الجلسة الخامسة -توازن الأجسام في الفراغ

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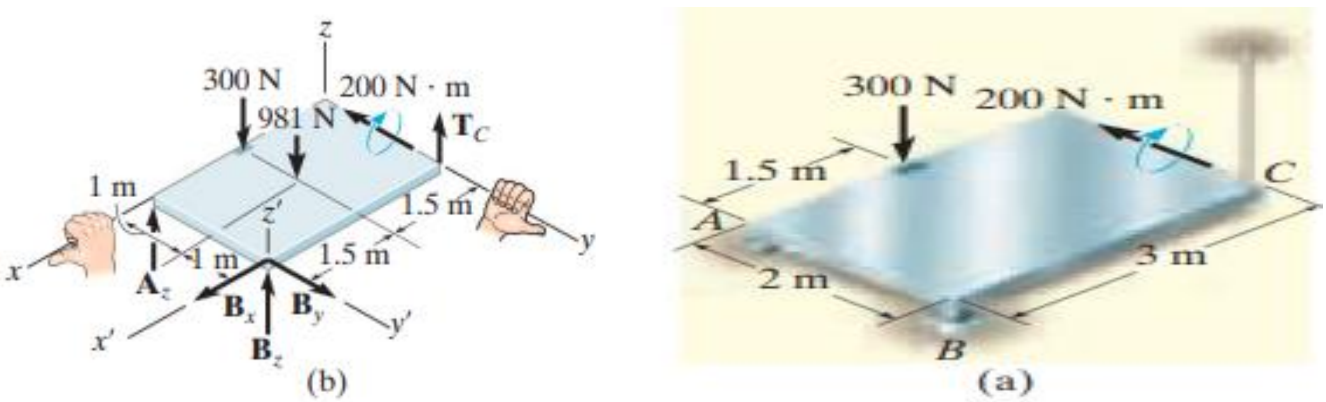
EXAMPLE 1:The homogeneous plate shown in Fig has a mass of 100 kg and is subjected to a force and couple moment along its edges. If it is supported in the horizontal plane by a roller at A, a ball-and-socket joint at B, and a cord at C, determine the components of reaction at these supports.



$$\Sigma F_x = 0; \qquad B_x = 0$$

$$\Sigma F_y = 0; \qquad B_y = 0$$

$$\Sigma F_z = 0; \qquad A_z + B_z + T_C - 300\text{ N} - 981\text{ N} = 0$$

$$\Sigma M_x = 0; \quad T_C(2\text{ m}) - 981\text{ N}(1\text{ m}) + B_z(2\text{ m}) = 0 \quad (2)$$

$$\Sigma M_y = 0; \quad 300\text{ N}(1.5\text{ m}) + 981\text{ N}(1.5\text{ m}) - B_z(3\text{ m}) - A_z(3\text{ m}) - 200\text{ N}\cdot\text{m} = 0 \quad (3)$$

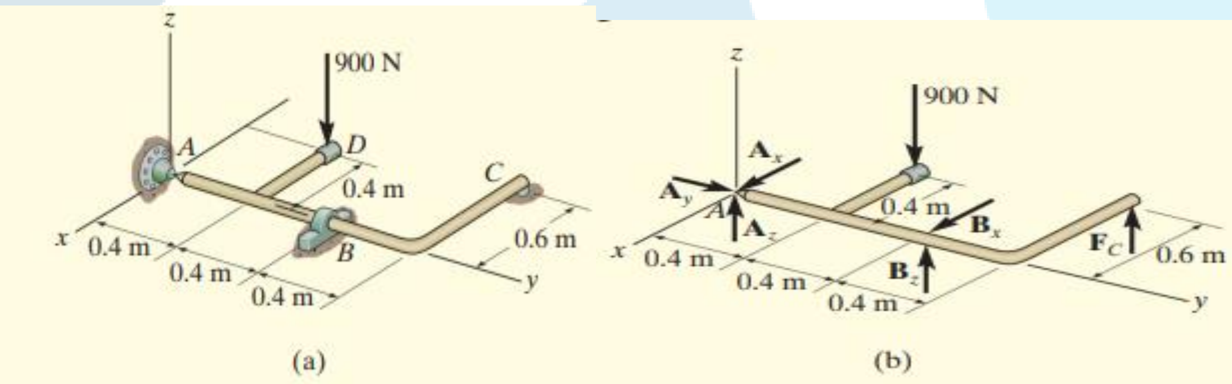
العزوم حول x` ,y`

$$\Sigma M_{x'} = 0; \quad 981\text{ N}(1\text{ m}) + 300\text{ N}(2\text{ m}) - A_z(2\text{ m}) = 0 \quad (4)$$

$$\Sigma M_{y'} = 0; \quad -300\text{ N}(1.5\text{ m}) - 981\text{ N}(1.5\text{ m}) - 200\text{ N}\cdot\text{m} + T_C(3\text{ m}) = 0 \quad (5)$$

$$A_z = 790\text{ N} \quad B_z = -217\text{ N} \quad T_C = 707\text{ N}$$

EXAMPLE2 :Determine the components of reaction that the ball-and-socket joint at A, the smooth journal bearing at B, and the roller support at C exert on the rod assembly in Fig. a



$$\Sigma F_y = 0; \qquad A_y = 0$$

$$\Sigma M_y = 0; \qquad F_C(0.6\text{ m}) - 900\text{ N}(0.4\text{ m}) = 0$$
$$F_C = 600\text{ N}$$

$$\Sigma M_x = 0; \qquad B_z(0.8\text{ m}) + 600\text{ N}(1.2\text{ m}) - 900\text{ N}(0.4\text{ m}) = 0$$
$$B_z = -450\text{ N}$$

$$\Sigma M_z = 0; \qquad -B_x(0.8\text{ m}) = 0 \quad B_x = 0$$

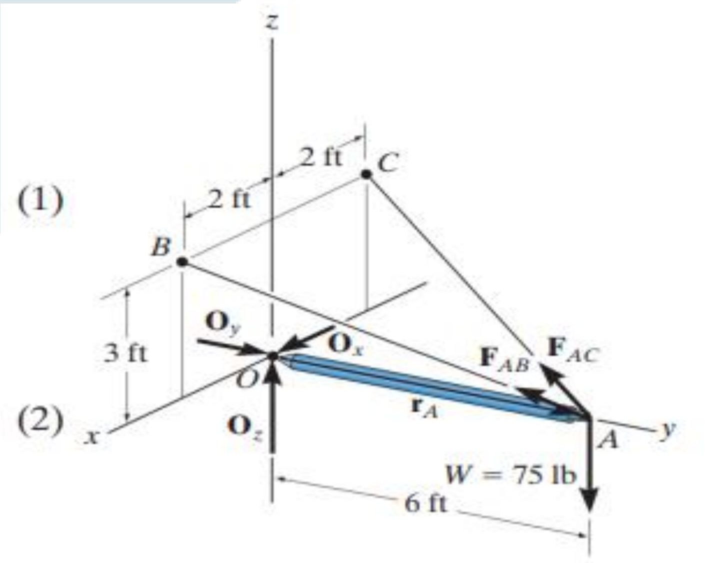
Thus,

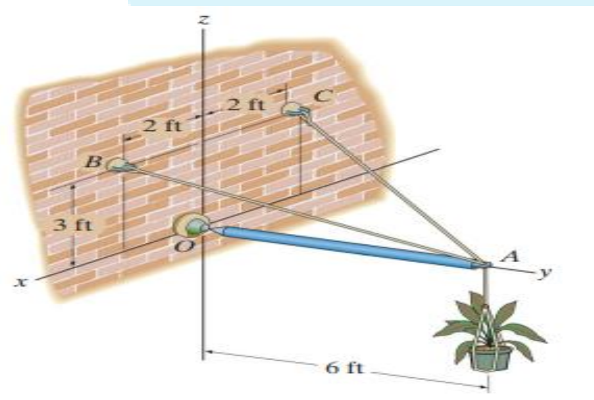
$$\Sigma F_x = 0; \qquad A_x + 0 = 0 \qquad A_x = 0$$

Finally, using the results of B_z and F_C.

$$\Sigma F_z = 0; \qquad A_z + (-450\text{ N}) + 600\text{ N} - 900\text{ N} = 0$$
$$A_z = 750\text{ N}$$

EXAMPLE 3 :The boom is used to support the 75-lb flowerpot in Fig. a. Determine the tension developed in wires AB and AC.





(a)

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left(\frac{\{2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\ \text{ft}}{\sqrt{(2\ \text{ft})^2 + (-6\ \text{ft})^2 + (3\ \text{ft})^2}}\right)$$
$$= \tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left(\frac{\{-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\ \text{ft}}{\sqrt{(-2\ \text{ft})^2 + (-6\ \text{ft})^2 + (3\ \text{ft})^2}}\right)$$

$$= -\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}$$

$$\Sigma\mathbf{M}_O = \mathbf{0}; \qquad \mathbf{r}_A \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{W}) = \mathbf{0}$$

$$(6\mathbf{j}) \times \left[\left(\tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}\right) + \left(-\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}\right) + (-75\mathbf{k})\right] = \mathbf{0}$$

$$\left(\tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450\right)\mathbf{i} + \left(-\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC}\right)\mathbf{k} = \mathbf{0}$$

$$\Sigma M_x = 0; \qquad \tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450 = 0 \qquad (1)$$

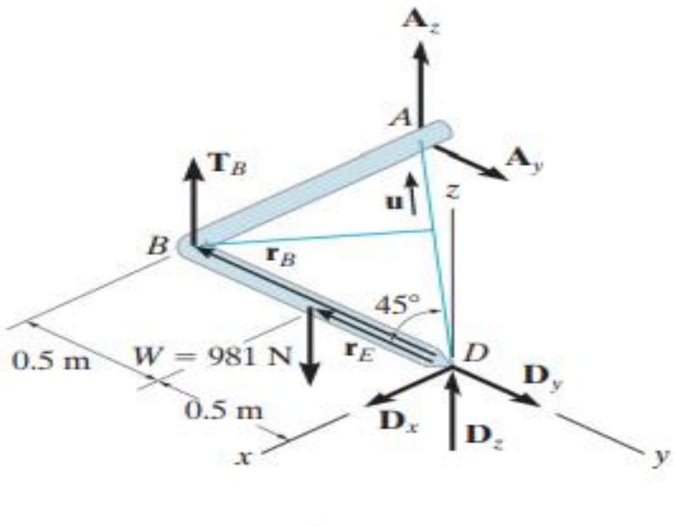
$$\Sigma M_y = 0; \qquad 0 = 0$$

$$\Sigma M_z = 0; \qquad -\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC} = 0$$

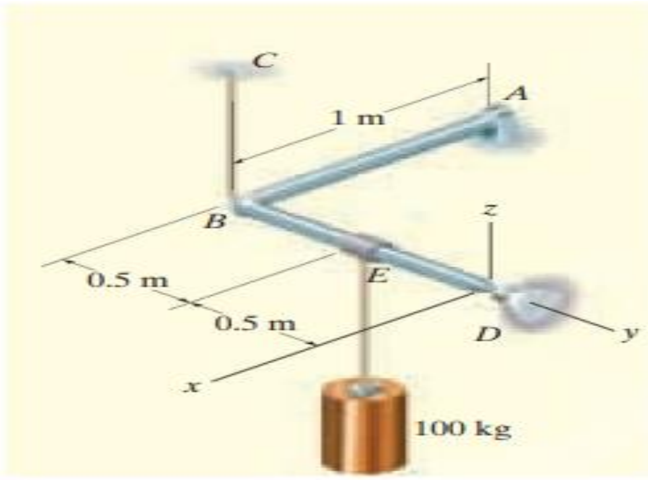
$$F_{AB} = F_{AC} = 87.5\ \text{lb}$$

EXAMPLE 4 : The bent rod in Fig. a is supported at A by a journal bearing, at D by a ball-and-socket joint, and at B by means of cable BC. Using only one equilibrium equation, obtain a direct solution for the tension in cable BC. The bearing at A is

capable of exerting force components only in the z and y directions since it is properly aligned on the shaft. In other words, no couple moments are required at this support.



(b)



(a)

The cable tension TB may be obtained directly by summing moments about an axis that passes through points D and A. The direction of this axis is defined by the unit vector u, where:

$$\mathbf{u} = \frac{\mathbf{r}_{DA}}{r_{DA}} = -\frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$$
$$= -0.7071\mathbf{i} - 0.7071\mathbf{j}$$

$$\Sigma M_{DA} = \mathbf{u} \cdot \Sigma(\mathbf{r} \times \mathbf{F}) = 0$$

$$\mathbf{u} \cdot (\mathbf{r}_B \times \mathbf{T}_B + \mathbf{r}_E \times \mathbf{W}) = \mathbf{0}$$

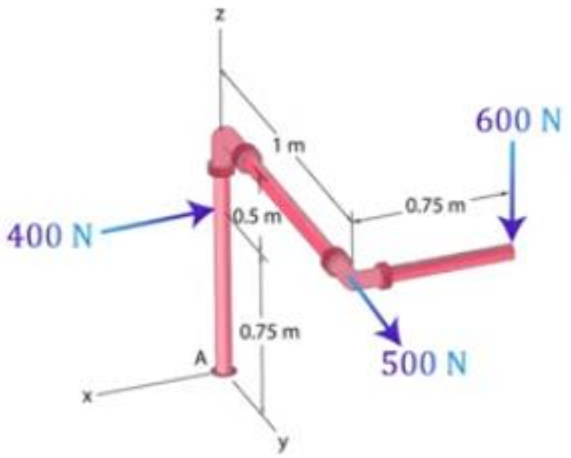
$$(-0.7071\mathbf{i} - 0.7071\mathbf{j}) \cdot \left[(-1\mathbf{j}) \times (T_B\mathbf{k}) + (-0.5\mathbf{j}) \times (-981\mathbf{k}) \right] = \mathbf{0}$$

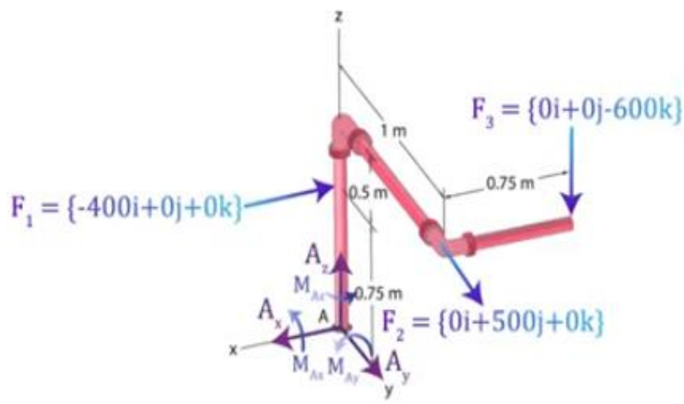
$$-0.7071\mathbf{i} - 0.7071\mathbf{j}) \cdot [(-T_B + 490.5)\mathbf{i}] = \mathbf{0}$$

$$-0.7071(-T_B + 490.5) + 0 + 0 = 0$$

$$T_B = 490.5 \text{ N} \qquad \textit{Ans.}$$

Example 5 : Determine the reactions at fixed support A .





$$\sum F_y = 0$$
$$A_y + 500 = 0$$

$$\sum F_z = 0$$
$$A_z - 600 = 0$$

$$\sum F_x = 0$$
$$A_x - 400 = 0$$
$$A_x = 400 \text{ N}$$

$$F_1 = \{-400i+0j+0k\}$$
$$F_2 = \{0i+500j+0k\}$$
$$F_3 = \{0i+0j-600k\}$$

$$\mathbf{r}_1 = \{0\mathbf{i} + 0\mathbf{j} + 0.75\mathbf{k}\}$$
$$\mathbf{r}_2 = \{0\mathbf{i} + 1\mathbf{j} + 1.25\mathbf{k}\}$$
$$\mathbf{r}_3 = \{-0.75\mathbf{i} + 1\mathbf{j} + 1.25\mathbf{k}\}$$

$$\mathbf{M}_{A1} = \mathbf{r}_1 \times \mathbf{F}_1$$

$$\mathbf{M}_{A1} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.75 \\ -400 & 0 & 0 \end{vmatrix}$$

$$\mathbf{M}_{A1} = \{0\mathbf{i} - 300\mathbf{j} + 0\mathbf{k}\}$$

$$\mathbf{M}_{A2} = \mathbf{r}_2 \times \mathbf{F}_2$$

$$\mathbf{M}_{A2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1.25 \\ 0 & 500 & 0 \end{vmatrix}$$

$$\mathbf{M}_{A2} = \{-625\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}\}$$

$$\mathbf{M}_{A3} = \mathbf{r}_3 \times \mathbf{F}_3$$

$$\mathbf{M}_{A3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.75 & 1 & 1.25 \\ 0 & 0 & -600 \end{vmatrix}$$

$$\mathbf{M}_{A3} = \{-600\mathbf{i} - 450\mathbf{j} + 0\mathbf{k}\}$$

y

$$\sum \mathbf{M} = 0$$

$$\mathbf{M}_{A1} + \mathbf{M}_{A2} + \mathbf{M}_{A3} = 0$$

$$(-625 - 600 + M_{Ax})\mathbf{i} + (-300 - 450 + M_{Ay})\mathbf{j} + (M_{Az})\mathbf{k} = 0$$

$$(-625 - 600 + M_{Ax})\mathbf{i} + (-300 - 450 + M_{Ay})\mathbf{j} + (M_{Az})\mathbf{k} = 0$$

i components

$$-625 - 600 + M_{Ax} = 0$$

$$\boxed{M_{Ax} = 1225\ \text{N} \cdot \text{m}}$$

j components

$$-300 - 450 + M_{Ay} = 0$$

$$\boxed{M_{Ay} = 750\ \text{N} \cdot \text{m}}$$

k components

$$\boxed{M_{Az} = 0\ \text{N} \cdot \text{m}}$$