جَامعة المَنارة
MANARA UNIVERSITY

Steel Structures 2
Summer Sem.
2026-2025

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Lecture 9-10

- Flexural Members

- ✓ -I- Laterally Restrained Beams
- ✓ II- Laterally Unrestrained Beams

- Beam-Column Members II

Beam-Column Members

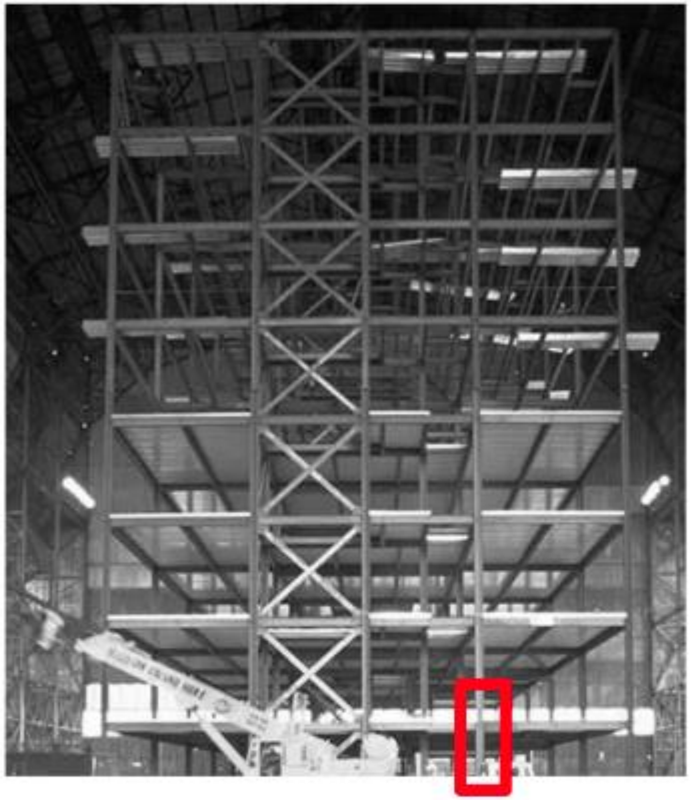
Introduction: Beam-Column Members

EXAMPLE 3

Safety check of a beam-column of the first storey of the building illustrated in the figure. The member, composed by a HEB 320 cross section in steel S 355, has a length of 4.335.

The relevant geometric characteristics of HEB 320 cross section are: $A = 161.3\ cm^2$; $W_{pl,y} = 2149\ cm^3$, $I_y = 30820\ cm^4$, $i_y = 13.82\ cm$; $I_z = 9239\ cm^4$, $i_z = 7.57\ cm$; $I_T = 225.1\ cm^4$ and $I_W = 2069 \times 10^3\ cm^6$.

The mechanical characteristics of the material are: $f_y = 355\ MPa$, $E = 210\ GPa$ and $G = 81\ GPa$.

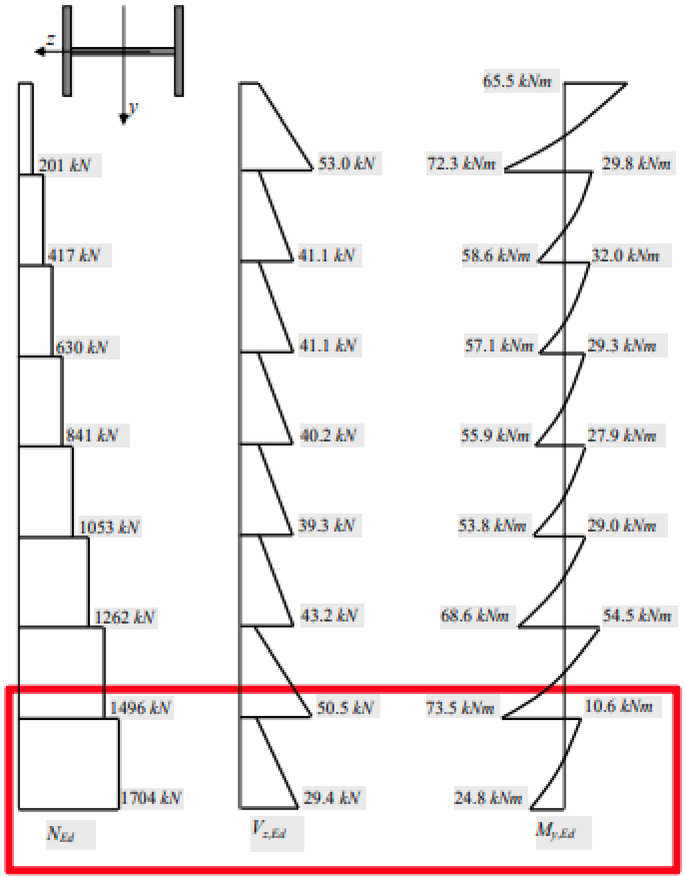


Introduction: Beam-Column Members

EXAMPLE 3

The design internal forces obtained through the structure analysis (second order) for the various load combinations are illustrated in the figure. Two simplification are assumed for the subsequent design verifications: i) the shear force is sufficient small so can be neglected; ii) the shape of the bending moment diagram is linear.

Design values are: N_{Ed} **= 1704 kN;** $M_{y,Ed}$ **= 24.8 kNm** at the base cross section.



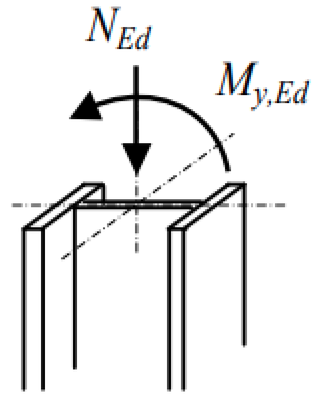
i) Cross section classification

As the compression force is high, the cross section is classified under compression only (conservative approach). As the section HEB 320 is a stocky section, even under this load condition, is **class 1.**

EXAMPLE 3

ii) Cross section resistance

The design internal forces: $M_{y,Ed}$ = **24.8 kNm** | N_{Ed} = **1704 *kN*** (compression).



$$N_{pl,Rd} = Af_y/\gamma_{M0} = 161.3\times10^{-4}\times355\times10^3/1.0 = 5726.2\,\text{kN}$$

As, $N_{Ed} = 1704\,kN < N_{pl,Rd} = 5726.2\,\text{kN}$, the axial force resistance is verified.

Since, $N_{Ed} = 1704\text{kN} > 0.25N_{pl,Rd} = 1431.5\text{kN}$,

in accordance with clause 6.2.9.1(4) of EC3-1-1, it is necessary to reduce the **plastic bending resistance** (to $M_{N,y,Rd}$):

$$M_{pl,y,Rd} = \frac{W_{pl,y}f_y}{\gamma_{Mo}} = \frac{2149\times10^{-6}\times355\times10^3}{1.0} = 762.9\,kNm$$

$$a = \frac{A-2bt_f}{A} = \frac{161.3-2\times30\times2.05}{161.3} = 0.24$$

$$M_{N,y,Rd} = M_{pl,y,Rd}\frac{1-n}{1-0.5a}$$

$$n = \frac{N_{Ed}}{N_{pl,Rd}} = \frac{1704}{5726.2} = 0.30$$

As, $M_{y,Ed} = 24.8\,kNm < M_{N,y,Rd} = 606.9\,\text{kNm}$, the bending resistance is verified.

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Introduction: Beam-Column Members

EXAMPLE 3

iii) Verification of the stability of the member

In this example the Method 2 is applied. As the member is susceptible to torsional deformations (thin-walled open cross section), it is assumed that lateral-torsional buckling constitutes the relevant instability mode. Since $M_{z,Ed}$ = 0, the following conditions must be verified:

$$\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}} + k_{yy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} \leq 1.0$$

$$\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}} + k_{zy}\frac{M_{y,Ed}}{\chi_{LT} M_{y,Rk}/\gamma_{M1}} \leq 1.0$$

Step 1: characteristic resistance of the cross section

$$N_{Rk} = A f_y = 161.3 \times 10^{-4} \times 355 \times 10^{3} = 5726.2\,\text{kN}$$

$$M_{y,Rk} = W_{pl,y}\, f_y = 2149 \times 10^{-6} \times 355 \times 10^{3} = 762.9\,\text{kNm}$$

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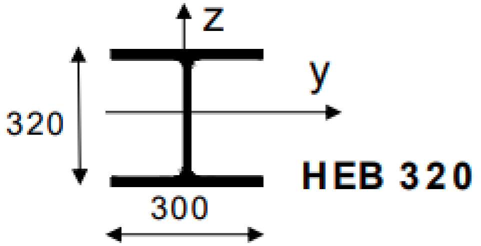
Introduction: Beam-Column Members

EXAMPLE 3

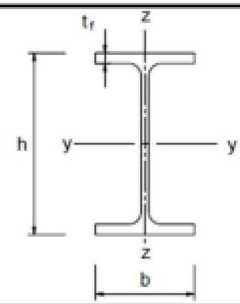
***Step 2*:** reduction coefficients due to flexural buckling, χ_y and χ_z



$$\frac{h}{b}=\frac{320}{300}=1.07<1.2 \quad \text{and} \quad t_f=20.5\,mm<100\,mm$$

$$\Rightarrow \begin{array}{l} \textit{flexural buckling around } y - \textit{curve } b(\alpha=0.34) \\ \textit{flexural buckling around } z - \textit{curve } c(\alpha=0.49). \end{array}$$

Table 6.2: Selection of buckling curve for a cross-section

Cross section		Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled sections		h/b > 1,2	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
			40 mm < $t_f \le 100$	y – y z – z	b c	a a
		h/b ≤ 1,2	$t_f \le 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c

<u>Plane *xz*</u> - $L_{E,y}$ = 4.335 *m*.

$$\bar{\lambda}_y=\frac{L_{E,y}}{i_y}\frac{1}{\lambda_1}=\frac{4.335}{13.82\times10^{-2}}\times\frac{1}{93.9\times0.81}=0.41$$

$$\Phi_y=0.62 \quad \Rightarrow \quad \boxed{\chi_y=0.92}$$

<u>Plane *xy*</u> - $L_{E,z}$ = 4.335 *m*

$$\bar{\lambda}_z=\frac{L_{E,z}}{i_z}\frac{1}{\lambda_1}=\frac{4.335}{7.57\times10^{-2}}\times\frac{1}{93.9\times0.81}=0.75$$

$$\Phi_z=0.92 \quad \Rightarrow \quad \boxed{\chi_z=0.69}$$

$$\phi=0.5\left[1+\alpha\left(\bar{\lambda}-0.2\right)+\bar{\lambda}^2\right]$$

$$\chi=\frac{1}{\Phi+\left(\Phi^2-\bar{\lambda}^2\right)^{0.5}}$$

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DESIGN OF BEAM-COLUMNS

EXAMPLE 3

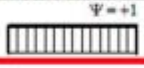
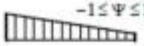
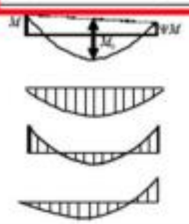
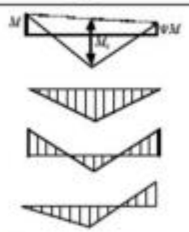
Step 3: calculation of the χ_{LT} using the alternative method applicable to rolled or equivalent welded sections (clause 6.3.2.3 of EC3-1-1)

The correction factor k_c, according to Table 6.6 of EC3-1-1, with Ψ = 10.6/(-24.8) = - 0.43, is given by:

$$k_c = \frac{1}{1.33 - 0.33\,\Psi} = \frac{1}{1.33 - 0.33 \times (-0.43)} = 0.68$$

$$f = 1 - 0.5 \times (1 - k_c) \times \left[1 - 2.0 \times (\bar{\lambda}_{LT} - 0.8)^2\right]$$

$$= 1 - 0.5 \times (1 - 0.68) \times \left[1 - 2.0 \times (0.39 - 0.8)^2\right] = 0.89$$

Diagram of bending moments	k_c
	1.0
	$\frac{1}{1.33 - 0.33\Psi}$
	0.94 0.90 0.91
	0.86 0.77 0.82
Ψ - ratio between end moments, with -1≤Ψ≤1.	

The modified lateral-torsional buckling reduction factor is given by:

$$\chi_{LT,\text{mod}} = 0.99/0.89 = 1.11 > 1.00$$

So, $\boxed{\chi_{LT,\text{mod}} = 1.00}$ must be adopted.

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Diagram of bending moments	k_c
Ψ = +1	1.0
−1 ≤ Ψ ≤ 1	$\frac{1}{1.33 - 0.33\Psi}$
M, ΨM, M_s	0.94 0.90 0.91
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Ψ - ratio between end moments, with -1≤Ψ≤1.	

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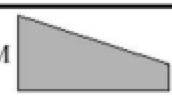
EXAMPLE 3

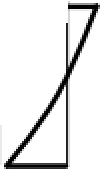
***Step 4*:** interaction factors k_{yy} and k_{zy}.

The equivalent factors of uniform moment C_{my} and C_{mLT} are obtained based on the bending moment diagram, between braced sections according to the z direction in case of C_{my} and laterally in case of C_{mLT}. Assuming the member braced in z direction and laterally just at the base and top cross sections, the factors C_{my} and C_{mLT} must be calculated based on the bending moment diagram along the total length of the member.

Since the bending moment diagram is assumed linear, defined by:
$M_{y,Ed,base}$= -24.8 kNm;
$M_{y,Ed,top}$ = 10.4 kNm, from Table B.3 of EC3-1-1, is obtained:

Table B.3: Equivalent uniform moment factors C_m in Tables B.1 and B.2

Moment diagram	range	C_{my} and C_{mz} and C_{mLT}	
		uniform loading	concentrated load
M  ψM	$-1 \le \psi \le 1$	$0{,}6 + 0{,}4\psi \ge 0{,}4$	



$$\psi = M_{y,Ed,top}/M_{y,Ed,base} = (10.6)/(-24.8) = -0.43 \quad \Rightarrow$$

$$\boxed{C_{my} = C_{mLT} = 0.60 + 0.4 \times (-0.43) = 0.43} \quad (> 0.40)$$

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EXAMPLE 3

Because the member is susceptible to torsional deformations, the interaction factors k_{yy} and k_{zy} are obtained from Table B.2 of EC3-1-1, through the following calculations:

$$k_{yy} = C_{my}\left[1+\left(\bar{\lambda}_y - 0.2\right)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right] = 0.43\times\left[1+\left(0.41-0.2\right)\times\frac{1704}{0.92\times 5726.2/1.0}\right] = 0.46;$$

As $k_{yy} \le C_{my}\left(1+0.8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) = 0.54$, then $\boxed{k_{yy} = 0.46}$

$$k_{zy} = \left[1-\frac{0.1\bar{\lambda}_z}{\left(C_{mLT}-0.25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] =$$

$$= \left[1-\frac{0.1\times 0.75}{\left(0.43-0.25\right)}\frac{1704}{0.69\times 5726.2/1.0}\right] = 0.82$$

As $k_{zy} \ge \left[1-\frac{0.1}{\left(C_{mLT}-0.25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] = 0.76$

then $\boxed{k_{zy} = 0.82}$

Step 5: Finally, the verification of equations 6.61 and 6.62 of EC3-1-1 yields:

$$\frac{1704}{0.92\times 5726.2/1.0} + 0.46\times\frac{24.8}{1.00\times 762.9/1.0} = 0.34 < 1.0$$

$$\frac{1704}{0.69\times 5726.2/1.0} + 0.82\times\frac{24.8}{1.00\times 762.1/1.0} = 0.46 < 1.0$$

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Section O.K.