Composite Steel-Concrete Structures
Sem. 2
2025-2026

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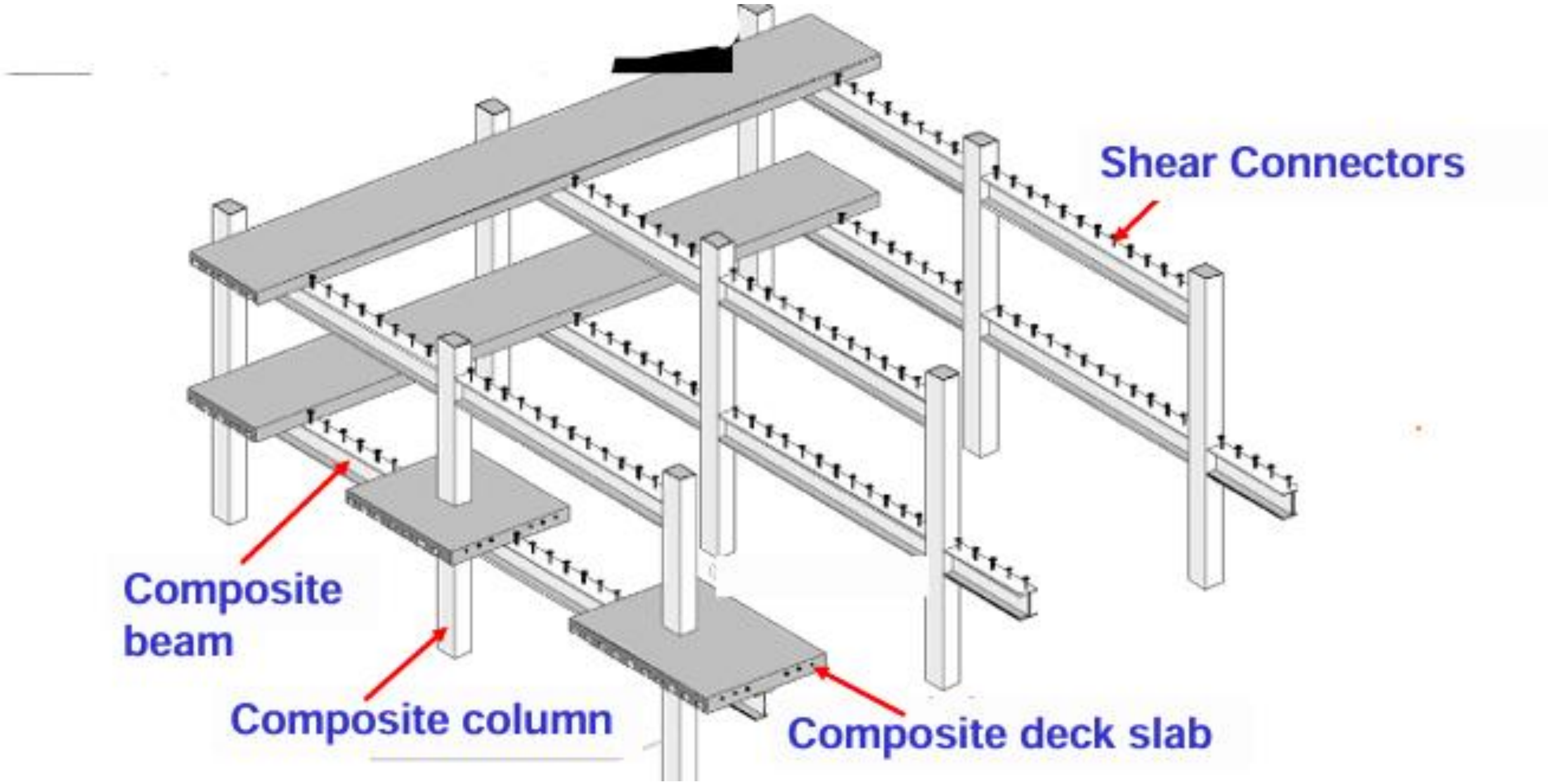
Lecture 7-8

- 4 Assessment of the shear connection for partial shear connection (cont.)

Deflection of simply supported composite beams

- Creep and shrinkage of the concrete
- The deformation of the shear connectors.



Shear Connectors
Composite beam
Composite column
Composite deck slab

1 Plastic moment resistance in sagging bending; Longitudinal shear not critical (lec 3-4)

Case 1. Neutral axis in the concrete slab-
Case 2. Neutral axis in the upper steel flange.
Case 3. Neutral axis in the web of the steel section.

2 Plastic moment resistance in hogging bending; longitudinal shear not critical (lec 5-6)

Case 4. Neutral axis in the upper flange of the steel section
Case 5. Neutral axis in the web of the steel section.

4 Assessment of the shear connection for partial shear connection (cont.) (lec 7-8)

Case 6. Plastic neutral axis in the flange of the steel section
Case 7. Plastic neutral axis in the web of the steel section

4 Assessment of the shear connection for partial shear connection (cont.)

Partial shear connection gives the structural engineer **great design freedom**.

On both economical and technical grounds, he can then choose between a minimum size of steel beam with a relatively large number of studs,

or a heavier steel beam with fewer studs.

The choice depends, for example, on the relative prices of studs and steel sections.

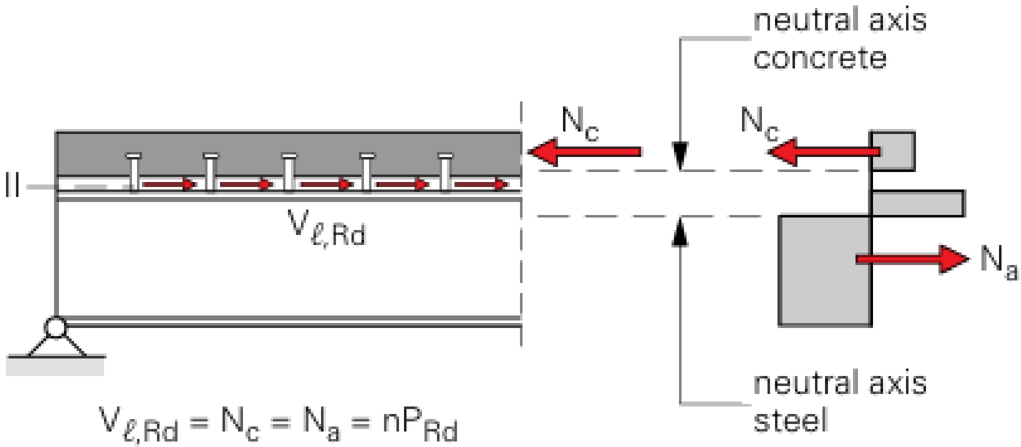
Other factors may also be relevant, such as the construction height and the assembly weight.

4 Assessment of the shear connection for partial shear connection (cont.)

If plastic design is permitted and the studs have sufficient deformation capacity, a stress distribution as shown in figure is achieved in the most heavily loaded section.

The resulting tension force in the steel section is equal to the sum of the resistances of the studs n P_{Rd}.

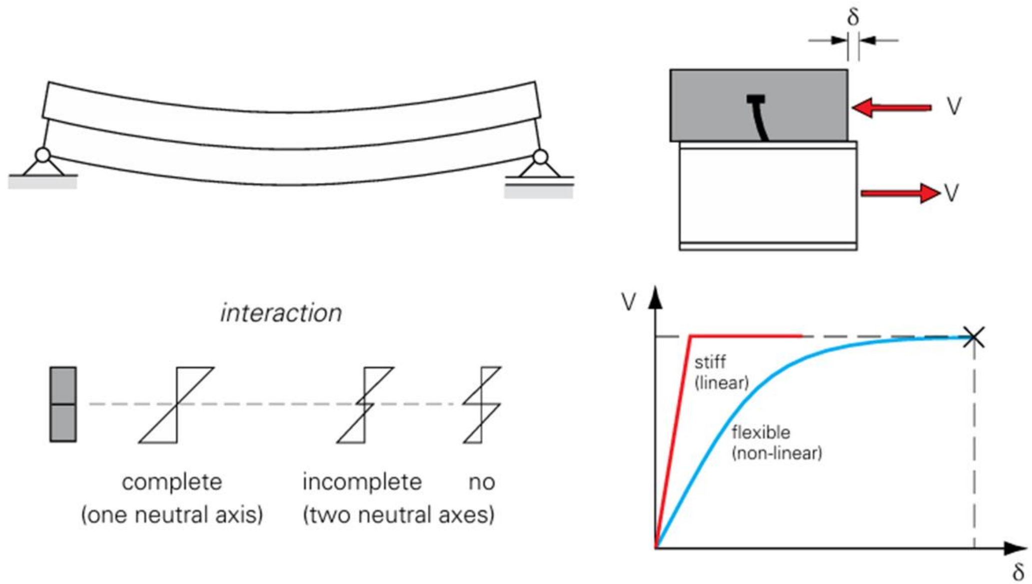
This results in the greatest possible bending moment resistance of the composite section.



4 Assessment of the shear connection for partial shear connection (cont.)

Note that the plastic neutral axes in the concrete and steel do not coincide (as can also be seen from the middle strain diagram in fig. a).

a)The stiffness of the shear connectors (headed studs) determines the level of interaction between the concrete slab and the steel section.



To make this possible, slip must occur and the connectors must have sufficient deformation capacity (ductility) to undergo the slip.

4 Assessment of the shear connection for partial shear connection (cont.)

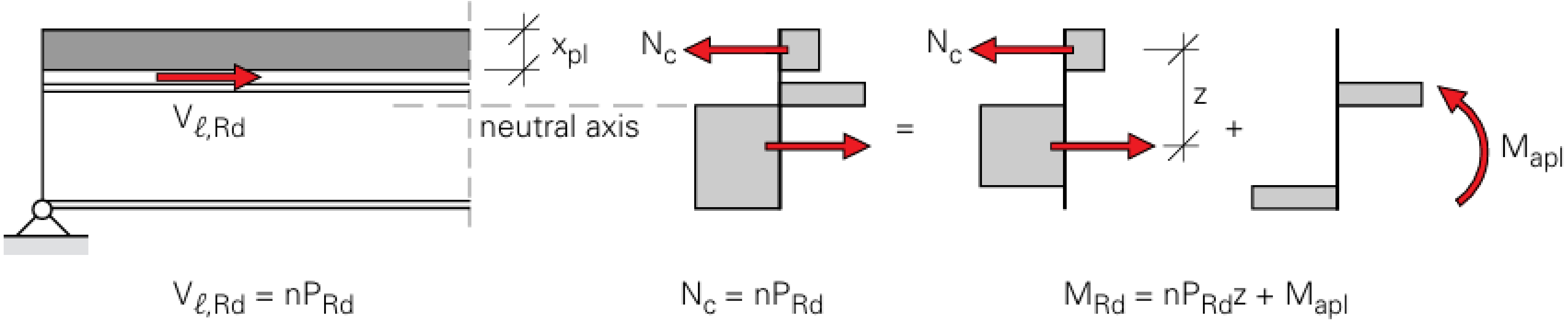
The calculation of moment resistance can be simplified by dividing the resistance into different components (fig. b). Depending on the size of the force $N_c = n\, P_{Rd}$, two possibilities arise with regard to the position of the plastic neutral axis in the steel.

Both can be considered as variations of cases 2 and 3 discussed above.

The compression force $N_c = A\, f_{yd}$ in the concrete slab is replaced by the compression force $N_c = n\, P_{Rd}$,

and the height of the concrete compression zone is calculated accordingly

4 Assessment of the shear connection for partial shear connection (cont.)



b)Division of plastic moment into two components.

- Case 6. Plastic neutral axis in the flange of the steel section. From longitudinal force equilibrium it follows that:

$$N_c = nP_{Rd} > \left(h_a - 2t_f\right)t_w f_{yd}$$

$$x_{pl} = \frac{nP_{Rd}}{b_{eff}0{,}85f_{cd}} \tag{2.24}$$

$$M_{Rd} = N_c\left(\frac{1}{2}h_a + h_p + h_c - \frac{1}{2}x_{pl}\right) + \frac{1}{2}\left(Af_{yd} - N_c\right)\left(h_a - \frac{Af_{yd} - N_c}{2b_a f_{yd}}\right)$$

4 Assessment of the shear connection for partial shear connection (cont.)

• Case 7. Plastic neutral axis in the web of the steel section. From longitudinal force equilibrium it follows that:

$$N_c = nP_{Rd} < (h_a - 2t_f)t_w f_{yd}$$

$$x_{pl} = \frac{nP_{Rd}}{b_{eff} 0{,}85 f_{cd}} \qquad (2.25)$$

$$M_{Rd} = N_c\left(\frac{1}{2}h_a + h_p + h_c - \frac{1}{2}x_{pl}\right) + W_{a,pl}f_{yd} - \frac{N_c^2}{4t_w f_{yd}}$$

Equations (2.24) and (2.25) define a relationship between M_{Rd} (= M_{Ed}) and the number of studs n, as shown by the curve ABC in figure.

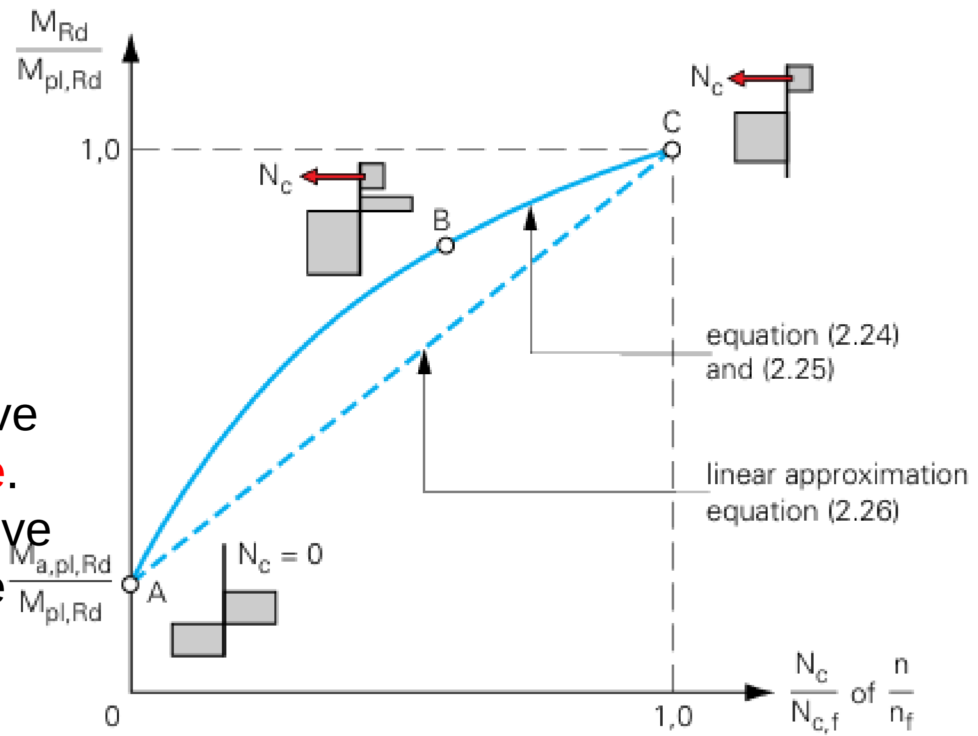
For simplicity, a linear relationship, represented by the straight line AC, is often used in design.

When the load on the beam, and thus M_{Ed}, is known the number of studs determined by the linear relationship follows:

4 Assessment of the shear connection for partial shear connection (cont.)

$$n = \frac{M_{Rd} - M_{a,pl,Rd}}{M_{pl,Rd} - M_{a,pl,Rd}} \, n_f \qquad (2.26)$$

Fig a: Relationship between M_{Rd} and the number of headed studs n.



N_c: Design value of the compressive normal force in the concrete flange.

$N_{c,f}$:Design value of the compressive normal force in the concrete flange with full shear connection

4 Assessment of the shear connection for partial shear connection (cont.)

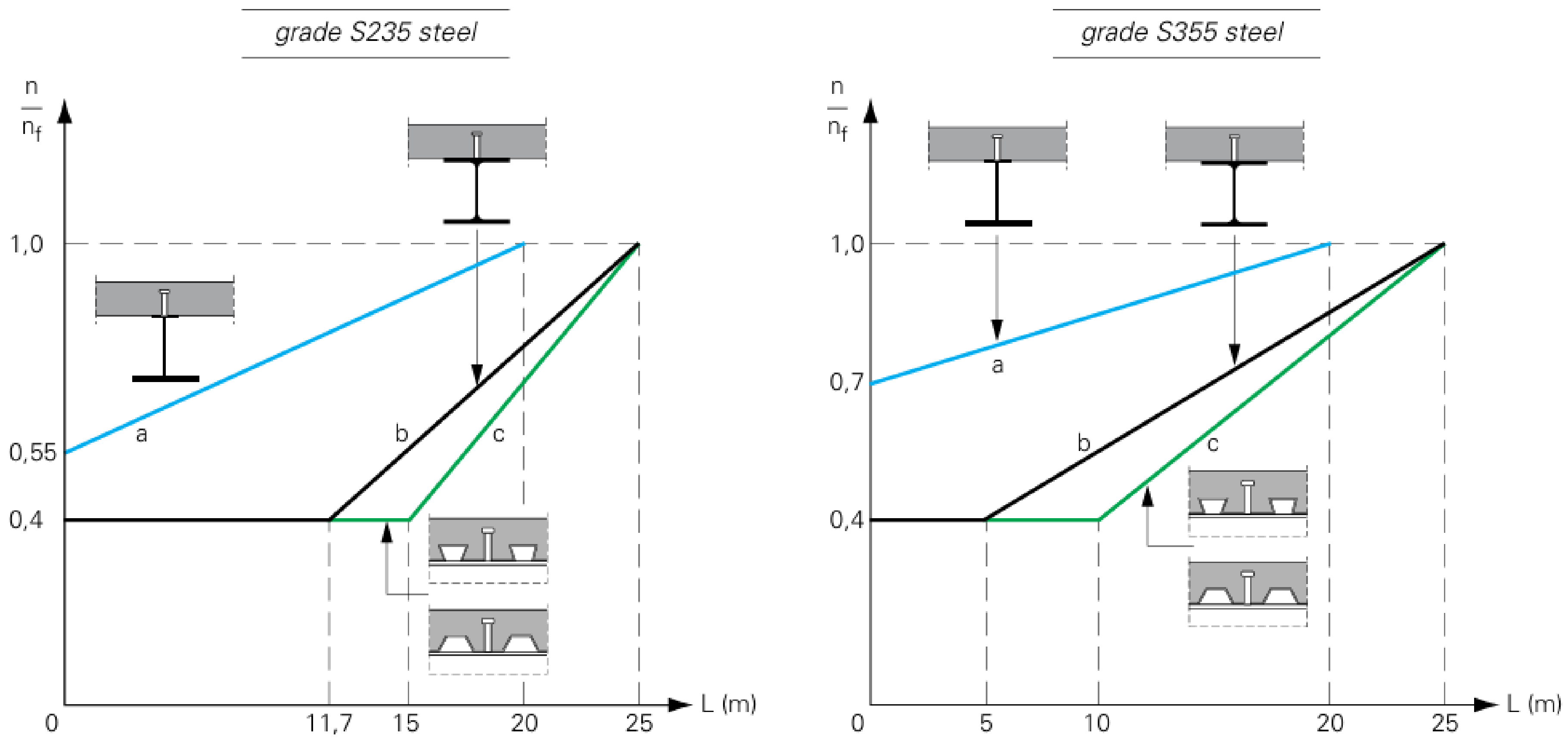
As noted above, the condition to use this Eurocode design model is that the connectors have sufficient ductility.

In EN 1994-1-1, cl. 6.6.1.1(5) is stated that a connector may be taken as ductile if the characteristic slip capacity δ_{uk} is at least 6 mm.

So you need to have sufficient connectors to keep slip below that level.

Therefore, for application of this design model in EN 1994-1-1, cl. 6.6.1.2 (fig.), a lower limit is set for the ratio n/n_f (minimum degree of shear connection).

4 Assessment of the shear connection for partial shear connection (cont.)



Minimum required number of headed studs, depending on the span length and cross section (case a, b or c), to allow the use of plastic theory.

4 Assessment of the shear connection for partial shear connection (cont.)

The given limits are based on parameter calculations and are depending on the span (expressed in meters) and the steel grade.,

The limits also depend on any asymmetry of the steel section (case a) or the use of composite floors (case c) (fig.):

$$\text{case a:} \quad \eta = \frac{n}{n_f} \geq 1 - \frac{355}{f_y}(0{,}30 - 0{,}015L) \quad \text{and} \quad \eta \geq 0{,}4 \quad \text{if} \quad 3A_t = A_b \qquad (2.27)$$

$$\text{case b:} \quad \eta = \frac{n}{n_f} \geq 1 - \frac{355}{f_y}(0{,}75 - 0{,}03L) \quad \text{and} \quad \eta \geq 0{,}4 \quad \text{if} \quad A_t = A_b \qquad (2.28)$$

$$\text{case c:} \quad \eta = \frac{n}{n_f} \geq 1 - \frac{355}{f_y}(1{,}0 - 0{,}04L) \quad \text{and} \quad \eta \geq 0{,}4 \quad \text{if} \quad A_t = A_b \qquad (2.29)$$

A_t is the area of the top flange and A_b the area of the bottom flange of the steel section. For asymmetric beams with $A_t < A_b < 3A_t$, linear interpolation between equation (2.27) and (2.28) is permitted.

Cases a and b apply to beams with both composite and solid concrete slabs.

Case c applies exclusively to beams with composite slabs with ribs perpendicular to the beam, for which all the **following conditions** must also be met:

4 Assessment of the shear connection for partial shear connection (cont.)

Case c applies exclusively to beams with composite slabs with ribs perpendicular to the beam, for which all the **following conditions** must also be met:

– not more than one stud per rib with d = 19 mm and h ≥ 76 mm;

– steel sheeting with $b_0/hp \geq 2$ and $h_p \leq 60$ mm;

– the linear approximation according to equation (2.26) is adopted:

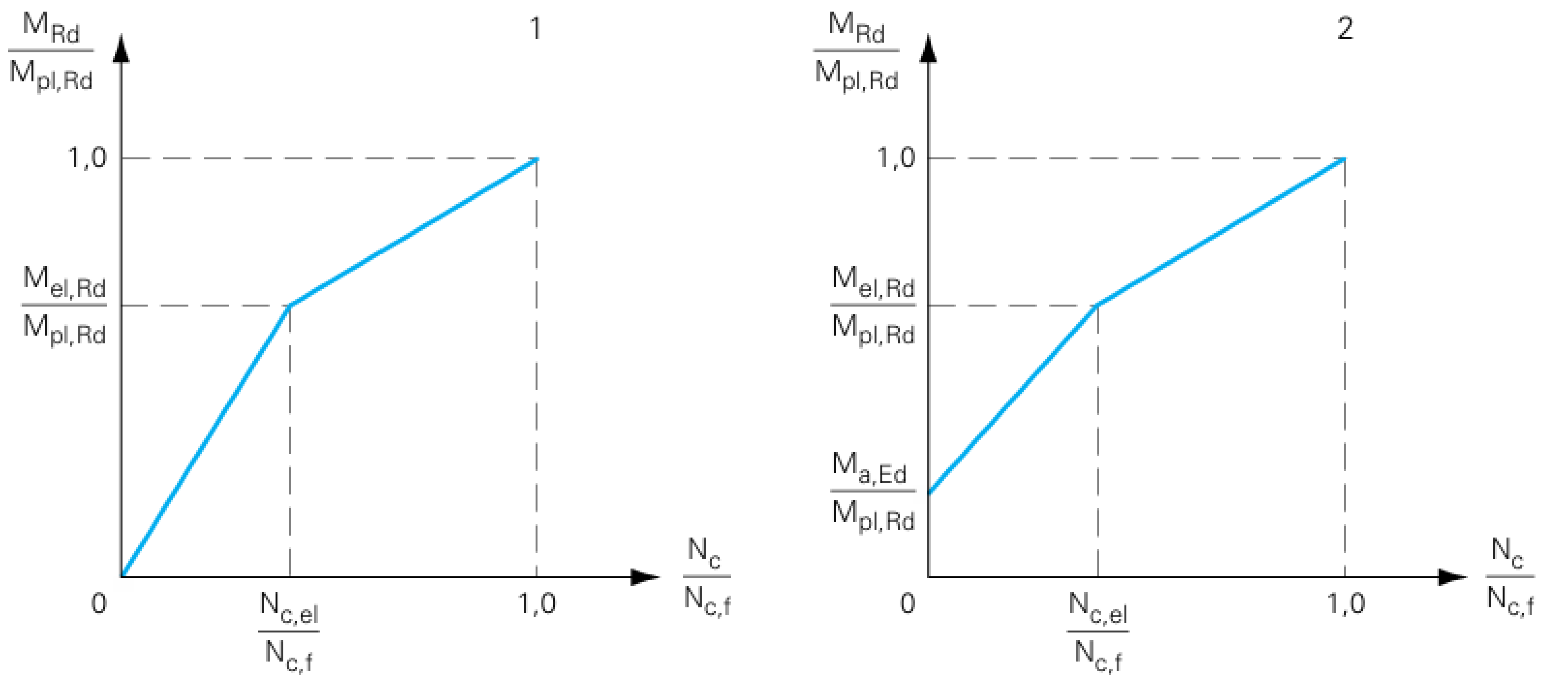
this is the line AC in figure a.

When plastic design is not possible, or when the shear connectors do not have sufficient deformation capacity, the assessment of the shear connection shall be carried out according to linear or nonlinear elasticiy theory.

This type of assessment is covered by EN 1994-1-1, cl. 6.2.1.5 and cl. 6.2.1.4.

For cross sections in class 1 and 2, the simplified relationship between M_{Rd} and N_c as shown in figure may be used.

4 Assessment of the shear connection for partial shear connection (cont.)



Simplified relationship between M_{Rd} and N_c in the case that the concrete slab is in compression.

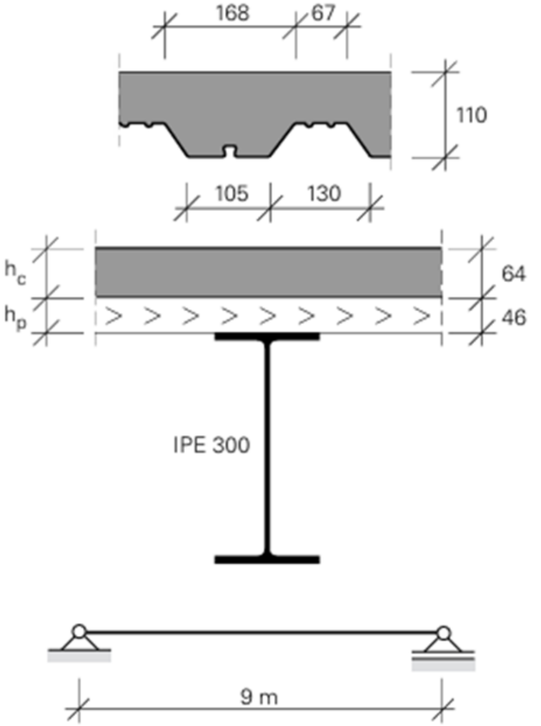
Example 2.5

- *Given*. The composite beam in example 2.1, with studs Ø19 mm, lenght h_{sc} = 85 mm and f_u = 450 N/mm^2.
- *Question*. Calculate the design value of the moment resistance M_{Rd} assuming there is one stud in every other rib.

- *Answer*. From example 2.4 followd P_{Rd} = 73,7 kN with 19 ribs per half span. One stud in every other rib means 9 connectors per half span. So:

$$N_c = nP_{Rd} = 9 \cdot 73{,}7 = 663\ \text{kN}$$

$$\frac{n}{n_f} = \frac{N_c}{N_{cf}} = \frac{663}{1264} = 0{,}52$$



The nonlinear equation (2.24) or (2.25) is used. In this case, the minimum degree of connection according to equation (2.28), case b, must be satisfied:

$$\eta = \frac{n}{n_f} \geq 1 - \frac{355}{f_y}(0{,}75 - 0{,}03L) = 1 - \frac{355}{235}\cdot(0{,}75 - 0{,}03\cdot 9) = 0{,}27 \quad \text{and} \quad \eta \geq 0{,}4 \text{ (critical)}$$

$$N_c = 663 > (h_a - 2t_f)t_w f_{yd} = (300 - 2\cdot 10{,}7)\cdot 7{,}1\cdot 235\cdot 10^{-3} = 465 \text{ kN}$$

Equation (2.24) applies here (case 6), so:

$$x_{pl} = \frac{nP_{Rd}}{b_{eff}0{,}85f_{cd}} = \frac{663\cdot 10^3}{2250\cdot 0{,}85\cdot 16{,}7} = 21 \text{ mm}$$

$$M_{Rd} = N_c\left(\frac{1}{2}h_a + h_p + h_c - \frac{1}{2}x_{pl}\right) + \frac{1}{2}\left(Af_{yd} - N_c\right)\left(h_a - \frac{Af_{yd} - N_c}{2b_a f_{yd}}\right)$$

$$= 663\cdot\left(150 + 46 + 64 - \frac{1}{2}\cdot 21\right)\cdot 10^{-3} + \frac{1}{2}\cdot(1264 - 663)\cdot\left(300 - \frac{(1264 - 663)\cdot 10^3}{2\cdot 150\cdot 235}\right)\cdot 10^{-3}$$

$$= 165 + 88 = 253 \text{ kNm}$$

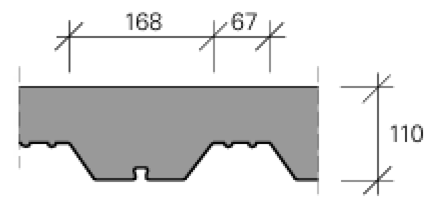
In this case: $M_{Rd}/M_{pl,Rd} = 253/303 = 0{,}83.$

So even when the number of studs is halved, a moment resistance of more than 80% of the design plastic resistance moment $M_{pl,Rd}$ is still achieved!

Example 2.1

- *Given*. A statically determinate beam IPE 300 with f_{yd} = 235 N/mm^2 and a span L = 9,0 m. Key dimensions of the composite floor are h_c = 64 mm and h_p = 46 mm. The ribs of the steel sheeting are perpendicular to the beam axis (fig. 2.17). The concrete is of class C25/30 with f_{cd} = 16,7 N/mm^2.
- *Question*. Determine the plastic moment resistance $M_{pl,Rd}$.

- **Answer**. One row of shear studs is assumed (b_0 = 0). The effective width is b_{eff} = 2·9000/8 =2250 mm, therefore:

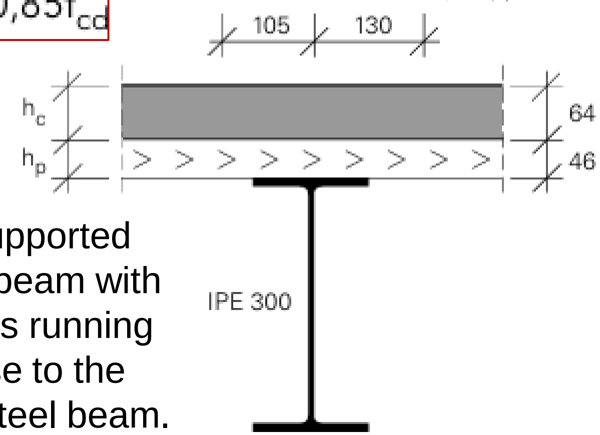


$$h_c b_{eff} 0{,}85 f_{cd} = 64 \cdot 2250 \cdot 0{,}85 \cdot 16{,}7 \cdot 10^{-3} = 2044 \text{ kN}$$

$$A f_{yd} = 53{,}8 \cdot 10^2 \cdot 235 \cdot 10^{-3} = 1264 \text{ kN} < 2044 \text{ kN}$$

$$A f_{yd} < h_c b_{eff} 0{,}85 f_{cd}$$

The neutral axis is thus in the concrete slab, (case 1)

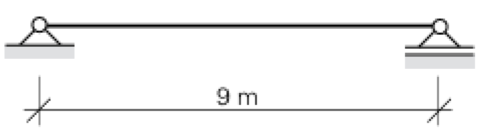


$$x_{pl} = \frac{A f_{yd}}{b_{eff} 0{,}85 f_{cd}} = \frac{53{,}8 \cdot 10^2 \cdot 235}{2250 \cdot 0{,}85 \cdot 16{,}7} = 40 \text{ mm} < h_c = 64 \text{ mm}$$

$$M_{pl,Rd} = A f_{yd} \cdot \left(\frac{1}{2} h_a + h_p + h_c - \frac{1}{2} x_{pl} \right) =$$

$$= 53{,}8 \cdot 10^2 \cdot 235 \cdot \left(\frac{1}{2} \cdot 300 + 46 + 64 - \frac{1}{2} \cdot 40 \right) \cdot 10^{-6} = 303 \text{ kNm}$$

Simply supported composite beam with decking ribs running transverse to the supporting steel beam.



Deflection of simply supported composite beams

The deflection of simply supported beams is influenced by a number of factors, including:

- creep and shrinkage of the concrete
- and the deformation of the shear connectors.

These last factors are discussed in this section.

1 Creep and shrinkage of concrete

Concrete deforms over time under the influence of a constant load, whereas steel does not.

When concrete is subjected to a long-term service load an elastic strain initially occurs. Over time, this strain increases until a certain limit value is reached.

This phenomenon is called creep of the concrete. The ratio of the deformation $\varepsilon_{cc}(t,t_0)$ at time t and instantaneous deformation $\varepsilon_{cc}(t_0)$ is defined as the creep coefficient $\varphi(t,t_0)$.

The following factors influence the creep deformation:

– composition of the concrete;

– maturity of the concrete;

– dimensions of the concrete element;

– humidity;

– temperature..

Furthermore, the size of the creep deformation depends on the maturity of the concrete when first loaded (t_0), the load duration and the stress level. EN 1992, cl. 3.1.4 provides rules for the determination of the creep coefficient.

The deflection of a composite beam increases due to the creep of the concrete, although the flexural stiffness of the steel section limits this effect.

Therefore, for certain applications of composite beams, it is easier to take into account the effect of creep than it is with concrete structures.

EN 1994-1-1, cl. 5.4.2.2(11) provides an approximate method that may be applied for beams in buildings that are not primarily intended for storage and are not prestressed.

The simplified method does not require a separate calculation for longterm and short term load.

The deflection is calculated for both types of loading with $E_c = E_{cm}/2$.

This method is usually accurate enough for buildings, particularly given the rather arbitrary nature of deflection limits and the accuracy with which other influences can be taken into account.

If a greater degree of accuracy is required, the same methodology can be followed as is used for concrete structures in accordance with EN 1992-1-1.

EN 1994-1-1, cl. 7.3.1(8) states that :

– unless explicitly stated otherwise – **deflection** due to shrink age need only be calculated for composite beams with a ratio of span to overall depth greater than 20.

The curvature due to shrinkage can be calculated as follows:

$$\kappa_s = \frac{\varepsilon_{cs}\left(h + h_c + 2h_p\right)}{2(1 + nr)I_c} \quad \text{with} \quad r = \frac{A_a}{b_{eff}h_c} \tag{2.33}$$

The shrinkage is **constant** over the entire length of the beam. Therefore, the curvature is constant and the **deflection** is:

$$\delta_s = \frac{1}{8}\kappa_s L^2 \tag{2.34}$$

2 Deformation of shear connectors

Deformation of the shear connectors causes slip at the interface between the concrete slab and the steel beam.
However, the effects of slip, leading to incomplete interaction may be ignored according EN 199411, cl. 7.3.1 provided that:

– either not less shear connectors are used than half the number for full shear connection ($n/n_f \geq 0,5$),
or the forces resulting from an elastic behaviour and which act on the shear connectors in the serviceability limit state do not exceed P_{Rd};
– when using a slab with ribs transverse to the beam, the depth of the ribs h_p does not exceed 80 mm.

If these two conditions are not met, the effects of slip must be taken into account. However, EN 199411 **does not provide any calculation** rules for this.

However, some countries provide in their National Annex rules how to take slip into account.

For instance the Dutch National Annex, cl. 7.3.1 contain an additional rule, which applies to $n/nf \geq 0{,}4$ when the first condition is not met. In that case no testing or an accurate calculation is required, but the increase in deflection due to incomplete interaction may be determined as follows, depending on the method of execution:

propped
$$\frac{\delta}{\delta_c} = 1 + 0{,}5(1 - \eta)\left(\frac{\delta_a}{\delta_c} - 1\right) \quad (2.35)$$

unpropped
$$\frac{\delta}{\delta_c} = 1 + 0{,}3(1 - \eta)\left(\frac{\delta_a}{\delta_c} - 1\right) \quad (2.36)$$

Where:

δ_a deflection of the composite beam without concrete (steel beam alone);

δ_c deflection of the composite beam in the event of full shear connection;

η degree of shear connection ($\eta = n/nf$).

However, both equations have been found to overestimate the effect of slip.

Example 2.10

- *Given*. The composite beam in example 2.1 with a self-weight of q_{sw} = 6,5 kN/m (see fig. 2.17).
- *Question*. Determine the elastic part of the load/deflection diagram for cases when the beam is propped and unpropped during pouring of the concrete. The ratio n between the modulus of elasticity of steel and concrete for all loads the composite beam is subject to is n = 15.

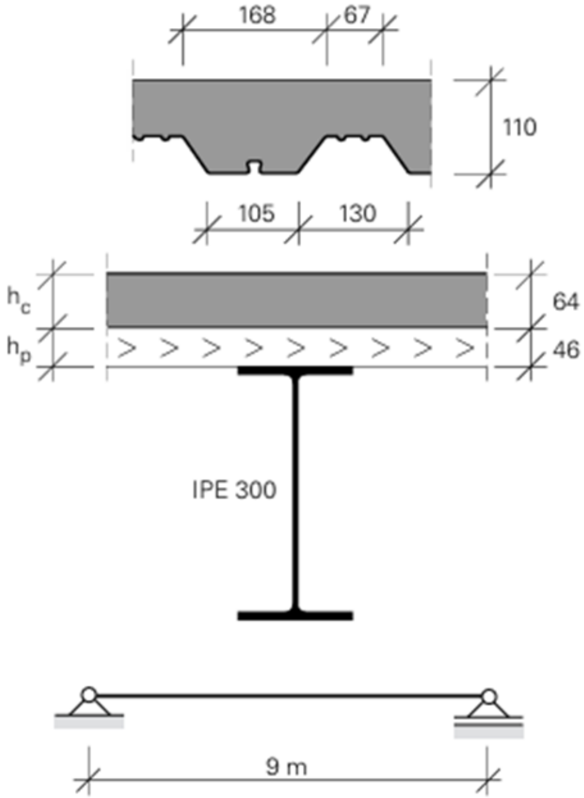
Answer. The second moment of area of the composite beam is given by equation (2.15):

$$r = \frac{A_a}{b_{eff} h_c} = \frac{53{,}8 \cdot 10^2}{2250 \cdot 64} = 0{,}037$$

$$I_c = \frac{A_a\left(h_c + 2h_p + h_a\right)^2}{4(1 + nr)} + \frac{b_{eff} h_c^3}{12n} + I_a$$

$$= \frac{53{,}8 \cdot 10^2 \cdot (64 + 2 \cdot 46 + 300)^2}{4 \cdot (1 + 15 \cdot 0{,}037)} + \frac{2250 \cdot 64^3}{12 \cdot 15} + 8360 \cdot 10^4$$

$$= (17985 + 328 + 8360) \cdot 10^4 = 26673 \cdot 10^4 \text{ mm}^4$$



The location of the neutral axis is given by equation (2.14):

The location of the neutral axis is given by equation (2.14):

$$x_{el} = h - \frac{\frac{1}{2}A_a h_a + \frac{1}{n} b_{eff} h_c \left(h_a + h_p + \frac{1}{2} h_c\right)}{A_a + \frac{1}{n} b_{eff} h_c}$$

$$= 410 - \frac{\frac{1}{2} \cdot 53{,}8 \cdot 10^2 \cdot 300 + \frac{1}{15} \cdot 2250 \cdot 64 \cdot (300 + 46 + 32)}{53{,}8 \cdot 10^2 + \frac{1}{15} \cdot 2250 \cdot 64} = 114 \text{ mm}$$

Unpropped beam during execution:

The load by the self weight of the concrete slab and of the steel beam is q_{sw} = 6,5 kN/m. Hence:

$$\delta_1 = \delta_{sw} = \frac{5 q_{sw} L^4}{384 E_a I_a} = \frac{5 \cdot 6{,}5 \cdot 9000^4}{384 \cdot 2{,}1 \cdot 10^5 \cdot 8360 \cdot 10^4} = 32 \text{ mm}$$

$$M_{sw} = \frac{1}{8} q_{sw} L^2 = \frac{1}{8} \cdot 6{,}5 \cdot 9^2 = 65{,}8 \text{ kNm}$$

$$\sigma_{a1} = \frac{M_{sw} 0{,}5h}{I_a} = \frac{65{,}8 \cdot 10^6 \cdot 0{,}5 \cdot 300}{8360 \cdot 10^4} = 118 \text{ N/mm}^2$$

As additional load is applied the stress in the steel section will increase.
The end of the elastic part of the load/defection diagram is reached when the steel starts yielding.
The increase of the stress in the steel section from that due to self weight up to yield is:

$$\sigma_{a2} = f_{yd} - \sigma_{a1} = 235 - 118 = 117 \text{ N/mm}^2$$

The corresponding additional bending moment and load are therefore:

$$M_{el,extra} = \frac{\sigma_{a2} I_c}{h - x_{el}} = \frac{117 \cdot 26673 \cdot 10^4}{410 - 114} \cdot 10^{-6} = 105 \text{ kNm}$$

$$q_{el,extra} = \frac{8M_{el,extra}}{L^2} = \frac{8 \cdot 105}{9^2} = 10{,}4 \text{ kN/m} \quad \left(q_{total} = 10{,}4 + 6{,}5 = 16{,}9 \text{ kN/m}\right)$$

and the additional deflection:

$$\delta_2 = \frac{5M_{el,extra}L^2}{48E_a I_c} = \frac{5 \cdot 105 \cdot 10^6 \cdot 9000^2}{48 \cdot 2{,}1 \cdot 10^5 \cdot 26673 \cdot 10^4} = 16 \text{ mm} \quad \left(\delta_{total} = 16 + 32 = 48 \text{ mm}\right)$$

The resistance of the beam follows from example 2.1:

$$q_{pl} = 8M_{pl,Rd}/L^2 = 8 \cdot 303/9^2 = 29{,}9 \text{ kN/m}.$$

The load/deflection diagram is shown in figure 2.57.

Propped beam during execution:

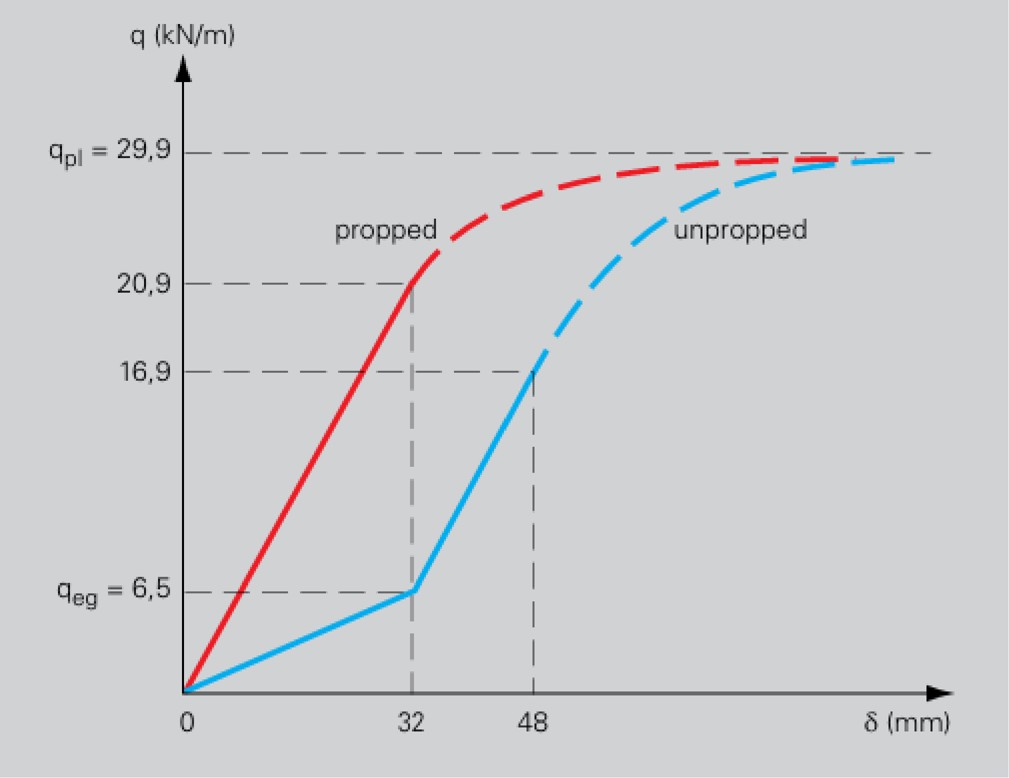
The deflection for this situation follows from:

$$M_{el} = \frac{f_{yd} I_c}{h - x_{el}} = \frac{235 \cdot 26673 \cdot 10^4}{410 - 114} \cdot 10^{-6} = 212 \text{ kNm}$$

$$q_{el} = \frac{8M_{el}}{L^2} = \frac{8 \cdot 212}{9^2} = 20{,}9 \text{ kN/m}$$

$$\delta_3 = \frac{q_{el}}{q_{el,extra}} \cdot \delta_2 = \frac{20{,}9}{10{,}4} \cdot 16 = 32 \text{ mm}$$

Assume that one stud is placed in each alternate rib.
As determined in example 2.5 the degree of shear connection is $n/n_f = 0{,}52 > 0{,}5$.
The depth of the sheeting is $h_p = 46$ mm < 80 mm.
The load/deflection diagram is shown in figure 2.57.



2.57 Relationship between distributed load q and deflection δ.

Example 2.1

- *Given*. A statically determinate beam IPE 300 with f_{yd} = 235 N/mm^2 and a span L = 9,0 m. Key dimensions of the composite floor are h_c = 64 mm and h_p = 46 mm. The ribs of the steel sheeting are perpendicular to the beam axis (fig. 2.17). The concrete is of class C25/30 with f_{cd} = 16,7 N/mm^2.
- *Question*. Determine the plastic moment resistance $M_{pl,Rd}$.

- **Answer**. One row of shear studs is assumed ($b_0 = 0$). The effective width is $b_{eff} = 2 \cdot 9000/8 = 2250$ mm, therefore:

$$h_c b_{eff} 0{,}85 f_{cd} = 64 \cdot 2250 \cdot 0{,}85 \cdot 16{,}7 \cdot 10^{-3} = 2044 \text{ kN}$$

$$Af_{yd} = 53{,}8 \cdot 10^2 \cdot 235 \cdot 10^{-3} = 1264 \text{ kN} < 2044 \text{ kN}$$

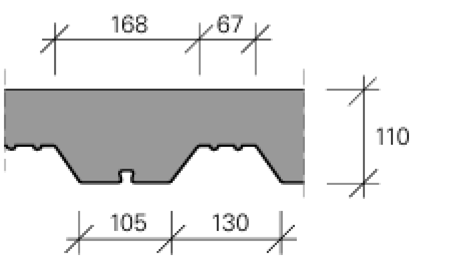
$$Af_{yd} < h_c b_{eff} 0{,}85 f_{cd}$$

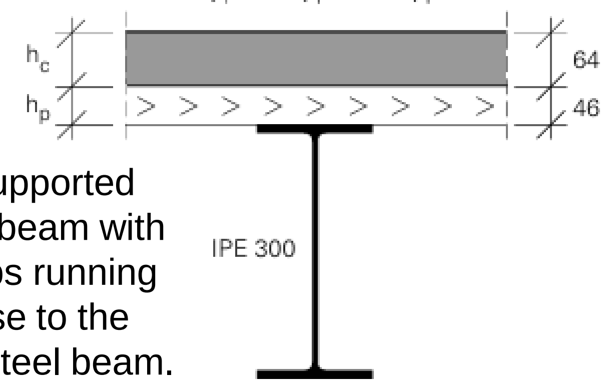
The neutral axis is thus in the concrete slab, (case 1)

$$x_{pl} = \frac{Af_{yd}}{b_{eff} 0{,}85 f_{cd}} = \frac{53{,}8 \cdot 10^2 \cdot 235}{2250 \cdot 0{,}85 \cdot 16{,}7} = 40 \text{ mm} < h_c = 64 \text{ mm}$$

$$M_{pl,Rd} = Af_{yd} \cdot \left(\frac{1}{2} h_a + h_p + h_c - \frac{1}{2} x_{pl}\right) =$$

$$= 53{,}8 \cdot 10^2 \cdot 235 \cdot \left(\frac{1}{2} \cdot 300 + 46 + 64 - \frac{1}{2} \cdot 40\right) \cdot 10^{-6} = 303 \text{ kNm}$$





Simply supported composite beam with decking ribs running transverse to the supporting steel beam.