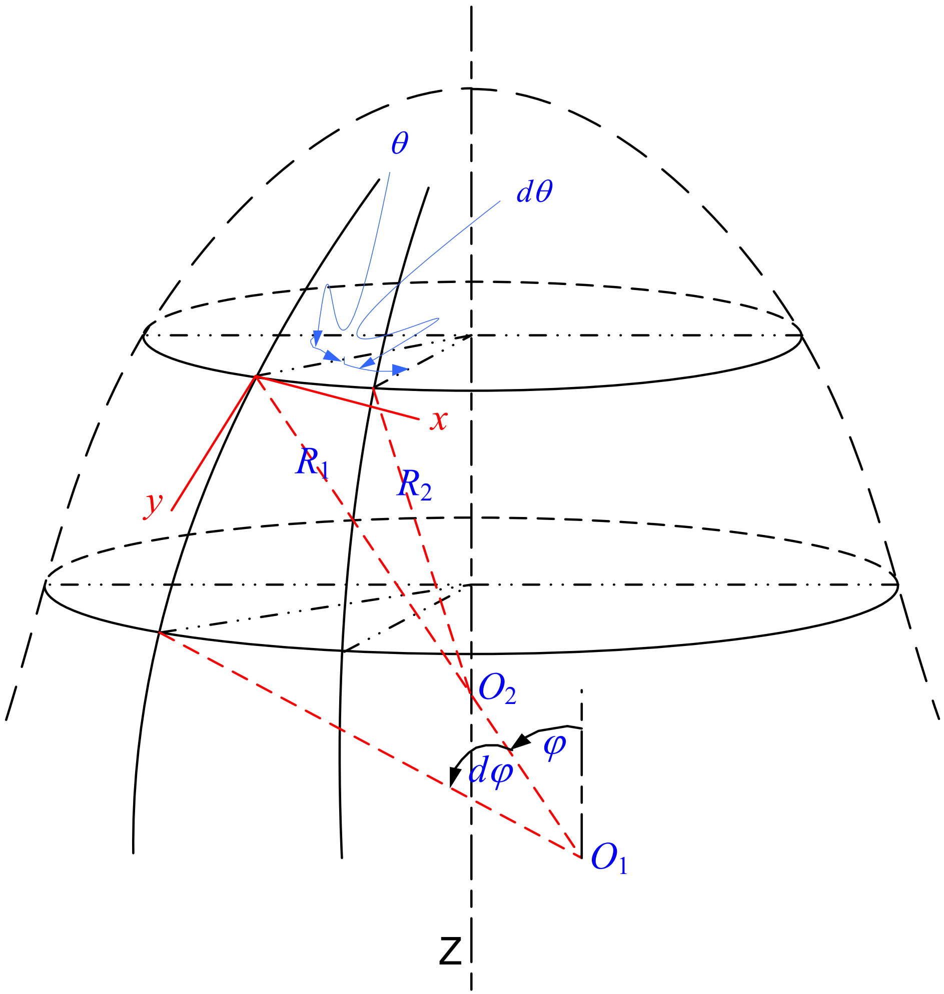
Good membrane behavior & less sensitive to bending

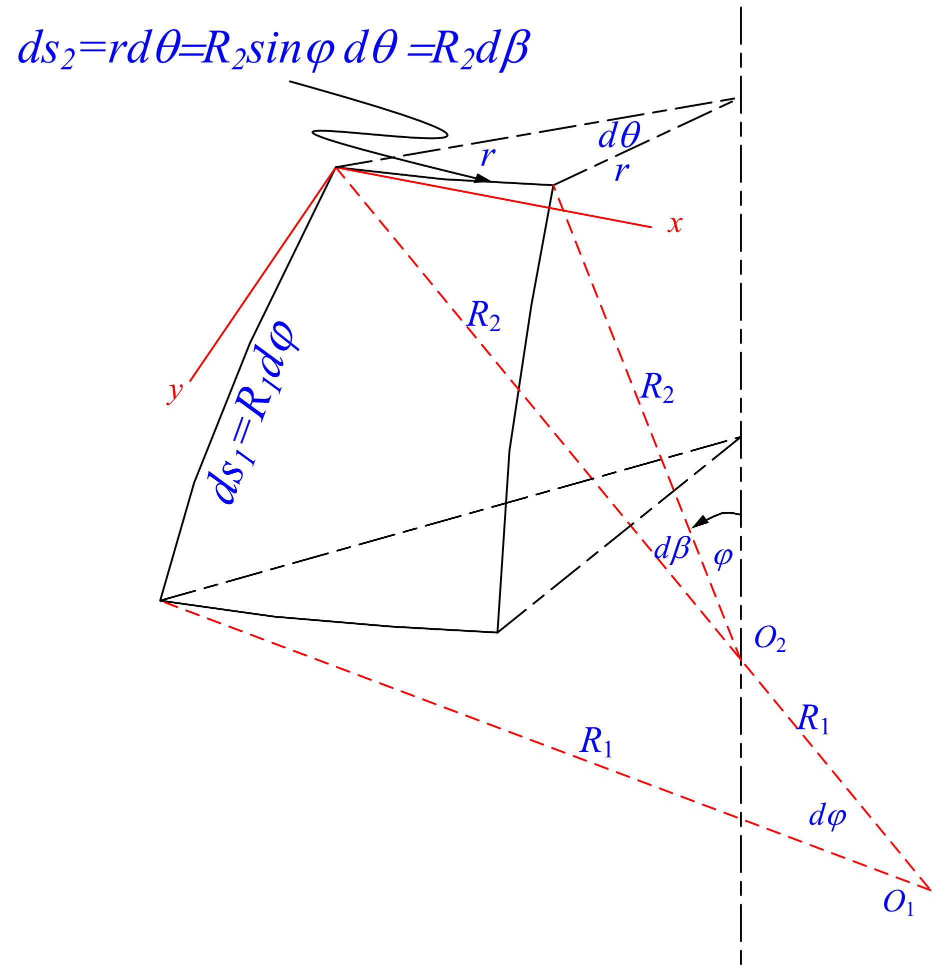
More sensitive to bending

Membrane Theory for Shells of Revolution

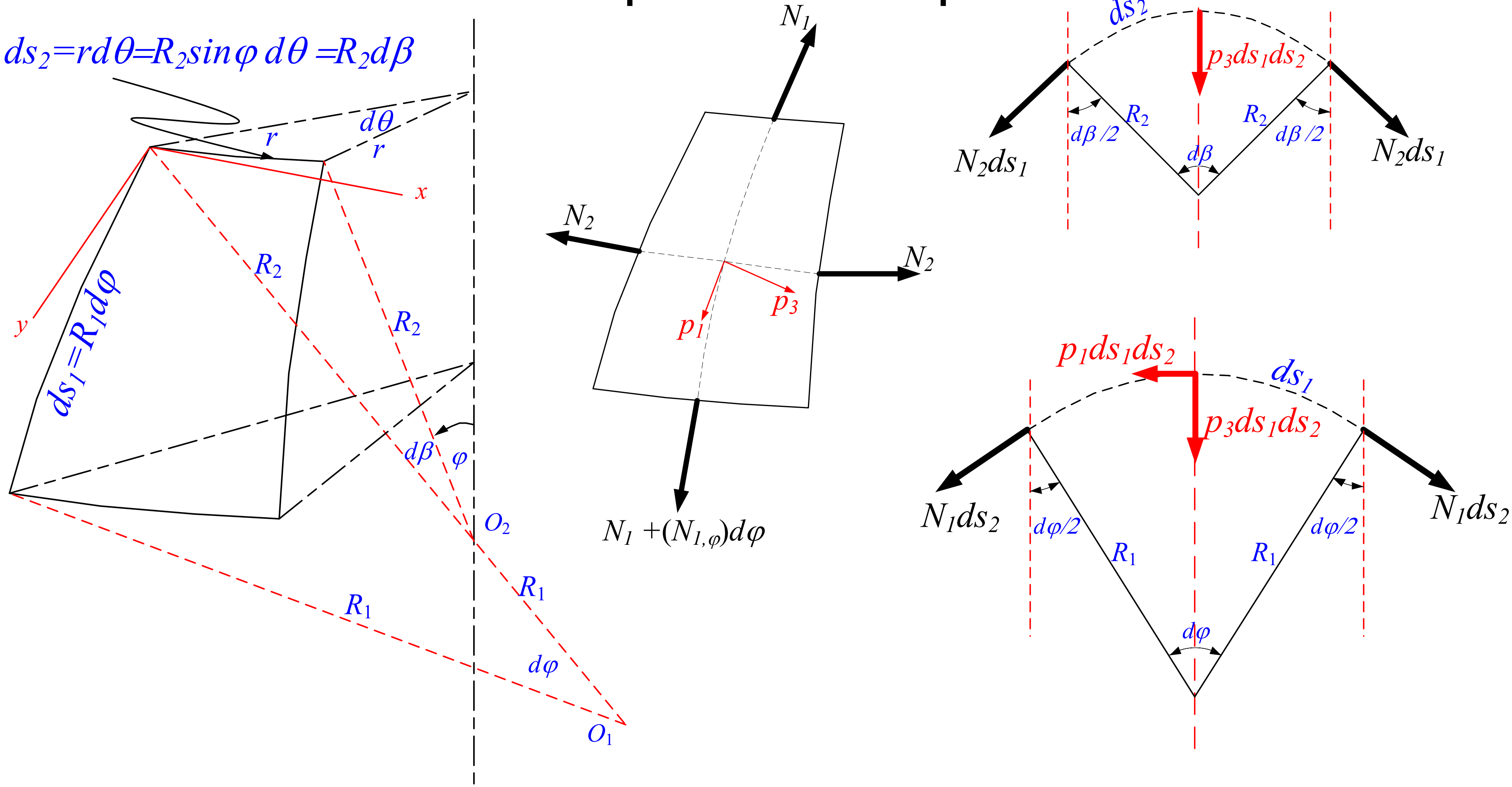


INTERNAL FORCES IN SYMMETRICALLY LOADED SHELLS OF REVOLUTION





Normal & Meridional Equilibrium Eqs.



N_1 *the meridional force*

N_2 *the tangential force*

$$\frac{N_1}{R_1}+\frac{N_2}{R_2}+p_3=0$$

Normal Eq. Eq.

$$\frac{1}{R_1}N_{1,\varphi}+\frac{N_1-N_2}{R_2}\cot\varphi+p_1=0$$

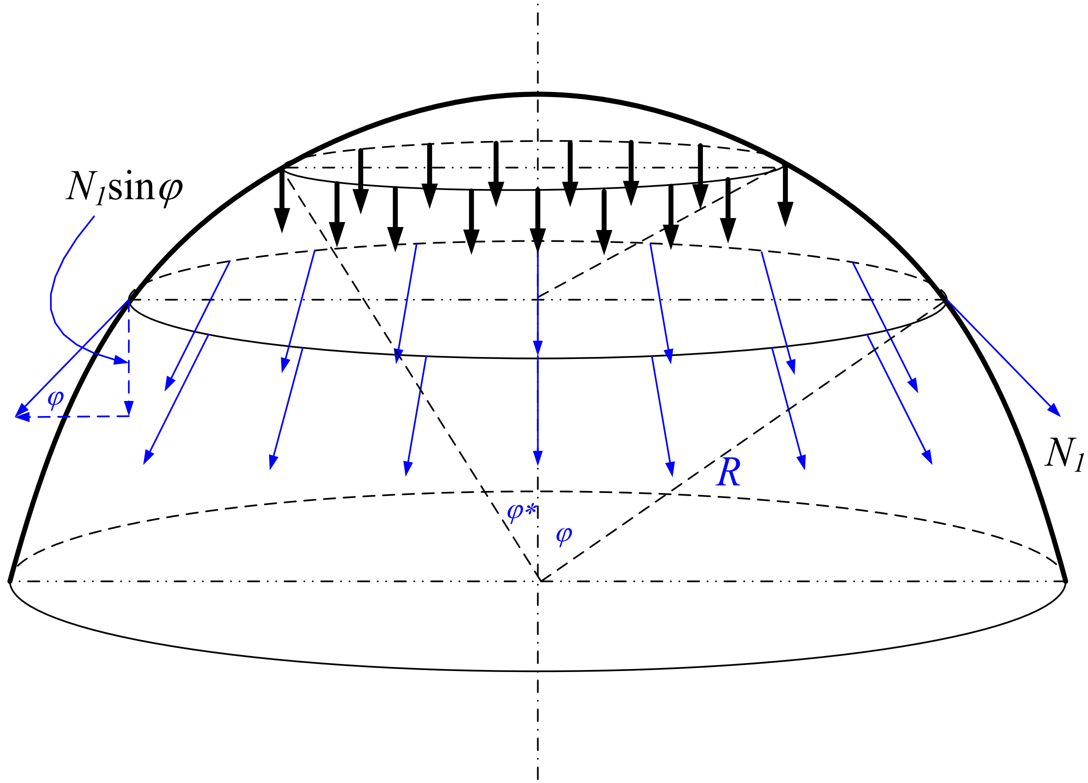
Meridional Eq. Eq. not used

Application of the Membrane Theory to the Analysis of Shell Structures

1. Axisymmetrically loaded dome roofs

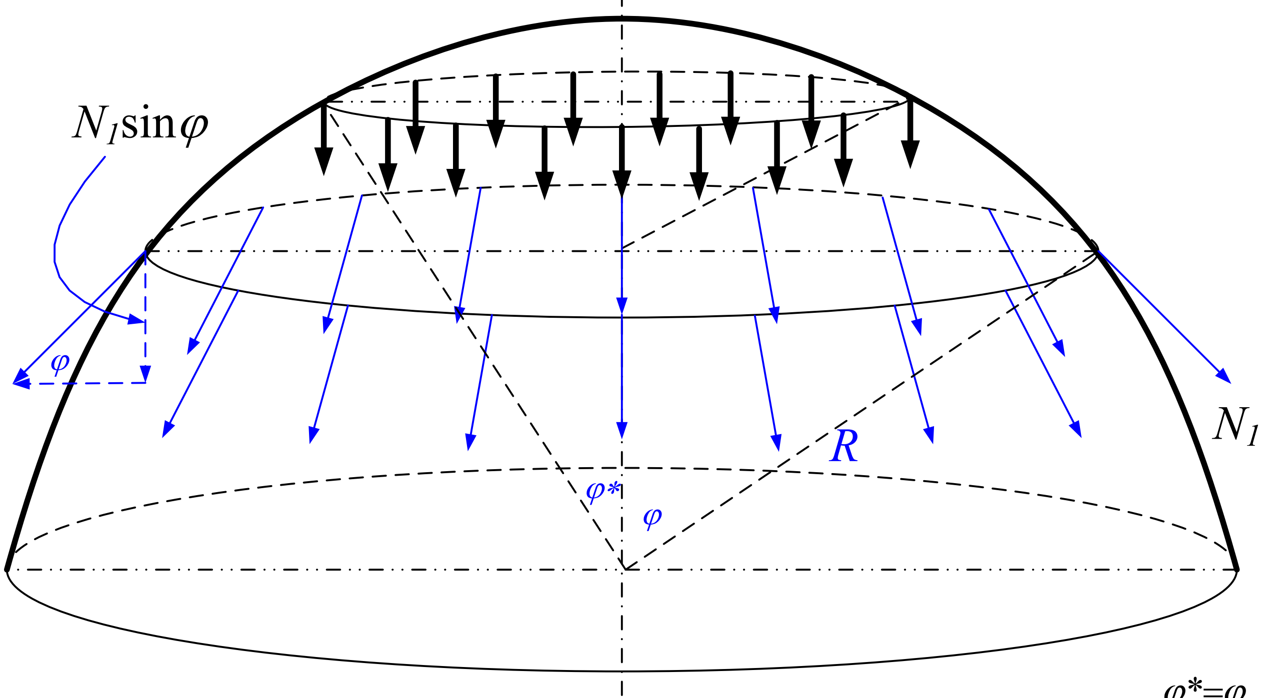
(a) Spherical domes under self-weight

First, we consider a dome with radius *R* closed at its apex and subjected to a dead load of *p* per unit area of the middle surface (e.g., own weight, weight of cladding, etc.).



Resolve the DL *p*, into $p_3 = p\cos\varphi$ & $p_1 = p\sin\varphi$

From geometry: $R_1 = R_2 = R$



Normal Eq. Eq.

$$\frac{N_1}{R_1} + \frac{N_2}{R_2} + p_3 = 0$$

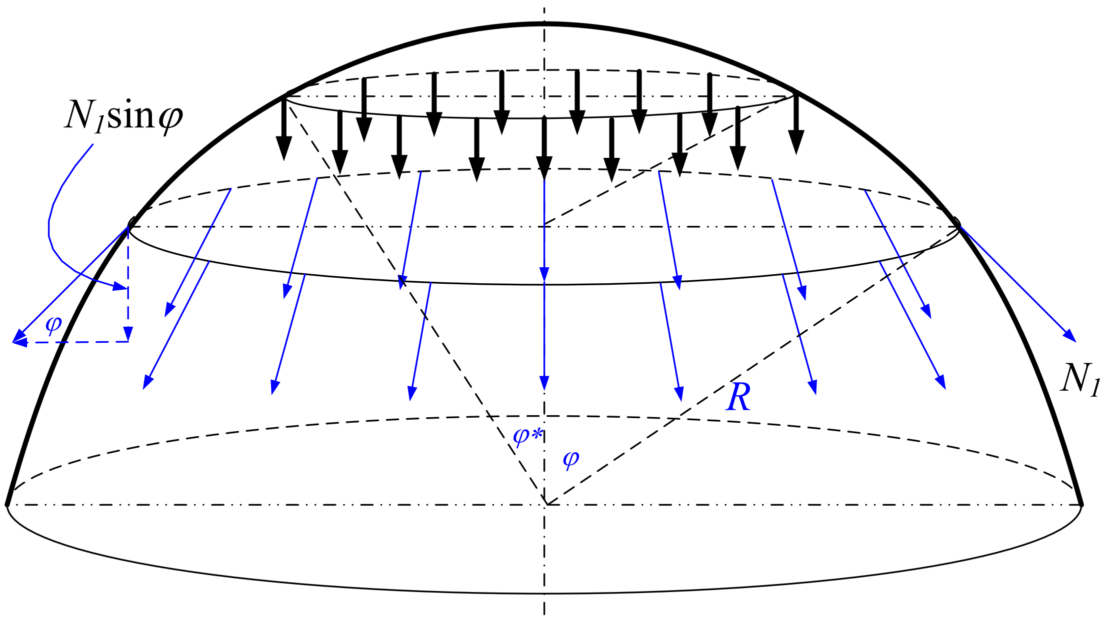
Overall Vertical Eq. Eq.: $N_1 \sin\varphi (2\pi R \sin\varphi) + \int_{\varphi^*=0}^{\varphi^*=\varphi} p(2\pi R \sin\varphi^*) R d\varphi^* = 0$

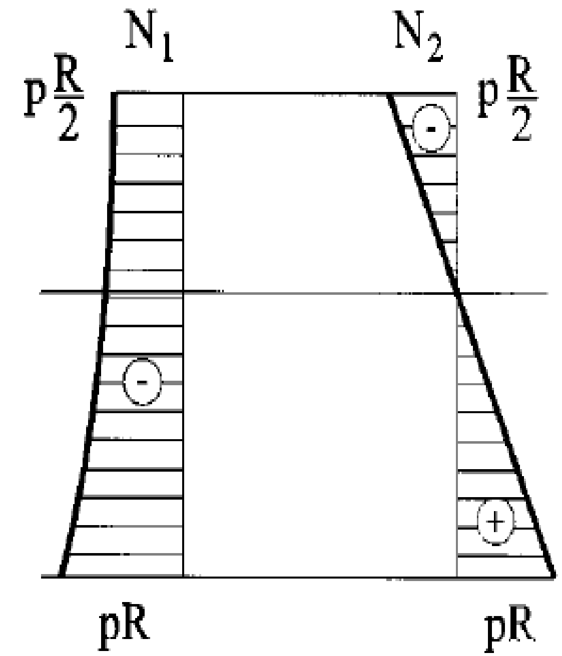
$$N_1 = -\frac{pR}{\sin^2\varphi} \int_{\varphi^*=0}^{\varphi^*=\varphi} \sin\varphi^* d\varphi^* = -\frac{pR(1-\cos\varphi)}{\sin^2\varphi} = -\frac{pR}{1+\cos\varphi}$$

Substituting in the normal Eq. Eq.

$$N_2 = -pR\cos\varphi + \frac{pR}{1+\cos\varphi} = -pR(\cos\varphi - \frac{1}{1+\cos\varphi})$$

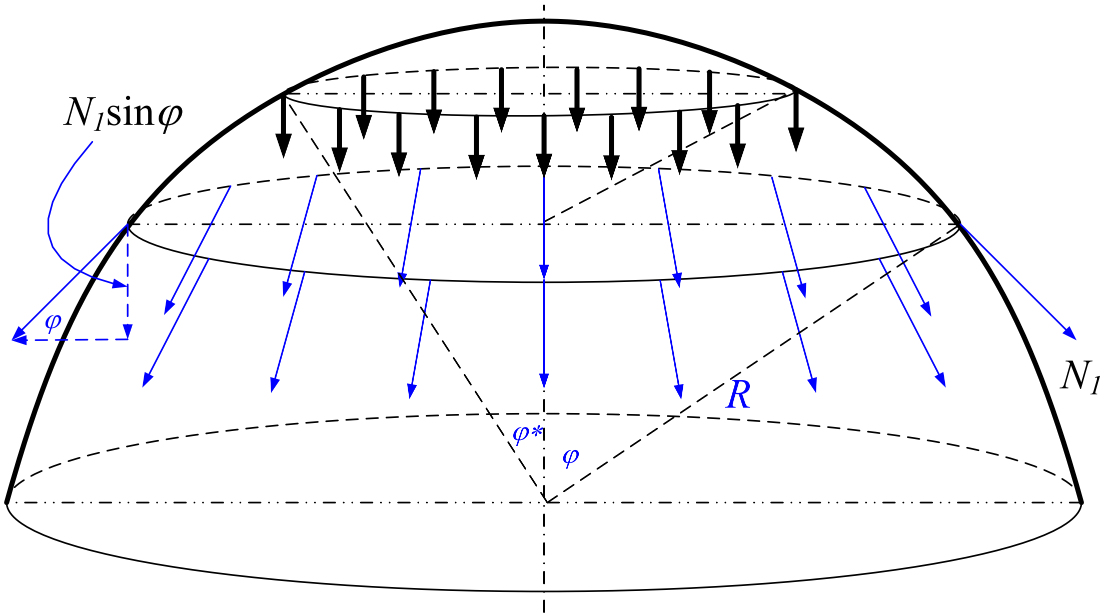
At the crown of the dome, where $\varphi = 0$, we have : $N_1 = N_2 = -pR/2$

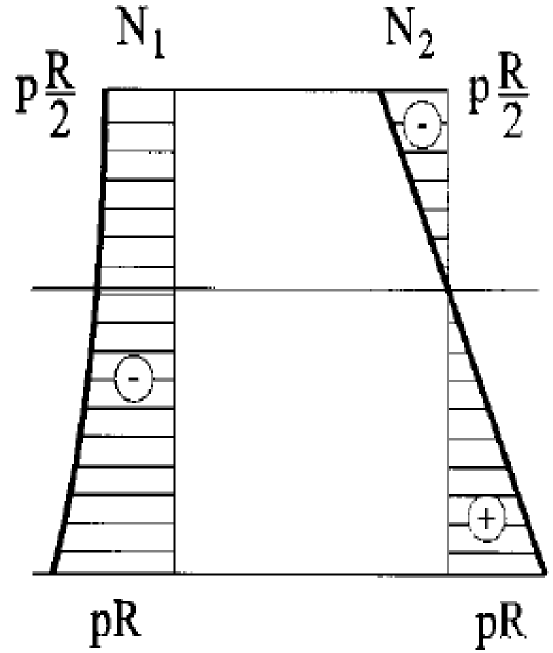




The meridional force, N_1, is compressive along the meridian of the dome, increasing from the apex to the bottom of the dome.

The circumferential compressive force N_2, near the crown, decreases gradually with φ and changes sign. At $\varphi = 90$, $N_1 = -pR$ & $N_2 = pR$



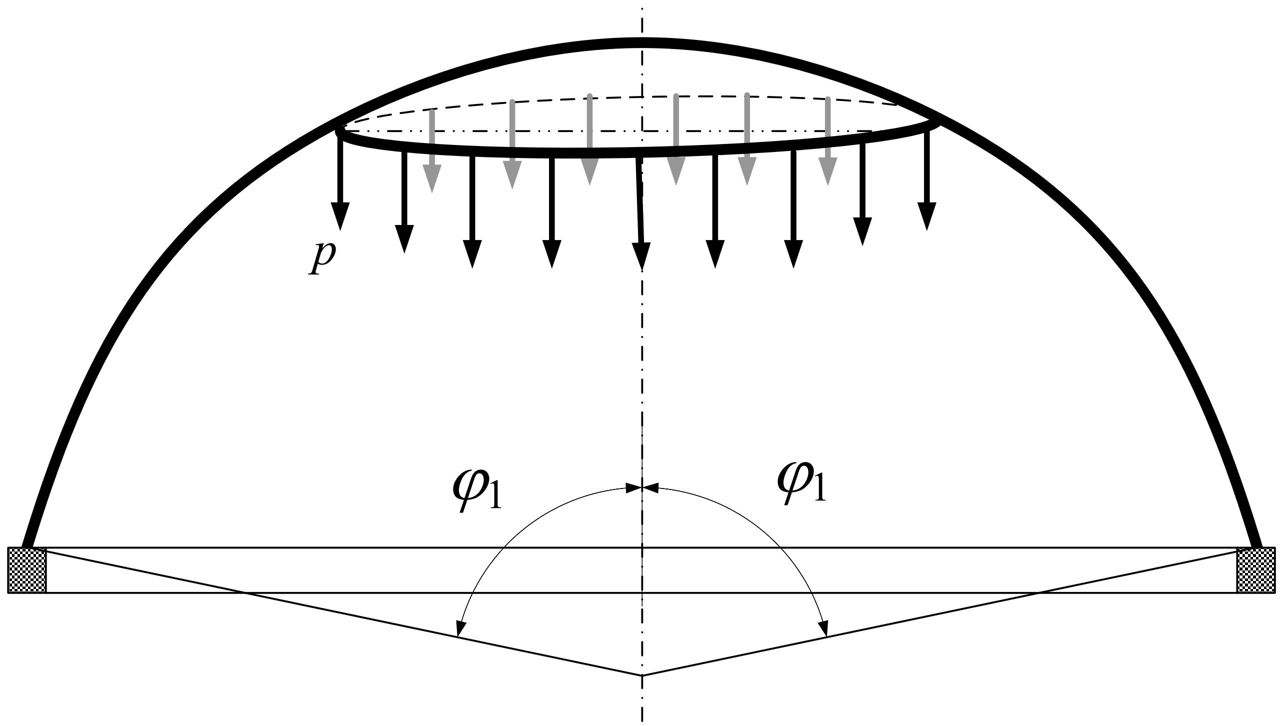


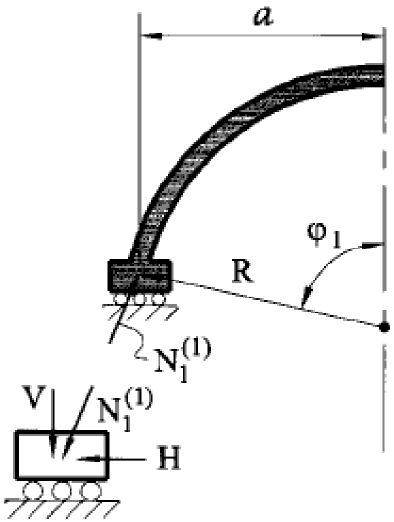
The Change in sign of the circumferential compressive force N_2, takes place when $N_2 = 0$

$$N_2 = -pR(\cos\varphi - \frac{1}{1+\cos\varphi}) = 0$$

$$\cos\bar{\varphi} - \frac{1}{1+\cos\bar{\varphi}} = 0 \Rightarrow \cos^2\bar{\varphi} + \cos\bar{\varphi} - 1 = 0 \Rightarrow \bar{\varphi} = 51^0 49'$$

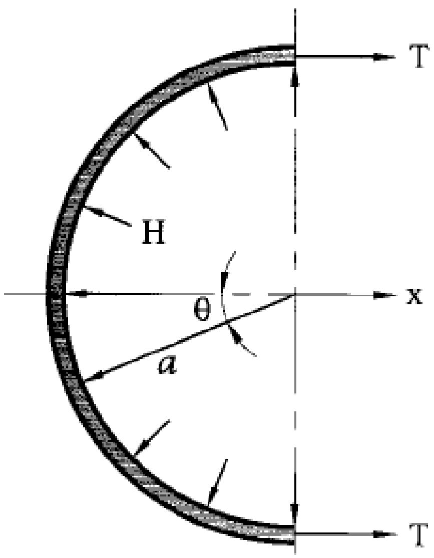
Spherical domes whose opening angle is less than [2(51°49‘)] are free from tensile stresses.


p
φ1
φ1



$$V = N_1^{(1)} \sin\varphi_1, \quad H = -N_1^{(1)} \cos\varphi_1 .$$

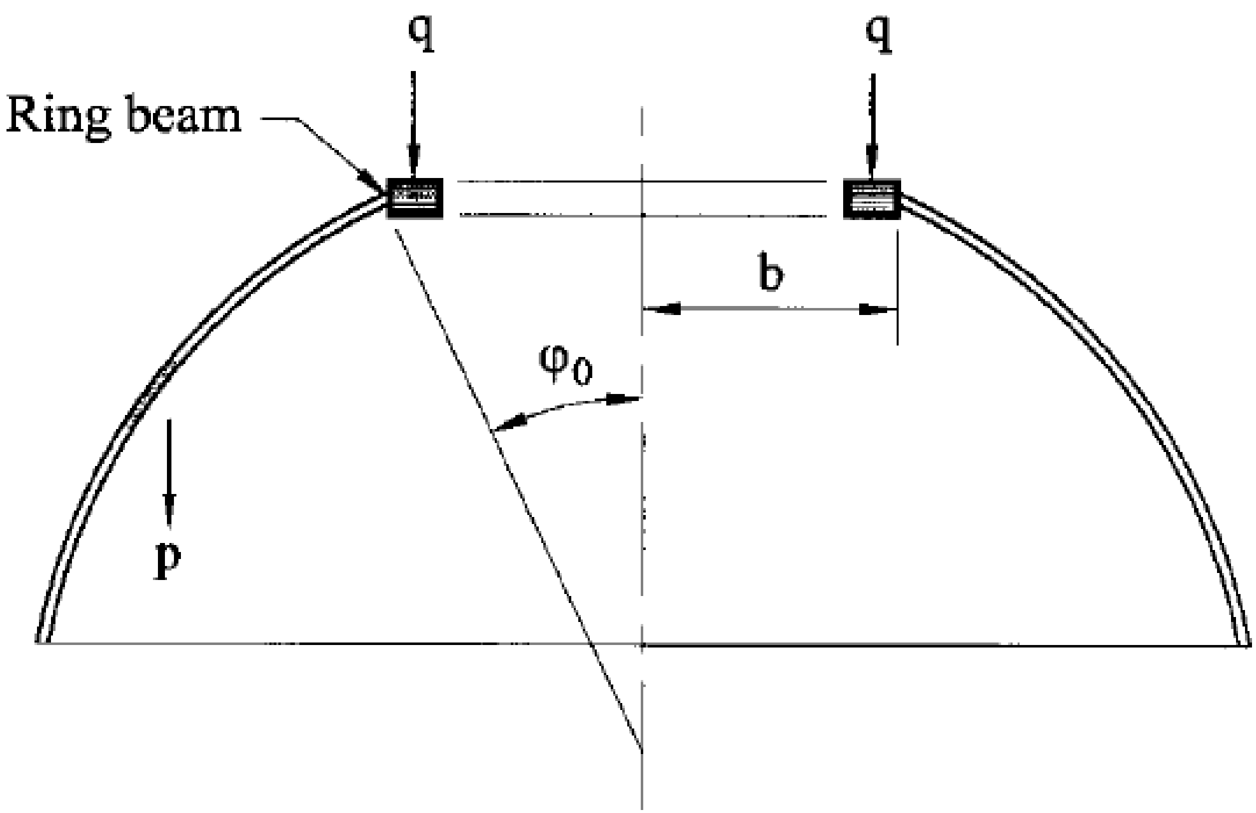
$$2T = \int_{-\pi/2}^{\pi/2} -N_1^{(1)} \cos\varphi_1 \cos\theta (a d\theta) \quad \text{or} \quad T = -N_1^{(1)} a \cos\varphi_1$$



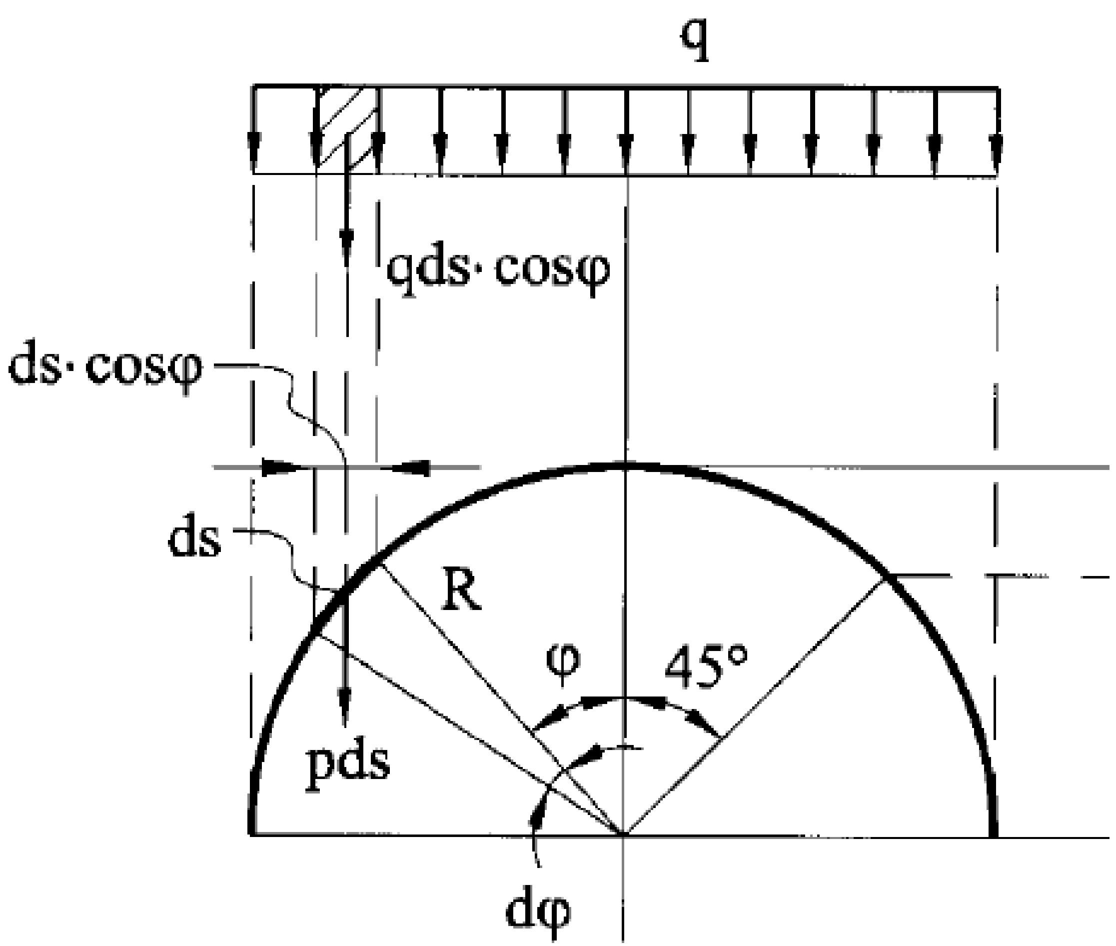
$$T = -N_1^{(1)} R \cos\varphi_1 \sin\varphi_1$$

$$T = pR^2 \frac{\cos\varphi_1 \sin\varphi_1}{1 + \cos\varphi_1}$$

Membrane forces distribution in a spherical dome under self-weight, which has a skylight at its top



(b) Spherical domes: live or snow loads



$$pR\mathrm{d}\varphi = (qR\mathrm{d}\varphi)\cos\varphi \quad \text{or} \quad p = q\cos\varphi$$

$$p_3 = q\cos^2\varphi, \quad p_1 = q\sin\varphi\cos\varphi$$