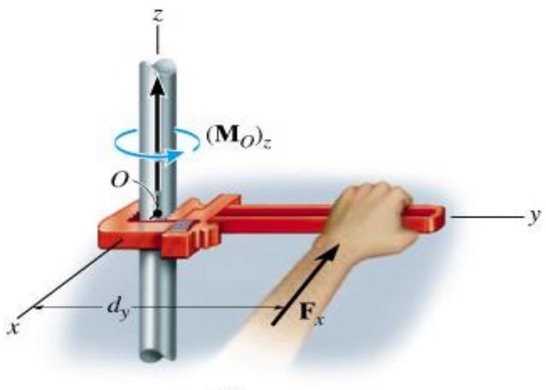
المحاضرة الثالثة – عزم القوة ،عزم المزدوجة ، استبدال نظام القوى والعزوم بقوة محصلة وحيدة وعزم مزدوجة .

Moments, Couples, and Force Couple Systems

The tendency of a force to rotate a rigid body about :3.1 Introduction to Moments any defined axis is called the Moment of the force about the axis



(a)

MOMENT OF A FORCE - SCALAR FORMULATION :

M = F . d (N.m)

The moment, M, of a force about a point provides a measure of the tendency for rotation (sometimes called a torque)

$$M = F \cdot d$$

The Moment of Force (F) about an axis through Point (O) or for short, the Moment of F about O, is the product of the magnitude of the force and the perpendicular distance between Point (O) and the line of action of Force (F).

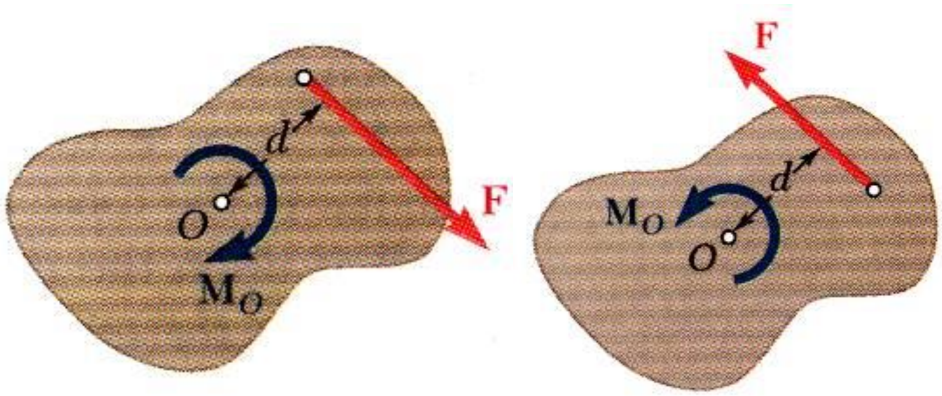
$$M_O = F.d$$

The units of a Moment are:

- N·m in the SI system
- lbs.ft or *lbs.in* in the US Customary system

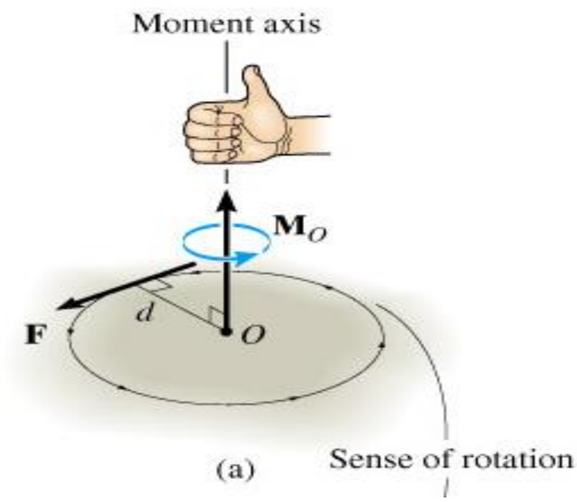
Properties of a Moment :

- Moments not only have a magnitude, they also have a sense to them.
- The sense of a moment is clockwise or counter-clockwise depending on which way it will tend to make the object rotate



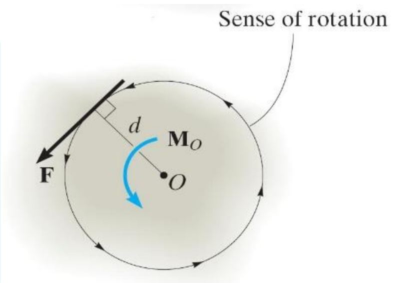
(b) $M_O = -Fd$ (a) $M_O = +Fd$

- The sense of a Moment is defined by the direction it is acting on the Axis and can be found using Right Hand Rule



Varignon's Theorem :

- The moment of a Force about any axis is equal to the sum of the moments of its components about that axis
- This means that resolving or replacing forces with their resultant force will not affect the moment on the object being. In the 2-D case, the magnitude of the moment is $M_o = F\,d$



As shown, d is the perpendicular distance from point O to the line of action of the force.

In 2-D, the direction of M_O is either clockwise or counter-clockwise, depending on the tendency for rotation.

Cross Products :

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

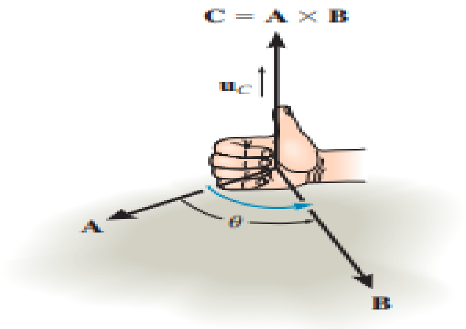
Magnitude : $C = A.B \sin\theta$

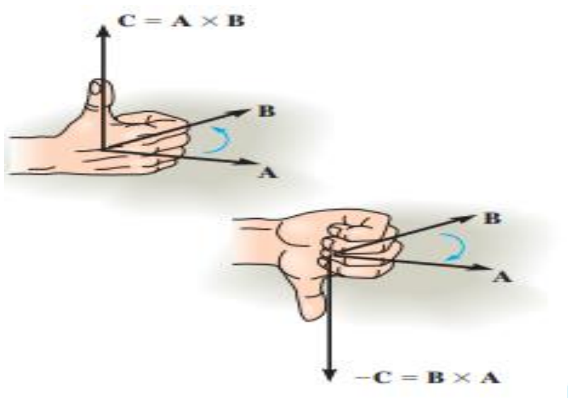
Direction : Vector C has a direction that is perpendicular to the plane containing A and B, such that C is specified by the right –hand rule , curling the fingers of the right hand from vector A (cross) to vector B , the thumb points in the direction of C.

$$C = (A.B \sin\theta\ ).U_C$$

Where the scalar $A.B \sin\theta$ defines the magnitude of C

and the unit vector U_C defines the direction of C .



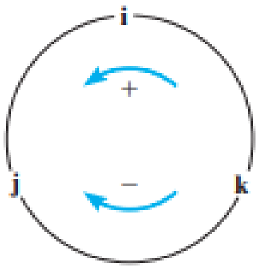


$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

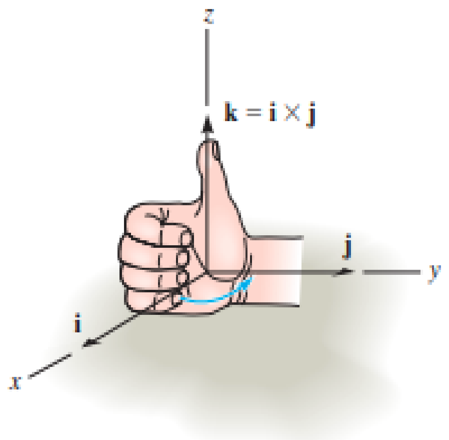
$$a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$

Cartesian Vector Formulation.



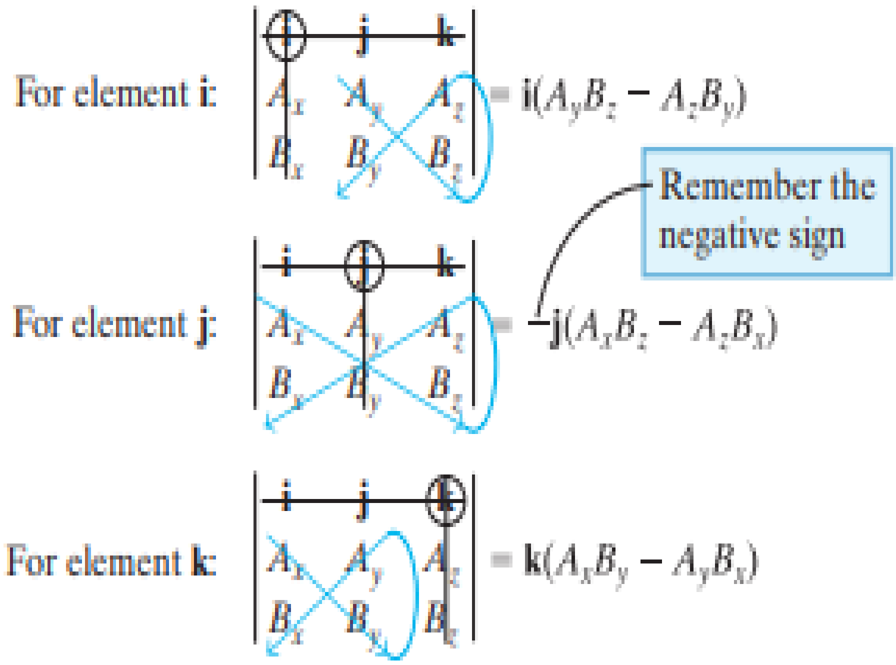
$$\mathbf{i}\times\mathbf{j}=\mathbf{k} \quad \mathbf{i}\times\mathbf{k}=-\mathbf{j} \quad \mathbf{i}\times\mathbf{i}=\mathbf{0}$$
$$\mathbf{j}\times\mathbf{k}=\mathbf{i} \quad \mathbf{j}\times\mathbf{i}=-\mathbf{k} \quad \mathbf{j}\times\mathbf{j}=\mathbf{0}$$
$$\mathbf{k}\times\mathbf{i}=\mathbf{j} \quad \mathbf{k}\times\mathbf{j}=-\mathbf{i} \quad \mathbf{k}\times\mathbf{k}=\mathbf{0}$$



$$\begin{aligned}\mathbf{A}\times\mathbf{B} &= (A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k})\times(B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k})\\ &= A_xB_x(\mathbf{i}\times\mathbf{i})+A_xB_y(\mathbf{i}\times\mathbf{j})+A_xB_z(\mathbf{i}\times\mathbf{k})\\ &\quad + A_yB_x(\mathbf{j}\times\mathbf{i})+A_yB_y(\mathbf{j}\times\mathbf{j})+A_yB_z(\mathbf{j}\times\mathbf{k})\\ &\quad + A_zB_x(\mathbf{k}\times\mathbf{i})+A_zB_y(\mathbf{k}\times\mathbf{j})+A_zB_z(\mathbf{k}\times\mathbf{k})\end{aligned}$$

$$\mathbf{A}\times\mathbf{B}=(A_yB_z-A_zB_y)\mathbf{i}-(A_xB_z-A_zB_x)\mathbf{j}+(A_xB_y-A_yB_x)\mathbf{k}$$

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

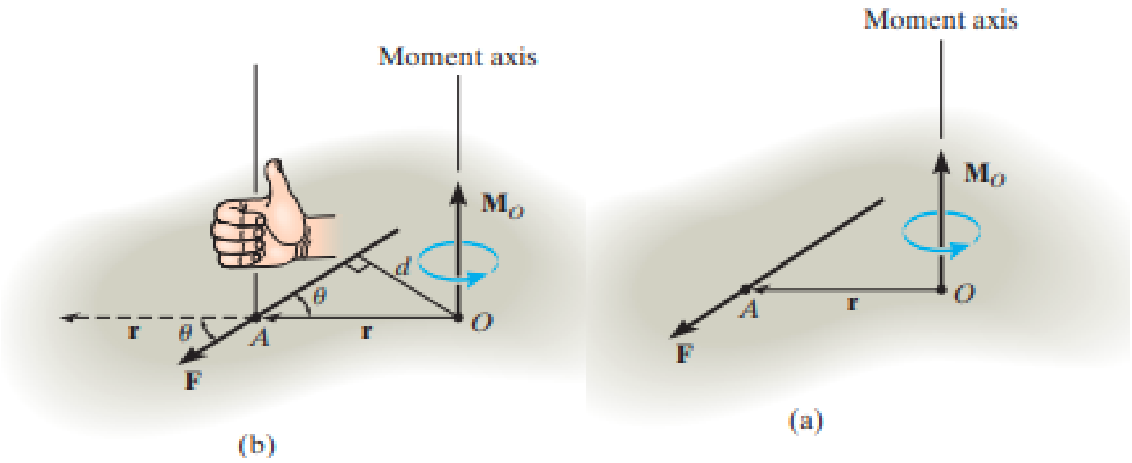


Moment of force -vector formulation :

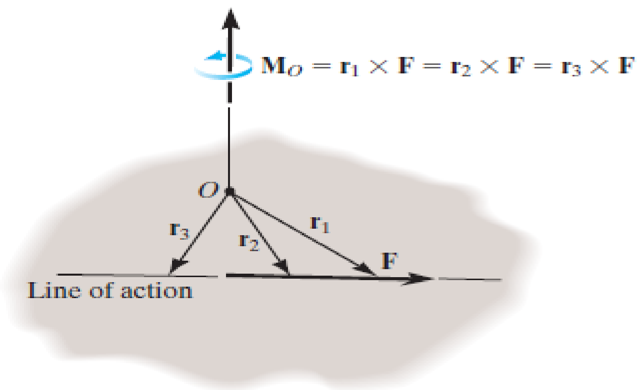
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Magnitude: $M_O = r.F sin\theta = F(r sin\theta) = F.d$

Direction :**The direction and sense of MO in are determined by the right-hand rule as it applies to the cross product. Thus, sliding r to the dashed position and curling the right-hand fingers from r toward F, "r cross F," the thumb is directed upward or perpendicular to the plane containing r and F and this is in the same direction as MO**

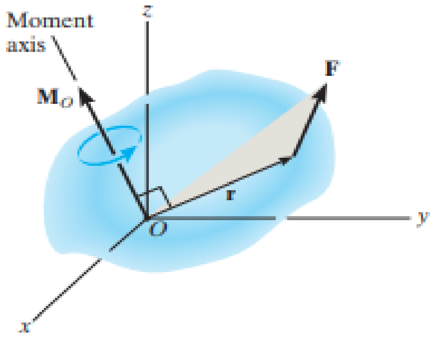


Principle of Transmissibility:



$$\mathbf{M}_O = \mathbf{r}_1 \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} = \mathbf{r}_3 \times \mathbf{F}$$

Cartesian Vector Formulation: If we establish x, y, z coordinate axes, then the position vector r and force F can be expressed as Cartesian vectors



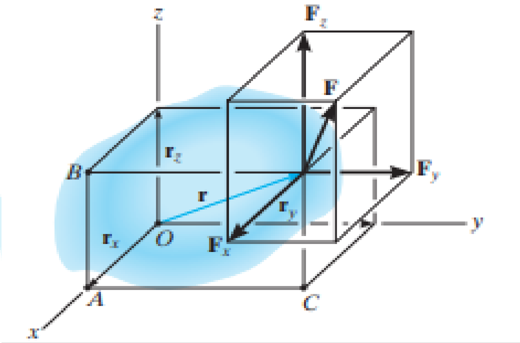
(a)

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

-rx, ry, rz represent the x, y, z components of the position vector drawn from point O to any point on the line of action of the force.

- Fx, Fy, Fz represent the x, y, z components of the force vector.

$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} - (r_x F_z - r_z F_x)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

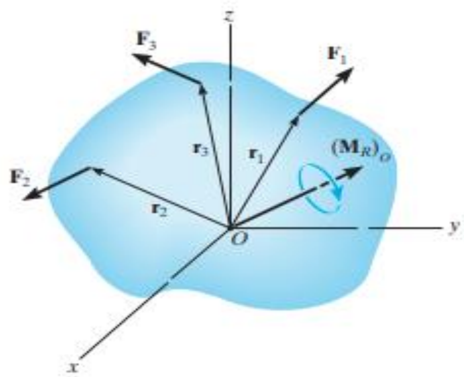


Resultant Moment of a System of Forces:

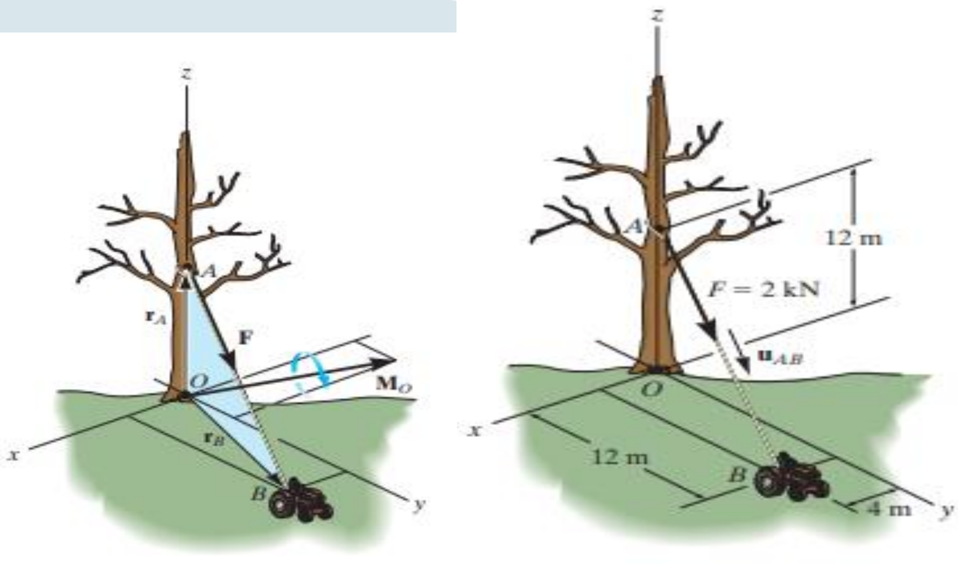
If a body is acted upon by a system of forces, the resultant moment of the forces about point O can be determined by vector addition of the moment of each force.

This resultant can be written symbolically as:

$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$



Example 1: Determine the moment produced by the force F in about point O. Express the result as a Cartesian vector.



(b) (a)

As shown in Fig b, either rA or rB can be used to determine the moment about point O. These position vectors are:

$$\mathbf{r}_A = \{12\mathbf{k}\}\ \text{m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\ \text{m}$$

Force **F** expressed as a Cartesian vector is

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\ \text{kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\ \text{m}}{\sqrt{(4\ \text{m})^2 + (12\ \text{m})^2 + (-12\ \text{m})^2}}\right]$$

$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$

Thus

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

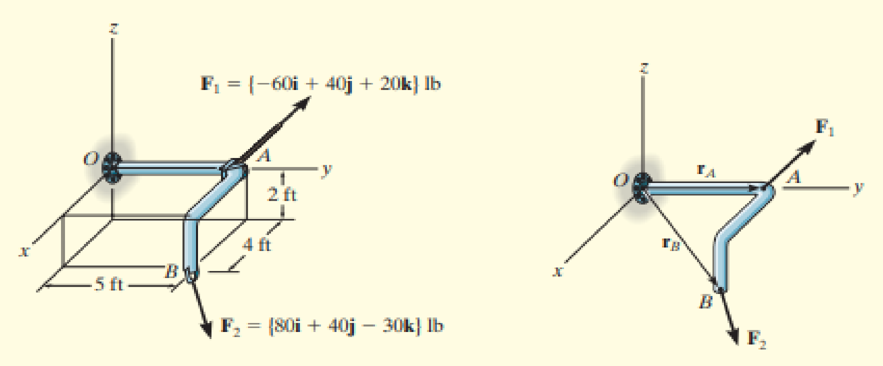
$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [12(-1.376) - 0(1.376)]\mathbf{i} - [4(-1.376) - 0(0.4588)]\mathbf{j} + [4(1.376) - 12(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN}\cdot\text{m} \qquad \textit{Ans.}$$

Example 2 :Two forces act on the rod shown in Fig. a. Determine the resultant moment they create about the flange at O. Express the result as a Cartesian vector.



(a) (b)

$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$

$$= \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= [5(20) - 0(40)]\mathbf{i} - [0]\mathbf{j} + [0(40) - (5)(-60)]\mathbf{k}$$

$$+ [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k}$$

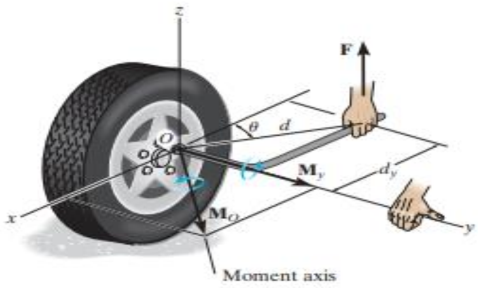
$$= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\}\ \text{lb} \cdot \text{ft}$$ *Ans.*

Moment of force about a specified axis:

Scalar Analysis. To use a scalar analysis in the case of the lug nut in Fig. a, the moment arm, or perpendicular distance from the axis to the line of action of the force, is $d_y = d \cos \theta$. Thus, the moment of F about the *y* axis is:

$M_y = F\, d_y = F(d \cos \theta)$. According to the right-hand rule, M_y is directed along the positive *y* axis as shown in the figure. In general, for any axis a, the moment is:

$$M_a = F.d_a$$



(a)

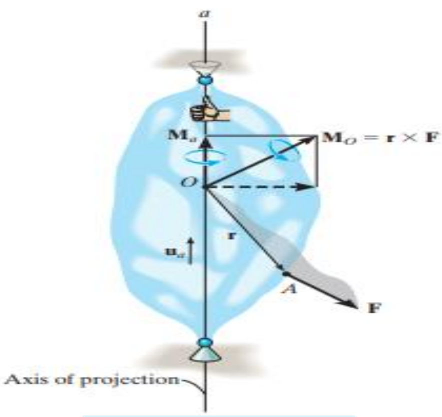
Vector Analysis:

we must first determine the moment of the force about any point O on the *y* axis, MO = r x F. The component My along the *y* axis is the projection of MO using the dot product :$M_y = j.M_O = j.(r \times F)$

We can generalize this approach by letting ua be the unit vector that specifies the direction of the a axis. Then the moment of F about a point O on the axis is MO = r x F, and the projection of this moment onto the a axis is Ma = ua . (r x F). This combination is referred to as the scalar triple product.

$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$



where uax , uay , uaz represent the x, *y*, z components of the unit vector defining the direction of the a axis. rx, ry, rz represent the x, *y*, z components of the

position vector extended from any point O on the a axis to any point A on the line of action of the force Fx, Fy, Fz represent the x, y, z components of the force vector.

Provided Ma is determined, we can then express Ma as a Cartesian vector, namely

$$\boldsymbol{M_a} = M_a \boldsymbol{U_a}$$

Important Points

• The moment of a force about a specified axis can be determined provided the perpendicular distance da from the force line of action to the axis can be determined. Ma = F .da.

• If vector analysis is used, Ma = ua . (r x F), where ua defines the direction of the axis and r is extended from any point on the axis to any point on the line of action of the force.

• If Ma is calculated as a negative scalar, then the sense of direction of Ma is opposite to ua.

• The moment Ma expressed as a Cartesian vector is determined from Ma = Ma.ua

Force Couples

- A Couple is defined as two Forces having the same magnitude, parallel lines of action, and opposite sense
- In this situation, the sum of the forces in each direction is zero, so a couple does not affect the sum of forces equations

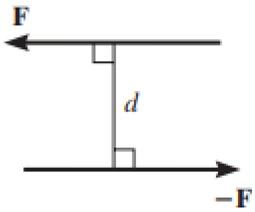
A force couple will however tend to rotate the body

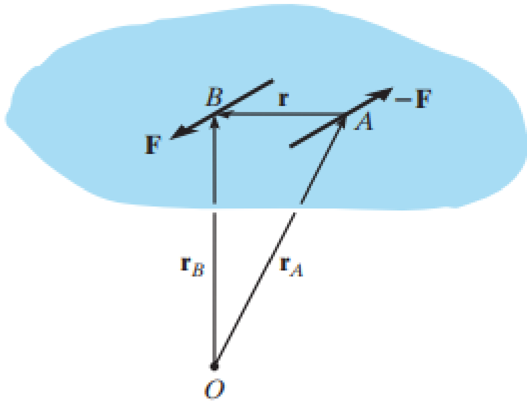
$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

However $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

This result indicates that a couple moment is a free vector, i.e., it can act at any point since M depends only upon the position vector r directed between the forces and not the position vectors rA and rB, directed from the arbitrary point O to the forces





Scalar Formulation. The moment of a couple, M,, is defined as having a magnitude of $M = F.d$

where F is the magnitude of one of the forces and d is the perpendicular distance or moment arm between the forces. The direction and sense of the couple moment are determined by the right-hand rule, where the thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces. In all cases, M will act perpendicular to the plane containing these forces.

Vector Formulation. The moment of a couple can also be expressed by the vector cross product using

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

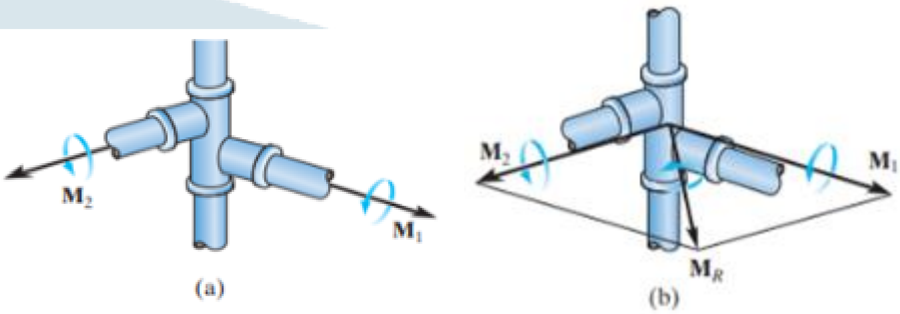
Equivalent Couples. If two couples produce a moment with the same magnitude and direction, then these two couples are equivalent .

Resultant Couple Moment. Since couple moments are vectors, their resultant can be determined by vector addition

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

If more than two couple moments act on the body, we may generalize this concept and write the vector resultant as

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$



Important Points

• A couple moment is produced by two noncollinear forces that are equal in magnitude but opposite in direction. Its effect is to produce pure rotation, or tendency for rotation in a specified direction.

• A couple moment is a free vector, and as a result it causes the same rotational effect on a body regardless of where the couple moment is applied to the body.

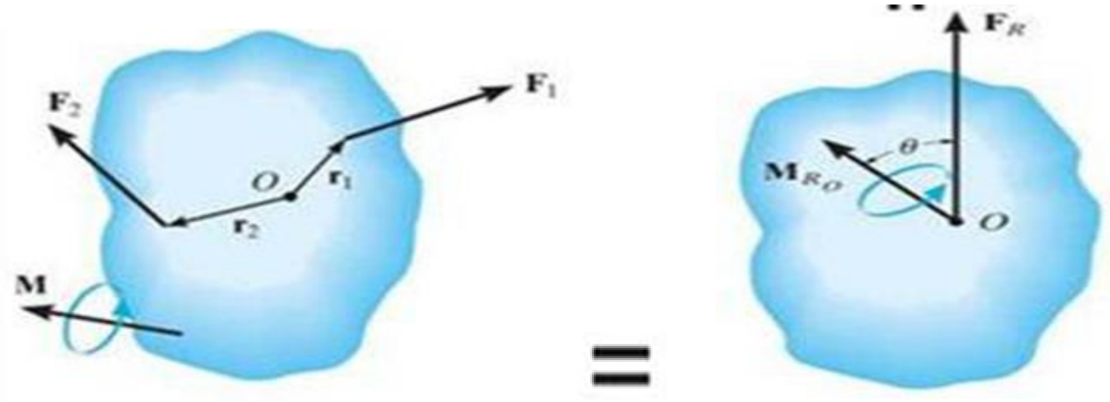
• The moment of the two couple forces can be determined about any point. For convenience, this point is often chosen on the line of action of one of the forces in order to eliminate the moment of this force about the point.

• In three dimensions the couple moment is often determined using the vector formulation, M = r x F, where r is directed from any point on the line of action of one of the forces to any point on the line of action of the other force F.

• A resultant couple moment is simply the vector sum of all the couple moments of the system.

SIMPLIFICATION OF FORCE AND COUPLE SYSTEM

When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect .The two force and couple systems are called equivalent systems since they have the same external effect on the body.



When several forces and couple moments act on a body,

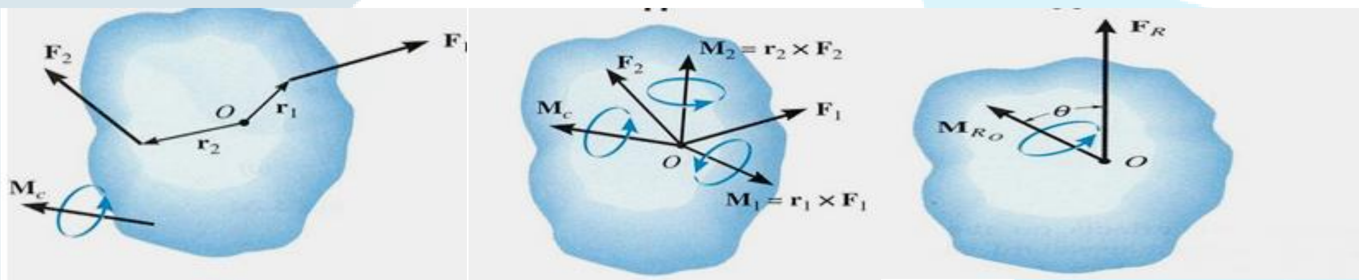
you can move each force and its associated couple moment

to a common point O .Now you can add all the forces

and couple moments together and find one resultant force-couple

moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$



If the force system lies in the x-y plane (a 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations:

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

Important Points

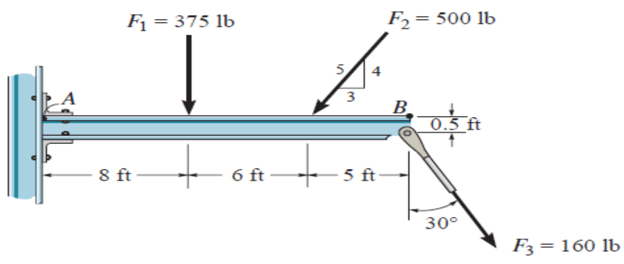
- Force is a sliding vector, since it will create the same external effects on a body when it is applied at any point P along its line of action. This is called the principle of transmissibility.
- A couple moment is a free vector since it will create the same external effects on a body when it is applied at any point P on the body.
- When a force is moved to another point P that is not on its line of action, it will create the same external effects on the body if a couple moment is also applied to the body. The couple moment is determined by taking the moment of the force about point P.

Further Simplification of a Force and Couple System

Procedure for Analysis The technique used to reduce a coplanar or parallel force system to a single resultant force follows a similar procedure outlined in the previous section.

- **Establish the x, *y*, *z*, axes and locate the resultant force FR an arbitrary distance away from the origin of the coordinates. Force Summation.**
- **The resultant force is equal to the sum of all the forces in the system.**
- **For a coplanar force system, resolve each force into its x and *y* components. Positive components are directed along the positive x and *y* axes, and negative components are directed along the negative x and *y* axes. Moment Summation.**
- **The moment of the resultant force about point O is equal to the sum of all the couple moments in the system plus the moments of all the forces in the system about O.**
- **This moment condition is used to find the location of the resultant force from point O.**

EXAMPLE 3:Determine the moment about point *A* of each of the three forces acting on the beam.



$$\circlearrowleft + (M_{F_1})_A = -375(8)$$

$$= -3000 \text{ lb}\cdot\text{ft} = 3.00 \text{ kip}\cdot\text{ft} \quad \textbf{\textit{(Clockwise)}}$$

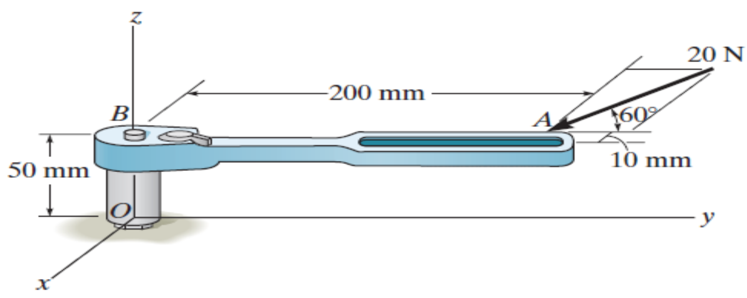
$$\circlearrowleft + (M_{F_2})_A = -500\left(\frac{4}{5}\right)(14)$$

$$= -5600 \text{ lb}\cdot\text{ft} = 5.60 \text{ kip}\cdot\text{ft} \quad \textbf{\textit{(Clockwise)}}$$

$$\circlearrowleft + (M_{F_3})_A = -160(\cos 30^\circ)(19) + 160 \sin 30^\circ(0.5)$$

$$= -2593 \text{ lb}\cdot\text{ft} = 2.59 \text{ kip}\cdot\text{ft} \quad \textbf{\textit{(Clockwise)}}$$

EXAMPLE4 :The 20-N horizontal force acts on the handle of the socket wrench. Determine the moment of this force about point *O*. Specify the coordinate direction angles of the moment axis.



$$\mathbf{F} = 20\,(\sin 60°\mathbf{i} - \cos 60°\mathbf{j}) = \{17.32\mathbf{i} - 10\mathbf{j}\}\text{ N}$$

$$\mathbf{r}_{OA} = \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\text{ m}$$

$$\mathbf{M}_O = \mathbf{r}_{OA} \times F$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0.05 \\ 17.32 & -10 & 0 \end{vmatrix}$$
$$= \{0.5\mathbf{i} + 0.8660\mathbf{j} - 3.3641\mathbf{k}\}\text{ N}\cdot\text{m}$$
$$= \{0.5\mathbf{i} + 0.866\mathbf{j} - 3.36\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$M_O = \sqrt{(M_O)_x^2 + (M_O)_y^2 + (M_O)_z^2} = \sqrt{0.5^2 + 0.8660^2 + (-3.3641)^2}$$
$$= 3.5096\text{ N}\cdot\text{m}$$

$$\alpha = \cos^{-1}\left[\frac{(M_O)_x}{M_O}\right] = \cos^{-1}\left(\frac{0.5}{3.5096}\right) = 81.81° = 81.8°$$

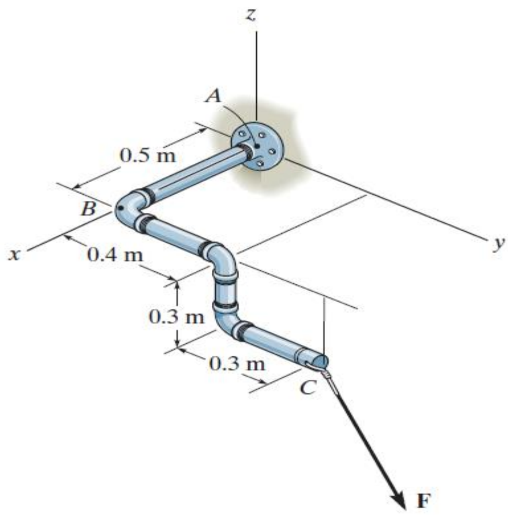
$$\beta = \cos^{-1}\left[\frac{(M_O)_y}{M_O}\right] = \cos^{-1}\left(\frac{0.8660}{3.5096}\right) = 75.71° = 75.7°$$

$$\gamma = \cos^{-1}\left[\frac{(M_O)_z}{M_O}\right] = \cos^{-1}\left(\frac{-3.3641}{3.5096}\right) = 163.45° = 163°$$

EXAMPLE 5 : The pipe assembly is subjected to the force of **F** = {600**i** + 800**j** - 500**k**} N. Determine the moment of this force about point *A*.

$$\mathbf{r}_{AC} = \{0.5\mathbf{i} + 0.7\mathbf{j} - 0.3\mathbf{k}\}\ \text{m}$$

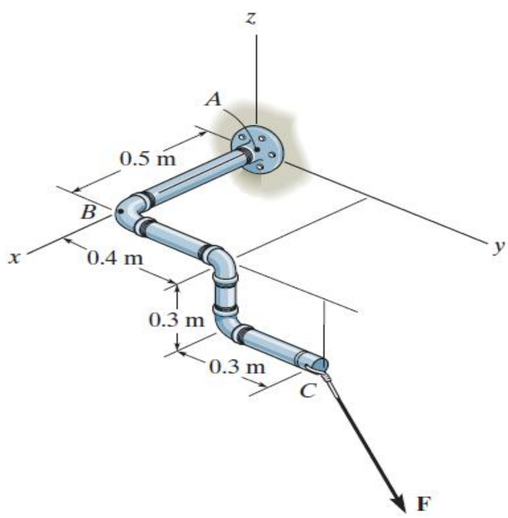
$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.5 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix} = \{-110\mathbf{i} + 70\mathbf{j} - 20\mathbf{k}\}\ \text{N} \cdot \text{m}$$



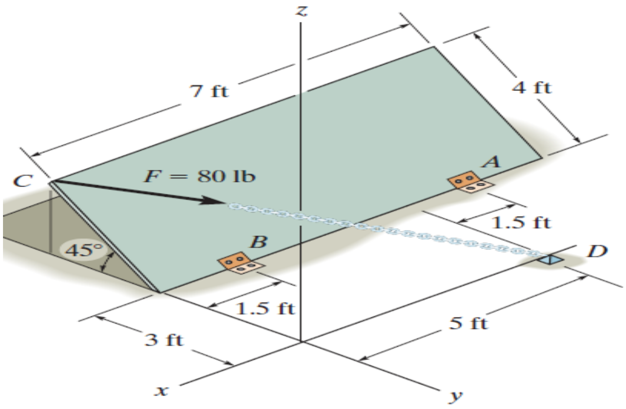
EXAMPLE4 :The pipe assembly is subjected to the force of **F** = {600**i** + 800**j** - 500**k**} N. Determine the moment of this force about point *B*.

$$\mathbf{r}_{BC} = (0.5 - 0.5)\mathbf{i} + (0.7 - 0)\mathbf{j} + (-0.3 - 0)\mathbf{k} = \{0.7\mathbf{j} - 0.3\mathbf{k}\}\ \text{m}$$

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix} = \{-110\mathbf{i} - 180\mathbf{j} - 420\mathbf{k}\}\ \text{N} \cdot \text{m}$$



EXAMPLE6 : Determine the moment of the force **F** about the door hinge at *B*. Express the result as a Cartesian vector.



Moment of *F* About Point *B*.

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & -2.8284 & 2.8284 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

$$= \{-82.9496\mathbf{i} - 96.77\mathbf{j} - 52.78\mathbf{k}\} \text{ lb} \cdot \text{ft}$$

$$= \{-82.9\mathbf{i} - 96.8\mathbf{j} - 52.8\mathbf{k}\} \text{ lb} \cdot \text{ft}$$

$$\mathbf{r}_{BC} = [0 - (-1.5)]\mathbf{i} + [-(3 + 4\cos 45°) - (-3)]\mathbf{j} + (4\sin 45° - 0)\mathbf{k}$$

$$= \{1.5\mathbf{i} - 2.8284\mathbf{j} + 2.8284\mathbf{k}\} \text{ ft}$$

$$\mathbf{r}_{BD} = [-5 - (-1.5)]\mathbf{i} + [0 - (-3)]\mathbf{j} + (0 - 0)\mathbf{k} = \{-3.5\mathbf{i} + 3\mathbf{j}\} \text{ ft}$$

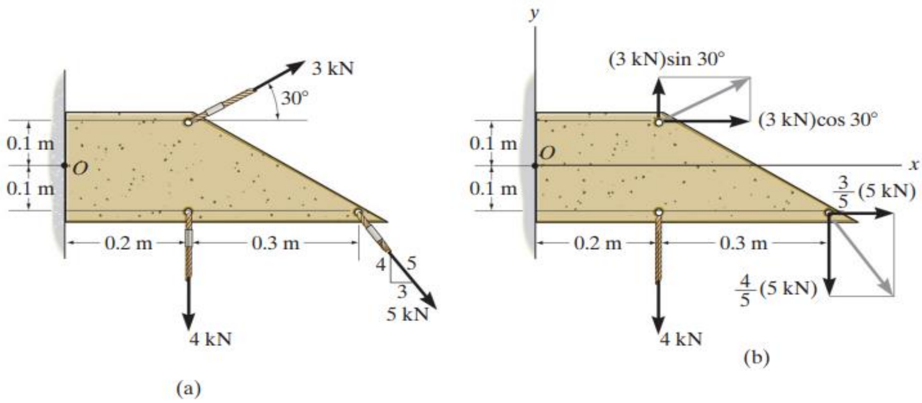
$$\mathbf{r}_{CD} = (-5-0)\mathbf{i} + \{0 - [-(3 + 4\cos 45°)]\}\mathbf{j} + (0 - 4\sin 45°)\mathbf{k}$$

$$= \{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}\} \text{ ft}$$

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{CD}}{\mathbf{r}_{CD}}\right) = 80\left(\frac{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{(-5)^2 + 5.8284^2 + (-2.8284)^2}}\right)$$

$$= \{-48.88\mathbf{i} + 56.98\mathbf{j} - 27.65\mathbf{k}\} \text{ lb}$$

EXAMPLE7 : Replace the force and couple system shown in Fig. a by an equivalent resultant force and couple moment acting at point O.



Force Summation

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = (3\text{ kN})\cos 30^\circ + \left(\tfrac{3}{5}\right)(5\text{ kN}) = 5.598\text{ kN} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = (3\text{ kN})\sin 30^\circ - \left(\tfrac{4}{5}\right)(5\text{ kN}) - 4\text{ kN} = -6.50\text{ kN} = 6.50\text{ kN}\downarrow$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{(5.598\text{ kN})^2 + (6.50\text{ kN})^2} = 8.58\text{ kN}$$

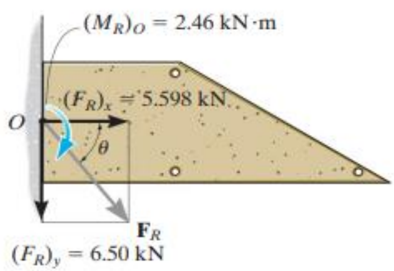
$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{6.50\text{ kN}}{5.598\text{ kN}}\right) = 49.3^\circ$$

Moment Summation

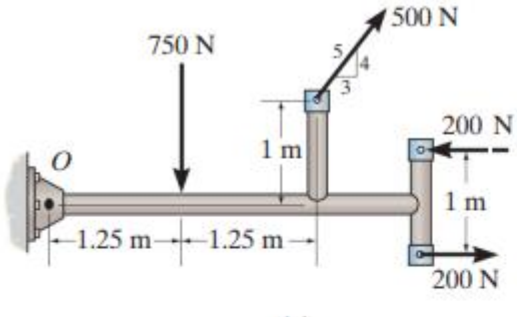
$$\circlearrowleft + (M_R)_O = \Sigma M_O;$$

$$(M_R)_O = (3\text{ kN})\sin 30^\circ(0.2\text{ m}) - (3\text{ kN})\cos 30^\circ(0.1\text{ m}) + \left(\tfrac{3}{5}\right)(5\text{ kN})(0.1\text{ m}) - \left(\tfrac{4}{5}\right)(5\text{ kN})(0.5\text{ m}) - (4\text{ kN})(0.2\text{ m})$$

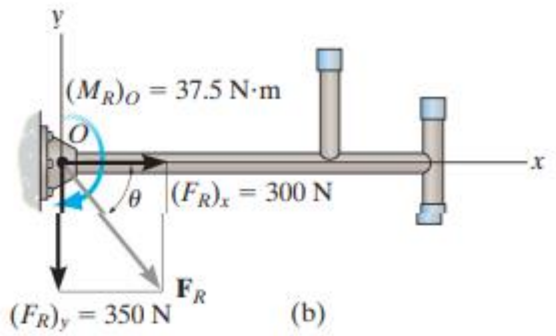
$$= -2.46\text{ kN}\cdot\text{m} = 2.46\text{ kN}\cdot\text{m} \circlearrowright \qquad \textit{Ans.}$$



Example 8 :Replace the force and couple system acting on the member in Fig. a by an equivalent resultant force and couple moment acting at point O.



(a)



(b)

Force Summation

$$\xrightarrow{+} (F_R)_x = \Sigma F_x;\ (F_R)_x = \left(\tfrac{3}{5}\right)(500\text{ N}) = 300\text{ N}\rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;\ (F_R)_y = (500\text{ N})\left(\tfrac{4}{5}\right) - 750\text{ N} = -350\text{ N} = 350\text{ N}\downarrow$$

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(300\text{ N})^2 + (350\text{ N})^2} = 461\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350\text{ N}}{300\text{ N}}\right) = 49.4°$$

Moment Summation

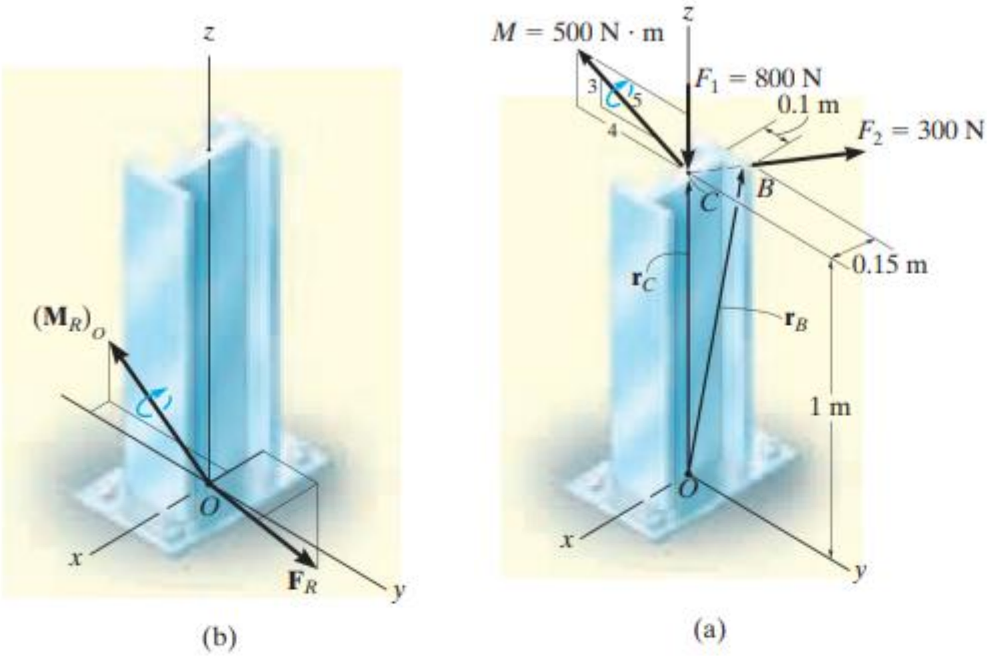
$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright$$

EXAMPLE9 :The structural member is subjected to a couple moment M and forces F1 and F2 in Fig. a. Replace this system by an equivalent resultant force and couple moment acting at its base, point O.



SOLUTION :(VECTOR ANALYSIS) The three-dimensional aspects of the problem can be simplified by using a Cartesian vector analysis. Expressing the forces and couple moment as Cartesian vectors, we have:

$$\mathbf{F}_1 = \{-800\mathbf{k}\} \text{ N}$$

$$\mathbf{F}_2 = (300 \text{ N})\mathbf{u}_{CB}$$

$$= (300 \text{ N})\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right)$$

$$= 300 \text{ N}\left[\frac{\{-0.15\mathbf{i} + 0.1\mathbf{j}\} \text{ m}}{\sqrt{(-0.15 \text{ m})^2 + (0.1 \text{ m})^2}}\right] = \{-249.6\mathbf{i} + 166.4\mathbf{j}\} \text{ N}$$

$$\mathbf{M} = -500\left(\tfrac{4}{5}\right)\mathbf{j} + 500\left(\tfrac{3}{5}\right)\mathbf{k} = \{-400\mathbf{j} + 300\mathbf{k}\} \text{ N} \cdot \text{m}$$

Force Summation:

$$\mathbf{F}_1 = \{-800\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = (300\ \text{N})\mathbf{u}_{CB}$$

$$= (300\ \text{N})\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right)$$

$$= 300\ \text{N}\left[\frac{\{-0.15\mathbf{i} + 0.1\mathbf{j}\}\ \text{m}}{\sqrt{(-0.15\ \text{m})^2 + (0.1\ \text{m})^2}}\right] = \{-249.6\mathbf{i} + 166.4\mathbf{j}\}\ \text{N}$$

$$\mathbf{M} = -500\left(\tfrac{4}{5}\right)\mathbf{j} + 500\left(\tfrac{3}{5}\right)\mathbf{k} = \{-400\mathbf{j} + 300\mathbf{k}\}\ \text{N}\cdot\text{m}$$

Moment Summation

$$(\mathbf{M}_R)_o = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

$$(\mathbf{M}_R)_o = \mathbf{M} + \mathbf{r}_C \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

$$(\mathbf{M}_R)_o = (-400\mathbf{j} + 300\mathbf{k}) + (1\mathbf{k}) \times (-800\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.1 & 1 \\ -249.6 & 166.4 & 0 \end{vmatrix}$$

$$= (-400\mathbf{j} + 300\mathbf{k}) + (0) + (-166.4\mathbf{i} - 249.6\mathbf{j})$$

$$= \{-166\mathbf{i} - 650\mathbf{j} + 300\mathbf{k}\}\ \text{N}\cdot\text{m}$$