# الجلسة الثانية – توازن الجسيم

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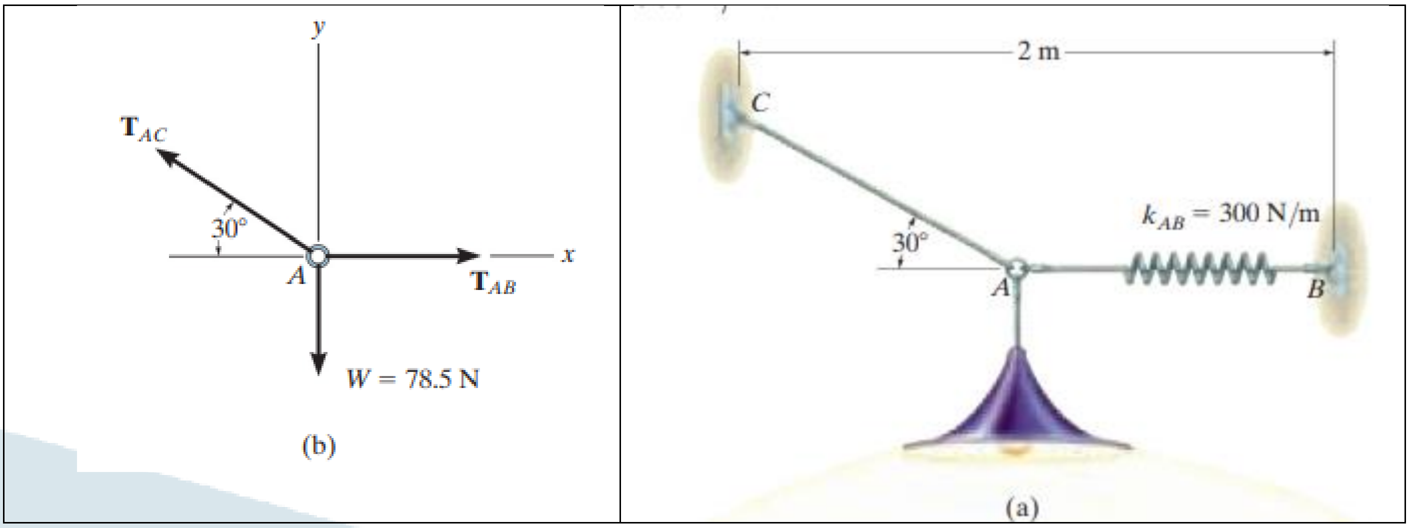
مسألة (2) : أوجد الطول اللازم للحبل $AC$،من أجل تعليق مصباح كتلته $8Kg$ في وضعية التوازن.

الطول الأصلي للنابض $L'_{AB}=0.4m$،ثابت صلابة النابض

$$k_{AB} = 300N/m$$




الحل : إذا علمنا قيمة القوة المؤثرة في النابض ، نستطيع حساب استطالة النابض عن طريق المعادلة : $F=K.S$، عنها نستطيع حساب طول الحبل من الشكل الهندسي .

مخطط الجسم الحر:مخطط الجسم الحرللجسيم $A$ مبيّن في الشكل$(b)$

وزن المصباح : $W=8(9.81)=78.5\,N$

كتابة معادلات التوازن :

$$\sum F_x = 0\,, T_{AB} - T_{AC}.\cos 30 = 0 \quad (1)$$

$$(2) \quad = 0 \sum F_Y = 0\,, T_{AC}.\sin 30 - 78.5$$

بحلّ المعادلتين ينتج :

$$T_{AC} = 157N, T_{AB} = 136N$$

استطالة النابض AB : $T_{AB} = K_{AB}.S_{AB}$

$$L_{AB} = L'_{AB} + S_{AB}$$ الطول النهائي للنابض :

$$L_{AB} = 0.4m + 0.453m = 0.853m$$

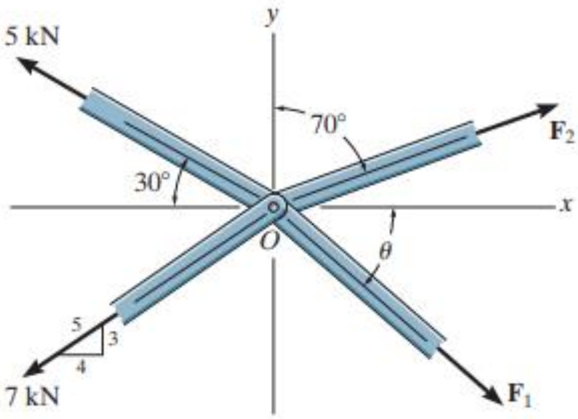
المسافة الكلية من $C$ إلى $B$:

$$2m = l_{AC}.\cos 30 + 0.853m$$

يكون الطول المطلوب للحبل AC :

$$l_{AC} = 1.32m$$

**مسألة 3:عناصر من جائز شبكي , متمفصلة عند النقطة O. أوجد مقدار القوة F1 والزاوية $\theta$ من أجل التوازن . F2=6KN.**



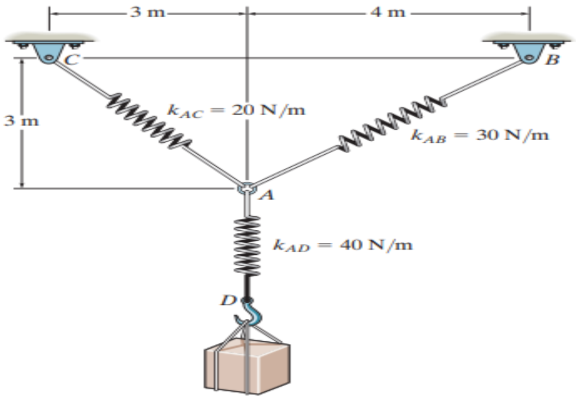

$$(1)\sum F_x = 0 \Rightarrow F_1\cos\theta + F_2 \sin 70 - 5\cos 30 - 7\left(\frac{4}{5}\right) = 0$$

$$(2)\sum F_y = 0 \Rightarrow F_1\sin\theta + F_2 \cos 70 + 5\sin 30 - 7\left(\frac{3}{5}\right) = 0$$

بقسمة المعادلة (2) على (1):

$$\theta = 4.69\ .\ F1 = 4.31KN$$

**مسألة4 : احسب مقدار الاستطالة لكل نابض من أجل تعليق صندوق كتلته 2Kg في وضعية التوازن .**




$(40) \Rightarrow\ x_{AD}$= 0.4905m$F_{AD}\ = 2(9.81) = x_{AD}$

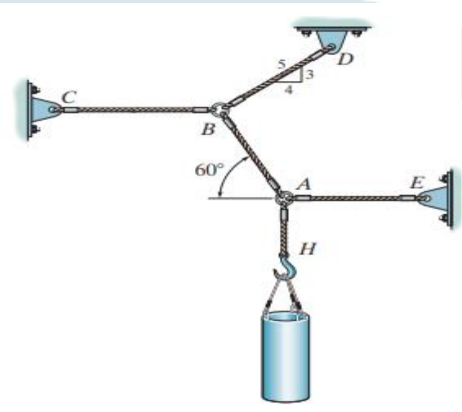
(1) $) -F_{AC}(\frac{1}{\sqrt{2}})$=0 $\sum Fx = 0 \Rightarrow F_{AB}(\frac{4}{5}$

$)+F_{AC}\left(\frac{1}{\sqrt{2}}\right) - 2(9.81) = 0 \quad (2)\sum FY = 0 \Rightarrow F_{AB}(\frac{3}{5}$

=15.86KN, $x_{AD}$=$\frac{15.86}{20}$ =0.793m$F_{AC}$

= 14.01N, $x_{AB} = \frac{14.01}{30}$ =0.467 m$F_{AB}$

- مسألة 5:اسطوانة كتلتها 30 Kg. احسب القوة المؤثرة في كل حبل من أجل التوازن




العقدة A:

$$\sum F_x = 0 \Rightarrow T_{AE} - T_{AB}\cos 60 = 0 \qquad (1)$$

$$\sum F_y = 0 \Rightarrow T_{AB}\sin 60 - 30(9.81) = 0 \ (2)$$

العقدة B:

$$\sum F_y = 0 \Rightarrow -T_{BC} + T_{AB}.cos60 + T_{BD}(\frac{4}{5}) = 0\ (3)$$

$$\sum F_y = 0 \Rightarrow -\mathrm{T_{AB}}.\sin 60 - \mathrm{T_{BD}}(\frac{4}{5}) = 0\quad (4)$$

**294** N = =**30(9.81)**$T_{HA}$

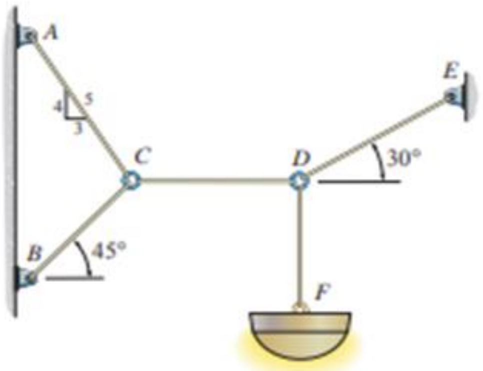
$\mathrm{T_{AE}} = 170\mathrm{N}$

340 N$T_{AB}$ =

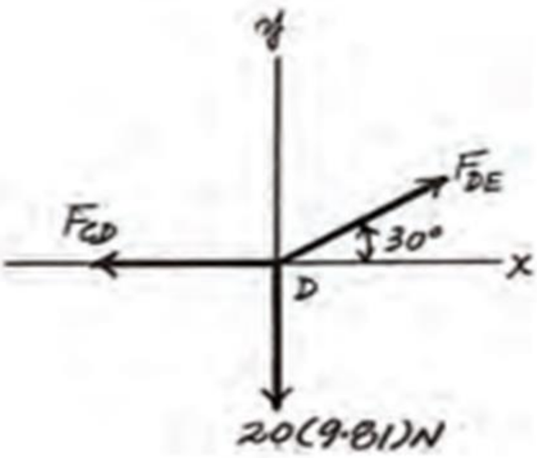
$T_{BD} = 490\mathrm{N}$

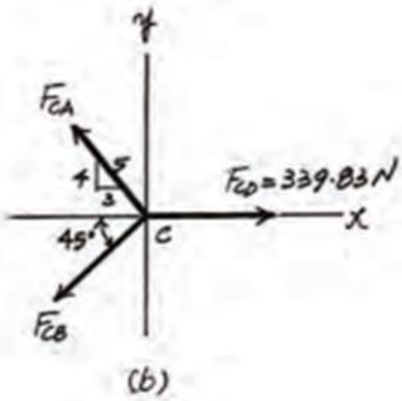
$T_{BC} = 562\mathrm{N}$

**مسألة 6 : احسب القوة المؤثرة في كل حبل من أجل تعليق** مصباح كتلته Kg20 **في وضعية التوازن .**




**نبدأ الحل عند الجسيم D ونرسم مخطط الجسم الحر للجسيم ونكتب معادلات التوازن:**



$$\sum F_x = 0 \Rightarrow -F_{CD} + F_{DE}\cos30 = 0 \qquad (1)$$
$$\sum F_y = 0 \Rightarrow F_{DE}\sin60 - 30(9.81) = 0\ (2)$$
$$F_{DE} = 392N\ , F_{CD} = 339.8N$$

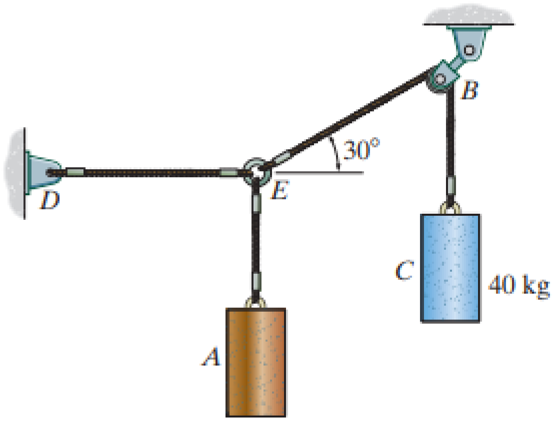
نستخدم النتائج وننتقل إلى دراسة العقدة C:

$$\sum F_x = 0 \Rightarrow 339.8 - F_{CD}.\cos45 + F_{CA}(\frac{3}{5}) = 0 \qquad (1)$$
$$\sum F_y = 0 \Rightarrow F_{CA}(\frac{4}{5}) - F_{CB}\sin45 = 0\ (2)$$

عن طريق حل المعادلتين (1) و (2) ينتج :

**FBC= 275 N , FAC=243N**

**مسألة 9**: إذا كانت الكتلة الاسطوانة C 40Kg، احسب كتلة الاسطوانة A لتعليق المجموعة في وضعية التوازن.

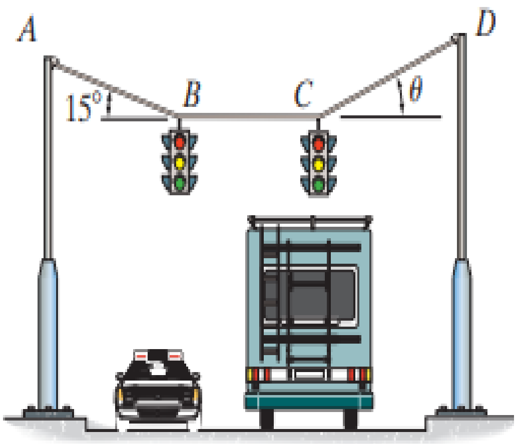



$$+\uparrow \Sigma F_y = 0; \quad (392.4\ \text{N})\sin 30^\circ - m_A(9.81) = 0$$
$$m_A = 20\ \text{kg} \qquad \textit{Ans.}$$

**مسألة10** : احسب قوى الشد في الكبلات AB و BCو CD، اللازمة لتعليق اشارات المرور

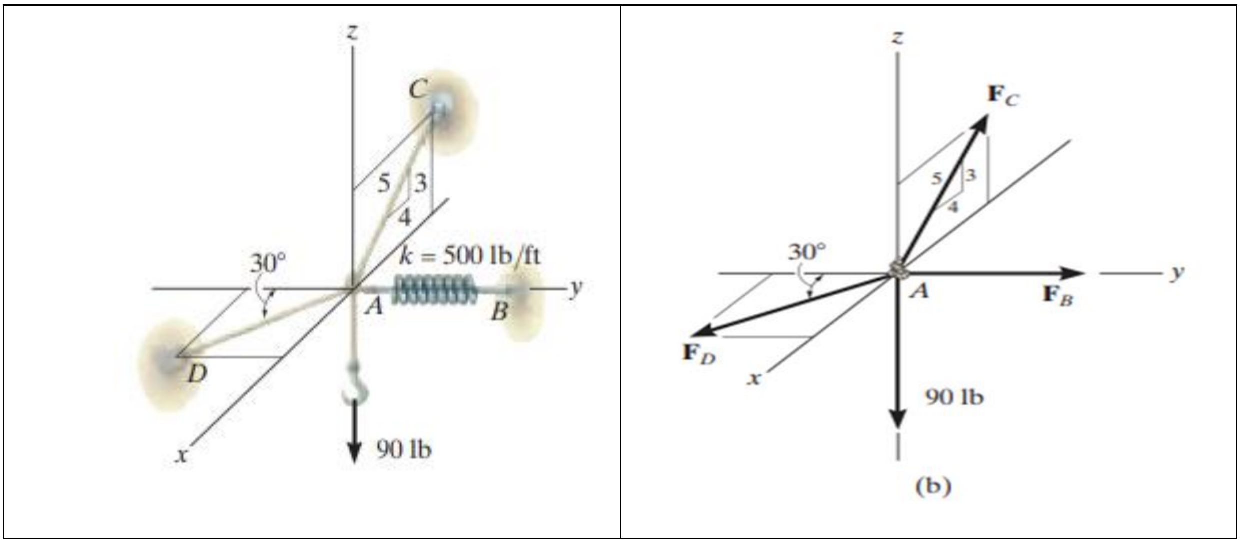
ذات الكتل 10kg و15 kg عند النقطتين B وC على التوالي . واحسب قيمة الزاوية $\theta$.

$$+\uparrow \Sigma F_y = 0; \quad T_{AB} \sin 15^\circ - 10(9.81)\text{ N} = 0$$
$$T_{AB} = 379.03\text{ N} = 379\text{ N}$$
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T_{BC} - 379.03\text{ N} \cos 15^\circ = 0$$
$$T_{BC} = 366.11\text{ N} = 366\text{ N}$$
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T_{CD} \cos\theta - 366.11\text{ N} = 0$$
$$+\uparrow \Sigma F_y = 0; \quad T_{CD} \sin\theta - 15(9.81)\text{ N} = 0$$
$$T_{CD} = 395\text{ N}$$
$$\theta = 21.9^\circ$$




## EXAMPLE11:

**A 90-lb load is suspended from the hook shown in Fig. a. If the load is supported by two cables and a spring having a stiffness k = 500 lb/ft, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.**




Free-Body Diagram: **The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig**

Equations of Equilibrium: **By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be used. Considering components Directed along each positive axis as "positive," we have:**

$$\Sigma F_x = 0; \qquad F_D \sin 30^\circ - \left(\tfrac{4}{5}\right) F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -F_D \cos 30^\circ + F_B = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad \left(\tfrac{3}{5}\right) F_C - 90 \text{ lb} = 0 \qquad (3)$$

$$F_C = 150 \text{ lb}$$

$$F_D = 240 \text{ lb}$$

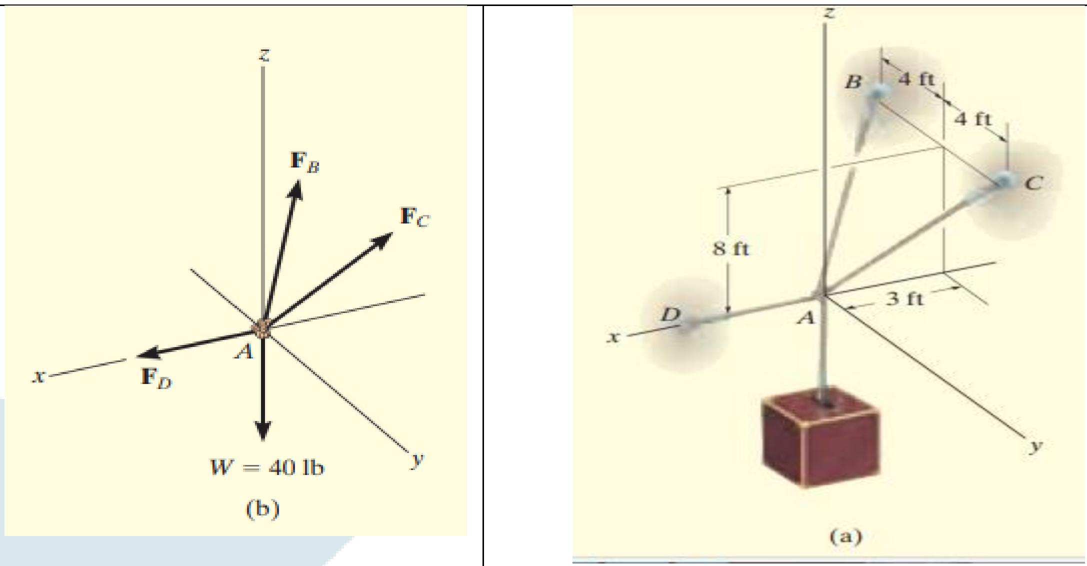
$$F_B = 207.8 \text{ lb} = 208 \text{ lb}$$

$$F_B = k s_{AB}$$

$$207.8 \text{ lb} = (500 \text{ lb/ft})(s_{AB})$$

$$s_{AB} = 0.416 \text{ ft}$$

EXAMPLE12 : **Determine the force in each cable used to support the 40-lb crate shown in Fig. a**



(b) (a)

Equations of Equilibrium: **First we will express each force in Cartesian vector form. Since the coordinates of points B and C are B(-3 ft, -4 ft, 8 ft) and C(-3 ft, 4 ft, 8 ft), we have**

$$\mathbf{F}_B = F_B\left[\frac{-3\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}}\right]$$
$$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$
$$\mathbf{F}_C = F_C\left[\frac{-3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}}\right]$$
$$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$
$$\mathbf{F}_D = F_D\mathbf{i}$$
$$\mathbf{W} = \{-40\mathbf{k}\}\text{ lb}$$

Equilibrium requires :

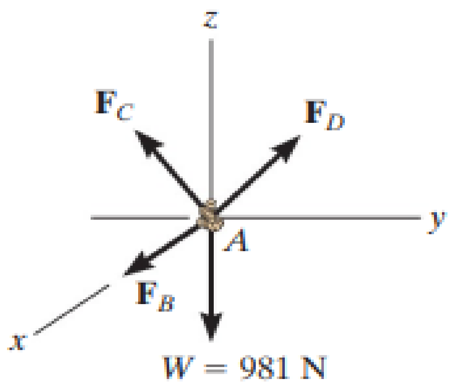
$$\Sigma\mathbf{F} = \mathbf{0};\quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$
$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$
$$-0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = \mathbf{0}$$

**Equating the respective i, j, k components to zero yields**

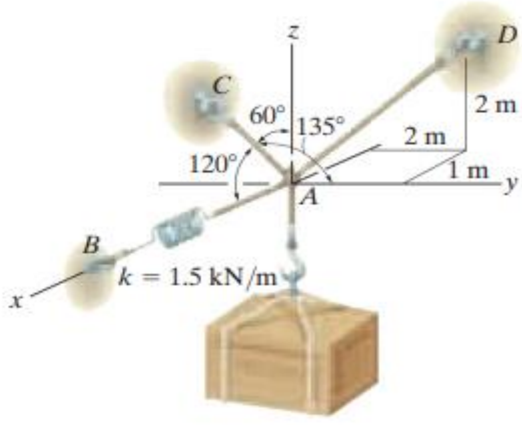
$$\Sigma F_x = 0;\quad -0.318F_B - 0.318F_C + F_D = 0 \quad (1)$$
$$\Sigma F_y = 0;\quad -0.424F_B + 0.424F_C = 0 \quad (2)$$
$$\Sigma F_z = 0;\quad 0.848F_B + 0.848F_C - 40 = 0 \quad (3)$$

$$F_B = F_C = 23.6\text{ lb}$$
$$F_D = 15.0\text{ lb}$$

EXAMPLE13:**Determine the tension in each cord used to support the100-kg crate shown in Fig. a**



(b)



(a)

Solution :

Free-Body Diagram: **The force in each of the cords can be determined by investigating the equilibrium of point A. The free-body diagram is shown in Fig. b.**

**The weight of the crate is W = 100(9.81) = 981 N**

Equations of Equilibrium:

**Each force on the free-body diagram is first expressed in Cartesian vector form.**

$$\mathbf{F}_B = F_B\mathbf{i}$$

$$\mathbf{F}_C = F_C\cos 120°\mathbf{i} + F_C\cos 135°\mathbf{j} + F_C\cos 60°\mathbf{k}$$
$$= -0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}\right]$$
$$= -0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k}$$

$$\mathbf{W} = \{-981\mathbf{k}\}\text{ N}$$

**Equilibrium requires :**

$$\Sigma \mathbf{F} = 0; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$

$$F_B\mathbf{i} - 0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$-0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k} - 981\mathbf{k} = \mathbf{0}$$

**Equating the respective i, j, k components to zero :**

$$\Sigma F_x = 0; \qquad F_B - 0.5F_C - 0.333F_D = 0$$

$$\Sigma F_y = 0; \qquad -0.707F_C + 0.667F_D = 0$$

$$\Sigma F_z = 0; \qquad 0.5F_C + 0.667F_D - 981 = 0$$

$$F_C = 813 \text{ N}$$

$$F_D = 862 \text{ N}$$

$$F_B = 694 \text{ N}$$

**EXAMPLE14 :Determine the magnitude of forces F1, F2, F3, so that the particle is held in equilibrium**

$$\Sigma F_x = 0; \quad \left[\left(\tfrac{3}{5}\right)F_3\right]\left(\tfrac{3}{5}\right) + 600 \text{ N} - F_2 = 0 \qquad (1)$$

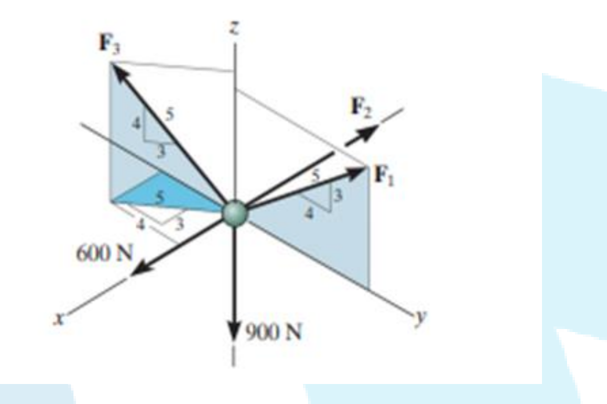
$$\Sigma F_y = 0; \quad \left(\tfrac{4}{5}\right)F_1 - \left[\left(\tfrac{3}{5}\right)F_3\right]\left(\tfrac{4}{5}\right) = 0 \qquad (2)$$

$$\Sigma F_z = 0; \quad \left(\tfrac{4}{5}\right)F_3 + \left(\tfrac{3}{5}\right)F_1 - 900 \text{ N} = 0 \qquad (3)$$

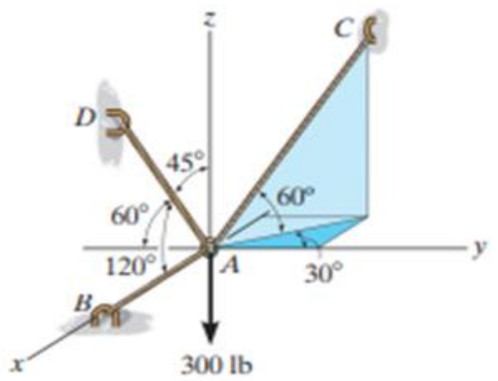
$F_3 = 776$ N *Ans.*

$F_1 = 466$ N *Ans.*

$F_2 = 879$ N *Ans.*




**EXAMPLE15 :Determine the tension developed in cables AB, AC, and AD.**




$$\mathbf{F}_{AC} = F_{AC}\{-\cos 60^\circ \sin 30^\circ\,\mathbf{i} + \cos 60^\circ \cos 30^\circ\,\mathbf{j} + \sin 60^\circ\,\mathbf{k}\}$$
$$= -0.25F_{AC}\,\mathbf{i} + 0.4330F_{AC}\,\mathbf{j} + 0.8660F_{AC}\,\mathbf{k}$$
$$\mathbf{F}_{AD} = F_{AD}\{\cos 120^\circ\,\mathbf{i} + \cos 120^\circ\,\mathbf{j} + \cos 45^\circ\,\mathbf{k}\}$$
$$= -0.5F_{AD}\,\mathbf{i} - 0.5F_{AD}\,\mathbf{j} + 0.7071F_{AD}\,\mathbf{k}$$
$$\Sigma F_y = 0;\quad 0.4330F_{AC} - 0.5F_{AD} = 0$$
$$\Sigma F_z = 0;\quad 0.8660F_{AC} + 0.7071F_{AD} - 300 = 0$$
$$F_{AD} = 175.74 \text{ lb} = 176 \text{ lb} \qquad \textit{Ans.}$$
$$F_{AC} = 202.92 \text{ lb} = 203 \text{ lb} \qquad \textit{Ans.}$$
$$\Sigma F_x = 0;\quad F_{AB} - 0.25(202.92) - 0.5(175.74) = 0$$
$$F_{AB} = 138.60 \text{ lb} = 139 \text{ lb} \qquad \textit{Ans.}$$